SEMANTIC INTERPRETATION OF THE CLASSICAL / INTUITIONIST LOGICAL DIVIDE THROUGH THE LANGUAGE OF SCIENTIFIC THEORIES

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**Abstract.** Double negations are easily recognised in both the so-called “negative literature” and the original texts of some important scientific theories. Often they are not equivalent to the corresponding affirmative propositions. In the case the law of double negation fails they belong to non-classical logic, as first, intuitionist logic. Through a comparative analysis of the theories including them the main features of a new kind of theoretical organization governed by intuitionist logic are obtained. Its arguing proceeds through doubly negated propositions and *ad absurdum* arguments. Then, the final doubly negated predicate is translated in the corresponding affirmative one, from which all the consequences are deduced in order to test them against reality. According to this model the true formalization of intuitionist logic is Kolmogorov’s 1932 paper. The above analysis also shows that modal words play an auxiliary or substitutive role to intuitionist propositions. In addition, it is shown that Vasiliev’s paraconsistent logic is represented through doubly negated propositions. By means of these three kinds of logic one can attribute a specific kind of logic to each of the three distinct steps of the process of building a scientific theory from the experimental data to its final theoretical system. It is also shown that the definition of negation is of a structural kind, rather than of an objective kind or a subjective kind: it has to be referred to the organization of the theory to which it belongs.

**Keywords**: Doubly negated propositions, Intuitionist logic, Alternative organization to the axiomatic one, Kolmogorov’s 1932 paper, Structural foundation of logic, Paraconsistent logic, Structural definition of negation

**1. The relevance of doubly negated propositions**

Surprisingly, the linguistic studies on double negation are *very* *few in number* with respect to the plethora of studies on a single negation. One reason is that surely, natural languages make use of doubly negated propositions in an unclear way. Moreover, most studies on double negations, by assuming speaker’s subjective viewpoint characterize this kind of linguistic figure as an understatement, or a unfrankly speaking; moreover, most scholars analyzing a double negation incorrectly focus the attention on the former negative word of a double negations and then, in a subordinate way, the negative, following word.

In last decades, Laurence Horn devoted several important papers to this kind of linguistic figure. He remarked that almost all scholars of the English language had a “dogma”, i.e. to avoid any double negation (evaluated as “a Latinate phenomenon”); because they considered as “immature” many languages Spanish, Italian, German, Russian, etc. (Horn 2002, pp. 79ff). Even Jaakko Hintikka neglected the double negation. (Hintikka 1968, p. 47)

Together with a minority of linguists, Horn considered three main cases; *i*) two negations are “in contradictory opposition”, i.e. they apply the Law of double negation, which pertains to classical logic; in such a case “Two negations affirm,” as it is commonly said; *ii*) the two negations are “in contrary opposition”, so that the law of double negation fails (in the following, the negative words will be underlined): “Two negations do not affirm”; for instance: “It is not impossible” ≠ “It is possible”; “A not un happy marriage” ≠ “A happy marriage”; *iii*) the latter negation reinforces the former one for adding psychological emphasis; e.g. “I cannot go no further”; “I cannot get no satisfaction”.

Within a literary text the occurrence of the law of the excluded middle is rarely manifested (by dilemmas); hence, almost never we can reveal through this law what kind of logic an author assumes. Instead it can be easily manifested by recognizing in the text all propositions each including two negations, and then by examining whether the meaning of each doubly negated proposition is different from that of the corresponding affirmative proposition because the latter one lacks of evidence, i.e. it is idealistic in nature (DNP). For instances of ancient DNPs, let us recall Hippocrates’ maxim: “First, do not harm” (≠ “Do the good”); or Decalog’s proposition: “Thou do not kill” (≠ “Thou save life”); or the double nature of Christ as affirmed by the Chalcedonian Council (451): “without mutation and without confusion”, “without separation and without division”; “Not-Other” as Cusanus’ best name for God (1462).

Although many Anglo-Saxon linguists condemn their use, DNP are easily recognised in relevant texts of the so-called “negative literature” (i.e. theology, ethics and epistemology), as the next point 7) will show.

Previous my investigations on the occurrences of DNPs within original texts suggested the following rules for recognizing them:

1) By definition of a DNP one has to exclude doubly negated propositions concerning facts of which we have evidence (e.g. “I have nothing else than 10$”, which corresponds to the proposition obtained by applying to it the LDN: “I have 10$”, a fact which is easily verified); i.e. the rhetoric or psychological use of doubly negated propositions.

2) A single word may be composed by two negations; e.g. in-nocent (≠ righteous).

3) The word “only”, being equivalent to “nothing else”, works as a DNP.

4) Modal words “must”, “possible”, “necessary”, etc. work as DNPs, owing to the S4 translation between modal logic and intuitionist logic. (Hughes, Cresswell 1996, pp. 224ff.). These words will be dotted underlined.

5) The words of a comparison as “more... than...” or “less... than...” work as a DNP, because this comparison is not equivalent to “equal”.

6) An interrogative negative proposition, whose answer is implicitly intended as a negative one, works as a DNP. E.g. “Am I stupid ?”

7) A proposition may include in an implicit way a second negation; which may be recognized by understanding the entire discourse: Court's judgment: “Acquitted for insufficient evidence [of guilty]”; Karl Popper: “Science is fallible [owing to negative experimental results]”; Hans Jonas: “The ethics of the fear [of mankind’s suicide]”; hence, in order to discover the complete meaning of a negative proposition sometimes one has to refer to the context which may add an implicit second negation.

