Mathematical Cognition: Brain and Cognitive Research and Its Implications for Education

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Abstract

Mathematical cognition is one of the most important cognitive functions of human beings. The latest brain and cognitive research have shown that mathematical cognition is a system with multiple components and subsystems. It has phylogenetic root, also is related to ontogenetic development and learning, relying on a large-scale cerebral network including parietal, frontal and temporal regions. Especially, the parietal cortex plays an important role during mathematical cognitive processes. This indicates that language and visuospatial functions are both key to mathematical cognition. All of those advances have important implications for basic mathematical education.

Keywords

numerical cognition, brain and cognitive science, educational application

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Numbers are one of the most important symbol systems for human beings. Understanding and appreciating numbers, acquiring a certain level of arithmetic knowledge and numeracy skills, and acquiring basic mathematical literacy are essential for individuals to live, study and work normally. In the new century, as

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technology, economy and society progress, and as the world becomes increasingly computerised and digitised, mathematical literacy is of even greater importance to a competent citizen[1]. For this reason, mathematical literacy has received worldwide attention, with countries and international organisations such as the USA, the European Union and the OECD attaching great importance to it, organising special research teams and investing considerable resources in the development and development of mathematical literacy.

The development and science of mathematical literacy must be based on the study of mathematical cognition, its development and its brain mechanisms. In the last 30 years, research findings in cognitive psychology, neuropsychology, developmental psychology and animal psychology have revealed that mathematical cognition is a cognitive system with complex components and structures, based on the evolution of the lineage and the development of the individual. In particular, since the 1990s, with the flourishing of brain and cognitive science, researchers have conducted more in-depth and systematic research on the brain mechanisms of mathematical cognition, establishing a preliminary correspondence between the cognitive and brain levels. These findings have undoubtedly provided valuable insights for current basic mathematics education. In the light of recent advances in brain and cognitive science and some of our recent research findings [2] [3] [4], this article will discuss the components and structure of mathematical cognition, evolution and development, the structural and functional basis of the brain, impairment and its brain mechanisms, learning and brain plasticity, and analyse their implications for basic mathematics education.

I. Components and Structures of Mathematical Cognition

Broadly speaking, all mathematics-related thinking activities can be considered as mathematical cognition. Current research findings indicate that mathematical cognition consists of three basic components: (1) number processing, i.e. how individuals encode different number symbols, such as Arabic numbers (1, 2, 3, etc.), verbal numbers (one, two, three, etc.), into understandable cognitive representations; (2) arithmetic knowledge, i.e. the various types of computation-related knowledge that individuals acquire through learning, such as multiplication knowledge; (3) calculation, i. e. the knowledge that individuals acquire through performing calculations, especially mental arithmetic; However, the relationship between these three components, i.e. the structure of mathematical cognition, has remained an important question for researchers to date.

A large body of current research suggests that the three components of mathematical cognition are relatively independent cognitive processing modules. For example, studies of brain-damaged patients have found that some patients can understand numbers despite their loss of numeracy, while others can use Arabic numerals to complete tasks despite their inability to do so in verbal numerical conditions[5][6]. Some researchers have suggested that this phenomenon indicates that arithmetic knowledge and computation require the use of numerical processing as a basis. They suggest that all forms of numbers will be entered into the same numerical representation and further support the storage and computation of arithmetic knowledge[7]. Other researchers have argued that numbers do not have just one quantitative representation, but three representations: phonological, morphological and denotational. This multi-representational view sees the three representations as relatively independent cognitive modules, with different forms of numbers being entered into different modules. For example, Arabic numerals are entered into glyphic representations, verbal numerals are entered into phonological representations, and quantitative stimuli that are not in symbolic form, such as dots, are entered into semantic representations. In addition, arithmetic knowledge and computation are associated with different modules and show some representational specificity[8][9]. For example, multiplication may rely on phonological representations as it is learned more by rote, whereas subtraction requires more numerical operations and is therefore more likely to be associated with semantic representations.

In recent years, research in brain and cognitive science has continued to suggest that the structure of human mathematical cognition may be more consistent with the idea of multiple representations. For example, some studies of brain-injured patients have found that there is a relative separation between the phonological and morphological representations of numbers, with damage to one representation not affecting the processing of the others [10]. Recent functional brain imaging studies also suggest that the processing of these representations may have different cortical support networks, with less overlap between them [11]. Thus, although not fully supported by all studies, the idea of multiple representations is still accepted by most researchers.

