DEFINING A GENERAL STRUCTURE OF FOUR INFERENTIAL PROCESSES BY MEANS OF FOUR PAIRS OF CHOICES CONCERNING TWO BASIC DICHOTOMIES

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**Abstract.** In previous papers I have characterized four ways of reasoning in Peirce’s philosophy, and four ways of reasoning in Computability Theory. I have established their correspondence on the basis of the four pairs of choices regarding two dichotomies, respectively the dichotomy between two kinds of Mathematics and the dichotomy between two kinds of Logic. In the present paper I introduce four principles of reasoning in theoretical Physics and I interpret also them by means of the four pairs of choices regarding the above two dichotomies. I show that there exists a meaningful correspondence among the previous three fourfold sets of elements. This convergence of the characteristic ways of reasoning within three very different fields of research - Peirce’s philosophy, Computability theory and physical theories - suggests that there exists a general-purpose structure of four ways of reasoning. This structure is recognized as applied by Mendeleev when he built his periodic table. Moreover, it is shown that a chemist~~,~~ applies all the above ways of reasoning at the same time. Peirce’s professional practice as a chemist applying at the same time this variety of reasoning explains his stubborn research into the variety of the possible inferences.

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**Keywords: D**ichotomy on the kind of Mathematics, Dichotomy on the kind of Logic, Peirce’s four ways of reasoning of Computability Theory, Four prime physical principles, General structure of ways of reasoning, Mendeleev’s ways of reasoning, Chemical origin of Peirce’s reasoning.

**1. Introduction**

The aim of this paper is to define four main ways of reasoning (in the following: WoRs; in particular, inductive and abductive reasoning).

In the first part of the present paper I will summarize what I have shown in previous papers: *i*) Peirce’s writings on the inference process of abduction may be best interpreted by means of intuitionist Logic. *ii*) Beyond the declared inference processes - deduction, induction, abduction - Peirce‘s writings on both the criticisms of Descartes’ philosophy and the characterization of the main logical features of a computer implicitly made use of a fourth inference process, which I called limitation. (Drago 2013)

I will examine WoRs through the most basic notions possible. Recent research on the foundations of both Mathematics and Logic has suggested two dichotomies; one regarding the two formal kinds of Mathematics - either classical or constructive -, corresponding to the two kinds of the philosophical notion of infinity; another one on the two formal kinds of mathematical Logic - either classical or intuitionist -, corresponding to the two kinds of the philosophical notion of the organization of a theory. These dichotomies are traced back to Leibniz’s two labyrinths, which the human mind encounters in its reasoning (Leibniz 1710). They constitute the foundations of science. (Drago 1994; Drago 2012) By means of the four pairs of choices regarding the above two dichotomies I have characterizes as well-defined logical processes both Peirce’s four inferential processes and the four WoRs of Computability Theory (= CT) - i.e. recursion, minimalization, oracle and undecidabilities. Moreover, these two sets of fourfold WoRs prove to be in a mutual correspondence; in particular, the inference process of an abduction corresponds to CT’s WoR of an oracle; as such, it is accurately defined in both mathematical and logical terms. (Drago 2007; Drago 2016)

In the following two parts of present paper I will obtain three main results. *i*) Recently, I have recognized within theoretical Physics (classical Chemistry included) four prime principles of reasoning, i.e., causality, extremants, physical existence of a mathematical object, impossibilities, all characterized by means of the four pairs of choices regarding the two dichotomies.(Drago 2015) There exists a semantic correspondence among these prime physical principles of reasoning and ~~correspond to~~ the two fourfold WoRs of both CT and Peirce’s philosophy. In particular, in a physical theory an abduction corresponds to the claim to attribute physical existence to a mathematical object (e.g. in geometrical optics, to claim that a straight line represents a light beam). *ii*) All of this is evidence of the same structure of four WoRs which is common to the three fields of research, i.e. Peirce’s philosophy, CT and theoretical Physics. *iii*) The various well-founded physical theories – built up over the centuries - enjoy this structure of reasoning and this constitutes evidence for its adequately representing the scientific WoRs about the real world. This structure was substantially reiterated by CT and was anticipated by Peirce’s philosophical reflection. Hence, this structure represents not only the WoRs of a variety of formal scientific theories, but also a philosophical conception. This convergence on the four WoRs, obtained from three very different fields of research, constitutes sufficient evidence for its both philosophical and logical completeness. *iv*) I will recognize Peirce’s and CT’s WoRs in Mendeleev’s reasoning when he built the Table of elements, in particular the abduction inference. *v*) Yet, CT differs from a common physical theory, which argues mainly through a single prime principle of reasoning, because it argues by means of all WoRs at the same time. *vi*) A classical chemist also reasons in the latter way. This fact provides an interpretation of the great work performed by the professional chemist Peirce in discovering all possible WoRs, as well as his insistence on the notion of abduction, really an essential inference process of Physical-Chemistry, but completely ignored by theoretical physicists, owing to its elusive nature, which will be explained later.

**2. Two dichotomies as the foundations of physical theories**

I exploit two decisive results obtained by the investigations into the foundations of science. Half a century ago two basic philosophical notions received clear-cut formal definitions. The notion of infinity, in which philosophers had distinguished actual infinity (AI) and potential infinity (Pl), has been formalized as two well-defined formal systems; on one hand, traditional classical mathematics, which since the 17th century has relied upon Al (e.g. through notions as infinitesimals and Zermelo's axiom); and, on the other, constructive mathematics, relying on (almost) only PI. (Markov 1962; Bishop 1967)

A more laborious historical process led to a formal definition of the philosophical organization of a theory. Aristotle suggested that a scientific theory has to be organised through a pyramidal system of deductions drawn from a small number of axioms. Of course, this organization is governed by classical logic. For a very long time the mainstream maintained that classical logic is unique. Eventually, in last Century the logicians recognized a plurality of kinds of logic, all formalized in mathematical terms; in particular, intuitionist logic was recognized as being on a par with classical logic. Moreover, by means of a comparative analysis of some theories - pertaining to Logic, Mathematics and Physics – which exhibit a different organization from the deductive one, e.g. classical Chemistry, I discovered that each of these theories makes use of propositions of a particular kind. They are doubly negated propositions which are not equivalent to the corresponding affirmative propositions owing to the lack of evidence for the contents of the latter ones (DNPs).[[1]](#footnote-1)