8) The negative character of a word may depend in another way on the context; for example, in science the words “change”, “different”, “variation”, etc. all work as negative words, because each of them represents a problem which a theoretical scientist has to explain.

**2. A DNPs as sharp divide between classical logic and non-classical kinds of logic**

In the second half of 20th Century scholars of mathematical logic privileged the double negation law with respect to the excluded middle law in the role of differentiating classical logic from most kinds of non- classical logic, first of all intuitionist logic, and then the minimal one, the positive one, etc. (Prawitz and Melmnaas 1968; Prawitz 1976)

Let us recall that at the beginning of modern science Galileo Galilei claimed that a scientific proposition has to be supported by experimental evidence. Yet later the birth of infinitesimal analysis plus Newton’s idealistic notions (absolute space and time, his version of the inertia principle, etc.) introduced into science some idealizations with a partial experimental support or even without it; their marvelous results led scientists to allow theories including idealistic notions, provided that at last an anchorage to experimental evidence was preserved. Against this attitude, in last century inside both Logic and Mathematics respectively Intuitionists (Brouwer 1975, Dummett 1977) and Constructivists (Bishop 1967, pp. 1-10) claimed a requirement, i.e. a proposition is true if it is supported by factual evidence; in the case a proposition does not satisfy such a requirement and there is no evidence of its falsity, it has to be replaced by its doubly negated proposition according to the “negative translation” (Troelstra and van Dalen 1988, pp. 56ff.).

**3. The relevance of DNPs in the theoretical arguments of scientific texts**

Also several scientific theories have been presented by their respective authors through texts containing numerous DNPs. They are the following ones: classical chemistry, S. Carnot thermodynamics, L. Carnot’s mechanics, Lagrange’s mechanics, Avogadro’s atomic theory, S. Carnot’s thermodynamics, Lobachevsky’s theory of parallel lines, Galois’ theory, Einstein’s 1905 paper on special relativity, Einstein’s 1905 paper on quanta, Kolmogorov’s minimal logic, Markov’s theory of constructive numbers, Computer science. (Drago 2012) Let us examine the most representative ones; we will see that *their original presentations* rely in an essential way on DNPs.

*3.1 DNPs in Lavoisier’s Chemistry (1789)*

At the end of the eighteenth Century a number of a new kind of scientific theories born. Let us consider the first theory which exited out Newtonian paradigm, Antoine-Laurent Lavoisier’s chemistry (1789). In particular; he put aside Isaac Newton’s suggestion of gravitational force as representing inter-molecular interactions too, because considered as the universal force. Rather, he suggested building a new theory through a strict experimental method.

Already at the early times of such a theory some characteristic propositions of Chemistry were DNPs. (Drago and Oliva 1990) It is manifest why: chemists had to speak about unknown objects, atoms and molecules, through propositions which at that time lacked of direct experimental evidence of them. In the beginning of his well-known dialogue Robert Boyle suggested four basic propositions; three out of them are DNPs: "It not seems absurd..."; "It is not even impossible..."; "I will not deny peremptorily..."

.Having disproved the old division of matter in four elements, (air, water, earth and fire), Lavoisier put a great problem, i.e. whether matter is divisible at a finite end or not. He applied the following methodological principle: "A mathematical divisibility *ad infinitum* does not apply to the matter of which the world is made". Let us remark that the proposition: "A mathematical division *ad finitum* applies to the matter" did not hold true because at that time no one could decide by experimental means at what finite extent matter is divisible.

Moreover, let us consider the famous Lavoisier’s definition of a chemical element: "We will call an element… what we cannot yet decompose";(Lavoisier 1862-92, p. 7) it is a DNP, because the terminating end of this decomposition process was unknown, hence an element could not be defined as a determined “simple” component of matter.

In addition, Lavoisier suggested a new method for this chemical research. It relied upon a principle concerning the weights of substances involved in a chemical reaction. First, he stated it as a DNP and then as a possibility (which means: « It is not the case that it is not… », i.e. one more DNP): "Nothing is created [= not transformed], [...] a possible principle is that in each [chemical] operation the quantity of matter before and after the operation is the same". (Lavoisier 1862-1892, p. 15) Notice that the content of the former proposition was not assured, because experimental evidence for stating it was not enough; being aware of that, Lavoisier correctly qualified this proposition as a "possible" principle. It is well-known that for a long time this way of theorizing was fiercely opposed by those chemists that considered theoretical chemistry as decidable from the “assured” notions and principles of Newtonian mechanics.

However, a century after Lavoisier, Mendeleev eventually acquired a proper way of completing the theory through his periodic table, just by reasoning through DNPs, in particular the word “equivalent” (= not different) for representing the relations between the various elements under a chemical aspect or a physical one.(Drago 2014) But the reaction of scientists to this theoretical novelty, including so many features divergent from the Newtonian model of a theory, was to separate chemistry from theoretical physics (commonly represented by mainly Newtonian mechanics).

*3.2 DNP within the inertia principle of Lazare Carnot’s mechanics (1803)*

Let us consider the inertia principle that constitutes the principle differentiating modern science from ancient science.[[1]](#footnote-1) In Lazare Carnot’s mechanics (1803) it is the first of seven “hypotheses” (notice that through the word “hypothesis” he does not attributes an absolute validity to these principles).