II. Evolution and development of mathematical cognition

The main difference between mathematical cognition and language lies in the fact that the former has a strong evolutionary basis in the species. Numerous animal experiments have shown that animals such as orangutans, apes, dolphins, parrots, rats and even amphibians are able to understand the quantitative properties of stimuli and can even perform some arithmetic processing. In one experiment, for example, hungry rats were placed in a box with two levers and had to press one of the levers a certain number of times before they could press the other lever for food. It was found that after a period of practice, the rats were able to differentiate between different amounts of stimuli and respond accordingly. In addition, the rats were able to integrate and abstract stimuli from different sensory channels (e.g. visual and auditory) and extract the numerical characteristics of the stimuli, indicating that they were able to understand the quantitative characteristics of external stimuli[12]. However, these findings also suggest that animals' understanding of number is an imprecise representation, highlighted by the distance effect and magnitude effect, i.e. as the distance between numbers decreases and the number itself increases, the accuracy of the animal's number representation decreases.

In developmental psychology studies, it has been found that as early as infancy, humans already show quantitative understanding similar to that of animals. For example, one study found that at around six months of age, infants can distinguish between large quantitative differences, such as the difference between 8 and 16 dots[13]. It has also been found that at almost the same age, infants can determine the correctness of simple arithmetic relationships such as " 1 + 1 = 2", "1 + 1 = 1", "2-1=1", "2-1=2", etc.[14]. Similarly to animals, infants' number representations have both distance and size effects. For example, in the former study, infants' ability to discriminate between quantities depended on the ratio of the two quantitative stimuli, with almost all infants unable to discriminate between 8 and 10 points. In the latter study, however, it was found that as the number of items in the experiment increased, the infants' correct judgement declined rapidly.

In addition, some researchers have found that humans continue to have such generalised quantitative representations even in adulthood. For example, one study found that subjects also showed significant distance and size effects when judging the magnitude of two single-digit numbers, e.g., it was easier to judge 3 and 7 than 3 and 4, with significantly less reaction time and a significantly higher correct rate; and when distances were equal, it was easier to judge large numbers such as 7 and 9 than small numbers such as 3 and 5 [15]. This feature is very similar to the number representations of animals and infants, and thus suggests that human mathematical cognition has a clear evolutionary origin.

Of course, human mathematical cognition is not limited to its evolutionary basis. Current research in brain and cognitive science also suggests that human mathematical cognition consists of at least two subsystems, a general representation system based on evolution and a precise representation system based on long-term learning experience. From the perspective of multiple representations, the approximate representation system is mainly concerned with the semantic representation of numbers, while the precise representation system is related to symbolic features such as sounds and shapes. While the approximate representational system shows less age-related changes during the development of the individual, the precise representational system undergoes significant changes with age. Our findings suggest that the cognitive development of early childhood mathematics involves four main areas: number, quantity, shape and pattern perception. This development is influenced not only by the evolutionary basis of mathematical cognition, but also, and more importantly, by ecological factors such as the family and the kindergarten, which contribute more directly to this development[2]. The emphasis on early informal mathematical learning experiences has therefore become the consensus of current researchers. The development of mathematical cognition is now beginning to be the focus of research in the brain and cognitive sciences, and is likely to provide a richer body of research over time.

III. The structural and functional basis of mathematical cognition in the brain

As early as the end of the 19th century, some neuropsychological studies found that after damage to specific parts of the brain, patients' mathematical cognition would be impaired, but not their other functions such as language and memory. This has led researchers to wonder whether there are specific areas of mathematical cognition within the brain. However, it is difficult to pinpoint the exact cortical location because of differences in the depth and size of the damage. With the maturation of imaging techniques with high spatial resolution, such as PET and fM RI, the structural and functional basis of mathematical cognition has been investigated in more depth in recent years.

Current findings consistently suggest that the human parietal cortex, particularly the area around the bilateral intraparietal sulcus, is strongly associated with mathematical cognition. Some researchers have found that this area is significantly activated when subjects compare the size of numbers, and that its activation tends to decrease monotonically as the distance between numbers increases [16]. This "distance effect" on cortical activity suggests that the intraparietal sulcus may be responsible for a large amount of processing and manipulation. Furthermore, it has also been found that this area is activated after seeing a number even if the subject does not perform any number manipulation [17], suggesting that its activation is highly automatic. Our results also show that Chinese people also use this area during digital processing, suggesting that its function is not affected by cultural differences [18].

Using single-cell recording techniques, some researchers have found that neurons in the parietal cortex of apes show significant selective firing in response to different quantitative stimuli, demonstrating sensitivity to quantitative differences [19]. This cortical network correspondence is strong evidence for a close link between human and animal numerical abilities.

