# The Case for Comparability

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#### Abstract

We argue that all comparative expressions in natural language obey a principle that we call Comparability: if x and y are at least as F as themselves, then either x is at least as F as y or y is at least as F as x. This principle has been widely rejected among philosophers, especially by ethicists, and its falsity has been claimed to have important normative implications. We argue that Comparability is needed to explain the goodness of several patterns of inference that seem manifestly valid, that the purported failures of Comparability would have absurd consequences, and that the influential arguments against Comparability are less compelling than they may have initially seemed.

#### 1 The Logic of Comparatives

Many important questions of philosophical interest are posed using comparative constructions: those involving 'more' and 'less', the comparative morpheme '-er', the equative particle 'as' (e.g., in 'as much as'), and their cognates. We may want to know, for example, what makes an outcome better or worse; what we have most reason to do; which of some hypotheses is more probable; which facts or properties are more fundamental than others; what makes an institution more or less just; what makes one thing more beautiful than another.

Philosophers since at least Aristotle (*Topics*, Book III) have observed that comparative constructions conform to a general logic. For example, for any gradable adjective 'F' (such as 'flat', 'silly', 'good', or 'beautiful'), the following schema is widely assumed to be valid:

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**Comparative Transitivity** If x is more F than y and y is more F than z, then x is more F than z.

The case for the validity of Comparative Transitivity, as we see it, is based on certain ingrained patterns of usage. For example, 'The soup is better than the salad and the salad is better than the dessert, but the soup isn't better than the dessert' seems bad much as outright contradictions like 'The soup is and isn't better than the salad' seem bad. And this reaction isn't based on some kind of reflection distinctive to the topic of *goodness*. As Wheeler (1972, p. 320) observes: 'if someone invents the word "glof" and says the truths "John is glofer than Mary" and "Mary is glofer than Fred", we can know that John is glofer than Fred even though we don't know what "glof" means.' This suggests that Comparative Transitivity really is part of the logic of comparatives, not just an over-hasty generalization from an impoverished range of examples.

When we say that Comparative Transitivity is valid, we mean that its instances always express necessary truths, so long as any ambiguity or context-sensitivity they harbor is resolved uniformly (Dorr, 2014). We are not committing ourselves to their having any such disputed status as analyticity, or being such that anyone who fully understood them would have to accept them. Such a claim would be bold indeed. For example, Rachels (1998) and Temkin (2012) have given much-discussed arguments against certain instances of Comparative Transitivity, maintaining that there are cases where one thing is better than a second and the second is better than some third thing without the first thing being better than the third. We will not engage with their arguments here: following most others in this literature, we find the appearances in favour of the validity of Comparative Transitivity far more compelling than the alleged counterexamples (see Binmore & Voorhoeve, 2003; Broome, 2004; Huemer, 2013; Nebel, 2018; Pummer, 2017). But Rachels and Temkin seem to understand what they are saying very well indeed. Thus, when Temkin (2012, p. 495) insists that views that violate Comparative Transitivity 'represent substantive normative positions', we agree, but emphatically reject the assumption that logical disputes must be non-substantive. (Indeed, our claims of validity are compatible with the view of Williamson (2007) that no sentences whatsoever are analytic in the sense alleged.)

When we talk of the logic of comparatives, we mean to include principles about the "equative" forms '(at least) as *F* as' and 'equally *F*' as well as principles about 'more', 'less', and '-er'. Here are three plausibly valid schemas relating these forms:

**Strict Comparison** *x* is more *F* than *y* if and only if *x* is at least as *F* as *y* and *y* is not at least as *F* as *x*.

**Equality** *x* and *y* are equally *F* if and only if *x* is at least as *F* as *y* and *y* is at least as *F* as *x*.

**Reversal** *x* is less *F* than *y* if and only if *y* is more *F* than *x*.

Given these principles, Comparative Transitivity and the analogous transitivity principles for 'equally *F*' and 'less *F*' all follow from

**Equative Transitivity** If *x* is at least as *F* as *y* and *y* is at least as *F* as *z*, then *x* is at least as *F* as *z*.

Another noteworthy consequence of Strict Comparison and Reversal is that sentences of the form 'x is more/less F than x' and 'x is both more F and less F than y' are logically inconsistent: just like sentences of the form 'x is F and x is not F', they are necessarily false on any uniform interpretation. And indeed, our treatment of such sentences is analogous in revealing ways. Just like 'The soup is delicious and also not delicious', 'The soup is both more delicious and less delicious than the dessert' sends us looking for a plausible *non*-uniform interpretation where the two occurrences of 'delicious' mean different things.

Here is another plausible principle about equatives:

**Restricted Reflexivity** If x is at least as F as something or something is at least as F as x, then x is at least as F as x.

One might be tempted to think that the simpler and stronger schema 'x is at least as F as x' is also valid. However, that schema would commit us to the truth of dubious sentences like 'This song is at least as hairy as itself' and 'The number five is at least as blue as itself'. Restricted Reflexivity by contrast leaves us free to reject these sentences, on the grounds that no song is as hairy as anything else and no number is as blue as anything else.

The meaning of a comparative construction, we assume, is determined compositionally by combining the meaning of its base form 'F' with that of the comparative-forming element ('more', '-er', 'as', etc.). This naturally suggests that we might find logical connections between the positive form (without overt modifiers) and the comparative. Here is one plausible example:

**Monotonicity** If *x* is *F* and *y* is at least as *F* as *x*, then *y* is *F*.

Some might reject Monotonicity on the grounds that 'The coffee is expensive' may express something true and 'The sandwich is expensive' something false when the coffee costs \$6 and the sandwich \$7. But we can account for this by claiming that the two uses of 'is expensive' are naturally interpreted in different ways: the first means 'is expensive for a coffee', and the second means 'is expensive for a sandwich'.<sup>1</sup>

The standard treatment of the positive form in semantics (going back to Bartsch & Vennemann, 1972) analyzes sentences like 'Kara is healthy' as containing a phonologically null degree modifier 'POS' playing the same role that 'very' plays in 'Kara is very healthy'. This suggests generalizing Monotonicity to the following schema, where 'V' can be replaced by any positive degree modifier such as 'very', 'extremely', 'somewhat', 'pretty', 'quite', and so on:

# **Modified Monotonicity** If *x* is *V F* and *y* is at least as *F* as *x*, then *y* is *V F*.

An adequate logic of comparatives should also have something to say about the superlative forms '*F*-est' and 'most *F*'. For example, they are plausibly subject to an analogue of Monotonicity:

**Superlative Monotonicity** If *x* is one of the most *F K*s and *y* is a *K* and *y* is at least as *F* as *x*, then *y* is one of the most *F K*s.

The appearances that favor all of these principles suggest that there is a general logic that unifies the cluster of expressions that include 'more *F* than', 'as *F* as', 'most *F*', 'very *F*', and '*F*'. This logic, moreover, appears to go beyond adjectives. For example, our schemas extend naturally to comparative constructions based on nouns ('at least as much coffee', 'more reason'), verbs ('resemble more', 'like at least as much'), and adverbs ('dance at least as gracefully', 'prefer more strongly'). Subsuming these different forms under unified generalizations would require a general semantic analysis of comparative constructions, which we cannot provide here (see Wellwood, 2019). In what follows, we will continue to work with formulations of the schemas involving adjectives, but will treat sentences stated using non-adjectival comparatives as if they were honorary instances of

<sup>&</sup>lt;sup>1</sup>It might be objected that 'The \$6 coffee is expensive and the \$7 sandwich is not expensive' can be made true without equivocation on 'expensive', on the grounds that 'is expensive' also has a reading paraphrasable as 'is expensive for the kind of thing it is' (see, e.g., John Hawthorne, 2007). In support of such a reading, one can note that 'Everything on that menu is expensive' is naturally understood to require the coffee to be expensive for a coffee, the sandwich to be expensive for a sandwich, etc. In response, we would claim that if 'x is expensive' can mean 'x is expensive for what it is', 'x is at least as expensive as y' can (though not as easily) mean 'x is at least as expensive for what it is as y is for what *it* is'. We suggest that this reading of 'at least as expensive as' is required for a uniform interpretation of an instance of Monotonicity in which the two positive occurrences of 'expensive' have the "expensive for what it is" reading. For those who do not accept this, we note that the schema 'If x is F for a K and y is a K and y is at least as F as x, then x is more F than y' does not raise the same issue, and could play the same role in our argument as Monotonicity.

those schemas, since it would be implausible to suppose that the schemas stated using adjectives are valid but that their analogues for other syntactic categories are not.<sup>2</sup>

# The Thesis of Comparability

In this paper, we will be taking for granted the validity of the above schemas and arguing, on that assumption, for the validity of the following, more controversial schema:

**Comparability** If *x* is at least as *F* as *x* and *y* is at least as *F* as *y*, then either *x* is at least as *F* as *y* or *y* is at least as *F* as *x*.

Given Strict Comparison, Equality, and Reversal, this is equivalent to the more naturalsounding

**Trichotomy** If *x* is at least as *F* as *x* and *y* is at least as *F* as *y*, then either *x* is more *F* than *y*, *x* is less *F* than *y*, or *x* and *y* are equally *F*.

We state Comparability and Trichotomy in conditional form rather than just endorsing their consequents because we do not want to be committed either way as regards the truth of odd disjunctions like 'Either the square root of two is at least as tall as the Eiffel Tower, or the Eiffel Tower is at least as tall as the square root of two' or 'Either my left arm is more expensive than this laptop, this laptop is more expensive than my left arm, or my

**Mixed Strict Comparison** *x* is more *F* than *y* is *G* if and only if *x* is at least as *F* as *y* is *G* and *y* is not at least as *G* as *x* is *F*.

**Mixed Equative Transitivity** If *x* is at least as *F* as *y* is *G*, and *y* is at least as *G* as *z* is *H*, then *x* is at least as *F* as *z* is *H*.

Since 'x is more F than y' and 'x is at least as F as y' are obviously equivalent to 'x is more F than y is F' and 'x is at least as F as y is F', respectively, these principles will imply their non-mixed versions as special cases. The monotonicity principles, by contrast, also have natural mixed generalizations, but these do not seem to be valid. For example, the natural generalization of Monotonicity would be

**Mixed Monotonicity** If *x* is *F* and *y* is at least as *G* as *x* is *F*, then *y* is *G*.

This looks hard to defend: some people are wide but not tall, despite being taller than they are wide (Bale, 2008, p. 4). For Restricted Reflexivity, finally, it is unclear what a mixed generalization would even look like, though in note 1 below we introduce a schema for mixed comparatives that has Restricted Reflexivity as a special case.

<sup>&</sup>lt;sup>2</sup>It is also desirable to generalize the logic to cover "mixed" comparatives of the form '*x* is more/less *F* than *y* is *G*' and '*x* is at least as *F* as *y* is *G*' (see Bale, 2008; Hamann, Nerbonne, & Pietsch, 1980; Kamp, 1975; Paoli, 1999; von Stechow, 1984). Some of our basic schemas have natural mixed generalizations:

left arm and this laptop are equally expensive.' Comparability and Trichotomy do not require these disjunctions to be true, unless 'The square root of two is at least as tall as itself' and 'My left arm is as expensive as itself' are true.<sup>3</sup>

It is worth emphasizing that, as with the other schemas, the validity of Comparability only requires its instances to be true when any ambiguous or context-sensitive elements in them are interpreted uniformly. For example, 'Either this seminar is at least as long as Fifth Avenue or Fifth Avenue is at least as long as this seminar' seems very dubious, even though 'This seminar is at least as long as itself' and 'Fifth Avenue is at least as long as itself' both seem true. Plausibly, this is because there are two meanings of 'at least as long as' in play in the latter sentences, one involving time and the other involving space. This diagnosis can also be used to account for oddities like 'Either this cup of coffee is at least as good as the latest *Star Wars* movie, or the latest *Star Wars* movie is at least as good as this cup of coffee,' by appealing to the context-sensitivity of 'good'. Either there is no uniform interpretation of 'good' on which the antecedent of Comparability is satisfied for this example, or such an interpretation is extremely difficult to access.

We stipulatively use 'x is *F*-assessable' to mean that x is at least as *F* as itself. Given Restricted Reflexivity, this is equivalent to the more cumbersome, but perhaps more intuitive, claim that x is either at least as *F* as something or such that something is at least as *F* as it. We use 'x and y are *F*-comparable' to mean 'Either x is at least as *F* as y or y is at least as *F* as x', and 'x and y are *F*-incomparable' to mean 'x and y are both *F*-assessable but are not *F*-comparable'. Using these definitions, Comparability can be restated succinctly as 'No two things are *F*-incomparable'. These stipulations are not supposed to reflect the ordinary meanings of 'assessable', 'comparable', and 'incomparable'. (Thus, we characterize Chang (2002) as a proponent of incomparability even though, on her view, all pairs of items that are incomparable in our technical sense are comparable in her preferred sense.)

It is dialectically appropriate for us to take the earlier schemas for granted in defending Comparability, since most opponents of Comparability in the philosophical literature

<sup>&</sup>lt;sup>3</sup>As discussed in note 1 above, it is natural to look for some principle about mixed comparatives that implies Comparability as a special case. We propose the following:

**Mixed Comparability** If (either x at at least as F as something is G or something is at least as G as x is F) and (either y at at least as G as something is F or something is at least as F as y is G), then (either x at at least as F as y is G or y is at least as G as x is F).

Given the equivalence of 'x is as F as y' with 'x is as F as y is F', this implies both Restricted Reflexivity and Comparability. We get something logically equivalent to Restricted Reflexivity by setting x = y and F = G; setting just F = G, we get a principle which is equivalent to Comparability given Restricted Reflexivity.

accept those schemas. Indeed, as we will see, some of the most influential objections to Comparability presuppose the validity of at least certain instances of Equative Transitivity, Strict Comparison, and Equality. Some readers may be tempted to deny the validity of even these three schemas. But even such readers may be willing to grant that there is some good status possessed by the above schemas, or restrictions of the schemas to a certain class of expressions narrower than our broad category of comparatives. We hope to convince such a reader that Comparability has a similar status.