*First hypothesis*: A body, once put in a state of rest, would not be able by itself [= if not by others] to leave that state, and, once set in motion, would not be able by itself [if not by others] to change either its speed or its direction. (L. Carnot 1803, p. 49)

(where the words “leave” and "change" have to be considered negative words because oblige a theoretical physicist to produce an explanation on why these changes occur).

This double negation is not merely a way of speaking, without importance for physics, because, unlike figures of speech, is not equivalent to an affirmative proposition enjoying an experimental meaning; if the two negations are removed we have an abstract word, with no physical meaning; surprisingly enough, we obtain exactly the affirmative word of Newton's version of this principle based on the verb "perseveres" or "continues"; both these words have animistic (or ethical) meanings, not an experimental one. Moreover, L. Carnot’s theory was based upon the problem of looking for “the in-altered quantities during a collision” (L. Carnot 1783, pp. 18 and 43), solved by means of spatial symmetries and the principle of virtual works (L. Carnot 1803, p. x). Here we see the great logical distance between two distinct types of theorization of Newton’s and L. Carnot’s modern mechanics.[[2]](#footnote-2)

But later L. Carnot’s theory was ignored because misinterpreted as the foundation of no more than technical physics, an engineers’ subject.

*3.3 DNPs in Sadi Carnot’s thermodynamics (1824)*

In order to found a thermodynamic theory Sadi Carnot put the problem of the maximum in the heat/work conversions. In order to solve it his booklet suggested a new scientific method according to which one argues in a surprising way, which was illustrated in details by the paper (Drago, Pisano 2000).

Here he assumed no axiom (not even an axiom on the nature of heat, which implicitly was the caloric), but only the common knowledge about heat phenomena. The numerous DNPs of the theoretical part of his book compose seven *ad absurdum* arguments (AAAs); one of them is his well-known theorem about the maximum efficiency in allheat/work conversions. The conclusion is a doubly negated predicate:

no change of temperature inside the bodies employed for obtaining the motrice power of heat occurs without a change in the volume”. (S. Carnot 1924, p. 23)

In the subsequent part of the theory the author makes use of the corresponding affirmative proposition as a hypothesis (Let us call PSR this logical step). From this hypothesis the author draws in a deductive way all consequences and then tests them with respect the gas behaviour and the efficiencies of his contemporary heat engines. Present thermodynamic textbooks substantially repeat the same argument.

Surprisingly, *via* (the rejection of) an absurdity he generated concrete knowledge on the behaviour of material machines. This way of arguing is surely different from a deductive one, deriving strict consequences from *a priori* principles; it is rather another manifestation of the power of the logical arguing, yet in an inductive way.

 His work resulted as “too difficult” to his scientific friends (Robelin 1832). Later, it was almost ignored along 25 years. Eventually, Rudolf Clausius and Lord Kelvin re-formulated his theory by reducing the original S. Carnot’s way of reasoning to only his unavoidable AAA.

*3.4 DNPs in Lobachevsky’s theory of parallel lines (1840)*

Let us consider a clear instance of a theory presenting numerous DNPs, NikolaiLobachevsky’s theory of parallel lines. His main text (Lobachevsky 1840) puts the fundamental problem of how much parallel lines through a point outside a given straight line exist.

The first 22 propositions and the theorems proving them include numerous DNPs, which have been listed and analyzed by the paper (Bazhanov and Drago 2010). After fifteen preliminary propositions concerning notions and theorems of common knowledge, the proposition no. 16 presents the problem of how many are the parallels lines. It is expressed by the following words: “… in the ignorance whether the parallel lines are not only one…” (Lobachevsky 1840)**.** Then he explores the case of two parallels lines. In order to obtain evidence for this guess, first he suggests a new, more general definition of parallelism than the common one; this definition is then corroborated by two theorems proving that the new definition enjoys two well-known properties of traditional parallel lines. Then he proves four theorems, all AAAs except one.

A crucial step occurs at the end of the proposition no. 22, which is proved by an AAA. Having obtained equivalent evidence to that of Euclid’s hypothesis in allpoints and figures of the space, he concludes that his supposition of two parallel lines “can likewise be admitted without leading to any contradiction in the results”. This proposition is a DNP since it is not equivalent to: “the theory derived from this supposition is consistent”, whose proof was beyond his capability). Incidentally, in all his writings about non-Euclidean geometries Lobachevsky never stated “There exist two parallel lines”; instead he always wrote a DNP, like the previous one.

After the proposition no. 22 he then derives consequences from exactly the corresponding affirmative proposition of previous DNP; that means that through PSR he has implicitly changed it into an affirmative proposition. This proposition represents the hypothesis of two parallel lines, from which one can deduce consequences to be tested with reality. From this new axiom he deductively derives fourteen theorems of his non-Euclidean geometry, i.e. what we know as the hyperbolic geometry. (Bazhanov and Drago 2010)

Lobachevsky never considered as proven his new geometry. He looked for an astronomical confirmation, yet he was unsuccessful. Moreover, he had no decisive mathematical proof of the truth of his geometry, but only an analogy: the translation of the Euclidean spherical trigonometry into the hyperbolic spherical trigonometry through the mere change of the real number representing an angle ** into the imaginary number *i*. For this reason he called his geometry “imaginary”, a word to be meant in the same sense of the imaginary numbers; i.e. numbers which are not real, yet through them it is possible to obtain results about real numbers.