Some studies have also shown that the parietal lobe is not the only brain region that supports mathematical cognition. An fMRI study showed that when subjects performed two different arithmetic tasks, one for exact calculation and one for estimation of results, the activation patterns of the brain regions showed significant differences. The exact calculation activated more of the left prefrontal and angular gyrus regions associated with language function, whereas the estimation results triggered more activation in the bilateral parietal cortex [20]. This difference in activation patterns suggests that the cortical support network for mathematical cognition in humans is distributed, supporting the idea that there are multiple representations of mathematical cognition with multiple subsystems.

In the light of current research findings, some researchers have suggested that the main processes of digit processing are supported by a large network of prefrontal, parietal and temporal regions [21], as shown in Figure 1. Of these, the bilateral parietal areas, especially the area around the intraparietal sulcus, are mainly associated with the semantic representation of numbers, which has a strong evolutionary basis and is more visuospatial in character than a precise linguistic representation. The prefrontal lobe, especially the left inferior frontal gyrus, has a large overlap with brain areas associated with verbal working memory, reflecting a link between numerical processing and language function. The bilateral temporal lobes, especially the bilateral syrinx, are mainly associated with the processing of numerical forms. From the present findings, it appears that Arabic numeral stimuli may be processed in the right cingulate gyrus and then transmitted through the right middle temporal gyrus to the right parietal area, whereas verbal numerals may be processed more through the left channel. According to the multiple representations view, the above-mentioned brain areas are responsible for processing not only the morphological, phonological and semantic features of numbers, but also for different arithmetical and computational tasks, such as estimation, which are mainly carried out bilaterally in the parietal areas, and the storage and retelling of mathematical facts, which depend on the involvement of the prefrontal areas. Some numerical manipulation tasks, such as complex numbers, negative numbers and fractions, require the processing of numerical forms and are therefore more closely linked to temporal areas, especially the cingulate gyrus.

IV. Disorders of mathematical cognition and their brain mechanisms

In general, there are two main types of mathematical cognitive disorder: acquired dyscalculia, which is a mathematical cognitive disorder caused by brain damage, and dyscalculia, which is a mathematical cognitive disorder that occurs during an individual's development without apparent brain damage. In this section, we focus on developmental dyscalculia and its brain mechanisms.

The current research findings indicate that developmental dyscalculia has two basic characteristics: (1) a significant delay in the development of numerical processing and computation; and (2) no significant impairment in cognitive functions other than numerical processing and computation, such as intelligence and speech. In the International Classification of Diseases, 10th Revision (ICD-10) and the Diagnostic and Statistical Manual of Mental Disorders, 4th Edition (DSM-IV), developmental dyscalculia is defined as a mismatch between the development of mathematical ability and the development of general intelligence. As can be seen, developmental dyscalculia is highly idiosyncratic and is characterised by impairments in the core components of mathematical cognition. Developmental dyscalculia is a worldwide problem in terms of its prevalence, which is approximately the same across countries, at around 6% [22]. This far exceeds estimates and is on a par with the prevalence of developmental dyslexia. As a result, developmental dyscalculia has been a focus of interest for researchers in this field. Our first survey of developmental dyscalculia in Chinese children found a prevalence of 5.5% and three basic types of impairment: auditory, visual and semantic [4]. This suggests that impairments in developmental dyscalculia may also be diverse, supporting the idea of multiple representations of mathematical cognition.

Early studies of the brain mechanisms underlying developmental dyscalculia have focused on the relationship between dyscalculia and unilateralisation of brain function due to the lack of fine-tuned localisation tools. In the early days, because developmental dyscalculia was not associated with language impairment, and because language function was primarily in the left hemisphere of the brain, researchers often attributed the impairment to developmental problems in the right hemisphere of the brain. In recent years, researchers have begun to refine the cortical networks associated with developmental dyscalculia, using techniques with high spatial and temporal resolution such as ERP, PET and fMRI. For example, some studies using M RS found that neurometabolic levels in the left temporoparietal region were significantly lower in patients with developmental dyscalculia than in normal subjects [23]. Another morphological brain analysis using VBM also showed that low weight new-borns with dyscalculia had significantly lower grey matter density in the left intraparietal sulcus [24]. This suggests that developmental dyscalculia is more likely to be associated with abnormalities in the parietal region, which in turn suggests that the parietal cortex plays a key role in the development of mathematical cognition. In recent years, some studies have begun to establish a correspondence between subtypes of developmental dyscalculia and developmental abnormalities in different brain regions, in order to further explore the causes of the disorder.