It is beyond the scope of this paper to explain *why* Comparability and the other schemas are valid. The natural assumption is that it is somehow due to the meanings of 'more', 'as', 'most', and the various degree modifiers—much as the validity of the inference 'If no *F* is *G*, then it is not the case that some *F* is *G*' is due to the meanings of 'no' and 'some'. But it is a controversial question exactly what kind of semantic values these comparative-forming words should be assigned.<sup>4</sup> Fortunately, our arguments will not require us to take a stand on this question.

Our main arguments for Comparability will be given in sections 3 and 4. Section 2 considers the influential arguments against Comparability from the literature, and section 5 responds to them in the light of our positive arguments for Comparability. However, before getting into considerations specific to Comparability, we should further clarify what we take ourselves to be committed to in classifying it and the other schemas as "valid". These clarifications may be skipped without losing the main thread of our argument. But they will allow us to address some possible objections to Comparability which, if successful, would tell against the other schemas as well.

# Clarifications

To begin with, we emphasize that in claiming that the schemas are valid, we are not claiming that it is impossible to use, either by stipulative fiat or as a sort of local idiom, an expression of the form 'more *F* than' ('*F*-er than', 'at least as *F* as', etc.) in a way that violates the schemas. One can, with diligent effort, impose a use on 'more westerly' on which 'Tokyo is more westerly than New York', 'New York is more westerly than Istanbul', 'Istanbul is more westerly than Tokyo', and 'Nothing is more westerly than itself' are all true, in apparent violation of Comparative Transitivity. Similarly, one might

<sup>&</sup>lt;sup>4</sup>There is a wide range of possible semantic explanations of the validity of the schemas, since most semantic accounts of comparatives—for example, those of Bale (2008), Kennedy (1999), Klein (1982), and van Rooij (2011)—are structured in such a way as to validate all of them. For a recent and especially general account of the meanings of the comparative and equative morphemes that entails the validity of Comparability, see Wellwood (2019).

fall into the practice of using 'larger than' to stand for a certain mathematical relation that is not asymmetric, or is even reflexive, and which therefore makes for apparent counterexamples to Strict Comparison.<sup>5</sup> And logicians sometimes use 'is at least as strong as' to mean entails (and 'is stronger than' to mean entails and is not entailed by), making any pair of propositions neither of which entails the other an apparent counterexample to Comparability. We doubt that such examples show that there is no such thing as the logic of comparatives, or that it does not include these schemas—any more than the possibility of stipulatively or idiomatically using 'some shmugs' and 'all shmugs' as synonyms of 'at least three dogs' and 'at least four dogs' respectively shows that there is no such thing as the logic of quantifier-words, or that it does not include schemas like 'If some Fs are not *G* then it is not the case that all *F*s are *G*'. We take the cases to be analogous. Just as the stipulated meanings for 'some shmugs' and 'all shmugs' are not derived compositionally by combining a meaning for 'shmugs' with the standing meanings of 'some' and 'all', the problematic meanings for 'more westerly', 'larger', and 'stronger than' are not derived compositionally by combining meanings for the base forms 'westerly', 'large', and 'strong' with the standing meanings of 'more' and '-er.' We will therefore not consider these expressions to be comparatives in the sense we are concerned with, and so do not treat the result of substituting them into the schemas as generating genuine instances of those schemas.<sup>6</sup>

These stipulative and idiomatic uses are not the only cases where expressions that look like comparatives fail to be genuine comparatives according to our refined definition. Another kind of example involves cases where the role of F is played by a complex expression containing a quantifier. If Hilary is more interested in mathematics but less interested in physics than Logan, then both (1a) and (1b) are false:

- (1) a. Hilary is at least as interested in every subject as Logan.
  - b. Logan is at least as interested in every subject as Hilary.

Since people are at least as interested in every subject as themselves, we have a superficial

<sup>&</sup>lt;sup>5</sup>"ProofWiki", an online compendium of mathematical proofs, defines '*S* is larger than *T*' (where *S* and *T* are sets) to mean that there exists a bijection from *T* to a subset of *S*, so that every set is larger than itself; they use 'strictly larger' for the irreflexive relation (https://proofwiki.org/w/ index.php?title=Definition:Larger\_Set&oldid=443640).

<sup>&</sup>lt;sup>6</sup>This is often obscured by the ways in which the ordinary pattern of uses of comparatives tends to reassert itself even in contexts where officially some other stipulation is in play. For example, philosophers will sometimes say that one theory is "much stronger than" or "only a little stronger than" another, and will be tempted to assume unreflectively that if  $T_3$  is much stronger than  $T_1$  and  $T_2$  is only a little stronger, then  $T_3$  is stronger than  $T_2$ . This reasoning does not fit naturally with the stipulation that 'at least as strong as' means 'entails', and can lead to all sorts of confusions.

failure of Comparability. Likewise for Comparative Transitivity, since both (2a) and (2b) are true but (2c) is false:

- (2) a. Hilary is more interested in some subject than Logan.
  - b. Logan is more interested in some subject than Hilary.
  - c. Hilary is more interested in some subject than Hilary.

And similarly for Strict Comparison: if Hilary is more interested in mathematics than Logan but they are instead equally interested in every other subject, then (1a) is true and (1b) is false, but 'Hilary is more interested in every subject than Logan' is also false.

Rather than concluding from these examples that not only Comparability but also Comparative Transitivity and Strict Comparison are invalid, it seems more reasonable to deny that the meaning of 'more/as interested in every/some subject' is the comparative or equative of 'interested in every/some subject'. If it were, then we would expect such expressions to obey Monotonicity; but if Hilary is interested in mathematics and Lindsay isn't interested in anything, but more interested in physics than Hilary, then both (3a) and (3b) are true but (3c) is false:

- (3) a. Hilary is interested in some subject.
  - b. Logan is more interested in some subject than Hilary.
  - c. Logan is interested in some subject.

This is not surprising. Famously, quantifiers can behave, via some special syntactic or semantic mechanism, as if they took wide scope even when they occur in deeply embedded positions (see Barker & Shan, 2014; Ruys & Winter, 2011). This allows 'Hilary is at least as interested in some/every subject as Logan' to be equivalent to 'For some/every subject, Hilary is at least as interested in that subject as Logan'. Whatever mechanism is responsible for this equivalence, it presumably implies that the meaning of 'at least as interested in every subject' is not the result of operating with the ordinary meaning of '[at least] as' on a meaning for 'interested in every subject' of the same type as that of a single adjective. This expression is thus not a genuine comparative in our sense, and the counterexamples are merely apparent.<sup>7</sup>

<sup>&</sup>lt;sup>7</sup>We also need to be careful about readings of 'is as/more F' that involve *implicit* quantification over times or situations (thanks to Tim Williamson for raising this point). For example, suppose that on most nights when Venus and Mars are both visible they are equally bright, but on some nights Venus is brighter than Mars; Mars is never brighter than Venus. Then there is a reading on which 'Venus is at least as bright as Mars' is true, amounting to something like 'Most nights, Venus is at least as bright as Mars'. 'Mars is at least as bright as Venus' is false and 'Venus is brighter'.

A related kind of merely apparent counterexample involves conjunctions and disjunctions rather than quantifiers within the internal argument of the adjective. All the apparent counterexamples from the previous paragraph remain if we replace 'every subject' and 'some subject' throughout with 'math and physics' and 'math or physics', respectively. We can get parallel effects using conjunctions and disjunctions of adjectives themselves, like 'edifying and/or entertaining' (Bale, 2020). Suppose *Reasons and Persons* is more edifying than *The Lord of the Rings* but less entertaining. Then (4a) and (4b) both seem false:

- (4) a. *Reasons and Persons* is at least as edifying and entertaining as *The Lord of the Rings*.
  - b. *The Lord of the Rings* is at least as edifying and entertaining as *Reasons and Persons*.

Since both books are (necessarily) at least as edifying and entertaining as themselves, this appears to conflict with Comparability. Variants of this example generate apparent counterexamples to Strict Comparison and (using 'or' rather than 'and') to Comparative Transitivity (as noted by Bale, 2020) as well as Monotonicity. Unsurprisingly, we maintain that in these examples, 'at least as interested in math and/or physics' and 'as least as edifying and/or entertaining' do not function as genuine comparatives in our sense. Some special mechanism lets 'and' and 'or' function as if they scoped above 'at least as' and 'more', so that (e.g.) 'at least as *F* and *G*' is equivalent to 'at least as *F* and at least as *G*'. Whatever the mechanism is (we mention some possibilities in note 1), it does not involve generating a meaning for the complex adjectival phrase of the same type as that of a bare adjective and then operating on this with the standard meaning of 'at least as' or 'more'.

Bale (2020) argues for the competing view that 'and' in 'more F and G' is just a standard Boolean conjunction operator taking F and G as arguments.<sup>8</sup> A satisfactory defence of

<sup>8</sup>On Bale's semantics (building on Cresswell, 1976), an adjectival phrase *F* expresses the same binary relation among objects as '... is more *F* than ... is'. This allows 'more *F* and *G*' to be equivalent to 'more *F* and more *G*' with a standard Boolean meaning for 'and', scoping below 'more'. For 'or', by contrast, he favours a set-forming semantics à la Alonso-Ovalle (2006). Bale does not explicitly discuss 'as', and in fact we see no easy way for a semantics where adjectives

than Mars' are both false on the corresponding readings, so we have an apparent violation of Strict Comparison. If instead each planet was brighter than the other half the time, we would have an apparent violation of Comparability. If each of the three brightness orderings Jupiter > Mars > Venus, Mars > Venus > Jupiter, and Venus > Jupiter > Mars is instantiated on one night in three, then arguably 'Jupiter is brighter than Mars' and 'Mars is brighter than Venus' are true and 'Jupiter is brighter than Venus' false, in apparent violation of Comparative Transitivity. In effect, the sentences are interpreted as if prefixed by 'At most times'. While it is not obvious how this quantificational element arises compositionally, we take it that our restriction to "genuine comparatives" will exclude such cases.

our proposal would require a more thorough treatment of his arguments than we have space to provide here; it would not be a disaster for us if we ended up having to restrict Comparability to exclude instances where F is a conjunction, since such a restriction would be needed for Strict Comparison anyway, and our thesis is conditional on it and the other schemas. But as prima facie motivation for positing the kind of special mechanism we need, we note that conjunctions of other sorts of constituents also make for apparent exceptions to plausible logical principles. Consider plural nouns. If all Republicans and only some Democrats signed a measure, then (5a) is true but (5b) seems false:

- (5) a. Not all Republicans and Democrats are signatories.
  - b. Some Republicans and Democrats are not signatories.

It seems hasty to conclude that the logic of quantification does not include the Aristotelian schema 'If not all *Fs* are *Gs*, then some *Fs* are not *Gs*'. Conjunctions of plural nouns also give rise to apparent violations of the analogue of Comparability for comparative quantifiers of the form 'more/as many *Fs*'. For example, if Avery interviewed more Democrats but fewer Republicans than Brooklyn, (6a) and (6b) both have prominent false readings.<sup>9</sup>

- (6) a. Avery interviewed at least as many Republicans and Democrats as Brooklyn did.
  - b. Brooklyn interviewed at least as many Republicans and Democrats as Avery did.

Similar examples give rise to apparent violations of the analogue of Strict Comparison, which seems especially unimpeachable. But it is hard to see how a view that treats 'Republicans and Democrats' as a constituent with the same kind of meaning as 'Republicans' (analogous to Bale's treatment of conjunctions of adjectives) could even begin to explain how we get the semantic effect of two occurrences of 'not all'/'some'/'at least as many'/ 'more' in these examples. Whatever special mechanism gives the effect of such doubling in the case of nouns should generalize easily to conjunctions of adjectives.<sup>10</sup>

express binary relations among objects to deliver both the equivalence with 'more' and the one with 'as'.

<sup>&</sup>lt;sup>9</sup>These sentences also have other readings, which may be true in this case. In (6a), 'as many Republicans and Democrats' can mean 'as many Republicans and Democrats in total', and maybe it can also mean 'as many people who are both Republicans and Democrats' (compare 'Some friends and colleagues are coming to dinner'). See Champollion (2016) for an account of these readings.

<sup>&</sup>lt;sup>10</sup>One possible mechanism involves ellipsis: a second occurrence of 'more' or 'at least as' is present semantically but unpronounced. Bale argues against treating 'more *F* and *G*' as an

One final caveat: some philosophers reject (or at least decline to accept) the validity of instances of the Law of the Excluded Middle, 'Either *P* or it is not the case that *P'*. Particularly germane in the present context are views according to which the *vagueness* of *P* can make instances of this schema unacceptable. These philosophers have reason to be suspicious of Comparability and many of its consequences. For example, they should not accept the validity of Trichotomy. For the combination of Trichotomy with the seemingly valid 'If either *x* and *y* are equally *F* or *y* is more *F* than *x*, then *x* is not more *F* than *y'* (a trivial consequence of Strict Comparison and Equality) seems to imply 'If *x* and *y* are at least as *F* as themselves, then either *x* is more *F* than *y* or *x* is not more *F* than *y*.' But it would be strange for anyone who thought that vagueness required giving up Excluded

elliptical version of 'more *F* and more *G*' on the grounds that 'more *F* than *a* is and *G* than *b* is' seems ungrammatical whereas 'more *F* than *a* is and more *G* than *b* is' is fine (and likewise for 'less'). Insofar as this sort of argument is good, it generalises to the case of nouns: for example, while 'many Republicans and Democrats protested' can mean 'many Republicans and many Democrats protested', 'many Republicans protested and Democrats protested' cannot mean 'many Republicans protested and many Democrats protested is sort of *Republicans* than Brooklyn did and Democrats than Charlie did' sounds no better than '*Reasons and Persons* is more edifying than *The Lord of the Rings* is and entertaining than *The Silmarillion* is'. We are inclined to reject ellipsis in both cases.