Later, after his death Lobachevsky’s geometrical work was slowly taken into account; however it was characterized by most as “metageometry”, a word which in that time was similar to “metaphysics”. Eventually Lobachevsky’s work was accepted, but only its deductive part, i.e. the part after proposition no. 22.

*3.5 DNPs in Kolmogorov’s 1924/25 paper*

In this paper Andrej Kolmogorov wanted to solve a basic problem of the debate between intuitionists and formalists:

Brouwer's writings have revealed that it is illegitimate to use the principle of the excluded middle [≠ it is legitimate the principle of included middle] in the domain of transfinite arguments. Our task here will be to explain why this illegitimate use [of the principle of the excluded middle] has not yet led to contradictions [≠ has led to compatible conclusions] and also why the above illegitimacy will often gone unnoticed [≠ legitimacy is noticed].

Only [= Nothing else] the finitary conclusions of mathematics can have significance in applications. But the transfinite arguments are often used to provide a foundation for finitary conclusions. Brouwer considers, therefore, that even those who are interested only [= nothing else] in the finitary results of mathematics cannot ignore [≠ know] the intuitionistic critique [= rejection] of the principle of excluded middle [≠ acceptation of the included middle].

We shall prove that all the finitary conclusions obtained by means of a transfinite use of the principle of excluded middle [≠ the finite use of the principle of included middle] are correct and can be proved even without its help.

The natural question [= It is unknown] is whether the transfinite premises that are used to obtain correct finitary conclusions have any meaning [≠ it is known that finite premises… have meaning] (Kolmogorov 1924/25, p. 416)

One easily verifies that the thread of the logical arguing is preserved by the list of the above DNS's, if read in sequence. (One obtains the same result in the last part of the paper: the mere list of DNS's is enough for representing the logical core of the paper).

Remarkably, he made use of a rigorous way of arguing by defining a language composed by “pseudo-truths”; each of them is defined as "a judgment asserting its double negation statement", (Kolmogorov 1924/25, p. 416) i.e. "It not true that it is not…” that is exactly a DNP.

Subsequently, Kolmogorov in an essential way argues through pseudotruths. Thus, the entire paper proceeds by arguing just through DNPs. In such a way he obtains a theorem of universal validity (there exists no inconsistency in the use of LEM in finite sets); his conclusion holds true for all possible implications. After that, an intuitionist scholar is allowed to make use of LEM in a finite context. (Drago 2005)

Yet along some years this objective way to deal with the foundations of logic has remained largely unknown outside the USSR, even to Arendt Heyting.

**4. The relationships among intuitionist logic, modal logic and Vasiliev’s paraconsistent logic through the DNPs**

Through the DNPs of intuitionist logic we can investigate also some other kinds of logic. In particular, we are now in a position to explain the relationships among the kinds of logic underlying theoretical research in science.

An analysis of above texts about the occurrences of modal words shows that these words play a substitutive or auxiliary role to intuitionist logic. Let us first recall that an affirmative proposition including a modal word, as e.g. “must”, “possible”, “necessary”, etc., works as DNPs, owing to the S4 translation between modal logic and intuitionist logic. (Hughes and Cresswell 1996, pp. 224ff.). Moreover, in this case of a proposition including a modal word e.g. “possible”, one can replace it by the corresponding DNP, i.e. “It is not true that it is not the case…” Actually, a modal word is more agreeable and more directly comprehensible than a turn of two negative words, whose contents moreover are not those determined by the classical negations, because the negations of these words are not the exactly contrary of the positive words.

Moreover it is currently written: “It is im-possible a motion without an end”, although it is enough to write: “It is absurd a motion without an end”, or equivalently “A motion without an end does not exist”. In sum, often linguistic reasons attribute to modal logic an auxiliary or substitutive role to intuitionist logic: either a modal word strengthens the nature of a proposition as a DNP, or facilitates a quick comprehension of its content.

The relevance of the DNPs for the foundations of other kinds of logic is manifested by one more result concerning the foundations of an extreme kind of logic, i.e. paraconsistent logic as it was intended by its founder, Nikolai Vasiliev. A previous paper (Drago 2001) showed that this logic can be interpreted according to a doubly negative translation of the three characteristic Vasiliev's propositions: "S is A", "S is not A", "S is and is not A", ("indifferent judgement"). (Kline 1965, p. 316) One can consistently substitute for the word "is" the implication symbol →, and for the proposition A a DNP, i.e. the proposition ¬¬S. One obtains respectively: "*A*  *A*", "*A fails to*  *A*", "*A A and A fails to A*". The structure of Vasiliev's logic follows. Moreover, this substitution agrees with da Costa and Puga's formalisation of Vasiliev's paraconsistent logic.(da Costa and Puga, 1988) Incidentally, this fact shows that the DNPs are relevant also to a formalized language.

The latter three propositions can be interpreted as the three versions of a scientific principle: *i*) the first Vasiliev’s proposition as either an experimental proposition or an axiom-principle - whose nature may be also idealistic one -, belonging to a deductive organization of the theory, *ii*) the second proposition as a methodological, heuristic principle addressing to search for discovering a new scientific method, *iii*) the third proposition as a guess for a principle, whose role is not provisionally decided, since it is insufficiently supported by either empirical or/and theoretical evidence. In other terms, a theoretical scientist may consider a proposition as *i*) either an affirmative, even idealistic axiom, as it occurs in an AO theory; *ii*) or a methodological principle, i.e. a DNP, as it occurs in one of the above listed theories of the beginnings of sect. 3; *iii*) or a theoretical proposition without theoretical qualification with respect to an envisaged future theory. Hence, Vasiliev's logic tried to represents propositions inside all the three theoretical contexts, i.e., both kinds of organizations and moreover the informal context lacking a theoretical organization.