An instance of a DNP in theoretical Physics is the following one: “Motion without end is impossible”. (Dugas 1955, p. 121) In such a case the double negation law fails since this proposition is not equivalent to: “Every motion has an end", which, as a scientific law, is false; because nobody is capable of operatively determining, owing to the a priori unknown friction function, the final point of say the Earth's trajectory, or also of a ball struck with a cue on a billiard table before this end occurs. In the last century the scholars of mathematical logic achieved a crucial result; i.e. the validity or not of the double negation law represents the best discriminating mark between classical logic and most non-classical logics, above all intuitionist logic (Prawitz and Melmnaas 1968; Grize 1970, pp. 206-210; Dummett 1977, pp. l7-26; Troelstra and van Dalen 1988, pp. 56ff.). This failure of the double negation law qualifies the former proposition as belonging to non-classical logic, in particular, intuitionist logic.[[2]](#footnote-2)

This logic governs a different model of organization of a scientific theory, which I have obtained by means of a comparative analysis of all past scientific theories which present an organization other than a deductive one; in particular, Lobachevsky’s theory of non-Euclidean geometry (Lobachevsky 1955). Each of these theories is aimed at solving a basic problem by inventing a new scientific method by means of *ad absurdum* arguments. I called this model of organization a problem-based organization (PO), whereas I called AO the Aristotelian organization of a deductive kind. In such a way the philosophical notion of two kinds of organization of a theory is translated into a formal dichotomy between the two main kinds of mathematical logic. In sum, we have, on the one hand, classical logic, governing AO theories (e.g. Euclid's *Elements*) and, on the other, intuitionist logic, governing PO theories. (Drago 2012)

The following six kinds of analysis have corroborated these two dichotomies as the foundations of science:

*i*) A clear recognition of the foundations of Newton’s mechanics as constituted by the following two choices; the deductive organization, starting from his celebrated three principles, AO, and the use of an idealistic mathematics, i.e. infinitesimal analysis, hence the choice IA. (Drago 1988) *ii*) The rational re-construction of Lazare Carnot’s mechanics, which completed Leibniz’s effort to suggest an alternative theory to Newton’s mechanics;(Drago Manno 1989; Drago 2004) it is based on the problem of the impact of bodies (PO) and its mathematics is plain algebraic-trigonometric mathematics; hence, its two choices regarding the two dichotomies diverge from those of Newton’s. *iii*) The rational re-construction of Sadi Carnot’s thermodynamics, which was the first alternative physical theory to Newton’s mechanics (Drago and Pisano 2000); it manifests the alternative choices to Newton’s, makes use of elementary mathematics and is based on the problem of the highest efficiency in the conversion of heat into work. *iv*) The interpretation of the large number of the new theories developed at the time of French revolution; they differ from each other in the pairs of choices.(Drago 1989) *v*) The interpretation of the revolutionary role played by Einstein’s first paper on quanta, as manifesting a complete alternative to Newton’s foundations, an alternative that can be traced back to the difference between the fundamental choices of this theory and those of Newton’s (Drago 2013) *vi*) A systematic interpretation of all categories applied by the historians of Physics, in particular Koyré’s and Kuhn’s categories which translate the pairs of choices - AI&AO of Newton’s mechanics - into subjective terms. (Drago 2017).

*Viceversa*, each pair of choices determines one out four models of a scientific theory (MSTs). I baptized the MST of the choices AI&AO, upon which Newton’s mechanics relies, the Newtonian MST. Instead, Classical Chemistry, L. Carnot‘s Mechanics, S. Carnot’s Thermodynamics, Lobachevsky’s non-Euclidean Geometry, Einstein's first theory of quanta,[[3]](#footnote-3) etc. (Drago 1996) all belong to the Carnotian MST, whose choices are PI&PO; whereas Descartes’ theory of geometrical optics is representative of the Descartesian MST of the choices PI&AO; Lagrange’s theory of mechanics is representative of the Lagrangian MST whose choices are AI&PO.

Notice that the two dichotomies are more powerful categories than any category suggested by previous philosophers of science, most of whom suggested a single notion (i.e. causality, determinism, economy of thinking, extremants, probability, etc.); the dichotomies are instead two independent notions. Moreover, they are two very particular notions, i.e. dichotomies; as such, they allow four choices; hence, instead of a monist or at most a twofold scheme, a fourfold scheme constitutes the foundations of science. In addition, previous scholars looked for either philosophical, informal notions (e.g., space, time, set, determinism, causality, etc.) or formal notions (ruler and compass, infinitesimals, Euclidean geometry, calculus, Newton’s mechanics, etc.) as the foundations of science. Instead, the above dichotomies constitute, at the same time, philosophical notions (infinity, organization) and formal scientific notions (or even theories); indeed, each dichotomy is formalized in mathematical terms. Hence, this double faced nature allows their application to fields of reality in both formal and informal terms.

I add that these dichotomies can be traced back to a noble philosophical father, Leibniz. He stressed "two labyrinths of human mind”: 1) the notion of infinity: either actual infinity or potential infinity; 2) "either law or freedom". (Leibniz 1710, Preface) He was unable to decide whether each labyrinth is solvable or not. Subsequently, no one has resolved them by scientific means. This fact suggests that each labyrinth is actually a dichotomy for human reason. Of course, the first of Leibniz’s labyrinths concerns the same above dichotomy of the two kinds of mathematical infinity. The second labyrinth corresponds to the above dichotomy of the two kinds of organization, provided that this organization is considered from a subjective viewpoint: either obedience to a compulsory law derived from fixed principles, or the freedom to creatively discover a method for solving a given problem.