A second possible mechanism involves type-raising, implemented either as a primitive operation (Dowty, 1988; Partee & Rooth, 1983), or derived in some more general type logic (Kubota & Levine, 2020; Lambek, 1968; Morrill, 1994). 'Edifying' and 'entertaining' both raise from their base type to a higher type that can take the semantic value of 'more' as an argument; these higher-type denotations are combined with Boolean 'and' or 'or', and then take 'more' as an argument, yielding the same denotation as 'more edifying or more entertaining'. See Dowty (1988) for the application of this kind of type-raising to conjunctions of nouns.

A third possibility involves a nonstandard semantics for 'and' or 'or' such as the tuple-forming semantics of 'and' (Winter, 1995) or the set-forming semantics for 'or' (Alonso-Ovalle, 2006; Simons, 2005). On this kind of approach, the co-ordinated expression (e.g., 'edifying and entertaining') denotes some kind of compound entity, such as a set or ordered pair; the standard denotations of expressions taking such compound arguments, such as 'more' and 'as', are lifted to functions that apply their standard meanings pointwise to sets or tuples; and at some point, a silent operator is inserted to map a set or tuple of propositions, properties, or relations to the ordinary Boolean conjunction or disjunction of its elements.

Each of these approaches faces overgeneration worries (e.g., in Bale's examples (37) and (38)) which will need to be handled with care. Bale also suggests that a tuple-forming account of 'and' (in the style of Winter) will have trouble with the sentence 'Seymour is more handsome and talented than how handsome and talented Patrick is'; this presents a challenge to the second, type-raising approach as well. But this is not specific to adjectives: 'Avery interviewed more Republicans and Democrats than how many Republicans and Democrats Brooklyn interviewed' presents the same difficulty. An adequate treatment of these examples will also, plausibly, need to cover the more general, and notoriously difficult, topic of "respectively" readings—as in 'The table is more long and tall than the pool is deep and wide [respectively]' (see Gawron & Kehler, 2004; Kubota & Levine, 2020, ch. 5)—which challenge Bale's semantics too.

Middle to accept this schema as valid, since the comparative forms of adjectives are often vague. And since Trichotomy follows rather unproblematically from Strict Comparison, Equality, Reversal, and Comparability, any doubts about Excluded Middle should carry over to at least one of these schemas as well. Our own view is that Excluded Middle is valid even when vague expressions are in play, so this objection to Comparability does not trouble us. A defence of Excluded Middle, however, would take us too far afield from our topic, so we here simply assume its validity. (For a variety of views about vagueness that are compatible with the validity of Excluded Middle, see Bacon, 2018; Dorr, 2003; Fine, 1975; Graff, 2000; Keefe, 2000; Lewis, 1969; Williamson, 1994.) We suggest that those who decline to accept Excluded Middle focus not on Comparability but on the following classically-equivalent variant that avoids disjunction and is thus not obviously affected by doubts related to Excluded Middle:

**Conditional Comparability** If *x* and *y* are at least as *F* as themselves and *x* is not at least as *F* as *y*, then *y* is at least as *F* as *x*.

In a non-classical logic like that of Field (2003), where Conditional Comparability does not imply Comparability, Conditional Comparability may retain much of the philosophical significance of Comparability and be supported by variants of some of our arguments below. However we will not attempt to track which of our arguments could be adapted to any particular non-classical background logic.

# 2 The Debate about Comparability

Comparability seems to be taken for granted in most contemporary work by semanticists working on comparatives (see, e.g., Bale, 2008; Kennedy, 1999; Klein, 1982; van Rooij, 2011; Wellwood, 2019). However, we are not aware of explicit arguments for Comparability in the linguistics literature, and it is not obvious how indispensable the assumptions that lead to its validity are in the linguists' accounts of the phenomena they are primarily concerned to explain.

Many philosophers, by contrast, either argue explicitly against certain instances of Comparability, or rely on such failures in accounts of other phenomena, in ways that could easily be retooled as abductive arguments against Comparability.<sup>11</sup> Some of these

<sup>&</sup>lt;sup>11</sup>Perhaps the most influential examples are Chang (2002) and Raz (1985). Examples related to rational choice and preference include Handfield (2014), Hsieh (2005), Rabinowicz (2008, 2012), Raz (1999), and Schoenfield (2014). Examples related to moral decision-making include Hare (2016), Richardson (1994), and Sinnott-Armstrong (1985). Examples from population ethics include Black-orby, Bossert, and Donaldson (1996), Chang (2016), Frick (2017), Gustafsson (2020), Hájek and Ra-

arguments are specific to certain expressions of interest to some particular fields of philosophy, and cannot be addressed in this paper.<sup>12</sup> But it will be useful to consider some particularly influential arguments against Comparability which, if successful, would generalize to a wide class of comparative expressions. These arguments turn on the so-called "multidimensionality" which many comparative expressions exhibit. For example, in the case of 'good', it often seems deeply tendentious to characterise either of two things as at least as good as the other, because each is better than the other in some respects, and there seems to be no non-arbitrary basis for aggregating the different respects into a single overall judgment. For example, according to Parfit (2016, p. 113),

When two painful ordeals differ greatly in both their length and their intensity, there are no precise truths about whether, and by how much, one of these pains would be worse. There is no scale on which we could weigh the relative importance of intensity and length.

Sen (1997b, p. 5) makes a similar point about 'more unequal than':

Most statistical measures of the inequality level assume a high degree of measurement.... It is, however, possible to argue that the implicit notion of inequality that we carry in our mind is, in fact, much less precise and may correspond to an incomplete quasi-ordering. We may not indeed be able to decide whether

<sup>12</sup>It is worth mentioning a particularly influential argument in the case of 'good', which proceeds from a kind of "Fitting Attitudes" analysis of value relations. Rabinowicz (2008, 2012) argues that for x to be better than y just is for it to be required—or "unfitting not"—to prefer x to y, and for x and y to be equally good is for it to be required to be indifferent between them (see also Hájek & Rabinowicz, 2021). Given these claims, any case in which it is not required to prefer x to y, not required to prefer y to x, and not required to be indifferent between them (even though each is at least as good as itself) will be a counterexample to Trichotomy. In response, we could reject either Rabinowicz's Fitting Attitudes analysis of value relations or the possibility of that pattern of permissibility relations. Since such cases seem possible, we are inclined to reject Rabinowicz's analysis. His analysis seems to us independently implausible because it leads to violations of Strict Comparison. According to Rabinowicz, *x* is at least as good as *y* iff it is required to either prefer *x* to *y* or be indifferent between them. Plausibly, this disjunction may be required even if neither disjunct is, in which case x will be at least as good as y, and not vice versa, without x being better, contrary to Strict Comparison. Rabinowicz (2008, p. 43) seems willing to accept this consequence; we are not. This problem would seem to afflict any kind of Fitting Attitude analysis that is plausibly inconsistent with Comparability, not just Rabinowicz's.

binowicz (2021), Nebel (2019, 2020, 2022), Parfit (2016), Qizilbash (2007, 2018), Rabinowicz (2009a), and Thornley (forthcoming). Examples from formal epistemology include Bovens and Hartmann (2003), Bradley (2017), James Hawthorne (2009), Kaplan (1996), Konek (2019), Schoenfield (2012), and Smith (2014). Amartya Sen in particular has utilized incomparability across a wide variety of domains, including the theory of inequality, welfare aggregation, justice, and social and rational choice (Sen, 1970, 1980, 1997a, 1997b, 2009).

one distribution *x* is more or less unequal than another, but we may be able to compare some other pairs perfectly well. The notion of inequality has many aspects, and a coincidence of them may permit a clear ranking, but when these different aspects conflict an incomplete ranking may emerge.

Earlier still, Keynes (1921, p. 31) gives a similar justification for putative counterexamples to Comparability with respect to 'probable':

Consider three sets of experiments, each directed towards establishing a generalisation. The first set is more numerous; in the second set the irrelevant conditions have been more carefully varied; in the third case the generalisation in view is wider in scope than in the others. Which of these generalisations is on such evidence the most probable? There is, surely, no answer; there is neither equality nor inequality between them. We cannot always weigh the analogy against the induction, or the scope of the generalisation against the bulk of the evidence in support of it.

Kamp (1975, 140f.) argues along similar lines, and suggests that the phenomenon will arise for a wide range of adjectives:

Suppose for example that Smith, though less quick-witted than Jones, is much better at solving mathematical problems. Is Smith cleverer than Jones? This is perhaps not clear, for we usually regard quick-wittedness and problemsolving facility as indications of cleverness, without a canon for weighing these criteria against each other when they suggest different answers. ... Before any decision has been made it is true neither that Smith is cleverer than Jones nor that Jones is cleverer than Smith. [The claim that] for any objects  $u_1$  and  $u_2$  and adjective A, either  $u_1$  is at least as A as  $u_2$  or  $u_2$  is at least as A as  $u_1$ ...should fail to be true in general whenever we have two, largely independent, criteria for applicability of the adjective, but no clear procedure for weighing them.<sup>13</sup>

We take it that Parfit, Sen, Keynes, and Kamp are reasoning as follows. Consider a case in which *x* and *y* are both *F*, but in quite different ways. If Comparability is true, then

<sup>&</sup>lt;sup>13</sup>McConnell-Ginet (1973, p. 106) earlier makes a similar point using 'intelligent'. Picking up on these examples, Klein (1980) develops a semantics on which both 'Jude is at least as clever as Mona' and 'Mona is at least as clever as Jude' may be neither true nor false, but he uses super-valuationist machinery in evaluating complex sentences so that their disjunction, and instances of Comparability more generally, are nevertheless always true.

either one is more F than the other or they are equally F. Whichever it is, the comparison would have to be explained by some particular way of weighing between the relevant dimensions of F. But there is no reason to privilege any particular way of weighing between those dimensions. We should therefore deny that one of the items must be at least as F as the other.

There are doubtless subtle differences between the ways in which our various authors are thinking about this mode of argument, but we will lump them all together as "tradeoff arguments". While not all gradable expressions are subject to the kind of multidimensionality that drives these arguments, a great many are: for example one could easily run parallel arguments for 'funny', 'beautiful', 'interesting', 'useful', 'hairy', 'healthy', and 'tidy' (for some proposed diagnostics of multidimensionality, see Sassoon, 2013).

Another kind of argument, prominent in the axiological literature, is the "argument from small improvements".<sup>14</sup> Here, instead of citing the difficulties posed by tradeoffs among dimensions in the same way to support all three of '*x* is not more *F* than *y*', '*y* is not more *F* than *x*' and '*x* and *y* are not equally *F*', one starts with the first two of these claims (perhaps motivated as before by thoughts about the difficulty of tradeoffs), and then uses these as premises in a more complicated argument for the third claim (of non-equality). Here is an example from Chang (1998, pp. 23–24), based on Raz (1985):

Suppose we rationally judge that a particular career as a clarinetist is neither better nor worse than a particular career as a lawyer, say, with respect to goodness of careers. ... We can improve the clarinetist career a little with respect to goodness of careers, perhaps by increasing the salary by ten dollars. Are we thereby compelled to judge that the improved music career is better than the legal one? It seems rational to resist this conclusion. If it is rational, then the original careers cannot be equally good, since if they were, a small improvement in one must make it better than the other.

The general form of the argument can be reconstructed as follows:

(P1) x is not more F than y and y is not more F than x.

(P2)  $x^+$  is more *F* than *x*.

(P3)  $x^+$  is not more *F* than *y*.

(Conclusion) x is not at least as F as y and y is not at least as F as x.

<sup>&</sup>lt;sup>14</sup>The small improvement argument was first made, by De Sousa (1974), as an objection to the completeness axiom of expected utility theory. The axiological version is pressed, most influentially, by Raz (1985). We discuss the preference-theoretic version in Dorr, Nebel, and Zuehl (2021)

This argument is valid given Equative Transitivity and Strict Comparison.<sup>15</sup> So the challenge is to find particular instances of x, y, and  $x^+$  for which all three premises are plausible. As we saw in the quote from Chang, the strategy is to choose an x and y for which P1 is plausible not because of some very extensive similarities or some perfectly balanced competing considerations, but because x and y differ along multiple relevant dimensions in such a way that it would seem tendentious to characterize either as more F than the other. We then choose an  $x^+$  that is similar to x but slightly improved along some dimension. This makes P2 plausible, while keeping the pattern of relations between  $x^+$  and y similar enough to the pattern of relations between x and y that insofar as that pattern supports P1, it will provide a similar level of support to P3.

The small improvement argument is arguably an improvement over the tradeoff argument, since in the presence of the relevant kind of multidimensionality, the claim that the relevant items are equally F seems less immediately repugnant than the claims to the effect that one is more F than the other. (We discuss why this should be in section 5.)

We will come back to the tradeoff and small improvement arguments in section 5, after we have laid out our positive case for Comparability. We will suggest that there are good independent reasons for being suspicious of both arguments, so that overall, the considerations in favour of Comparability are more compelling.

Despite the broad influence of tradeoff and small improvement arguments, philosophers have certainly not been unanimously opposed to Comparability. Some have defended the validity of Comparability for specific adjectives on specific interpretations: for example, Regan (1997) appeals to requirements of practical reason to argue that Comparability holds for the specific sense of 'good' used by G. E. Moore. But there seems to be only one widely discussed argument that might have some chance of supporting Comparability in full generality, namely that of Broome (1997).

Broome's strategy is to argue that incomparability in any gradable expression is incompatible with vagueness. In doing so, he relies on the following "collapsing principle" about vague comparatives: if y is not more F than x and it is not determinately false that x is more F than y, then x is more F than y.<sup>16</sup> Unfortunately, this principle is subject to counterexample. For example, suppose that we are in the process of enlarging a statue

<sup>&</sup>lt;sup>15</sup>Suppose for contradiction that either *x* is at least as *F* as *y* or *y* is at least as *F* as *x*. By P1, each of *x* and *y* must at least as *F* as the other. But P2 implies that  $x^+$  is at least as *F* as *x*, so, by Equative Transitivity,  $x^+$  is at least as *F* as *y*. So by Strict Comparison the only way P3 could be true would be if *y* were also at least as *F* as  $x^+$ . But if this were the case, then by Equative Transitivity, *x* would have to be at least as *F* as  $x^+$ , which is ruled out by P2.