These three versions represent, yet in reverse order, the three steps of the process of a theory building: a guess, the adoption of it as a methodological principle for a research, and, after having completed the theory as an axiomatic organization, a principle-axiom, irrespectively of its idealization with respect to reality.

**5. From the DNPs the discovery of the alternative model of theoretical organization to the axiomatic one**

Along two millennia a clear presentation of a scientific theory required a systematic organization of a deductive kind from few axioms.

But this kind of organization cannot include a DNP. Indeed, a DNP cannot play the role of an axiom, because its content cannot be stated with certainty, except for stating a bound to our thinking (in set theory a DNP corresponds to Stone representation of intuitionist logic by means of open sets). In logical terms, being the contentof ¬¬*A* different from the content of *A,* no affirmative proposition can be derived from it*.* On the other hand, it is impossible to draw a DNP from an axiom, which, being an affirmative proposition, is equivalent to its doubly negated proposition; hence, the classical consequences of an affirmative axiom all are propositions of only classical nature. In sum, even one DNP obstructs a deductive organization of a theory.

Then, no theoretical organization? Most scholars seem maintain this opinion about the “negative” thinking. Instead, a comparative analysis on the texts of the above-mentioned theories shows the well-defined characteristic features of a common model of theory organization.

*i*) These theories presuppose no more than the common knowledge on the field at issue.

*ii*) They put a problem which is unsolvable through usual tools; eg, in Lavoisier’s chemistry: what are the elements of the matter; in Lazare Carnot’s mechanics: what are the invariants in a collision of bodies; in S. Carnot’s thermodynamics: what is the maximum efficiency of the heat/work transformations; in Lobachevsky’s geometry: how many parallel line exist; in Kolmogorov’s above-mentioned paper: why has the illegitimacy of the use of the law of excluded middle gone unnoticed.

*iii*) They are aimed at inventing a new method for solving the previously given problem.

*iv*) In order to solve the given problem the author argues by linking together the DNPs in order to constitute - through words as “otherwise we obtain an absurd result” - an AAA; which is of a weak kind, i.e. its final proposition is a DNP (notice that a common opinion holds that an *ad absurdum* argument concludes an affirmative proposition because it applies to its conclusion, a DNP, the double negation law, which however pertains to only classical logic). This DNP may work as a premise for a next AAA; hence, the AAAs may constitute a chain of AAAs.

*v*) The final AAA concludes a DNP suggesting a universal predicate, ¬¬*UT*, which represents a possible resolution for all cases of the starting problem.

*v*) At this point, an author of such a theory as a matter of fact performs the step PSR, i.e. he translates the above predicate¬¬*UT* into the corresponding affirmative predicate *UT* and the he considers this affirmative proposition as a valid hypothesis for resolving his problem. One can suppose that the author thinks to have already collected enough evidence to be justified in promoting his conclusion ¬¬*UT* to the corresponding affirmative proposition *UT*, although this change is not allowed by non-classical logic, which previously he adhered to.

*vi*) After this step the author assumes this proposition as a new axiom, from which all possible derivations by means of non-classical are drawn, to be subsequently tested against reality. (Drago 2012)

The logical step which in the above was denoted as PSR is explicitly manifested by three eminent scientists. Galilei:

Let us take it as a *postulate*, whose absolute truth will be stated [as a *methodological principle*] by seeing that further conclusions, built on this hypothesis, perfectly fit the [entire field of] experience. Having admitted [PSR] this… *[axiom-] principle*, the Author moves to consider the propositions, which are deductively concluded [from it]…” (Galilei 1958, p. 191; emphasis added).

Let's put aside that we would not have expected from the champion of experimentalism, Galilei, i.e., an use of the words "absolute truth"; here he is dealing with a postulate which, if considered as a hypothesis, gives valid conclusions; that is, it is a methodological principle. In the final proposition we discover that his hypothesis is changed into the concept of "principle"; which here is understood by Galilei as an axiom. So this period indicates a progressive intellectual dynamics represented by the sequence of three concepts: hypothesis, postulate (or methodological principle) and axiom principle. This dynamics corresponds to what a physicist usually does; he questions the experience on the basis of a hypothesis suggested by his intuition, then exploits it as a methodological principle, addressing his research to find out a systematic theory of the field of phenomena at issue; this research eventually leads him to translate this principle into a certain, affirmative principle of a theory, that then is explored for deductively developing a theory.

Also Lobachevsky verbally manifested this same translation, here concerning a mathematical subject:

[My *supposition* of two parallel lines] can likewise be *admitted* [as a *methodological principle*] without leading to any contradiction in the [previous] results and [PSR, as an *axiom*] founds a new geometry….” (Lobachevsky 1840, prop. 22; emphasis added).

Albert Einstein repeated similar words when, in the second page of his celebrated paper founding special relativity, he anticipated what he will establish.

We will raise the [informal] *conjecture* (the substance of which will be hereafter called [PSR] the "[*axiom-*] *principle* of relativity") to the state of a [plausible] *postulate*". (Einstein 1905, p. 891; emphasis added)

I conclude that there exists evidence of a scientific nature for an application of the logical step PSR to the conclusion of (the sequence of DNPs and AAAs of) a PO theory.