**3. Improving Peirce’s philosophical characterization of both the behaviour and WoR of a computer**

Among past philosophers Peirce was the only one with a background as a chemist and moreover one who worked as a scientist (he was mainly a geophysicist). He also was one of the few philosophers that developed their philosophical systems on the basis of scientific research. His philosophy was baptized by him as pragmatism, whose meaning *grosso modo* corresponds to the method of experimental science. In fact, his philosophical reflection was primarily concerned with the methods of inquiry and the growth of knowledge. In particular, Peirce was one of the ﬁrst philosophers to ponder on the “thinking machines” and among these philosophers he was certainly the most intelligent. Being a pragmatic philosopher, Peirce basically referred his thinking to operative processes; current computability theory (CT) also refers to an operative process (of calculation). Moreover, his reasoning was mainly aimed at solving problems, as CT also does. Furthermore, he made a great contribution to determining how to conceive CT. In a more specific terms, Peirce stressed the “impotencies” of such “thinking machines”. (Peirce 1887, pp. 168-169)

 His primary interest in investigating scientific research was to characterize the WoRs of man’s mind. Of course, he studied deduction and induction, adding a new inferential process, “abduction” (Fann 1970, p. 26), by which he meant mainly “the reasoning by which we are led to adopt a hypothesis” (2.102). Moreover, I have suggested that Peirce introduced, although he was unaware of it, a fourth process of reasoning, which, being of a limitative kind, I called “limitation”. Three of Peirce’s writings on a crucial philosophical subject - his criticism of Descartes’ basic tenets. (Peirce 1868a; Peirce 1896b; Peirce 1869) – actually makes use of a fourth kind of reasoning, establishing the “incapacities" of human reasoning. Moreover, some of Peirce’s reflections upon computers stressed their “impotencies” (Peirce 1887, pp. 168-169). On the basis of his writings on both Descartes’ tenets and computers’ impotencies (Drago 2014)

Although Peirce’s presentation of these WoRs is disputable because his writings did not even accurately define the two inferential processes of induction and abduction. I conclude that Peirce actually substantially suggested four WoRs: Deduction, induction, abduction and limitation (including both “incapacities” and “impotencies”). This framework is wider than the usual one, often reduced – as Peirce himself lamented (8.384) - to the deductive WoR only; or, at most, it is commonly enlarged to include elements of induction; while abduction is commonly ignored; at worst, any limitative reasoning is considered to be a useless constraint on the scientific research.

**4. A semantic correspondence between Peirce's four inferential processes and CT’s four mathematical WoRs**

Of course, Peirce’s framework of the four inferential processes is of a philosophical nature. In order to move towards a formal characterization of it, let us analyze his fourfold system from a new viewpoint; which is expressed by means of a formal language developed over millennia, i.e. mathematics.

Through this language CT has suggested four distinct techniques of calculation which the mind performs as processes of reasoning; i.e. recursion. minimalization. oracle[[4]](#footnote-4) and undecidabilities. Let us now compare Peirce's four inference processes with the four mathematical processes characterizing CT.

Notice that this comparison concerns, on one hand, formal WoRs of a mathematical or logical nature and, on the other, informal WoRs of a philosophical nature. The comparison will test whether a formal WoR is an instantiation of a more general WoR which is defined in philosophical terms; hence the correspondence to be established cannot be anything more than an equivalence of a semantic nature.[[5]](#footnote-5)

It is easy to see a correspondence between Peirce’s two inference processes, i.e. deduction and limitation, and two particular CT WoRs. Indeed, CT’s *recursion* represents a particular instance of deductive reasoning from the formula of the recursive function playing the role of an axiom, from which the n-th result is obtained by the n-th iteration of the same deduction process. Actually, Peirce (Peirce 1881) was one of the first mathematicians to suggest the mathematical definition of recursion as a specific instance of a deductive WoR.

Moreover, CT’s *undecidabilities* at a glance appear to be a formalization through exact mathematical tools of Peirce's notion of computers’ impotencies.

About Peirce’s two remaining inferential processes, i.e. induction and abduction, we have to take into account that he never accurately defined them. (Fann 1960, pp. 6, 9-10, 31) Thus in order to obtain accurate definitions of them I take advantage of the formal characterizations of CT’s two remaining mathematical WoRs, i.e. minimalization and oracle. Notice that the justification of a minimalization is given by a mathematical calculation generating it; whereas the justification of abduction is given by an *a posteriori* verification of a logical nature.[[6]](#footnote-6) Thus, I suggest that the Peirce’s two inference processes are mutually distinguished according to their kinds of justification, respectively an *a priori* mathematical one and an *a posteriori* logical justification. [[7]](#footnote-7)

After these qualifications of induction and abduction, Peirce’s inference of induction may be accurately defined as obtained by means of a specific mathematical process (continuity, infinitesimals, limits, extremants, involution, etc.), which are all included by CT’s mathematical WoR which produces a *minimalization* (or *maximalization*); whereas Peirce’s inference of abduction may be defined as an instantiation of a CT’s computing process obtaining from *an oracle* an answer suggesting an element that is a posteriori justified in logical terms, i.e. by its not contradicting its original mathematical context. (Drago 2014; Drago 2016)

Let us stress the elusive nature of abduction. It is a common opinion among scientific researchers that when a result is known as possible, because it has been already obtained by others, it is just a matter of time before the same result is obtained again. That means also that once the result of abduction is logically justified since it is shown to work, it is a matter of (a short) time before it is discovered that either an inductive or a deductive process obtain the same result. Of course, the latter two methods of discovery are considered more cogent than an abduction, whose purely logical proof may be open to metaphysical notions and considerations. For this reason, once a result is obtained by means of abduction, scientists promptly replace it with a more “respectable” inference. This explanation of the elusive nature of abduction holds true even more in the case of a physical theory, where a logical justification is commonly considered to be too abstract with respect to experimental reality; as a matter of fact, in the history of theoretical physics a physicist has never claimed a result by an argument based on abduction, if not as a mere guess, motivating the search for either a mathematical calculation, or a theoretical deduction, or an eminently experimental datum, whose evidence provides the correct justification of the result. It is for this reason that almost all scientists have avoided presenting abduction with impunity. This custom has excluded abduction from the commonly recognized experience of scientific reasoning belonging to the most important area of scientific inquiry.

Although the previous comparison of intuitive philosophical ideas, i.e. Peirce’s definitions of inferences, and formally defined mathematical ideas - i.e. CT’ mathematical processes -, allow only philosophical considerations, some considerations of this kind seem important: *i*) Peirce’s philosophical effort to qualify the potentialities of “thinking machines” not only anticipated better than anyone of his time a philosophy of CT, but also the inference process of limitation, and hence all kinds of inference processes. *ii*) Rather than a metaphysical basis - which Newton chose to give to his Mechanics (see his metaphysical notions of absolute space, absolute time and force-cause) -, or the basis of an empiricist philosophy - given by Lazare Carnot to his Mechanics (all its notions and also principles are of an empirical nature; Drago 2004) -, a pragmatist philosophy is CT’s philosophical basis. *iii*) This philosophy was formulated by the scientist-philosopher Peirce half a century before CT’s birth; hence he has to be considered the philosophical father of CT. *iv*) This philosophical basis of CT does not concern basic notions or principles – as in both Newton’s Mechanics and L. Carnot’s Mechanics - but that which is the main subject of CT, i.e. WoRs – which in science constitutes a higher level of conceptualization than notions and principles. *v*) The above illustrated correspondences provide Peirce's philosophical system of WoRs, abduction included, with exact definitions. *vii*) The correspondence of Peirce’s philosophical inference processes with scientific experience of CT’s WoRs along almost a century suggests a reasoning structure which is simultaneously informal and formal in nature.