<sup>&</sup>lt;sup>16</sup>Broome's wording lacks the 'determinately'; we find it hard not to read 'not false that' as logically redundant.

that was originally made of a certain quantity of bronze, b; we have moulded a new piece of bronze into the shape of a crown, which we are now in the process of soldering on to the top of b. Let  $b^+$  be the larger quantity of bronze comprising b together with our new bronze; suppose we are at a point in the process where it is neither determinately true nor determinately false that the new bronze is part of the statue. (This assumption should be acceptable to anyone who takes 'it is not determinately false that...' to be non-redundant.) If so it is also neither determinately true nor determinately false that the statue is heavier than b, and also neither determinately true nor determinately false that  $b^+$  is heavier than the statue. But evidently b is not heavier than the statue and the statue is not heavier than  $b^+$ . Given two applications of Broome's principle, these premises imply that the statue is heavier than b, and  $b^+$  is heavier than the statue. But we surely should not be committed to this conclusion, given that we are committed to its being neither determinately true nor determinately false. Indeed, the conclusion seems implausible: it's certainly not the case that only *some* of the new bronze is part of the statue.

There is a sizable literature on Broome's collapsing principle and how it might be revised to avoid such counterexamples (see Andersson, 2014; Andersson & Herlitz, 2018; Carlson, 2004, 2013; Constantinescu, 2012; Elson, 2014; Gustafsson, 2013, 2018; Qizilbash, 2007; Rabinowicz, 2009b). Suffice it to say that principles in the vicinity are highly controversial and, we believe, less independently plausible than Comparability itself. We will not rely on them.

We know of no other explicit arguments in favour of the validity of Comparability. In the next sections we supply two kinds of arguments. The first kind appeals to patterns of inference that strongly appear to be valid, and whose apparent validity (we argue) is best explained by the validity of Comparability. The second kind appeals to certain patterns of incomparability which would seem to be possible if Comparability were invalid, but which (we argue) have absurd consequences.

# 3 Direct Arguments for Comparability

Consider the following inferences:

- (7) a. Taylor's paper isn't as good as Kyle's. So Kyle's paper is better than Taylor's.
  - b. Sue doesn't admire Gandhi as much as she admires Martin Luther King, Jr. So Sue admires Martin Luther King, Jr., more than she admires Gandhi.
  - c. I didn't dance as beautifully as you did. So you danced more beautifully than I did.

Phenomenologically, these arguments feel valid; indeed the conclusions and the premises seem interchangeable. And this is not due to some special feature of the gradable expressions 'good', 'admire', and 'beautifully': for any gradable expression *F*, Not As *F* seems intuitively valid:

**Not As** F x is not as F as y. So, y is more F than x.

The strong impressions of validity pose a prima facie challenge for opponents of Comparability, since in any case where x and y are incomparable with respect to F-ness, the premise of Not As F would be true while the conclusion was false. But the examples above involve paradigmatically "multidimensional" comparatives, for which we would expect Comparability to fail if it were invalid in the way that has been claimed by its opponents. The pattern seems to be a completely general one: indeed, the appearance of validity remains strong when we use made-up comparatives (along the lines suggested by Wheeler, 1972, and mentioned in section 1).<sup>17</sup>

Two other valid-seeming argument-patterns are illustrated by the following examples:

- (8) a. Taylor's paper is no better than Kyle's. So, Kyle's paper is at least as good as Taylor's.
  - b. Alex had no more fun than Sam. So Sam had at least as much fun as Alex.
  - c. I spoke no more eloquently than you did. So you spoke at least as eloquently as I did.
- (9) a. Taylor's paper is no less interesting than Kyle's. So, Taylor's paper is at least as interesting as Kyle's.
  - b. Alex had no less reason to study than Robin had. So Alex had at least as much reason to study as Robin had.
  - c. I spoke no less eloquently than you did. So I spoke at least as eloquently as you did.

Again, the phenomena seem quite general: instances of the following schemas seem valid for any gradable expression *F*:

<sup>&</sup>lt;sup>17</sup>When we say that certain arguments *appear valid*, we mean that they initially seem (to the authors and, we expect, to most readers) to be such that, necessarily, if their premises are true, then so are their conclusions. We do not assume—indeed, we explicitly deny—that any argument which appears valid is valid or that any valid argument appears valid. But we take appearances ("judgments", "intuitions"...) of validity ("entailment", "implication", "consequence"...) to provide data that must be explained by a theory of logic and language (see, e.g. van Benthem, 1984; Hadley, 1978; Martin & Hjortland, 2021; Priest, 2016, 40; Sider & Braun, 2006; Winter, 2016).

**No More** F *x* is no more *F* than *y*. So, *y* is at least as *F* as *x*.

**No Less** *F x* is no less *F* than *y*. So, *x* is at least as *F* as *y*.

And again, this is surprising on the view that Comparability is invalid, since there is pressure to think that in a case where x and y are F-incomparable, the premises of No More F and No Less F would be true while the conclusions were false.

# Our Explanation

Our argument from this data to Comparability takes the form of an inference to the best explanation. The explanation we propose involves three claims:

- (i) a. 'As *F* as' is truth-conditionally equivalent to 'at least as *F* as'
  - b. 'No more/less *F* than' is truth-conditionally equivalent to 'not more/less *F* than'.
- (ii) All of these comparatives and equatives carry a presupposition of assessability:
  'x is [at least] as F as y' and 'x is more/less F than y' presuppose 'x is at least as F as x' and 'y is at least as F as y'.
- (iii) Comparability is valid (as are Strict Comparison and Reversal).

It is characteristic of presuppositions to "project through negation": a sentence and its negation presuppose the same things. So given (i) and (ii), '*x* is not as *F* as *y*' and '*x* is no more/less *F* than *y*' both have false presuppositions in the case where one or other of *x* and *y* fails to be *F*-assessable. Our three claims thus imply that Not As *F*, No More *F*, and No Less *F* are *presuppositionally valid*: whenever their premises neither entail nor presuppose anything false, their conclusions neither entail nor presuppose anything false. Whether or not we want to apply the technical term 'valid' to arguments with this status, it seems sufficient to account for the intuitive feeling of validity that arguments of these forms inspire: compare 'Taylor doesn't know that Sue is a spy; so not every spy is known by Taylor to be a spy', or 'Every animal I own is well-trained; so at least one animal I own is well-trained'.<sup>18</sup>

<sup>&</sup>lt;sup>18</sup>Our explanation can be easily generalized to account for the felt validity of instances of 'mixed' analogues of Not As *F*, No More *F*, and No Less *F*, like the following:

The plank isn't as long as the chasm is wide. So, the chasm is wider than the plank is long.

The chasm is no wider than the plank is long. So the plank is at least as long as the chasm is wide.

Note that a great many instances of Not As *F*, No More *F*, and No Less *F* will be valid *simpliciter* (necessarily truth-preserving), and not just presuppositionally valid, since the content of the presupposition of assessability will often be an obvious necessary truth. For example, it is presumably necessary that every paper is at least as good as itself, that everyone who dances does so at least as beautifully as themselves, etc. But, in some cases, there may be no specific grounds for assuming that the relevant objects are *F*-assessable, such as the inference from 'The thing he is thinking about isn't as tasty as the thing she is thinking about' to 'The thing she is thinking about is more tasty than the thing he is thinking about'. So we need the presupposition of assessability for a full account of the apparent validity of the schemas.

Claim (i-a) has been orthodoxy, among those who have discussed it, since at least Horn (1972) and Klein (1980). An apparent problem for this claim comes from the fact that 'x is as *F* as y' often looks interchangeable with 'x and y are equally *F*', whereas 'x is at least as *F* as y' never does. But as Horn and Klein observe, the stronger meaning associated with 'x is as *F* as y' looks to be an implicature—specifically, a *scalar implicature*, generated by the same pragmatic process that lets us use 'I ate most of the cake' to mean 'I ate most but not all of the cake', or use 'You are allowed to type your essay' to mean 'You are allowed but not required to type your essay' (see also Schwarzschild, 2008). This is confirmed by the acceptability of sentences like 'Jude is not only as tall as Mona, he is taller than Mona' (Horn, 1989, p. 386), where the implicature is explicitly "cancelled", and by the absence of this implicature in sentences like 'John is as tall as {any of his friends/anyone/he ever was}' (Horn, 1972, p. 51). Another hallmark of scalar implicatures is that the strengthened meanings normally disappear in negative environments: 'I didn't eat most of the cake'

The plank is no less long than the chasm is wide. So the plank is at least as long as the chasm is wide.

We simply need to appeal to Mixed Comparability (see note 1) and generalize our presupposition of *F*-assessability in terms of the antecedent of that schema. Say that *x* is *F*/*G*-assessable if and only if 'Either *x* is at least as *F* as something is *G* or something is at least as *G* as *x* is *F*' is true. So we propose that '*x* is as *F* as *y* is *G*', '*x* is more *F* than *y* is *G*', and '*x* is less *F* than *y* is *G*' all carry the presupposition that *x* is *F*/*G*-assessable and *y* is *G*/*F*-assessable. In other words: when the presuppositions of any of these sentences are satisfied, the antecedent of Mixed Comparability will be true. Given the validity of Mixed Comparability, the arguments above will thus be presuppositionally valid. And this explanation preserves our account of the apparent validity of Not As *F*, No More *F*, and No Less *F*.

The posited presupposition of F/G-assessability provides a plausible diagnosis of the oddity of certain mixed comparatives, such as 'She is as old as she is tall'. Plausibly, in the ordinary contexts where these sentences seem bad, they do so because the presupposition 'She is either at least as old as someone is tall or such that someone is at least as tall as she is old' is false (or hard to accommodate). This explains why 'She is not as old as she is tall' is just as odd.

won't normally seem true if I ate all of it; 'No students are allowed to type their essays' won't normally seem true if all students are required to type their essays. 'As *F* as' behaves similarly: 'No graduate student's paper was as good as Kyle's' wouldn't normally seem true in a case where some of the graduate students' papers were better than Kyle's and the remainder were worse than Kyle's. The way 'at least' lets us block the strengthened meaning is also seen in other cases of scalar implicature: compare 'I at least ate *most* of the cake' and 'You are at least *allowed* to type your essay.'

Claim (i-b), meanwhile, looks very hard to deny. 'No more F' presumably results from combining 'more F' with the familiar determiner 'no', which also occurs in other contexts like 'There is no butter in the batter' and 'We have no bananas'. Given the evident validity of 'If we do not have bananas, we have no bananas' and 'If is not the case that there is butter in the fridge, there is no butter in the fridge', it is hard to see how a reasonable general semantic account of 'no' could avoid validating 'x is not more/less F than y iff x is no more/less F than y'. Furthermore, we can observe that 'no' is generally equivalent to 'not any', and it seems clear that 'x is not any more/less F than y' is true iff 'x is not more/less F than y' is.

One difference between 'no more/less' and 'not more/less' is that the former often have an "evaluative flavour" which the latter lack (Nouwen, 2008). For example, 'Beethoven is no better than Bach', unlike 'Beethoven isn't better than Bach', carries the surprising suggestion that Bach isn't all that good; meanwhile, 'Beethoven is no less accomplished than Bach' suggests that Bach is quite accomplished. But this "evaluative" element is not plausibly built into the truth conditions of 'no more/less F than', any more than the assumption of shortness is built into the truth conditions of 'not as short as'.<sup>19</sup> Another difference is that, like 'as *F* as', 'no more *F*' and 'no less *F*' tend in some contexts to suggest 'equally *F*' (Nouwen, 2008; see also Horn, 1989, p. 243; Jespersen, 1966, p. 83). This, too, is very plausibly categorised as a scalar implicature. This observation may seem to provide a competing explanation of the apparent validity of No More F and No Less F: someone might suggest that these schemas only appear valid because the premises are taken to carry the strengthened meaning that *x* and *y* are equally *F*, which entails the conclusion of those schemas. But this is not at all promising: arguments like 'Taylor's paper is no better than Kyle's, so Taylor's paper isn't worse than Kyle's' do not have anything like the nice status of (8) and (9).<sup>20</sup>

<sup>&</sup>lt;sup>19</sup>Rett (2015) suggests that equative constructions involving negative antonyms (e.g., 'x is as short as y') *presuppose* that y (but not x) is short, and that this presupposition projects through negation; the comparative form carries no such presupposition. This is compatible with our claim that Not As *F* is presuppositionally valid, and with our basic explanation of it.

<sup>&</sup>lt;sup>20</sup>Indeed, the scalar implicature from 'no more/less' to equality provides an independent argu-

Turning next to (ii): the idea that comparatives and equatives carry a presupposition of assessability seems quite plausible even apart from its role in explaining the good standing of our schemas. Saying 'x is more F than y' or 'x is as F as y' addresses the question 'How F are x and y?', a question that intuitively takes for granted that each of x and y is either at least as F as something or such that something is at least as F as it. This explains the oddity of questions like 'Is the Eiffel Tower {as tall as/taller than} the square root of two?'. As further confirmation for this presupposition, we can look to other environments through which presuppositions project. Perhaps the most famous such an environment is the antecedent of a conditional: 'If Ryan's Ferrari broke down, he would take it to a special garage' presupposes that Ryan has a Ferrari; 'Even if Sue stops smoking, she will have many risk factors' presupposes that Sue smokes. Our posited presupposition of *F*-assessability passes this test. For example, in uttering (10), one seems to be assuming (perhaps inappropriately) that the children's choir was paid:

(10) If the performance by the children's choir was as expensive as the banquet, that was a very expensive wedding.<sup>21</sup>

Presuppositions also project through possibility modals like 'perhaps' and 'might': for example, 'Ryan's Ferrari might be parked in the garage' presupposes that Ryan has a Ferrari. The presupposition of *F*-assessability passes this test too. (11), for example, would seem defective if *Exterminator* were something other than a beer (say, a movie or a wine):

(11) *Exterminator* might not be as hoppy as *Jai Alai IPA*.