*vii*) Let us examine this step, PSR, in formal terms of the square of opposition. It corresponds to a change of the main thesis A (“S is P”) from its non-classical version (“not (S is not P)”) to the classical one. Through Dummett’s table (Dummett 1977, p. 29) it is apparent that previous translation is enough for changing the entire intuitionist predicate logic into the classical one (e.g. the similar change of thesis E is obtained by mere negation of thesis A; the change of thesis I is obtained by doubly negating thesis A).

This step apparently represents the step PSR, i.e. an application of Leibniz’ principle of sufficient reason; its antecedent is itself a universal DNP (“Nothing is without reason”) and the consequent is the corresponding affirmative proposition (“Everything has a reason”). An author of a theory performs on a specific predicate of this theory a logical translation which formally is the same translation performed by the application of this general principle, i.e. the logical translation occurring from the antecedent of Leibniz’ principle of sufficient reason to its consequent:

 *¬∃x ¬P(x) => ∀x P(x).*

Notice that the translation performed by this principle merely constitutes the inverse translation of the so-called ‘negative translation’. Yet, whereas the latter one can be always performed under some rules on the way to add the two negations, the application of PSR is valid, as Markov stated, under two requirements on the doubly negated predicate on which it is applied: it has to be 1) derived from an AAA and 2) decidable (Markov 1961, p. 5; Drago 2012) for the simple reason that for passing from a hypothetical world to a real word we have to be supported by reality criteria.

In sum, the sense of a PO theory is the following one: the conclusion of the final AAA suggests a surmise; but a surmise, being a DNP, cannot be accurately tested with reality; instead, the application of PSR translates this conclusion into an affirmation which may be tested. This move also changes the theoretical development, previously based upon non-classical logic, into a deductive development based upon classical logic. Then, an AO theory is developed in the aim of testing all the possible derivations from the new principle with reality. If the answers obtained are positive, the entire theory becomes effective; in sum, what was hypothetical is changed into a real theory.

I conclude that there exists a model of organization of a theory which is governed by intuitionist logic and therefore is radically different from the deductive model.[[3]](#footnote-3) I call it the model of a problem-based organization (PO). Being shared by several scientific theories playing prominent roles within the history of science, this new model has to be put on the same par of the deductive model. The discovery of this new kind of model substantiates the idea that the DNPs severs two entirely different (scientific) worlds, i.e. a world of inductive searching for discovering new methods for solving basic problems from a world of assured truths as those drawn from assured principles-axioms. (Drago 2007)

**6. The DNPs as a general divide of both the logical values of a proposition and the two logico-philosophical principles of a scientific theory**

Let us complete the review of the divides generated by DNPs in both the bottom and the top of scientific theories.

Let us notice that in order to analyze the logical value of a proposition within a general setting one cannot presuppose *a priori* – as most textbooks of logic do – that its value is only either true or false, since this assumption (the dichotomy law) leads directly to classical logic, the only logic which assumes that the truth of every proposition is sharply decided. Instead, within a general setting allowing also non-classical logic, one first of all has to examine the proposition at issue for recognizing the validity or not of the law of double negation, since this law constitutes the borderline between classical logic and non-classical logic. Only after having chosen one of the two specific kinds of logic, correctly a scholar investigates the logical value of the given proposition.

Moreover, let us remark that as a consequence of the existence of two models of theoretical organization, a single proposition may play very different roles: *i*) an affirmative proposition plays the role of a proposition which is true inside classical logic governing an axiomatic theory, but not inside non-classical logic, where an affirmative proposition is only partially true (being true only the corresponding DNP); *ii*) the same holds true for a negative proposition; *iii*) a doubly negated proposition may play the same role of an affirmative proposition if it belongs to classical logic; instead in non-classical logic it plays the role of a DNP. Hence, a correct analysis of a single proposition has to refer to the kind of logic which it belongs to; even more a correct analysis of the relation of two propositions, e.g. an inference, requires the same detailed inspection.[[4]](#footnote-4) Furthermore, the DNP of the PSR represents one of the two possible translations: between the intuitionist logic to the classical one, instead of the ‘negative translation’ Kolmogorov-Goedel-Kuroda, i.e. the translation of classical logic into the intuitionist logic; these two translations constitute a logical divide.

It is well-known that the general logico-philosophical principle governing classical logic is the non contradiction principle; instead, in intuitionist logic the opposition of a proposition *P* to its negative one, *¬P,* does not give a contradiction; from the previous section we learnt that in intuitionist logic a DNP is the relevant proposition and it at last refers to PSR.

These two logico-philosophical principles are mutually antagonist; the validity or not of the LDN mutually separates these two principles. As a consequence, also a kind of logic cannot be correctly interpreted without referring it to one of the two philosophical principles and in correspondence the two organizations of a theory, AO and PO.

**7. A structural foundation of intuitionist logic through the DNPs of Kolmogorov’s 1932 paper**

A question arise: what is the correct foundation of intuitionist logic? In order to answer the question we have o refer to the two intuitionist basic choices of its origins, i.e. the rejection of actual infinity (AI) for rather relying on no more than the potential infinity (PI) and the distrust in both the axiomatic method and the axioms of classical logic (Brouwer 1908). This distrust may be considered as an obscure intuition of a new systematic organization of a theory; actually, Brouwer suggested to systematically extracting all logical features from the reflections on all previous mathematical (and scientific) systems:

in arguments concerning empirical facts spanned upon mathematical systems, the logical principles are not directories, but regularities discovered afterwards in the accompanying language.(Brouwer 1908, p. 108)

The above comparison of some scientific theories proves that Brouwer’s kind of investigation effectively suggests new structures (i.e. the PO theoretical organization and its logical features) to the intuitionist approach.