From this correspondence one *may* suggest that Peirce’s four inferential processes represent all possible inferential processes; and *viceversa*, that for philosophical reasons CT’s four WoRs *may* represent a complete framework of formal WoRs. In the following Sect. 6 we will add decisive evidence for supporting these theses.

**5. Recognition of Peirce’s inference processes in Mendeleev’s WoRs aimed at formulating his Table of elements**

In the following we will recognize the previous system of four WoRs as the set of WoRs which Mendeleev made use of when formulating his table of the elements of matter. In the history of science this case-study is unique because Mendeleev not only reasoned in a variety of ways in order to obtain his result, but he also described his WoRs.[[8]](#footnote-8) An important publication by Mendeleev (one of Faraday’s Lectures, quoted by Scerri 2007, pp. 109-110) illustrates the method that Mendeleev exploited in order to construct his periodic table through physical and chemical experimental data. The specific WoR actually referred to by Mendeleev is in square brackets.

 1. The elements, if organized according to [the growing numbers of the] atomic weights [*deduction-recursion*], show an evident periodicity [*limitation*] of the properties.

 2. Elements that are similar in their chemical properties have atomic weights that are either nearly equal (eg, Platinum, Iridium, Osmium, etc.) [the similarity of elements is regularly represented by the atomic weight as well as all other properties, which means a contiguity of the values of each of their parameters; *deduction-recursion*] or [in the case of the same valence, in the sense of a variable with a *limited* range] that increase regularly [*recursion-deduction*] (eg, Potassium, Rubidium, Cesium).

 3. The organization of the elements, or groups of elements, in the order of their atomic weights, corresponds to their so-called valences [*limitation*], as well, to some extent, to their characteristic chemical properties - as is clear in another series [*deduction-recursion*] - in that of Lithium, Beryllium, Boron, Carbon, Nitrogen, Oxygen and Iron.

 4. The elements that are most widespread [in nature] have small atomic numbers [an experimental fact of geology, not chemistry].

 5. The value of atomic weight determines [*deduction*] the character of the elements, just as the value of the molecule determines the character of a compound.

 6. We must expect [owing to an *ad absurdum* argument, supporting an *abduction of the decisive hypothesis of* the periodicityfor constructing a theory conceived as a systematic table] the discovery of many elements that are still unknown [*abduction*], for example elements similar to Aluminium and Silicon, the atomic weight of which [*induction*] should be between 65 and 71.

 7. The atomic weight of an element can sometimes be corrected by the knowledge of the [atomic weights of the] adjoining elements [*induction*]. Therefore the atomic weight of Tellurium must be between 123 and 126, it cannot be 128.

 8. Certain characteristic properties of the elements can be predicted by their atomic weights [*induction*].

 Let us now interpret in more detail these reflections of Mendeleev’s through the above four WoRs (according to *grosso modo* the order of Mendeleev’s illustration).

. *Recursion-deduction*. In chemistry it corresponds to Prout’s hypothesis: each element can be obtained as a multiple of a same element (Hydrogen). Therefore, the elements can be listed as a series of growing values of a parameter, e.g. the atomic weight. However, in Mendeleev’s table the *atomic weight* expresses this growth in an irregular way;[[9]](#footnote-9) years later, the *atomic number* parameter and then the *number of electrons* will give the exact recursion (albeit the most elementary kind of recursion). In addition, the same kind of inference as before is applied to the elements sharing the same valence, i.e. belonging to the same chemical group. This WoR occurs at points 1 (first part), 2 (first and third part), 3 (second part), 5.

 *Induction through a limit process on (rational[[10]](#footnote-10), hence constructive) numbers.* This case occurs when we consider the series of the experimental determinations of the values of a parameter – e.g. the atomic weight - of the surrounding group of a given or supposed element *X*. From these experimental results~~,~~ we can obtain the value for the element *X* through a limit process (actually, by means of a limit process including very few approximating values). The series of values are considered as being no more than an approximation to the desired value. Through the use of this kind of inference each element is characterized by a list of approximatively determined values of its chemical-physical properties. All Mendeleev's *analogies*, formulated as “averages” on triads or octaves of elements, are such limit processes which represent inductions. This WoR occurs at points 6 (last part), 7 and 8.

 *Limitation*. *Valence* (of course, among the different valences enjoyed by an element we will consider the basic one). Its nature is of a limitation WoR because 1) it confines a variable to a finite interval; hence, by analogy with geometry, when the radius of curvature of the space is finite, the geometry is elliptic and its lines are periodic in nature; more precisely, valence defines the constraints within which a chemical element can combine itself with the other elements; or defines the constraints within which a chemical group is located; 2) it is defined by a DNP: it is not possible to consider as homologues two elements with different valences. It is by reflecting on the similarity of elements with the same valence that Mendeleev made "the crucial discovery" of periodicity [17, p. 105, p. 119]; which he then combined with the previous recursive progression; it is precisely in this way that he obtained his MT. This WoR occurs at points 1 (second part), 2 (second part), 3 (first part).

 *Abduction as an oracle of a decisive hypothesis for constructing a theory* (“the logic of [theory] pursuit”, as Achinstein (1993) put it); this inference may be what I have called "Peirce’s Principle". (Drago 2016). It concerns the table as a whole; it is of this nature the hypothesis that states the completeness of MT. By attributing importance to empty locations, an *ad absurdum* argument is implicitly stated: material reality would be absurd if it were to admit these voids in a series of material elements. This leads to the hypothesis that in the sequence of the elements it is impossible that, in that place, there does not exist a new element. To this proposition we apply a general principle of translation between two kinds of logic, i.e. the principle of sufficient reason, translating from intuitionist logic to classical logic.[[11]](#footnote-11) We then infer from the relations with its neighbouring elements, that this new element must have similar characteristics to those possessed by them. This WoR occurs at points 6 (first part).