Our posits (i) and (ii) thus both seem independently plausible, as well as being supported by their role in our proposed explanation of the apparent validity of Not As *F*, No More *F*, and No Less *F*.

One problem case for our proposed explanation comes from conjunctions of adjectives like 'suspenseful and exciting'. Recall that, on page 10, we set aside apparent instances

<sup>21</sup>Examples like this suggest that 'expensive' carries *some* kind of presupposition, to the effect that the relevant item was or could be bought. Whatever exactly this comes to, it presumably entails (and might be argued to be equivalent to) being 'expensive'-assessable. For the difficulty of pinning down precise presuppositions in such cases, see Magidor (2013, sec. 4.2).

ment for Comparability. In general, scalar implicatures involve strengthened meanings equivalent to the conjunction of the truth-conditional meaning ('I ate most of the cake') with the negations of the truth-conditional meanings of certain alternative sentences ('I ate all of the cake'). In the case of 'x is no more *F* than y', the only obvious stronger alternative is 'x is less *F* than y'. But the elimination of this alternative only yields the equality reading on the assumption that 'more', 'less' and 'equally' are exhaustive of the alternatives—which opponents of Comparability would deny.

of Comparability and the other schemas involving conjunctions of adjectives. However, when we plug conjunctive expressions into Not As *F*, No Less *F*, and No More *F*, we get much the same appearances of validity as our non-conjunctive examples enjoyed. The following inferences seem valid—at least, on their most natural readings, when uttered without any special emphasis or intonation:

- (12) a. *Alien* is no more suspenseful and exciting than *Predator*. So *Predator* is at least as suspenseful and exciting as *Alien*.
  - b. *Alien* is no less suspenseful and exciting than *Predator*. So *Alien* is at least as suspenseful and exciting as *Predator*.
  - c. *Alien* is not as suspenseful and exciting as *Predator*. So *Predator* is more suspenseful and exciting than *Alien*.

Given that we do not regard expressions like 'more suspenseful and exciting' as genuine comparatives, we do not accept the validity of the following apparent instance of Comparability:

(13) If *Alien* is at least as suspenseful and exciting as *Alien* and *Predator* is at least as suspenseful and exciting as *Predator*, then either *Alien* is at least as suspenseful and exciting as *Predator* or *Predator* is at least as suspenseful and exciting as *Alien*.

So, as it stands, our proposed explanation of the apparent validity of Not As *F*, No Less *F*, and No More *F* does not generalize to the conjunctive examples in (12).

The central observation we need to extend our explanation to these data is due to Szabolcsi and Haddican (2004). They note that when the 'and' in 'Mary didn't take math and physics' is unstressed, it is very hard to hear as 'It is not the case that: Mary took math and physics'. Instead it more or less has to mean 'Mary took neither math nor physics'. They show that this is the general pattern with conjunctions of definite noun phrases (see especially their twenty-six examples on pp. 235–36). Geurts (2005) and Schmitt (2013) observe that the pattern extends to other categories: for example, when the 'and' in 'Brown isn't tall and handsome' is unstressed, it seems equivalent to 'Brown is neither tall nor handsome'. It is not obvious what explains this effect. But for present purposes, it suffices to note that the tendency to hear 'x is no more/less F and G than y' and 'x is not as F and G as y' as equivalent, respectively, to 'x is neither more/less F nor more/less G than y' and 'x is neither as F nor as G as y' is another instance of this well-established pattern. And given Comparability and the presupposition of assessability, the inferences from these 'neither' sentences to the conclusions in the above inferences are presuppositionally valid. Thus, when combined with whatever mechanism is responsible for the "neither"

reading of negated conjunctions, our posits (i–iii) can explain the appearances involving conjunctions in the same way as those involving non-conjunctive comparatives.<sup>22</sup>

The explanation therefore seems to us to be quite compelling. But to conclude that it is the *best* explanation, we will need to look at some alternatives, a task to which we will now turn.

#### Alternative Explanations

What explanations might a denier of the validity of Comparability give for the apparent validity of Not As F, No More F, and No Less F? In the case of the latter two, one might naturally consider a view on which 'no more/less F' is not truth-conditionally equivalent to 'not more/less F', but rather has stronger truth-conditions, so that 'x is no more F than y' and 'x is no less F than y' are simply equivalent to 'x is at least as F as y' and 'y is at least as F as x' respectively. One could say something analogous about Not As F: perhaps, rather than expressing negation (as one would expect), the word 'not' in 'x is not as F as y' mean the same as 'y is at least as F as x'. On this view, the three argument-schemas are strictly valid, not just presuppositionally so.

But the semantic claims required by these competing explanations seem implausible and *ad hoc*. And, as the following examples illustrate, the implications in Not As *F*, No

One possible explanation of this data is that that stressed 'and' suggests a metalinguistic negation of the conjunction, but that 'no' and 'not any' (unlike 'not') rule out this metalinguistic interpretation. This seems plausible because the negation of stressed 'and' cries out for the sort of correction that characteristically follows metalinguistic negation. Compare 'Mary {didn't take/\*took no/\*didn't take any} math OR physics—she took both!' A referee objected that metalinguistic negation should also be ruled out by the negative polarity item 'ever', and yet '*Alien* was not ever more exciting AND suspenseful than *Predator*' is acceptable and conveys "not both". But metalinguistic negation is not always ruled out by negative polarity items; see Geurts, 1998; Israel, 2011; Pitts, 2011.

A different possible explanation would appeal to "intervention effects" on the licensing of negative polarity items like *any*. Linebarger (1980, p. 59) suggests that 'any' cannot take immediate scope within 'and': 'Mary didn't take calculus and any physics' sounds unacceptable (for complications, see Buccola and Crnič, 2021). 'Not any more/less *F* and any more/less *G*' is similarly odd, and in any case does not seem to have a "not both" reading: perhaps whatever explains this also explains the absence of such a reading for 'no/not any more/less *F* and *G*'.

<sup>&</sup>lt;sup>22</sup>Interestingly, 'no more/less *F* and *G*' can *only* be interpreted as "neither": in this case, by contrast with 'not more/less *F* and *G*', it seems impossible to use stress on 'and' to generate the "not both" reading. We do not have a conclusive explanation of this difference, but we note that this seems to be a very general issue beyond comparatives: 'Mary took no math and physics' can only mean that Mary didn't take any of either; stressed 'and' seems impossible here. The same effects also arise with 'not any', which is closely related to 'no': 'not any more/less *F* and *G*' can only be interpreted as 'neither any more/less *F* nor any more/less *G*'.

More *F*, and No Less *F* generalize to a wide variety of other "negative" contexts, exactly as one would expect if 'not' and 'no' were expressing regular negation:

- (14) a. None of Taylor's papers was as good as Kyle's. So Kyle's paper was better than any of Taylor's.
  - b. If she admired Gandhi any less than she admires Martin Luther King, Jr., she wouldn't have bought that biography. And she did buy the biography. So she must admire Gandhi at least as much as she admires Martin Luther King, Jr.'
  - c. I never danced any more beautifully than you did. So you always danced at least as beautifully as I did.

This makes semantic approaches based on the idea that 'not' and 'no' contribute something other than truth-conditional negation look quite unpromising.<sup>23</sup>

A more promising strategy for opponents of Comparability would be to appeal to some effect by which the premises of Not As F, No More F, and No Less F get associated with meanings that strengthen their standard truth-conditions by entailing that the items in question are comparable, and hence allow us to derive the conclusions of those schemas. Such a strengthened meaning does not seem to be generated by any general pragmatic mechanism, since no analogous strengthening is observed for other binary relations which uncontroversially do not obey the analogue of Comparability. Consider parthood. There are some things x and y such that x is not part of y and y is not part of x. But 'not part of' does not get pragmatically strengthened in anything like the way we are considering for 'not as F as', since there is no temptation to hear the following argument as valid:

(15) My carburettor is not part of your computer. So your computer is part of my carburettor.

So one would need a more narrowly tailored pragmatic strategy to explain our appearances of validity without implausibly over-generating.

One such strategy might appeal to the phenomenon of *negative strengthening* (Horn, 1989, sec. 5.3), the process whereby, for example, 'Cassidy is not happy' tends to convey

<sup>&</sup>lt;sup>23</sup>One objection to this, in the case of Not As *F*, is that (as noticed by an anonymous referee) the schema seems less immediately compelling when 'not' is replaced by 'it is not the case that': 'It is not the case that Taylor's paper is as good as Kyle's, so Kyle's paper is better than Taylor's' does not seem obviously valid. But, first, we do not assume that all (presuppositional) validities will appear obviously valid. And, second, such "external" negations may more naturally suggest a metalinguistic reading, which rejects the assertability of 'Taylor's paper is as good as Kyle's', rather than its truth (Horn, 1989, ch. 6; we thank the referee for this suggestion).

that Cassidy is rather unhappy. Negative strengthening is quite sensitive to the predicate being negated: for example, 'Cassidy is not sad' doesn't tend to suggest that Cassidy is rather happy. So one would need to tell a story about why the mechanism that drives this process gets to strengthen the particular premises of our schemas, does so regardless of the gradable expression 'F', and does not strengthen 'not part of' in the analogous way. It is not clear to us what that story might look like. But even if such a story could be given, our schemas behave quite differently than standard cases of negative strengthening. For example, 'Cassidy was almost happy about the news' does not tend to suggest that Cassidy was rather unhappy about the news, but 'Cassidy was almost as happy as Riley' seems to entail (Horn, 2009; Sevi, 1998) or at least implicate (Sadock, 1981; Ziegeler, 2000) that Cassidy was not as happy as Riley, which in turn (by Not As F) presuppositionally entails that Cassidy was less happy than Riley. Furthermore, negative strengthening tends to be asymmetrical with respect to antonyms: 'I'm not optimistic' suggests that one is rather pessimistic, but 'I'm not pessimistic' does not suggest that one is rather optimistic (Horn, 1989). So it is not clear why the phenomenon should apply to both 'no more' and 'no less', as it would have to in order to explain our data. Finally, negative strengthening is defeasible in a way that our inferences are not: 'I'm not happy, but I'm not unhappy either' is a perfectly fine thing to say; 'Cassidy is no less happy than Riley, but Cassidy is not at least as happy as Riley' is not.

Rather than appealing to some general pragmatic phenomenon, the opponent of Comparability might posit something distinctive about the conventional meanings of comparatives and equatives to generate the needed strengthening. The most obvious idea is to posit that sentences of the form 'x is [at least] as *F* as y' and 'x is more/less *F* than y' *presuppose* 'either x is at least as *F* as y or y is at least as *F* as x'. Given that presuppositions project through negation, the posited presupposition of comparability will secure for our schemas the very same status of presuppositional validity that we have claimed for them—though via a different presupposition. Where we posit a presupposition of *F*assessability, our opponents could posit the (stronger, according to them) presupposition of *F*-comparability.<sup>24</sup>

On our view, *F*-assessability entails *F*-comparability, so our opponents' presupposition will be satisfied whenever ours is. But our opponents deny this. Their view thus predicts a wider array of ways for speeches of the relevant form to suffer from presupposition

<sup>&</sup>lt;sup>24</sup>Magidor (2013, 145, n. 46) considers (without endorsing) the similar idea that "'*x* is smaller than *y*" triggers the presupposition that *x* and *y* are comparable in size', as one possible account of the oddity of 'My table {is/is not} smaller than the number three'. We have already suggested, in section 1, an alternative account of the oddity of such sentences (one which Magidor also considers), in terms of polysemy or context-sensitivity in comparatives like 'smaller'.

failure. But these predictions are not borne out when we look at sentences embedding comparatives and equatives in environments from which presuppositions project—for example, in the antecedent of a conditional. There is no hint that the presence of 'x is as F as y' or 'x is more/less F than y' in the antecedent of a conditional induces any non-trivial presupposition about the relation between x and y that is (as our opponents' view predicts) not guaranteed by their individual F-assessability. To see this, suppose that for all we know, Taylor's and Kyle's papers instantiate the pattern of relations that our opponents think sometimes give rise to incomparability—perhaps Taylor's paper is better written, but it contains more inaccuracies. Now consider (16):

(16) Even if Taylor's paper is {as good as/better than} Kyle's, she will only get an A–for the course.

The view we are considering—that Comparability sometimes fails, but is presupposed by the use of equatives and comparatives—should predict an utterance of (16) to be at serious risk of presupposition failure. It should seem that the speaker is making a contingent and, in this case, unwarranted assumption about how the papers compare, all things considered. And an audience who takes themselves to have evidence that the two papers are incomparable should, on this view, take the speaker to be under a serious misapprehension. But this is not the case: (16) seems perfectly acceptable. Indeed, it seems unproblematically true if Taylor's prior grades are not good enough for her to earn an A, no matter how their papers in fact compare. (We can make a similar point using other conditionals, including subjunctives like 'If I had danced as beautifully as you just danced, everyone would have been amazed'.)

Similarly, using the test of possibility modals, sentences of the form 'x might be as F as y' do not seem to presuppose anything about x and y that goes beyond their individual *F*-assessability:

(17) Taylor's paper might be {as good as/better than} Kyle's.