In the year 1930 Arendt Heyting produced an axiomatic of intuitionist logic. It was commonly considered the prominent novelty concerning intuitionist logic because the choice of the axiomatic organization (AO) for founding intuitionist logic pertained to the opposite program, Hilbert’s one. In the following I will be prove that instead it is the logical theory of Kolmogorov’s calculus of problems (Kolmogorov 1932) that is a foundation of intuitionist logic in agreement with the couple of choices of the original intuitionist program. His paper is analyzed in details by (Drago 2021).

*Kolmogorov’s rejection of all idealistic notions and rather choosing potential infinity (PI)* is manifested when he declares to be addressing his search to “the *concrete* areas of mathematics". (p. 329; my Italic) It is reiterated by the following words: “*Any proposition that is not without content should refer to one or more* ***completely determinate states of affairs accessible to our experience***….” (p. 332; my emphasis in bold).

Kolmogorov’s choice for PO, or equivalently the intuitionist way of arguing is recognized through two facts:

1) He neatly distinguishes from Heyting’s axiomatic system his objective calculus of problems performed through “rules” (pp. 328, 330, 331, fn. 6a). He underlines this point in fn.s no.s 6a-9.

One can introduce a corresponding symbolism [to these rules] and give the formal computational rules for the symbolical construction of the system… Thus in addition to theoretical logic one obtains a new *calculus of problems*. (p. 328)

2) A detailed analysis of the development of his paper shows the following features, which are the typical ones of a PO theory.

*i*) As in all PO theories, the background knowledge which is demanded to a reader of the paper is *the common knowledge*: “We do not define what a *problem* is; rather we explain this by simple examples.” (p. 328) Moreover, “we must assume that we have already solved the following two groups of problems *A* and *B*.”(p. 330) (which usually are instead intended as axioms).

*ii*) Again in the first page of his paper he presents a *crucial problem*: *a*) to “systematize the schemata of the solution of problems, for example, of geometrical construction problems.” (p. 328); in other words, to introduce “… a new *calculus of problems*.” (p. 328). In its turn, the resolution of this crucial problem is aimed to solve the general problem: *b*) to show that

*The calculus of problems is formally identical with the Brouwerian intuitionist logic... recently… formalized by Mr. Heyting…* (p. 328)

This proposition means that his problem was how to give one more formalization of this logic by means of a calculus of problems. But he also writes:

It will be shown that intuitionist logic should be replaced by the calculus of problems, for its objects are in reality not theoretical [read: idealistic] propositions, but rather problems. (p 328)

That means that his chief problem is to equip intuitionist logic with an objective semantics, in alternative to both Brouwer’s subjective basis and Heyting’s formal axiomatic.

*iii*) He states a *methodological principle*, i.e. a principle orienting his research. It is by him manifested in the second section of his paper; there he presupposes that the reader accepts “the intuitionistic epistemological assumptions…” (p. 328) According to Kolmogorov

The basic principle of the intuitionist critique of logical and mathematical theories is the following one: *Any proposition that is not without content should refer to one or more completely determinate states of affairs accessible to our experience*…. (p. 332);

Manifestly, it properly represents, rather than a critical attitude, the basic *methodological* principle for building intuitionist logic.

*iv*) The text includes twenty *DNP*s, which for brevity’s sake I omit. It would be easy to show that the mere sequence of these DNPs captures all the basic topics of the paper.

*v*) His arguing includes four AAAs. The first one shows that LEM “cannot be found on the [infinite] list of problems solved by [the reader]", otherwise a mathematician would "be omniscient", (p. 332) i.e. a clear absurd. From this AAA he obtains that in his calculus the law of excluded middle fails. A second AAA concerns the double negation law: this law

“cannot appear in our calculus of problems, for [read: otherwise]… the [non-intuitionist] formula (1) follows from it by means of 4.8.” (p. 332)

A third AAA is given for showing that when a classical propositional formula is false "the corresponding problem├ *p* cannot be solved":

Let us also remark that if a formula ├ *p* is false in the classical propositional calculus, the corresponding problem ├ *p* cannot be solved. Actually [read: Otherwise], from the proof of such a formula ├ p one can infer, by means of the previously assumed formulas and rules of computation of calculus of problems, the obviously contradictory formula [├ *p* & ├ ⌐*p*, hence] ├. ⌐*a*. (p. 332)

In my opinion, this proof is rather obscure; but it concerns the definition of negation, a subject which we will deal with in next section. The fourth AAA, located in fn. no. 5, shows that an alternative definition of negation is inconsistent.

These AAAs cover all the important topics of intuitionist logic: respectively law of excluded middle, double negation law, existence predicate and negation. The first AAA excludes the validity of the law of excluded middle (DNP 5: “… the formula (1) cannot be found on the [potentially infinite] list of problems…”; p. 332); subsequently, this DNP works as a premise to the second AAA, excluding in its turn the double negation law. Here we have an instance of a (short) chain AAAs. This fact shows that *there exists a specific way of reasoning within intuitionist logic; it is composed by a sequence of AAAs*.