 Table 1 summarizes both the suggested links between the four WoRs and Mendeleev’s illustration of his reasoning for constructing his MT.[[12]](#footnote-12)

Table 1: CORRESPONDENCES BETWEEN PEIRCE’S KINDS OF INFERENCES AND THOSE SUGGESTED IN MENDELEEV’S WORKS

|  |  |  |
| --- | --- | --- |
| **Mendeleev’s WoRs** | **Peirce’s inference processes** | **First physical principles** |
| *Series of the atomic weights of elements (Prout)* *(either in general or within a chemical group)* | Recursion-deduction  | Causality-deduction (Geometric optics, Newton's mechanics)  |
| *Atomic weight of an element obtained from a limit process performed on experimental data*  | Induction (idealistic or approximate) | Extremants  |
| *Valence that limits the groups of the elements. Periodicity of the properties of the elements* | Impotence | Limitation (eg, Impossibility of perpetual motion) |
| *Completeness of the table*  | Abduction (as Peirce's Principle) | Principle of sufficient reason (Solution of the basic problem of a PO theory) |
| *Best hypothesis of new element**("Analogy") through a mean of the properties of some neighbouring elements* | Abduction (as an oracle of the best hypothesis) | Existence (ray of light in Geometric optics, fields in Electromagnetism) |

 In sum, we have obtained that Peirce’s four inferential processes represent the WoRs which Mendeleev was aware of having applied when composing on a logical basishis systematic table. In particular, abduction plays a decisive WoR in his building the table.

 **6. Establishing through the four pairs of choices a correspondence among Peirce’ four inference processes, CT’s four WoRs and four prime physical principles**

In order to improve this kind of analysis, let us consider one more scientific instantiation of the possible WoRs, i.e. those accumulated by a variety of physical theories over the last few some centuries. It is easy to recognize, in correspondence to the four representative theories of the four couples of choices regarding the two dichotomies - and hence the above listed four MSTs -, four prime principles of reasoning, respectively: causality, as embodied by the notion of force-cause of Newton’s mechanics; extremants, as embodied by the principle of minimal action of Lagrange’s mechanics; physical existence of a mathematical element, as embodied by a straight line which is considered a light beam of geometrical Optics; limitation, as embodied in Thermodynamics by the principle of the impossibility of perpetual motion.(Drago 2011, Drago 2015)

As pertaining to a specific MST, each WoR is characterized by the pair of basic choices of its MST; these pairs are respectively: AI&AO, AI&PO, PI&AO, PI&PO.[[13]](#footnote-13) Within theoretical physics each of these principles of rational WoR combines an intuitive reality of reference, an operative method and a mathematical formalization (e.g. extremants).

Do the semantic contents of the four physical prime principles correspond to those of the four CT’s WoRs? We will obtain an affirmative answer by characterizing also CT’s Wors through the basic choices. First, let us investigate CT’s WoRs of undecidability. Notice that only when AI is excluded can there be an indecidability; otherwise, a suitable ideal element decides any question (e.g. the idealistic Zermelo's axiom solves the constructively impossible problem of composing a new infinite set from an infinite number of infinite sets). Moreover, only in the context of a PO undecidabilities exist; otherwise, the desired decision is nothing other than a theorem derived from some a priori axioms (as within the AO theory of Euclidean geometry the following question: "Are equal two triangles if their three sides are equal?” is decided by a theorem derived from the well-known axioms). Hence, the WoR of undecidabilities is characterized by the pair of choices PI&PO, i.e. the same pair of choices determining the prime physical principle of limitation in theoretical physics of the Carnotian MST, in particular in S. Carnot’s Thermodynamics. (Mach 1986, chp. XIX) It is easy to recognize that their semantic meanings are similar.

Recursion is eminently a tool of deductive reasoning: hence, it is determined by the choice AO. Moreover, at its birth (Goedel 1931) the mathematical technique of recursion was based on the choice PI; later, the notion of generalized recursion has been introduced; it manifestly appeals to ideal elements (e.g. the general recursive functions obtained by diagonalization processes on the totality of the primitive recursive functions; Davis et al. 1994, p. l05ff.). In sum, general recursion relies on the pair of choices AO&AI, i.e. the same pair of choices that in theoretical physics determines the (implicitly metaphysical) causal connection, as this pair does in Newton’s mechanics.

Unbounded minimalization relies on AI, because its process of calculation appeals to a mathematical technique which in general is a non-constructive one. Moreover, it is aimed at solving a (computation) problem which is not solvable through ordinary means, hence it relies on the choice PO.

By resolving a decision-making problem without any evidence, an oracle represents a non-constructive element which plays only the role of an axiom, hence it introduces an AO theory; its context, in order to avoid metaphysical detours, has to be decidable, i.e. the choice of this theory is PI.

I conclude that the correspondence established by the four pairs of basic choices allows us to conclude that CT’s four mathematical WoRs, are not only spontaneous inventions originating in a specific empirical context, but *constitute a foundational structure of CT.* The complexity of such a structure (including also an oracle, which is very different from the other WoRs) justifies why so many previous scholars were unsuccessful in recognizing it as a logical structure.

Moreover, the four foundational pairs of choices on the two basic dichotomies characterize a correspondence between CT’s four mathematical WoRs *with* the four prime physical principles; so that the two quadruples are put in a one-to-one correspondence.

In previous papers I showed that the four pairs of basic choices regarding the two dichotomies also characterize Peirce's four processes of inference. (Drago 2008; Drago 2014) This result further qualifies the results of Peirce’s philosophical investigation into inference processes (limitation included) as the anticipation of a well-defined philosophical system of WoRs; which represents “the logic of discovery”, whose existence Peirce so often stressed (e.g. when he stated that “each chief step in science has been a lesson in logic”; 5.363). Notice that the actualization of such a logic denies Reichenbach’s tenet: “Epistemology does not regard the process of thinking in their actual occurrence, this task is entirely left to psychology.” (Reichenbach 1938, p. 5).

In sum, we have obtained that *the three fourfold WoRs*, which include both formal and informal aspects, by sharing the same *four pairs of basic choices*, may be put in a mutual, one-to-one correspondence.[[14]](#footnote-14)

The following table summarizes all the above correspondences.

