**If there were no numbers, what would you think?**

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**Abstract:** Hartry Field has argued that mathematical realism is epistemologically problematic, because the realist is unable to explain the supposed reliability of our mathematical beliefs. In some of his discussions of this point, Field backs up his argument by saying that our purely mathematical beliefs do not 'counterfactually depend on the facts'. I argue that counterfactual dependence is irrelevant in this context; it does nothing to bolster Field's argument.

*1. Field's case against mathematical realism*

In Hartry Field's terminology (Field 1988, Field 1989: 25-30), a 'mathematical realist' is anyone with the following two characteristics. First, she thinks that mathematical statements generally accepted by mathematicians are true (setting aside the odd mistake, of course). Second, she takes these statements 'at face value'. That is, she thinks that these statements concern abstract objects: objects such as numbers, sets and functions which have no location in space or time, which are mind-independent, and which are inert, in the sense that they don't participate in causal processes. The mathematical realist is thus committed to the following claim:

*The Correlation*

Excepting a few mistakes, if P is a purely mathematical claim and mathematicians

believe that P is true, then P is true.

In a series of papers, Field has argued that the realist will have trouble explaining *The Correlation*. He uses this point as an objection to realism.[1] Field's argument, in summary, is this:

*The Main Argument*

1st Premise:     Mathematical realism is untenable in the absence of at least a sketch of an adequate explanation of *The Correlation*.

2nd Premise:    The realist is unable even to sketch an adequate explanation of *The Correlation*.

Conclusion:     Realism is untenable.

The first premise is motivated as follows. According to Field, if the realist is unable to find an explanation for *The Correlation*, at least in outline, she will be forced to accept that *The Correlation* is mere *coincidence*, which is untenable.

Sometimes, Field defends the second premise of *The Main Argument* with another argument:

*The Subsidiary Argument*

1st Premise:     To give an adequate explanation of *The Correlation*, one would have to show that mathematicians' purely mathematical beliefs 'counterfactually depend' on the mathematical facts. That is, one would have to show that, were the mathematical facts different, mathematicians would have correspondingly different beliefs. For example, one would have to show that, had the axiom of choice been false, mathematicians would not have believed it. (Field 1989: 238)[2]

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[1]      In more recent work, he concedes that there are certain forms of realism which might be safe from the objection. See Field 2005.

[2]      One might worry that, because purely mathematical truths are metaphysically necessary, it is at worst incoherent and at best frivolous to ask about what would have been the case had the mathematical facts been different. For responses, see (Field 1989: 233-38) and Clarke-Doane (2012: 320-21). Very briefly, Field and Clarke-Doane argue that, even if it is not 'metaphysically possible' for the purely mathematical facts to be different we can *coherently suppose* that the mathematical facts are different, and sensibly think about what follows from this supposition.

2nd Premise:     Mathematicians' purely mathematical beliefs do not counterfactually depend on the

mathematical facts: it is not the case that had the mathematical facts been different, the

mathematicians' beliefs would have been correspondingly different. For example, had

the axiom of choice been false, mathematicians would still have believed it.

Conclusion:     The mathematical realist is unable even to sketch an adequate explanation of *The*

*Correlation*.

Field does not give an explicit definition of 'counterfactual dependence', but Justin Clarke-Doane has done so on his behalf:

[O]ur beliefs of a kind, F, are [counterfactually dependent on the F-facts] if, had the [F-

facts] (as a group) been different (no matter how radically), our F-beliefs would have

been correspondingly different. (forthcoming).[3, 4]

The second premise of *The Subsidiary Argument* is easily understood. According to the realist, mathematical objects are inert. It follows that there are no nomological relations between mathematical facts and mathematicians' beliefs; it is thus hard to understand how those beliefs could counterfactually depend on those facts. In the next section, I will argue that the first premise of *The Subsidiary Argument* is not well motivated.[5]

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[3]     Clarke-Doane uses the term 'sensitive' in place of Field's term 'counterfactual dependent'. He also writes about 'truths' rather than 'facts'. I've modified his definition to ensure terminological consistency.
[4]     The word 'dependent' is potentially misleading here: our beliefs may be counterfactually 'dependent' on the facts even if the facts have no causal impact on us. In particular, it's possible for our current beliefs to be counterfactually 'dependent' on facts about the future. For example, my beliefs about what I will do next week counterfactually depend on the facts: in nearby worlds in which the facts about what I will do next week are different, I nevertheless have true beliefs about what I will do next week.
[5]     See Clarke-Doane (forthcoming) for another critique on *Field's Subsidiary Argument.*

I should stress that my target in this paper is only *The Subsidiary Argument*.[6] I am not criticizing *The Main Argument*. Field claims (1989: 238) that the latter constitutes a powerful challenge to mathematical realism, even if *The Subsidiary Argument* fails.