Based on these studies, the effective diagnosis and correction of developmental dyscalculia has become an important issue for researchers. There are currently two main approaches to the diagnosis of developmental dyscalculia: clinical diagnosis and standardised testing. The former is a clinical mathematical task that is age-appropriate and can be diagnosed as dyscalculia if the child fails to meet the requirements of his or her age group. This diagnostic approach tends to be too crude. For this reason, standardised tests of numeracy are more commonly used by researchers. The former are mainly specialised tests of mathematical ability, or numeracy subtests included in broad achievement tests, while the latter directly examine children's cognitive abilities in mathematics, such as the Neuropsychological Test Battery for Number Processing and Computation in Children developed by the European Group for the Study of Mathematical Cognition (ESMEC). The Neuropsychological Test Battery for Number Processing and Calculation in Children. In the last two years, some researchers have further argued that because developmental dyscalculia is associated with early brain abnormalities, it should be reflected primarily in basic mathematical cognitive abilities rather than in later mathematical achievement. This has led to the emergence of more effective diagnostic tools [25].

The correction and training of developmental dyscalculia is still in its infancy. Some researchers have pointed out that the correction and training of developmental dyscalculia must focus on restoring parietal function and promoting connections with other areas, such as the frontal lobe. On this basis, some experimental remediation tools are beginning to be used.

V. Learning and Facilitation of Mathematical Cognition and its Brain Mechanisms

Learning and brain plasticity has been an important theme in brain and cognitive science research. In recent years, with the growing understanding of the brain mechanisms underlying mathematical cognition, mathematical learning and facilitation has begun to receive widespread attention. Researchers are not only interested in the brain mechanisms underlying the acquisition of mathematical knowledge and abilities, but also in the structural and functional changes in the brain brought about by different learning styles and strategies.

Current research findings suggest that there are at least two ways in which individuals learn mathematics: mechanical learning and strategic learning. In mechanical learning, the individual stores arithmetic facts directly in his or her mind. For example, an fM RI study showed that with mechanical learning, activation in the parietal cortex shifts from the intraparietal sulcus to the language-related angular gyrus [28], suggesting that mechanical learning is closely linked to verbal function. In contrast, in strategic learning, individuals learn the rules of computation without directly storing arithmetic facts. In some recent studies, researchers have compared the differences between these two learning styles. In experiments, subjects learn artificially constructed arithmetic facts such as 4#5 = 7, 5 § 7 = 9, etc. These arithmetic facts include the following Each of these arithmetic facts contains different rules, e. g. #: {[(second number - first number) + 1] + second number}, §: {[(second number + first number) - 10] + second number}. During the learning process, one group of students learned by mechanical memory, while the other group learned their computational strategies. The fM-RI scans revealed that mechanical memory was more activated in the language-related left prefrontal and angular gyrus regions, whereas strategy learning elicited more activation in the right frontal, bilateral cingulate and cuneate regions [30]. This result suggests that mechanical memory and strategy learning have completely different neural mechanisms and that strategy acquisition may be more closely linked to visuospatial function.

In terms of the effectiveness of both learning styles, strategic learning may be more effective. Some cross-cultural studies of Chinese mathematical dominance have found that Chinese subjects are more likely to extract facts directly and less likely to be strategic in their calculations than in Western countries[27]. However, one of our fM RI studies found that Chinese subjects did not show activation of language areas in their calculations[17], suggesting that their mathematical advantage does not come from mechanical learning, but may be due to a more skillful mastery of strategies. Another fM RI study also showed a tendency for the higher-performing subjects to show reduced activation in language-related gyrus areas compared to the lower-performing subjects[29]. This further suggests that strategy learning is rarely based on language.

In recent years, specific forms of mathematics learning or training have also come to the attention of researchers. For example, in East Asian countries such as China, Japan and Korea, beadwork is often used as a means of mathematical training in basic mathematics education. One fMRI study showed that beadmakers have a greater ability to remember numbers and draw more on cortical networks of visuospatial memory than the general population [31]. Another fM RI study also found that beadmakers produced more activation in the left superior parietal region compared to the average person's calculation process [32]. This suggests that individuals are likely to make greater use of visuospatial functions during bead calculation, thus contributing to the acquisition of computational strategies.