Table 2: CORRESPONDENCE BETWEEN CT’s WORS AND THE PRIME PHYSICAL PRINCIPLES

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Couples of Basic Choices** | **AI&AO** | **AI&PO** | **PI&AO** | **PO&PI** |
| **Model of Scientific Theory** | Newtonian | Descartesian | Lagrangian | Carnotian |
| **Prime principles in theoretical Physics** | *Causality* | *Extremants* | *Physical existence of a mathematical object* | *Limitation* |
| **WoRs in Computability Theory** | *General recursion* | *Unbounded Minimalization* | *Oracle* | *Undecidabilities* |
| **Peirce’s four inferential processes** | *Deduction* | *Induction* | *Abduction* | *Incapacities* |
| **Nature of his inferential process**  | Conservative | Ampliative | Ampliative | Limitative |
| **General logic principles**  |  Principle of non-contradiction |  Principle of sufficient reason |

Notice that previous results manifest a progression from philosophical notions, which is what Peirce’s inference processes are, to prime physical principles, which combine formal and informal features, to CT’s WoRs which are instantiated by mathematical processes.

**7. CT’s use of four WoRs diverges from that of each physical theory**

However, the comparison between CT and a physical theory (apart from Chemistry) indicates a surprising feature, whereas the theoretical development of a physical theory essentially makes use of only one reasoning principle, CT makes use of all mathematical WoRs at the same time. E.g. within CT the undecidabilities, already characterized as a PI&PO WoR, occur together with the general recursion functions, representing an AI&AO WoR.

This situation was perhaps caused by a rooted prejudice shared by the dominant group of mathematicians: being very useful for practical purposes, CT’s theoretical content was negligible. In order to overcome such a prejudice CT’s scholars have enlarged the theoretical import of their theory as far as possible in order to obtain the greatest possible number of theoretical results. This endeavour then led scholars to make free use of WoRs that are more powerful than, in particular, the WoR of the theory of primitive recursive functions: general recursion, unbounded minimalization, and oracle, all representing WoRs including idealized elements.

The application of different WoRs causes radical variations in the formal definitions of the various notions - e.g. either primitive or general recursion; either bounded or unbounded minimalization, either a computation step obtained by means of an oracle or a computation through an effective computation. But it causes also variations in meanings which are very subtle, owing to the imprecision of common language using informal notions whose variations in meaning may be misinterpreted as tolerable variations; so that they are not recognized as significant. For instance, surely few people notice the difference between “computable functions” and “computed functions”, “computability theory” and “computations theory”, etc.; in other words, between the meaning of a modal word which simply alludes to some content and the meaning of its objective name. Yet, each previous variation implies a variation of the kind of logic, either classical or modal logic, and hence, *via* S4 model (Chellas 1980, pp. 76ff.), intuitionist logic; when the different notions above are combined together, their two representative kinds of logic imply a contradiction in the use of the double negation law.

Notice that whereas a contradiction within an axiomatic system may be recognized after an indefinite number of steps of reasoning (this is the reason why one cannot prove directly the consistency of say Arithmetic), a contradiction within a theoretical system which combines WoRs - which are incompatible also in informal (verbal) aspects - is even more elusive. As a consequence, while an actually inconsistent theory may survive for a long time without scholars stumbling on a formal contradiction, a theory mixing different WoRs may survive for a long time before scholars accurately define a contradiction generated by the radical differences in both the WoRs and the meanings of their basic notions.

 **8. Chemists’ simultaneous use of all WoRs**

After the birth of classical Chemistry, other chemical theories were developed: Stereochemistry, Chemical Kinetic, Chemical-Physics, Spectroscopy, etc.. How many chemical theories? Let us select the most representative ones by exploiting again the four pairs of basic choices. First, Classical Chemistry is characterized as PI and PO for the following reasons. Its organization is not the deductive one, AO, since it is not derived from a priori principles, but is based on a problem (“how many elements of matter are there?”) and moreover most classical chemists - in particular, Mendeleev -, wrote texts in which they made use of DNPs, hence non-classical logic. In addition, Classical Chemistry makes use of a very elementary mathematics, at most rational numbers; hence, PI. Second, the theory of Physical Chemistry derives from a non-chemical, but physical, notion of entropy, which therefore plays the role of an axiom; as a consequence, the organization of the theory is AO; moreover, it makes use of the same elementary mathematics as Thermodynamics, which no physicist would consider on a par with the second order differential equations of Mechanics.[[15]](#footnote-15) In sum, a PI&AO theory. Third, a minor, but very relevant, theory, i.e. Kinetic Chemistry. This theory is characterized by the choice AI, since it introduced the calculus into a field, Chemistry, which almost entirely ignored higher mathematics; certainly, its differential equations are only of the first order (and surely their solutions have constructive counter-parts), but at the time of its birth this theory was counted among the highly mathematized theories owing to its including infinitesimal analysis. Moreover, its organization is aimed at solving a problem, i.e. to studying the crucial notion of this theory, reaction speed. Hence, it is an AI&PO theory.

According to the characterizations - given in Sect. 6 - of the prime principles of reasoning the prime principle of Classical Chemistry is limitation, which in fact is constituted by the valence of each atom. The prime principle of Physical-Chemistry is that of attributing (chemical) existence to a mathematical (and even physical) notion, i.e. entropy. The prime principle of Kinetic chemistry is the extremant; indeed, this theory is aimed at accelerating as much as possible; the speed of chemical reaction. About a fourth prime principle, notice that at Peirce’s time chemists applied the prime principle of causality by reasoning about classical Mechanics, to which most of them - e.g. Mendeleev (Mendeleev 1905, I, xi, n. 2) and the same Peirce referred with the aim of obtaining an ultimate explanation of the foundations of Chemistry (Peirce wanted to explain the foundations of Chemistry through the notions of form and mechanical (“Boscovichian” 7.509) force; 1.428).

 The first consequence, at Peirce’s time, was that there existed three chemical theories that suggested reasoning according to three different WoRs; moreover, a chemist had unavoidably to refer to one more WoR, that of the paradigmatic theory of Mechanics, which relied on the choices AI and AO (the same choices of subsequent Quantum chemistry) whose prime principle is causality, hence deduction.