An utterance of (17) seems perfectly acceptable even if, for all we know, Taylor's and Kyle's papers instantiate tradeoffs of the kind that are characteristic of incomparability. It would seem bizarre for a hearer to object, 'Wait, why do you assume that either of the two papers is at least as good as the other?'<sup>25</sup>

<sup>&</sup>lt;sup>25</sup>Another standard environment for testing for presuppositions is polar questions, which carry the same presuppositions as their corresponding declarative sentences. On our view, 'Is Taylor's paper as good as Kyle's?' should presuppose nothing beyond the existence and uniqueness of the two papers. This seems right to us: it seems perfectly acceptable, in a sense in which 'Is Taylor's

These tests count against the view that uses of the equative and comparative forms carry a presupposition of comparability not entailed by the presupposition of assessability. At the very least, there does not seem to be positive evidence *for* the existence of such a presupposition. The only motivation for accepting it, so far as we can see, is a desire to explain the seeming validity of Not As *F*, No More *F*, and No Less *F* while rejecting Comparability. But, in the next section, we will offer arguments for Comparability that (we argue) cannot be adequately addressed by this supposed presupposition, so this explanation is not even strong enough to explain all of the problematic data. Moreover, and independently of these considerations, positing a presupposition, since it predicts that the utterances they produce in the course of giving tradeoff and small improvement arguments will be defective (due to presupposition failure), even if they do not involve the assertion of anything false. Proponents of those arguments have generally not thought of them as having the puzzling status of arguments whose premises presuppose the falsity of their conclusions, such as the following:

(18) Charlie doesn't regret that she ate the expired tuna salad yesterday. Charlie regrets everything she did yesterday. So, Charlie didn't eat the expired tuna salad yesterday.

Such arguments are bewildering in a way that opponents of Comparability do not find their own arguments to be. And we know of no other similarly influential arguments that have this status. So we doubt that the presupposition of *F*-comparability will ultimately seem attractive to the traditional opponent of Comparability.

A final response to the arguments of this section is to deny that Not As *F*, No More *F*, and No Less *F* are even presuppositionally valid. Some opponents of Comparability may claim not to perceive any appearances of validity to these schemas at all. We have little hope of convincing such a reader with the arguments of this section; they may find the arguments of section 4 more compelling. Others may admit that these schemas seem valid to them pre-theoretically, but claim that these appearances should be dismissed

Ferrari still parked outside?' is not acceptable unless we can reasonably assume that Taylor has a Ferrari. However, there is a potentially distracting factor here, in that the use of *any* question arguably presupposes (pragmatically) that the addressee is in some sense in a position to answer it. This presupposition may fail for reasons connected with vagueness: for example, in asking 'Is Harry bald?', one seems to be taking for granted that the question lacks a certain "defective" status that it would have if Harry were a borderline case for 'bald'. This phenomenon can make for *pragmatic* presupposition failure with polar questions under similar circumstances to those where our opponents would posit incomparability. As a result, polar questions are not so useful in testing for the presence of the alleged additional presupposition of comparability.

as misleading in the light of the arguments against Comparability. In reply, we submit that, while appearances of validity can of course be misleading, there should be a strong presumption that deeply ingrained dispositions to treat certain general argument-forms as if they were valid should not be dismissed as mistakes, without a compelling explanation for why we should systemically go wrong in these specific ways. After all, it is plausible that our dispositions to make or accept certain inferences play an especially central role in giving our words the meanings they have. Opponents of Comparability are thus saddled with a heavy explanatory burden: to explain how comparative constructions get to express relations that exhibit incomparability despite the entrenched facts of usage. It is not enough to say that people mistakenly treat Not As F, No More F, and No Less F as valid based on an incautious generalization from the fact that they are truth-preserving in most cases: there is an enormous range of generalizations that hold in most cases but which generate no corresponding appearances of validity (e.g., 'This thing is food, so it is not a pecan pie'). Furthermore, recall that we are arguing for the conditional thesis that Comparability is valid if the other schemas of section 1 are valid. Since the case for the validity of those schemas appeals crucially to similar appearances of validity, the proponent of this error-theoretic strategy would have to explain why we should trust *those* appearances—even in the face of apparent counterexamples such as those of Rachels (1998) and Temkin (2012)—but not the ones enjoyed by Not As F, No More F, and No Less F. This only adds to their explanatory burden. Our own burden is less great: it is not at all remarkable for philosophers to make errors, and we will see in section 5 that the influential arguments against Comparability are much less compelling than they may have seemed. Indeed, we will suggest there that apparent violations of Comparability are really just borderline cases of the kind that arise for any vague expression, and it is not at all remarkable for vagueness to be confused for something else.<sup>26</sup>

# 4 Chains of Incomparability

Our second family of arguments has a different structure. We will first argue that, if Comparability were invalid, then a certain pattern—which we call *incomparability-connectedness*—should be pervasive. We will then present some valid-seeming argument forms whose validity or presuppositional validity would rule out this pattern, and conclude on this basis that Comparability is valid.

<sup>&</sup>lt;sup>26</sup>This is a common refrain in the literature on apparent counterexamples to Comparative Transitivity: see, e.g., Nebel (2018), Thomas (2021), and Voorhoeve and Binmore (2006); against vagueness-based diagnoses of such arguments, see Pummer (2022).

## Incomparability-Connectedness and Strong Monotonicity

Consider a paradigm case of supposed incomparability, Raz and Chang's example of a career as a clarinetist (x) and a career as a lawyer (y). Supposedly, neither is better than the other, but  $x^+$ —an otherwise similar career as a clarinetist paying \$10 more than x—is better than x and still neither better nor worse than y. This is illustrated in Figure 1, where the arrow represents betterness and the squiggles represent incomparability. Let's say that in this case, x and  $x^+$  are two "incomparability steps" apart.



Figure 1. Small Improvement Case

If there exist such an x,  $x^+$ , and y, it is plausible that *any* two otherwise similar clarinetcareers differing by at most \$10 in annual salary are two incomparability-steps apart. For any two such clarinet-careers x' and  $x'^+$ , we will be able find a law-career y' whose pros and cons as against either of x' and  $x'^+$  are hard to weigh in the same way as the pros and cons of y as against x and  $x^+$ , so that the considerations that opponents of Comparability take to support the judgment that y is neither better nor worse than either x or  $x^+$  will also support the claim that y' is neither better nor worse than either x' or  $x'^+$ . So, the situation is as depicted in Figure 2. On the left, we have a range of clarinet-careers differing in salary;



Figure 2. Incomparability Connectedness

on the right, we have a range of law-careers that witness the fact that any two adjacent clarinet-careers are two incomparability-steps apart. The upshot, then, is that any two of the careers in the diagram are separated by some finite number of incomparability steps.

And this is obviously not specific to clarinet and law. Insofar as one accepts the judgments which support incomparability in the original case, one should find it plausible that *any* two possible careers (for a human being) are finitely many incomparability steps apart.<sup>27</sup>

Let's say that a set is *incomparability-connected* (with respect to *F*) when any two of its elements are finitely many *F*-incomparability steps apart, or equivalently, when it cannot be divided into two non-overlapping, non-empty subsets such that no member of one is *F*-incomparable with any member of the other. So, what we have seen is that opponents of Comparability are under pressure to think that the domain of all possible careers is incomparability-connected. Of course, one could in principle reject Comparability without accepting this claim. For example, one could think that the set of clarinetist careers with an annual salary less than \$60,000 is incomparability-connected, and that the set of clarinetist careers with an annual salary of at least \$60,000 is incomparability-connected, but that \$60,000 marks a "bottleneck" such that no career (in law or any other field) is incomparable both with something above that bottleneck and with something below it. But this seems bizarre: the case that two clarinetist careers differ by \$10 are separated by two incomparability steps is just as strong for pairs that straddle the supposed bottleneck as for those that don't.

The problem is that there are valid-looking principles which conflict with the idea that the domain of *F*-assessable things—or even a substantial subset of that domain— is incomparability-connected. These principles are natural strengthenings of ones we introduced in section 1; we take them to be supported by ordinary usage in a way that is similar to the principles of the previous section. For example, consider the following arguments:

- (19) a. Kara is healthy, and Sam is not healthy. So Kara is healthier than Sam.
  - b. Alex had fun at the fair and Cameron didn't. So Alex had more fun at the fair than Cameron.
  - c. Charlie likes broccoli and doesn't like chocolate. So Charlie likes broccoli more than chocolate.

The pattern seems quite general. Instances of Strong Monotonicity seem intuitively valid:

<sup>&</sup>lt;sup>27</sup>The diagram only displays relations of incomparability and betterness that are stipulated by the example. For example, since incomparability is not supposed to be transitive, it can happen that  $x^{++}$  is better than  $y^{--}$ . Note also that we do not assume that the law-careers all differ by the same amount of annual salary: perhaps the amount of law-salary required to generate incomparability is some nonlinear function of clarinet-salary.

## **Strong Monotonicity** *x* is *F*. *y* is not *F*. So, *x* is more *F* than y.<sup>28</sup>

We propose that this schema is *presuppositionally valid*.<sup>29</sup> We do not claim that it is valid *simpliciter*, since we do not want to be committed to the validity of instances like 'The Eiffel Tower is tall, and the square root of two is not tall, so the Eiffel Tower is taller than the square root of two'. Such arguments sound bizarre, plausibly because the premise 'The square root of two is not tall' falsely presupposes that the square root of two is 'tall'-assessable. This can be explained by the more general claim that uses of the unmodified form '*F*' carry a presupposition of *F*-assessability, which projects as usual through negation. The presuppositional validity of Strong Monotonicity will then follow from the strict validity of the analogous argument adding the *F*-assessability of the relevant objects as an extra premise:

**Qualified Strong Monotonicity** *x* is *F*. *y* is not *F*. Each of *x* and *y* is at least as *F* as itself. So, *x* is more *F* than *y*.

Note that given Comparability, the validity of Qualified Strong Monotonicity follows from that of Monotonicity (stated on page 3): if *x* is not more *F* than *y* and both are *F*-assessable, then by Comparability, *y* is at least as *F* as *x*, and so by Monotonicity, *y* is *F* if *x* is.<sup>30</sup>

<sup>29</sup>The putative counterexamples to Monotonicity discussed on page 3 can also be adapted to generate a challenge to Strong Monotonicity: for example, 'The coffee is expensive, and the sandwich isn't expensive' and 'The sandwich is more expensive than the coffee' both seem true when the coffee costs \$6 and the sandwich \$7. Our proposed response, involving different resolutions of the context-sensitivity of 'expensive', also applies in this case. However, as we anticipated in note 1, even if one didn't accept this response, the arguments of this section could still be run using a variant of Strong Monotonicity with the premises 'x is *F* for a *K*' and 'y is not *F* for a *K*'.

<sup>30</sup>Just like the schemas of section 3, apparent instances of Strong Monotonicity involving conjunctions of adjectives also sound compelling:

*Alien* is suspenseful and exciting. *Predator* is not suspenseful and exciting. So, *Alien* is more suspenseful and exciting than *Predator*.

Like the corresponding patterns noted on page 24, this can be explained in terms in terms of whatever makes it hard to escape a 'neither' reading for negated conjunctions without stress on the 'and'. With stress on 'and', the appearance of validity evaporates.

<sup>&</sup>lt;sup>28</sup>For similar principles, see Chisholm and Sosa (1966, p. 248) and van Benthem (1982, p. 198). Nebel (2018) states instances of Strong Monotonicity for 'good' and 'bad' and observes that they pose problems for incomparability in specific contexts, but fails to notice their more general implications; we thank Michael Rabenberg for first bringing them to our attention. Flanigan and Halstead (2018) propose a similar "dyadic–monadic" principle to defend the more limited claim that, given epistemicism about vagueness, options must be comparable. Gustafsson (2020) rejects Strong Monotonicity for 'good', precisely because it is incompatible with incomparability given further axiological assumptions; see also Thornley (forthcoming).

Of course the validity of Qualified Strong Monotonicity does not guarantee the validity of Comparability. Nevertheless, it poses a problem for opponents of Comparability, since, as we explained above, the considerations that would lead one to reject particular instances of Comparability strongly suggest that in many cases, the domain of *F*-assessable things is incomparability-connected. But, given Qualified Strong Monotonicity, any incomparability-connected set must either consist only of *F* things or only of things that are not *F*. If  $x^+$  is *F* and *x* is not, then there can be no *y* that is incomparable with both: if *y* is *F*, then it must be more *F* than *x*; if *y* is not *F*, then  $x^+$  must be more *F* than it.

Qualified Strong Monotonicity and incomparability-connectedness thus lead to the conclusion that either every *F*-assessable thing is *F* or no *F*-assessable thing is *F*. And this is absurd: obviously, some but not all possible careers are good. Of course, we may sometimes be able to get into a context in which the positive form applies to all or no members of some broad domain; this is particularly easy to do for "absolute" adjectives with natural endpoints, such as 'flat' and 'full'. But the argument for incomparability-connectedness works for *any* context, and it is simply not plausible that, for example, every contextual resolution of 'healthy' applies to either all or no living things.

Given the absurdity of this conclusion, opponents of Comparability have two options. First, they can retain Qualified Strong Monotonicity and thus deny that the domain of *F*-assessable things is incomparability-connected. Or, second, they can reject Qualified Strong Monotonicity, and find some alternative account of the seeming validity of Strong Monotonicity. We consider these options in turn.

#### Denying Incomparability-Connectedness

Let us first consider the possibility of denying that the domain of *F*-assessable things is incomparability-connected. On this view, the positive form '*F*' generates a bottleneck, such that no *F* thing is any finite number of incomparability-steps away from anything that isn't *F*, as in Figure 3.

As we have already complained, however, this view is ad hoc. If there is *F*-incomparability, it is best explained by the difficulty of making tradeoffs between dimensions of *F*—e.g., two careers that are each better than the other in different respects, where our use doesn't privilege any particular weighting of these respects in such a way that one of them gets to count as better overall. One would expect to be able to find such tradeoffs anywhere in the domain: if two good clarinet careers differing only by \$10 in annual salary are each incomparable with some law career, because better in some ways and worse in others, it seems extremely implausible that subtracting the same amount from each salary, until one



Figure 3. Positive Form Bottleneck

of the two clarinet careers is no longer good, should take us to a pair of clarinet careers that are not both incomparable with any law career. Each of the resulting clarinet careers will still be better in some ways and worse in others than many law careers, and in much the same pattern that generates incomparability everywhere else in the domain.

The bottleneck model is also quite dialectically awkward for proponents of the small improvement argument, since the mode of thinking that leads them to accept the premises of that argument for particular trios of objects does not seem to control in any way for the possibility that one of the objects is on the opposite side of a bottleneck from the other two (and thus comparable with both of them). Positing the bottleneck would thus require conceding that the relevant mode of thinking is unreliable, leaving it unclear what could justify accepting it in any particular case.