VI. Implications of Brain and Cognitive Science Research for Mathematics Education

In recent decades, basic mathematics education in China has made remarkable achievements and has accumulated a wealth of experience in teaching and learning. However, to date, we still lack an understanding of the scientific basis of mathematics education. In today's educational practice, students have a heavy burden of learning mathematics, their learning efficiency is not high, there is a disconnect between the acquisition of mathematical knowledge and the development of their abilities, the application of mathematics is lacking, students are not interested in mathematics, and the phenomenon of "rote learning" still exists widely. Therefore, it is necessary and urgent to strengthen the research on brain and cognitive science of mathematics cognition, to pay attention to some new developments in current research, and to better understand students' cognitive development, learning rules and effective learning styles in mathematics.

First, we need to understand the components and structure of mathematical cognition in a scientific way. The previous studies have shown that mathematical cognition is a complex cognitive system that not only contains multiple components, such as number processing, numerical knowledge and computation, but also has a complex pattern of how these components are organised. For example, the multiple representation perspective suggests that there may be differences in the internal structure of mathematical cognition in terms of representation and that there is a clear separation between different representational modules. These facts suggest that in the design of mathematics education curricula and classroom teaching, the components and structures of students' mathematical cognitive abilities, the different roles of different learning contents and task designs on the requirements and development of different components of mathematical cognition, and the relationships between the different components must be carefully analysed in order to design materials and select appropriate teaching methods in a more scientific way.

Secondly, we need to take a scientific view of the development of students' mathematical cognition. Current research has shown that there are two sets of mathematical cognitive systems in the individual: a general representation system given in the early years, and a precise representation system acquired through learning. The developmental pattern of children's numeracy is a gradual transition from the former to the latter. In this process, the general representation system provides an important basis for the development of the precise representation system, while at the same time building on the latter and playing an important role in everyday mathematical problem solving. Therefore, we should provide children with a rich informal mathematical learning environment and experiences at an early stage to facilitate their early cognitive development, and attach great importance to the synergy between the precise and approximate representational systems to avoid one-sidedness and to develop a well-rounded and balanced mathematical quality in children.

Thirdly, we need to take a scientific view of how mathematics is learned and taught. In mathematics education in China, there is still some 'language orientation' and 'practice orientation'. For example, when comparing the mathematics textbooks in Shanghai with those in Japan, we find that the amount of exercises and problems in China is almost three times that of Japan, despite the fact that both countries have the same amount of time for the same content[33] and that most mathematics teachers tend to use language as a teaching tool and adopt a lecture approach. At the same time, in some schools, there are still competitions and mechanical training in mathematics, with less attention paid to the formation of strategies and skills and the development of mathematical ways of thinking and abilities. Research in brain and cognitive science has shown that these training methods tend to promote mechanical memorisation of arithmetic facts and procedures, relying more on verbal functions and less on the development of computational strategies, and neglecting the role of visual-spatial functions, thus hindering the overall cognitive development of students.

Fourthly, we need to take a scientific view of the difficulties and barriers to learning mathematics. The aforementioned research findings amply demonstrate that dyscalculia is a common cognitive disorder among primary school children, highly specific and less related to their intellectual and linguistic development. This helps us to better understand the prevalence of 'bias' in students and to develop better educational responses. However, the impact of developmental dyscalculia should not be overestimated. Current research suggests that some children with developmental disabilities can gradually improve their impairments over the course of their development and eventually reach normal levels. Students with learning difficulties in mathematics should be accurately diagnosed and the causes of their impairment understood, while at the same time attention should be paid to developing appropriate and targeted programmes for learning mathematics.

Fifthly, we must also take a scientific approach to the assessment of mathematics education. The choice of assessment methods and approaches must be appropriate to the processing and developmental characteristics of children's mathematical cognition. The complexity of mathematical cognition in terms of its components and structure requires that there should not be a single method of assessment, but rather that reasonable assessment tools should be developed according to the components and structure of mathematical cognition and its brain mechanisms, so that assessment in mathematics education is more scientific and practical. It is also important to note that the developmental processes and patterns of different components of mathematical cognition differ, and therefore the selection of reasonable evaluation criteria and developmental evaluation in line with the laws of brain and cognitive science will be an important part of future research in mathematics education evaluation.

In conclusion, the current research findings in brain and cognitive science have provided many useful insights for mathematics education research and reform, and the establishment of a "brain-based and brain-friendly" approach to mathematics education and learning has become an important concern for researchers in brain and cognitive science, psychological science and educational science around the world. At present, as quality education and curriculum reform continue to progress, we should pay more attention to incorporating the findings of brain and cognitive science to explore new ways to improve the efficiency and quality of mathematics education and learning.

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