 The second consequence was that CT’s use of all WoRs at the same time is not a historical novelty, because already a century before CT the professional practice of chemists introduced this mode of reasoning.[[16]](#footnote-16)

**9. The origin of Peirce’s investigations on WoRs**

In total, Chemistry presents four WoRs, in agreement with the four prime physical principles. As a chemist, Peirce was familiar with all the four WoRs that a chemist of his time took into account (although he applied the WoR of deduction in Mechanics, rather than in Quantum Mechanics).[[17]](#footnote-17)

This fact explains: *i*) His insistence on his going beyond the causality principle alone, i.e. the deductive WoR that the mainstream attributed to Mathematics in its entirety and to paradigmatic Physics, i.e. Mechanics. *ii*) His great interest in discovering all possible WoRs. *iii*) His insistence on suggesting, beyond the two commonly accepted inference processes, i.e. deduction and induction, one more inference process, abduction. *iv*) At present time we know that this inference process is the prime principle of a very important theory, Physical Chemistry, in which he was the first Ph.D. at Harvard University; with respect to the empirical solutions given by classical chemistry the solutions obtained by Physical Chemistry represent the answers coming from an oracle, since they are essentially derived from the notion of entropy which is not of a chemical, but thermodynamic nature. Peirce revealed that his source of his ideas about the inference processes was Chemistry and in particular Physical Chemistry when he wrote:

“By a hypothesis [read: abduction] I mean…., it is merely a supposition about an observed object… but also any other supposed truth from which would result such facts as have been observed, as when van ‘t Hoff, having remarked that the osmotic pressure of one per cent solutions of a number of chemical substances was inversely proportional to their atomic weights, thought that perhaps the same relation would be found to exist between the same properties of any other chemical substance.”(6.254f.)

*v*) His belief that it is possible to accurately distinguish among all the inference processes, although he lacked evidence to support his thesis.

Yet, Peirce was unsuccessful in accomplishing his general program aimed at accurately defining the WoRs. Unfortunately, he was misled by his program of research into the foundations of Chemistry, looking for a mechanical explanation based on the notion of force (although at his time Energetists tried to dethrone the notion of force in favour of the notion of energy). Moreover, given that at Peirce’s time Chemistry was a young theory, no clear-cut borderline distinguished the different chemical theories; and, in addition, since in each chemical theory it was not clear what distinct roles are played by mathematics and principles, the above distinctions among the different chemical theories have to be taken as simply perceived, but not clearly understood.

In addition, Peirce’s WoRs on chemical elements were not similar to Mendeleev’s. As a matter of fact, Peirce meditated for a long time on the classification of chemical elements, so that he claimed to have suggested a table like Mendeleev’s (7.509). Yet, Peirce’s expressed even differing evaluations of Mendeleev’s Table which he seemed to be “in considerable doubt”, 7.222). He rarely refers to the WoRs employed for constructing this table; moreover, in such cases he mentions an undefined induction (once he writes “pure induction”); but never abduction![[18]](#footnote-18)

**10. Conclusions**

In conclusion, the two dichotomies have suggested structural categories of the reasoning. Through them we have mutually compared *i*) Peirce’s inference processes, *ii*) CT’s ways of formal reasoning through mathematical and logical tools, which are of an operative, hence objective, nature, *iii*) the physical prime physical principles which are of both an intuitive and a mathematical nature. We have characterized the correspondences of all of them. We have obtained a characterization of the structure of the WoRs; , through of CT’s, a formal qualification of this fourfold structure of the principles of reasoning.

We remarked a basic difference between a physical theory which makes use of one prime principle, and CT, which makes use of all WoRs at the same time. On the other hand, an examination of chemistry shows that it also makes use of all the WoRs at the same time. The fact that Peirce was educated as a chemist and hence made use of all WoRs at the same time explains his philosophical effort to inquire into inference processes, in particular the notion of an abduction, i.e. the characteristic WoR of the theory of Physical-Chemistry, in which Peirce was very competent.

More than a century after his effort, one may suggest that his profound genius in discovering very deep ideas concerning WoRs was facilitated by his implicit reference to a scientific field largely unknown and disregarded by the mainstream, i.e. the various chemical theories. However, we saw that in general Peirce had great difficulties in achieving clearly defined results (apart from his invention of mathematical recursion); he recognized only three of his four WoRs and never accurately defined them (apart from deduction). Moreover, all the correspondences (listed in Table 2) between his investigations and CT’s characteristic features lead us to qualify Peirce as the father not only of the philosophy of CT but also of its multiple manner of reasoning.