The cost of positing a single bottleneck may nevertheless seem worth paying to maintain the possibility of incomparability. But, as we will now show, there are other argumentforms on a similar footing to Strong Monotonicity, which would require the existence of a great many bottlenecks. Just as Monotonicity can be seen as a special case of Modified Monotonicity (stated on page 4), where the degree modifier is the unpronounced 'POS', Strong Monotonicity can be seen as a special case of the following schema, in which *V* can be any positive degree modifier, such as 'very', 'pretty', or 'somewhat':

**Strong Modified Monotonicity** *x* is *V F*. *y* is not *V F*. So *x* is more *F* than *y*.

Arguments of this form do indeed seem valid in the same way that instances of Strong Monotonicity do:

(20) Kara is very healthy, and Sam is not very healthy. So Kara is healthier than Sam.

There is a similarly compelling schema for superlatives:

**Strong Superlative Monotonicity** *x* is one of the most *F K*s. *y* is not one of the most *F K*s. So, *x* is more *F* than *y*.

For example:

(21) Parmesan is one of the best cheeses, and Grana Padano is not one of the best cheeses. So, Parmesan is better than Grana Padano.

Surely, if the right explanation of the apparent validity of Strong Monotonicity involves the validity of Qualified Strong Montonicity, the right explanation of the apparent validity of Strong Modified Monotonicity and Strong Superlative Monotonicity will likewise involve the validity of qualified versions which add the *F*-assessability of *x* and *y* as an extra premise. It is very plausible that '*x* is *V F*' and '*x* is one of the most *F K*s' both presuppose that *x* is *F*-assessable, just as '*x* is *F*' does: this explains, for example, why 'The square root of two is not very tall' and 'The square root of two is not one of the tallest things' sound odd. Given this, and the fact that presuppositions project through negation, the validity of the qualified schemas will secure the presuppositional validity of the unqualified versions.

The validity of the qualified versions of Strong Modified Monotonicity and Strong Superlative Monotonicity follows from Comparability in conjunction with other principles we discussed in section 1.<sup>31</sup> Of course, opponents of Comparability could also accept these schemas as valid. But if they do, they will be forced to multiply bottlenecks of the kind we considered above, on pain of accepting absurd conclusions. Plausibly, some but not all good clarinet careers are very good; some but not all very good clarinet careers are extremely good; some but not all extremely good clarinet careers are among the very best careers. The principles will thus drive us to a strange picture of the domain, on which all incomparabilities are confined within one of many narrow windows.

The situation looks worse still when we bear in mind the evident context-sensitivity of the degree-modified adjectives and plural superlatives. Clearly there is a lot of flexibility as regards where to draw the line between the things that count as 'very F' and the things that do not, or about how many Ks to include among 'the most F Ks'. These kinds of expressions seem to have further parameters of context-sensitivity over and above whatever context-sensitivity there might be in 'more F' and 'at least as F': even when the interpretation of the comparative and equative forms is nailed down, there is still plenty

<sup>&</sup>lt;sup>31</sup>For example in the case of Strong Superlative Monotonicity, suppose that x is one of the most F Ks and y is not one of the most F Ks, and that y is F-assessable. By Comparability and Strict Comparison, either x is more F than y or y is at least as F as x. The former case is just the conclusion of Strong Superlative Monotonicity; the latter, together with Superlative Monotonicity and the first premise of Strong Superlative Monotonicity, entails that y is one of the most F Ks, which contradicts the second premise.

of flexibility about the interpretation of the degree-modified and superlative forms. But given the qualified versions of Strong Modified Monotonicity and Strong Superlative Monotonicity, every possible resolution of the context-sensitivity of those forms will force an additional bottleneck as regards the extension of the comparative forms.

Indeed, it is rather tempting to think that whenever 'x is more F than y' has a true interpretation, we can find some candidate interpretation of 'V F', for some positive degree modifier V (e.g., 'very'), that makes 'x is V F and y is not V F' true, and is compatible with the given interpretation of 'more F'.<sup>32</sup> Similarly, whenever 'x is more F than y and x is a K' is true, we can find some candidate denotation for 'the most F Ks' that includes x but not y. If so, then given the qualified version of Strong Superlative Monotonicity, it would follow that any such x and y are separated by a bottleneck, so that 'not more F than' is transitive when restricted to the Ks. This means that no two Ks one of which is more F than the other are finitely many incomparability steps apart, so that even the simple pattern in Figure 1 is never instantiated.<sup>33</sup> Once one begins to posit bottlenecks, the modes of thinking that motivated doing so are in serious danger of generalizing so far as to rule out all incomparability whatsoever. There is thus considerable pressure on opponents of Comparability to try to preserve the initial picture on which the domain is incomparability-connected. This requires them to reject Qualified Strong Monotonicity and its cousins.

## Rejecting Qualified Strong Monotonicity

As we have seen, opponents of Comparability are driven to the view that Qualified Strong Monotonicity and its variants fail: in some cases, x is F and y is not F, and both are F-assessable, but x is not more F than y. The challenge for this view is to provide some alternative explanation for the appearances of validity enjoyed by inferences like

This principle strikes us as quite attractive, though it's clearly more theory-laden than the kind of argument we are choosing to rest our case on.

<sup>&</sup>lt;sup>32</sup>This picture of the relation between 'more' and the space of possible interpretations of 'V F' is enshrined in the influential semantics of Klein (1980), on which 'x is at least as F as y' is essentially treated as something like 'if x is F on a delineation, then y is also F on that delineation'. A similar view is suggested by McConnell-Ginet (1973).

<sup>&</sup>lt;sup>33</sup>We could try to capture this hunch in the form of an object-language schema by turning to numerical plural descriptions of the form 'the *n* most *F* Ks' (for which the analogue of Strong Superlative Monotonicity is just as plausible):

**Cardinal Superlatives** If *x* is more *F* than *y*, *x* and *y* are both *Ks*, and there are at most *n Ks*, then either *x* is the *Fst K*, or *x* is one of the two most *F Ks* and *y* is not, or *x* is one of the three most *F Ks* and *y* is not, ..., or *x* is one of the n - 1 most *F Ks* and *y* is not.

'Kara is healthy, and Sam is not healthy; so Kara is healthier than Sam.' However, the prospects for such a competing explanation seem even weaker than the prospects for a competing explanation of the apparent validity of the inference-patterns considered in section 3. In that case, there was the option of rescuing the presuppositional validity of the relevant schemas (Not As F, No More F, and No Less F) by positing an allegedly stronger presupposition of *comparability* for 'as F as', 'more F than', and 'less F than', going beyond our favoured presupposition of *F*-assessability. But it's hard to see what the analogous proposal in the case of Strong Monotonicity would even be, since its premises only use the positive form. One could say that 'x is F' carries the presupposition that x is *F*-comparable with everything that is *F*-assessable. However, we see no independent motivation for this view, and it seems completely alien to the way that opponents of Comparability are thinking of things: the considerations against the comparability of Mozart and Beethoven are not supposed to impugn utterances like 'Mozart was a good composer'. In addition, this presupposition will always be false if (as we are now taking opponents of Comparability to concede for many predicates F) the domain of F-assessable things is incomparability-connected, since that entails that every F thing is F-incomparable with something.

We thus see no plausible way to secure the presuppositional validity of Strong Monotonicity without accepting Qualified Strong Monotonicity as valid. True, by accepting the claim we considered in the previous section that 'more *F* than' presupposes *F*comparability, one can secure a different status for Strong Monotonicity: namely, that of *Strawson-validity*. An argument is Strawson-valid just in case, whenever its premises are true and the presuppositions of all the premises *and of the conclusion* are satisfied, the conclusion must be true (von Fintel, 1999). By contrast, presuppositional validity requires only the presuppositions of the premises to be satisfied. Strawson-validity has sometimes been thought of as an adequate substitute for validity proper (e.g., by Cariani & Goldstein, 2020; Mayr, 2018; Sharvit, 2017), similar to how we have been treating presuppositional validity. So our opponents might offer the Strawson-validity of Strong Monotonicity and its variants as an explanation of the apparent goodness of their instances. This would allow them to reject Qualified Strong Monotonicity and thus avoid having to posit implausible bottlenecks.

We have already argued against the presupposition of comparability in section 3. But even if we grant its existence, the status of Strawson-validity does not seem sufficient to account for the good appearances. Lots of arguments that are plausibly Strawson-valid seem quite tendentious, in a way that instances of Strong Monotonicity do not:

(22) a. Charlie regrets everything she did yesterday. So, Charlie regrets killing

Elvis Presley yesterday.

- b. Everything in the room matches the wallpaper. So the elephant in the room matches the wallpaper.
- c. No programmer in this company uses Emacs. So every programmer in this company has stopped using Emacs.

While Strawson-valid arguments will of course seem good in cases where the presuppositions of the conclusion can in fact be taken for granted, they do not seem compelling when we have no independent reason to think that those presuppositions are satisfied. But instances of Strong Monotonicity and its variants seem good even when we have no independent empirical evidence that the items in question are comparable; intuitively, we do not need to "go beyond" the premises to infer that the conclusion is true. Someone who knows that they like broccoli and that they don't like chocolate can simply infer that they like broccoli more than chocolate. Thus the view that the relevant argument-forms are Strawson-valid does not explain all that needs to be explained.

Could some other pragmatic mechanism be invoked to explain the good standing of the argument-forms? Some instances might be explained by the phenomenon of negative strengthening discussed on page 27. For example, perhaps 'Cassidy is happy and Riley is not happy, so Cassidy is happier than Riley' seems valid only because 'Riley is not happy' gets strengthened to convey that Riley is *rather unhappy* and all happy people are happier than all rather unhappy people. A parallel explanation could be given for the degree-modified versions. But many gradable expressions are not subject to negative strengthening. For example, 'not sad', 'not very sick', and 'not one of the worst movies' do not get strengthened to anything like 'rather happy', 'rather healthy', or 'a rather good movie'. So instances of Strong Monotonicity and its cousins involving such expressions cannot be explained in this way. Also, as we discussed, negative strengthening is highly defeasible, in a way that the inferences in question do not seem to be. For example, 'Cassidy is happy and Riley is neither happy nor unhappy' still seems to imply 'Cassidy is happy and Riley', as does 'Cassidy is very happy and Riley is happy but not very happy'.

Alternatively, it might be suggested that instances of Strong Monotonicity and its variants appear valid because their conclusions will be true whenever the premises are not only true but *known* (or *assertible*). The idea would be that we can know that something is or is not *F* only when it is not close to the boundary between *F* and non-*F* things, whereas incomparability between *F* and non-*F* things arises only near that boundary. But even if this postulated link between incomparability and knowledge were adequate to explain the

appearances of validity in the case where the premises are asserted, it will not predict the corresponding appearances in other environments, like conditionals ('If Kara is healthy and Sam is not healthy then Kara is healthier than Sam') and quantified sentences ('Any healthy person is healthier than anyone who isn't healthy'). Since the appearances do generalize in this way, the appeal to this kind of epistemic or pragmatic status is not sufficiently general.

The remaining option for our opponents is simply to dismiss the temptation to treat instances of Strong Monotonicity, Strong Modified Monotonicity, and Strong Superlative Monotonicity as valid as an error. We have already explained, at the end of section 3, why this sort of error-theoretic treatment of ordinary patterns of inference incurs a major explanatory burden that seems very hard to meet. Some readers have been tempted to dismiss the data we appealed to in that section, on the grounds that the relevant inferences are objectionably close to our conclusion of Comparability and should no longer seem valid to those who have been enlightened by the case against Comparability. Whatever the merits of that response to our earlier arguments, it is even less plausible when it comes to Strong Monotonicity and its cousins. It is simply *obvious* that, for example, a good career is better than a career that isn't good, just as it is obvious that a career that is better than a good career is also good; this sense of obviousness does not seem to rely on Comparability in any way.

If the arguments against Comparability were very strong, and the other options we have surveyed deemed inadequate, then perhaps the costs of this error theory would be worth accepting. But, as we will argue in the next section, the arguments are not nearly as strong as they may have initially seemed, and the judgments to which they appeal are not as compelling as those that favour our principles.

We conclude that the judgments that support the validity of Qualified Strong Monotonicity and its cousins are very hard to explain away. But as we have seen, a view where they are valid but Comparability is not will require positing bottlenecks in a way that seems unprincipled and inconsistent with the motivations for rejecting Comparability. The best option, we think, is to accept that Comparability is valid.

#### 5 Tradeoff and Small Improvement Arguments Revisited

This concludes our case for the validity of Comparability. Let us now revisit the influential arguments against its validity that we surveyed in section 2.

First: we admit that when one considers the multiplicity of dimensions and the difficulty of making tradeoffs, speeches like (23) can seem quite tempting: (23) George Carlin was not funnier than Richard Pryor, and Richard Pryor was not funnier than George Carlin. But they were not equally funny.

The tradeoff argument works by eliciting such speeches, taking them literally, and using them to draw the inference that there are cases of incomparability. We think they should be treated along the same lines as

(24) George Costanza is not bald, but he is also not *not* bald.

We are tempted to say things like (24) when it is vague whether or not someone is bald. But recall from section 1 that we are working on the assumption that the Law of Excluded Middle is valid in English; so we need an account of the acceptability of (24) that is compatible with the validity of 'George Costanza is either bald or not bald'. One plausible account of what's going on is that the first and second uses of 'not' in (24) are in effect weakened to 'not definitely', so that (24) becomes tantamount to

(25) George Costanza is not definitely bald, but he is also not definitely not bald.

(As is common in the vagueness literature, we use 'definitely bald' to mean 'bald, and not a borderline case of "bald"'). Proponents of Excluded Middle will of course take (25) to be consistent with 'George Costanza is either bald or not bald' being definitely true. This weakening is most plausibly regarded as a nonliteral use—perhaps a form of "metalinguistic negation" (Horn, 1989)—although one might also treat it as a genuine ambiguity in 'not'.<sup>34</sup> Either way, Excluded Middle can be preserved. Similarly, since "multidimensional" comparatives like 'funnier than' are certainly vague, we can claim that the operative reading of (23) is tantamount to

(26) Carlin was not definitely funnier than Pryor, and Pryor was not definitely funnier than Carlin, but they were not definitely equally funny.