*2. The error in The Subsidiary Argument*

The first premise of *The Subsidiary Argument* is that, to give an adequate explanation of *The Correlation*, one would have to show that mathematicians' purely mathematical beliefs counterfactually depend on the purely mathematical facts.  I'm going to argue that this premise is not well motivated, by looking at a story which shows that a correlation between belief and fact can be explicable even if the beliefs in question don't counterfactually depend on the facts:

> The people in Carlotta's department organize a lottery syndicate. They buy 100 tickets
> from the national lottery. Millions of tickets are sold; only a small number (several
> hundred) are randomly selected as winners. Carlotta understands that each ticket has
> only a tiny chance of winning, and she is a pessimist. So she forms the belief that the first
> ticket will not win, and the belief that the second ticket will not win, and the belief that
> the third ticket will not win, ... and so on. As it happens, Carlotta is right: none of the
> tickets will win. There is thus a perfect correlation between her beliefs of the form *ticket*
> *number n will not win* and the facts.

Nobody would regard this correlation as a mystery or a huge coincidence. It is easy to explain why it is that Carlotta's pessimistic beliefs are perfectly correlated with the facts: given the basic information

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[6]      It is worth pointing out in passing that relatives of *The Subsidiary Argument* have appeared in the metaethics literature. See for example, Ruse and Wilson (1986:186-7), Bedke (2009) and Joyce (2006: 183). I suspect the argument of the current paper could be modified and put to use in metaethics, but I'll leave this task to specialists.

provided, it is clear that the probability of Carlotta's beliefs all being true is very high. However, Carlotta's beliefs do not counterfactually depend on the facts in this case. Recall Clarke-Doane's definition of counterfactual dependence:

> [O]ur beliefs of a kind, F, are [counterfactually dependent on the F-facts] if, had the [F-facts] (as a group) been different (no matter how radically), our F-beliefs would have been correspondingly different.

Here the 'kind' is *beliefs about which lottery tickets are winners.* Now it is not the case that had the facts of this kind been different, even radically different, Carlotta's beliefs would have been correspondingly different. If one, or all, of the department's tickets had been winners, Carlotta would still believe of each that it is a loser. The example establishes that there can be an explicable, non-coincidental correlation between belief and fact, even if the beliefs in question do not 'counterfactually depend on the facts'.

This undermines the first premise in *The Subsidiary Argument*: We have no reason to think that one need show that mathematicians' mathematical beliefs counterfactually depend on the facts in order to explain *The Correlation*.

It is perhaps slightly strange for the mathematical realist to analogize her mathematical beliefs to Carlotta's pessimistic beliefs about the lottery: after all, Carlotta's beliefs of the form *ticket n is not a winner* are presumably unjustified, but the mathematical realist insists that her mathematical beliefs *are* justified. This disanalogy is of little relevance. The lottery example is supposed to undermine the first premise of *The Subsidiary Argument*. This premise does not concern the conditions for knowledge or justification.

*3. Other sensitivity conditions*

Those who have followed recent debates about the conditions for knowledge or epistemic justification will recognize Field's counterfactual dependence condition as a 'sensitivity' condition. Recent proponents of the view that there is a sensitivity condition on knowledge or justification have devised some interesting and elaborate new sensitivity conditions.[7] It might seem plausible that Field will be able to fix his argument by replacing his counterfactual dependence condition with one of these new, up to date sensitivity conditions.

This won't work. It is usually taken to be an important strength of sensitivity conditions for knowledge or justification that they explain why people like Carlotta in my lottery case do not have knowledge or justified belief (Pritchard 2011: 439). So Carlotta's beliefs are not sensitive, even according to the most up to date definitions of 'sensitive.' Therefore, even using the most up to date definitions, my example still shows that a correlation between belief and fact can be explicable even if the beliefs in question aren't sensitive.

Here's a more promising strategy for proponents of *The Main Argument:* they should forget about sensitivity conditions altogether. Perhaps they should consider appealing to some version of the 'safety' condition instead.[8] This response concedes my point, which is that counterfactual dependence (or sensitivity) is irrelevant in this discussion.[9]

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