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1. Notice that a single word, in particular a modal word, may be equivalent to a DNP; e.g. possible = it is not the case that it is not” (this kind of word will be underlined with dots). More in general, it is well-known that modal logic may be translated by means of its S4 model into intuitionist logic. (Chellas 1980, 76ff.) Notice that the current usage of the English language exorcises DNPs as pertaining to primitive languages. Moreover, some linguists maintain that those who speak by means of DNPs want to be, for instance, unclear. (Horn 2002, pp. 79ff.; Horn 2010, pp. 111-112) On the contrary, it is easy to show that the DNPs pertain to scientific research in Logic, Mathematics, Physics and classical Chemistry. In Logic the translation from classical logic to intuitionist logic is performed by doubly negating the propositions of the former logic. (Troelstra and van Dalen 1988, p. 56ff.) In Mathematics it is usual to develop a theory in order to make it "without contradictions” (here and in the following I underline the negative words belonging to a DNP for an easy inspection by the reader); owing to Goedel‘s theorems, it is impossible to state the corresponding affirmative proposition. i.e. the consistency of the theory at issue. In Mathematics and in theoretical Physics it is usual to study in-variant magnitudes; this adjective does not mean that the magnitudes remain ﬁxed. Moreover, substantial advances were achieved in Mechanics by means of the above mentioned, methodological principle of the impossibility of motion without end. In Chemistry, in order to solve the problem of what the elements of matter are, Lavoisier defined these unknown entities by means of a DNP: “If we link to the name of elements… the idea of last term arrived at by [chemical] analysis, all the substances which we were not able to decompose by any means are for us elements: (Lavoisier 1862-92, p. 7) where the word 'decompose' carries a negative meaning since it stands for ‘non-ultimate‘ or 'non-simple'. [↑](#footnote-ref-1)
2. As a matter of fact, Grzegorczyk (1964) independently proved that the production of new results by science experimental may be formalized through propositions belonging to intuitionist logic, that is, a logic using DNPs. [↑](#footnote-ref-2)
3. Einstein qualified his paper, suggesting the physical existence of light quanta (Einstein 1905a), as his “most revolutionary paper”.(Einstein 1905b) It explicitly presents the dichotomy of infinity in mathematics, and implicitly, yet in an almost rigorous way, presents the dichotomy in both the kind of organization and the kind of logic. Remarkably, it may be considered the most revolutionary paper in general, because it was the only paper to present the two dichotomies. (Drago 2013) Instead L. Carnot (1803) and Lobachevsky (1955) obtained the same result but through books, respectively the former a book founding an alternative Mechanics to Newton’s and the latter a book on non-Euclidean geometry. [↑](#footnote-ref-3)
4. It is roughly defined as a black box which is able to decide certain decision-making problems, otherwise unsolvable, through a single operation. It corresponds to the algebraic procedure of transcendental extension. (Odifreddi 1989, p. 175) It is more precisely defined as follows: “A number *m* that is replaced by *G(m)* in the course of a *G*-computation… is called an oracle to query to the *G*-computation” (where a *G*-computation is a computation of a partial recursive function *G* under a specific condition referring to total functions. Davis et al. 1994, pp. 197ff.). [↑](#footnote-ref-4)
5. CT accustomed scientists to comparing an informal notion of computability with formal notions (recursion, Diophantine equations, -calculus, etc.). It is proved that all the formal notions of computability are equivalent and “hence” (Turing-Church’s thesis) they may be equated to the informal notion. In our case the arguments will be looser than those of CT, because they are aimed at obtaining not mathematical results, but semantic results concerning different representations of WoRs. [↑](#footnote-ref-5)
6. E.g. it is an abduction that suggests the number √2√2 for solving the problem whether there exist two irrational numbers *a* and *b* such that *ab* is a rational number. Proof: either √2√2 is the desired rational number, or √2 elevated to √2√2 solves the problem. One more instance is Lobachevsky’ suggestion of a definition for a parallel line as that line that with the least displacement crosses the basic line; (Lobachevsky 1955, prop. 16) this definition is then justified by logical means , i.e. two theorems which make plausible this definition. In both cases the validity of the solution is verified by a logical argument. [↑](#footnote-ref-6)
7. Later, Peirce (1958, vol. 8 p. 58) called induction and abduction respectively “Quantitative and Qualitative induction”. (2.755; 6.526) As usual among Peirce’s scholars, a reference to (Peirce 1958) it will be given by a first number denoting the volume and a second number denoting the issue. [↑](#footnote-ref-7)
8. At MBR ’18 conference A. Rivadulla presented a new instance of abduction, i.e. Bessel’s discovering of the double nature of the star Syrius. During the discussion after his presentation I have remarked that Bessel’s words “non unfitting” constitute a DNP (“It is not non-usual that…”) which is logically equivalent to Peirce’s word “suspect”. It would interesting to re-visit all the cases of discovery of a new planet - they surely constituted instances of abductions - under the light of Peirce’ three statements. [↑](#footnote-ref-8)
9. Peirce suggested rendering uniform the increments of the atomic weights by supposing that they were inaccurate for several reasons; hence he added to the atomic weight of each element a weight of up to plus or minus 2,5. [↑](#footnote-ref-9)
10. Let us recall that the result of each measurement is only a rational number - because it is represented by a series of decimal digits truncated to the best approximation. [↑](#footnote-ref-10)
11. Peirce conceived abduction in a similar way to the application of the principle of sufficient reason: “Abduction “tries what *il lume naturale* … can do. It is really an appeal to instinct” (1.630) “Retroduction [read: abduction] goes upon the *hope* that there is sufficient affinity between the reasoner’s mind and nature’s to render guessing not altogether hopeless…” (1.121) [↑](#footnote-ref-11)
12. The text of Faraday’s Lecture (Mendeleev 1889) may be interpreted as concerning the same WoRs; recursion on atomic weights (p. 103); Limitation of the values of valence, “closed circle” (p. 104); induction (pp. 106-107); Abduction and induction (p. 117ff). [↑](#footnote-ref-12)
13. The 18th Century saw the origin of a paradigm of considering not only Newton’s mechanics, but theoretical physics as a whole as determined by the prime principle of force-cause. The considerable number of marvellous results thus obtained obscured the prime principles of all other theories, above all the prime thermodynamic principle of limitation together with the basic notion of entropy, which are at odds with the previous one. This paradigmatic view depreciated Thermodynamics as an immature, merely phenomenological, theory (see eg. Kuhn 1977). As a consequence, the paradigmatic view considered the so-called nomological deductive model of a scientific theory, which is derived from Newton’s mechanics, to be the only one. [↑](#footnote-ref-13)
14. E.g. CT’s oracle corresponds also in semantic terms to the prime principle attributing physical reality to a mathematical being; moreover, both are specifications - albeit within two different contexts - of the same philosophical notion of an abductive inference.

Their technical difference is an instance of the phenomenon of a radical variation of the (at least partially) formal representations of the philosophical notion of a WoR. [↑](#footnote-ref-14)
15. Van’t Hoff equation together with Maxwell’ relations among thermodynamic potentials - all first order differential equations – concern magnitudes of physical import, which are assumed as a priori axioms by a chemist. [↑](#footnote-ref-15)
16. By pertaining to a different level of explanation – i.e. electrons - from the chemical ones, Quantum Chemistry introduced the same argumentative habit of a physical theory, i.e. reasoning according to one principle, in this case deduction, leading to regarding as primitive the other (chemical) principles. Indeed, Quantum chemistry is an AO theory since it depends on Quantum mechanics which, not being a chemical theory, works as a principle-axiom; its mathematics (recall Dirac’s -function) is higher than PI mathematics; hence, its choice is AI. [↑](#footnote-ref-16)
17. In my opinion Peirce alluded to this situation when he wrote “Modern methods [of reasoning] have created modern science and this century has done more to create new methods than any former equal period.”(7.61) Moreover, Fann (1970, p. 23) adds that “he maintains that the producing of a method for the discovery of methods is one of the main problems of logic.”(3.364) “Though other definitions of logic occur in his writings, the one he used in his teaching at John Hopkins University (1879-1884) was that it is the art of devising methods of research, “the method of methods”.”(7.59) [↑](#footnote-ref-17)
18. Rather, he wanted to found a new theory of thinking which associates ideas in a similar way to the chemical way through which chemical elements associate themselves in compounds. It was called Phanerochemistry; later it became Semiotics. [↑](#footnote-ref-18)