This is consistent with Comparability as long as 'definitely' is non-redundant.<sup>35</sup>

<sup>&</sup>lt;sup>34</sup>One piece of evidence that some of the 'not's in (24) are interpreted in some special way (whether literal or not) is that the sentence becomes worse when they are "incorporated" as in 'Costanza is non-bald, but he is also not non-bald' (Brown and Garson, 2017). Furthermore, one cannot acceptably utter (24) with the same prosody on each conjunct ('Costanza is not BALD, but he is not not BALD') or with emphasis only on the second 'not' ('Costanza is not bald, but he is NOT not bald'); both sound like outright contradictions.

<sup>&</sup>lt;sup>35</sup>Even though philosophers who reject Excluded Middle would certainly reject Comparability, recall from page 13 that they might still accept Conditional Comparability, which might seem to be challenged by speeches like (23). But since many such philosophers are unwilling to accept outright contradictions, they will also need to explain away utterances like (24) in terms of some special use of 'not'. So they should take seriously the proposal that this special use is also in play in (23), blocking the argument from (23) against the validity of Conditional Comparability.

This diagnosis of the motivations that lead philosophers to find sentences like (23) obviously correct is strongly confirmed in many cases by an examination of the justifications offered for the utterances. For example, we have seen some say that 'there are no precise truths' (Parfit), that we 'may not ... be able to decide' (Sen), that there is 'no answer' (Keynes), or that it is 'perhaps not clear' (Kamp) whether one thing is more F than another. Such hedging is also common in the course of advancing some version of the small improvement argument. In one of the first statements of such an argument (quoted approvingly by Raz) Mackie (1977) suggests that there may be no 'objectively right and determinable answer' to the question of whether one thing is more F than another. In his own discussion of the argument, Raz imagines a 'small but definite improvement' to one of two options, suggesting a contrast to a possibly 'indefinite' comparison. Most tellingly, perhaps, is that proponents of incomparability rarely make outright assertions like (23). Parfit (1984, p. 431) says we 'might claim' that neither of two things is more F than the other, and that they aren't 'exactly' equally F. Chang (1998, p. 23) supposes that we 'rationally judge' that neither of two things is more F than the other. Carlson (2013, p. 449) gives a case in which neither of two things can 'be said to be' more F, though it is not 'plausible to claim that they are exactly' equally F. Such hedged assertions and assessments of assertibility and reasonableness are just what one would expect in the presence of vagueness. There is a marked contrast between the sentences typically used to express the putative counterexamples to Comparability and, say, the sentences used to express putative counterexamples to Comparative Transitivity—e.g., 'B is worse than A, *C* is worse than *B*, *D* is worse than *C*, ... and *Z* is worse than *Y*, yet *Z* is better than *A*' (Rachels, 1998).

We are far from the first to appeal to vagueness in explaining away the appearance of incomparability (see especially Broome, 1997). Proponents of incomparability have taken great pains to reject such appeals. Their arguments have been discussed at length elsewhere (Bronsther, 2019; Elson, 2017; Gustafsson, 2013; Williams, 2016). Our general impression is that the attempts to dismiss vagueness-based diagnoses rely on highly tendentious characterizations of vagueness. For example, Chang (2002, p. 682) argues that borderline cases have a distinctive phenomenology in which, 'insofar as we are willing to judge that the predicate applies, we are also willing to judge that it does not apply.' She suggests that, in putative cases of (what we are calling) incomparability, we are instead inclined to judge that one thing is not more F than another, but not similarly inclined to judge that it is. Chang admits that there might, in certain cases, be "perplexity" about the comparative judgments, but insists that this perplexity is distinct from the kind that characterizes borderline cases, in which 'it is perfectly permissible to resolve the

indeterminacy in favor of application or not' (683). In putative cases of incomparability, by contrast, she suggests that 'given a list of admissible ways in which the perplexity might be resolved, there is still a further question as to how the perplexity is to be resolved, for that resolution is not simply given by arbitrarily opting for one admissible resolution over another' (685). According to her, people who offer conflicting comparative judgments in putative cases of incomparability are involved in a 'genuine substantive disagreement' (685), but people who offer conflicting resolutions of vague predicates are not.

Here is a simple way to see that these arguments are not compelling. Consider two sequences of careers (or other items)  $x_1, \ldots, x_n$  and  $y_1, \ldots, y_n$  that get better and better and where, for every *i*, the proponent of incomparability is inclined to judge that  $x_i$  is neither better nor worse than  $y_i$ , which is neither better nor worse than  $x_{i+1}$ . Presumably, anyone with that inclination would also be initially inclined, for every *i*, to judge that it is not the case that ( $x_i$  is a good career and  $y_i$  is not), and also that it is not the case that ( $y_i$  is a good career and  $x_{i+1}$  is not). But these negated conjunctions lead to the absurd conclusion that it is not the case that  $(x_n \text{ is good and } x_1 \text{ is not})$ .<sup>36</sup> This pattern is characteristic of vagueness if anything is. But it is not plausible that there is a distinctive phenomenology to the negated conjunctions that is absent in the comparative judgments, or that one can arbitrarily choose a value of *i* at which one rejects the negated conjunctions while remaining at a loss as to how to resolve the perplexity raised by the corresponding comparisons, or that disagreement about the comparative judgments is more genuinely substantive than disagreements about the negated conjunctions.<sup>37</sup> Proponents of the tradeoff argument may continue to find it incredible that there should be a single correct way to weigh between the different dimensions of a multidimensional concept. But this seems no more incredible than familiar analogous propositions about vague predicatese.g., that a single cent can make the difference between a career that is good and a career that is not.

Our vagueness-theoretic diagnosis of the tradeoff argument could be extended to the small improvement argument. For a given choice of x, y, and  $x^+$ , that argument turns on the judgment that neither of x and y is more F than the other, nor is  $x^+$  more F than y.

<sup>&</sup>lt;sup>36</sup>At least, they do so in classical propositional logic, and in prominent nonclassical logics like intuitionism and the paraconsistent logic of Field (2003). They do not do so in the very weak logic of Fine (2017), in which vagueness is held to invalidate not only Excluded Middle but the rule of Conjunctive Syllogism ('*P*; not (*P* and *Q*); therefore not-*Q*').

<sup>&</sup>lt;sup>37</sup>Note also that the monadic predicate isn't the only one that is clearly susceptible to vagueness. We are also inclined, for every *i*, to judge that it's not the case that  $(y_{i+1} \text{ is better than } x_1 \text{ but } y_i \text{ is not})$ . But these judgments, which are suspiciously similar to the "small improvement" intuition, lead to the absurd conclusion that it's not the case that  $(y_n \text{ is better than } x_1 \text{ and } y_1 \text{ is not})$ .

These judgments may be rejected as an overly hasty inference from the recognition that, due to the vagueness in 'more *F*', neither '*x* is more *F* than *y*', '*y* is more *F* than *x*', nor ' $x^+$ is more *F* than *y*' is *definitely* true. On this view, when the 'not's in the small improvement argument are interpreted as expressing standard negation, the conjunction of its premises is definitely false, although none of its premises are definitely false (and perhaps all of the premises and the conclusion convey things that are definitely true, and consistent with Comparability, when 'not' is used nonliterally to mean 'not definitely').

It is worth noting, however, that small improvement arguments look like suspiciously powerful tools for arguing against claims of the form 'x and y are equally F' even when we *haven't* already ruled out 'x is more F than y' and 'y is more F than x'. Claims of the form 'x and y are equally F' are common in everyday life. For example, a hiring committee chair might utter

(27) These two candidates have equally good CVs, so we will have to look carefully at the writing samples.

A small-improvement-monger might object:

Really? So, you are committed to the view that if Candidate *A* had TAed for just one additional class, their CV would have been slightly better than Candidate *B*'s (since clearly in that case their CV would have been slightly better than it actually is)? Isn't that an implausibly strong thing to be committed to?

The objection sounds silly and pedantic. Raz himself says, of a person choosing between a legal career and a musical career, 'He is equally suited for both, and he stands an equal chance of success in both' (126).

One possible account of what is going wrong appeals to context-sensitivity. According to this account, comparative expressions can in some contexts take on "coarse-grained" meanings, where the domain of possible cases is "chunked" into some finite number of discrete sections. In a coarse-grained context, 'equally F' and 'at least as F as' will apply universally within each chunk, while 'more F than' will apply only across chunks. When we start paying attention to small differences in the relevant underlying factors, that generates pressure to change the context to a more fine-grained one, which means the extension of 'more F than' will expand to include certain pairs of items that were previously in the same chunk, and the extensions of 'equally F' and 'at least as F as' correspondingly shrink.<sup>38</sup> On this account, the small improvement argument can be diagnosed as turning

<sup>&</sup>lt;sup>38</sup>Schwarzschild and Wilkinson (2002) propose a "chunking" account for '[at least] as *F* as' and 'exactly as *F* as'. However they do not extend the account to 'more *F* than', so their theory

on a shift in the context. When we initially consider the disparate items x and y, the most natural resolution of context-sensitivity is a rather coarse-grained one that puts them in the same chunk, since, because of vagueness, there are no non-arbitrary grounds for choosing a resolution of context sensitivity that makes one but not the other of 'x is more F than y' and 'y is more F than x' true. Premise P1 ('x is not more F than y and y is not more F than x') is thus true in the context in which it is uttered. (Given the validity of Trichotomy, it follows that 'x and y are equally F' is also true in this context.) Likewise for P2 (' $x^+$  is not more F than y'). P3 (' $x^+$  is more F than x'), by contrast, invites us to attend to the small but one-sided differences between x and  $x^+$ , thus pushing us into a different, more fine-grained, context in which P3 is true. But in no context are all three premises true.

A variant of this diagnosis appeals to pragmatics rather than semantic context-sensitivity. On this account, when we say 'These things are equally F', we are very often speaking *loosely* (non-literally). Analogously, some philosophers argue that when we say 'The cities are 853 miles apart' or 'They arrived at 3pm' we are almost never speaking literally, since the literal truth conditions are ultra-demandingly exact (Hoek, 2018). Given that view, it is plausible that the same kind of looseness is in play when, at the beginning of the small improvement argument, people are inclined to accept 'Neither of *x* and *y* is more *F* than the other'. Note that this seems to be a different kind of nonliteral speech from the kind exemplified by 'He is not bald, but he is also not not bald', since it doesn't have anything special to do with negation. However, the two sources of nonliteralness could sometimes both be in play.

We need not commit ourselves to any particular diagnosis of the assertibility of ordinary attributions of 'equally F' like (27). It seems likely that, whatever the diagnosis, it can help to explain away the apparent truth of the key premises of the small improvement argument—which, as we have said, are rarely asserted outright in the first place—and potentially also the apparent acceptability of speeches like (23). In sum, we find the dominant arguments against Comparability to be much less compelling than our arguments for its validity.

surprisingly predicts the consistency of '*a* is more *F* than *b* and *b* as at least as *F* as *a*'. While they could give a pragmatic account of the oddity of this conjunction by positing that the first conjunct tends to push us into a more "fine-grained" context where the chunks are small enough to make the second conjunct false, it seems better to preserve Strict Comparison by applying "chunking" to comparatives as well as equatives.

## 6 Conclusion

The logic of comparatives in natural language is an interesting topic in its own right. And like other parts of logic, it is also of crucial importance for theorizing about a wide range of philosophical problems that are not in any way about language. For, as we noted at the outset, many interesting and important philosophical problems—problems about goodness, probability, confidence, preference, inequality, beauty, the strength of reasons, and so on—are formulated in natural languages using comparative expressions. The logic of comparatives—which, if we are right, includes Comparability—serves as an important tool for good reasoning about those problems. In other work (Dorr et al., 2021), we apply this tool to some disputed questions about preference, credence, and choice. We argue there, using Comparability, to the conclusions that everyone has complete preferences and real-valued credences, and that there is almost always some unique thing we ought to do, want, or believe.

Our arguments for Comparability do not, of course, show that relations that exhibit incomparability are uninteresting or theoretically unimportant. If you find the arguments convincing but were antecedently disposed to reject Comparability for some particular philosophically important term, you might be inclined to say 'So much the worse for natural language' and either abandon the original term for some technical surrogate, or redefine it in some stipulative way that does not conform to Comparability. Other constraints such as Comparative Transitivity and Strict Comparison could be evaded in the same way. Of course, we have no objection to the coining of new technical terminology, or to the stipulation of new technical meanings for ordinary expressions, so long as one does this in a way that does not court equivocation between the novel meanings and the originals. But even if one is careful to avoid equivocation, and even if one can somehow motivate interest in the new notions on their own terms, the philosophical puzzles stated using the original, natural-language expressions are interesting and important, and they remain unsolved.<sup>39</sup> If we are right, then the solutions to those problems—the ones that

<sup>&</sup>lt;sup>39</sup>For example, in response to our arguments, Hájek and Rabinowicz (2021, n. 15) suggest that a version of the "fitting attitudes" theory of 'better' (see note 2) should be treated as a Carnapian "explication" that may not capture the actual meaning of 'better'—because it violates Comparability—but is nonetheless theoretically fruitful. They suggest that the incomparabilities in this relation can help solve the problem of avoiding the "Repugnant Conclusion" of population ethics: the claim that, for any population of excellent lives, there is some better population of lives that are barely worth living. But if they are using 'better' as a term of art (obeying Comparative Transitivity but not Comparability), then at best, their approach is providing a way of avoiding some other Repugnant Conclusion\*, expressed using their technical sense of 'better'. It is not clear how repugnant this other conclusion really is. But even if it is repugnant, and even if Hájek and

originally gripped us—must be compatible with Comparability.

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Rabinowicz have successfully shown how to avoid it, there remains the problem of avoiding, or undermining the prima facie implausibility of, the original Repugnant Conclusion.

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