*vi*) Let us now consider the main problem tackled by the paper, i.e. to formally define a calculus of problems. His research is concluded by the proposition:

On the basis of the previously assumed postulates, one convinces oneself easily that these formal computations actually guarantee the solutions of the corresponding problems. (p. 331)

By including modal words (“convinces”, “easily”, “guarantee”) this proposition actually is a DNP. Its content is to appeal to a *subjective* feeling of both the author and the reader (“one convinces oneself easily…”); this feeling improperly replaces what the model of a PO requires, i.e. a doubly negated predicate resulting from an AAA proving the completeness of his calculus. However, this Kolmogorov’s DNP prepares an application of PSR on the result of his previous theoretical development. This application is not manifested, but, as a fact, just after one recognizes the result of its application:

all the formal rules of computation and a priori written formulas coincide with the rules of computation and axioms of Heyting’s first essay (p. 331)

This conclusion was also anticipated at the beginning of his paper, where he declared his main result:

*The calculus of problems is formally identical with the Brouwerian intuitionist logic, which has recently been formalized by Mr. Heyting* (p. 328).

In both cases he makes use of affirmative words (“coincide”, “is formally identical”), which are appropriate for representing the result of an application of PSR on a doubly negated predicate.

All in all, we see that Kolmogorov has applied the model of a PO but not entirely, because a final AAA and hence a DNP as final predicate are lacking; PSR is applied, but in an intuitive way. However, he obtains as result a correct affirmative proposition.

In sum, by a rational reconstruction of Kolmogorov’s paper we have obtained an almost completed foundation of intuitionist logic. It is based on the two choices PI and PO which are those of the original intuitionist program. Moreover, Kolmogorov’s formalization through DNPs, AAAs and the application of PSR represents the best theoretical organization of intuitionist logic.

In retrospect, we see that intuitionists’ persistence in developing a merely subjective viewpoint led them to disregard the global notion of the organization of an entire theory. Since Brouwer and his followers ignored the alternative theoretical organization, in the year 1930 the more faithful disciple of Brouwer, Heyting, made recourse to the theoretical organization of the opposite program, AO, notwithstanding this kind of foundation is extraneous to the original intuitionist program. It is not a case that Kolmogorov’s 1932 paper, closely approaching the model of a PO, remained almost ignored by the scholars whose dominant culture considered only the AO as possible.

In addition, Kolmogorov’s foundation goes beyond Brouwerian foundation based on the subjectivism of a personal thinking, as well as all foundations based on the objective meanings of crucial logical notions (eg inference, negation, etc.). It is a *structural* foundation because it refers to two basic choices which are the alternative ones to Hilbert’s, clearly AO and AI; i.e. it refers to the structure of the fundamental choices which have been emerged from the great debate on this subject in the first half of 20th Century.

Let us notice that Hilbert deserves the merit of having as first founded in a structural way, i.e. according to a particular couple of choices AI and AO, a kind of logic, classical logic. In addition, he deserves the merit of having declared, and as first having illustrated these choices. Yet, he illustrated them by denying other possible choice and hence any further structural foundation. (Hilbert 1925) Instead, in the above, we saw that the couple of alternative choices is possible; and, by implicitly relying on the alternative couple of choices Kolmogorov’s 1932 paper obtained a separate and independent foundation of one more kind of logic, intuitionist logic. In other words, by suggesting one more structural foundations of a kind of logic (the intuitionist one) Kolmogorov has unlocked the structural foundational work which as a fact Hilbert had started.

In addition, being both the alternative choices AO and PO, mutually incompatible, as well as the choices AI and PI, the different structural foundations of these kinds of logic result to be mutually incommensurable (also according to the original definition given by Feyerabend and independently Kuhn),. This incommensurability explains Hilbert’s obstruction (interpreted by Hans Freudenthal (1971, p. 393 I) as Hilbert’s incompetence of philosophy of science) to the alternatives to his choices. In other terms, Kolmogorov’s paper introduced an essentially pluralism of kinds of logic.

**8. The structural definition of logical negation through the DNPs**

As a consequence of the general divide operated by DNPs on the different kinds of logic, even the definition of a negation has to be referred to the theoretical organization which it belongs to. Within an AO governed classical logic all doubly negated propositions are exactly equated to the corresponding affirmations and hence negation is mirror opposite to the corresponding affirmation; instead, within a PO, governed by intuitionist logic, a DNP is true and hence both negation or affirmation are only partially true. Therefore the common basic notion of negation presents a radical variation in the value and hence also in the meaning. The mutual incommensurability of classical logic and non-classical logic gives reason why scholars wanting to define the whole meaning of a negation by only considering logical operations within a specific kind of logic (usually, classical logic) met insurmountable difficulties.(Horn and Wansing 2015)

But also inside intuitionist logic the question of the definition of negation is subtle; it represents the main basic problem of its philosophy. Brouwer suggested that a negative proposition exactly implies, through a specific construction, the absurdity.

Brouwer [from 1923a-c]… expressed negation as reasoning that leads to an absurdity, or, briefly, as an absurdity. (Franchella 1994, p. 258).[[5]](#footnote-5)

By remarking that this formalization implies to have in a preliminary way obtained the notion of negation in an AAA or in the law of contradiction, Kolmogorov (1924/25, §§3-6, pp. 420-422) distrusted in it. In his 1932 paper, after having stated the above quoted basic principle of intuitionist logic, he wrote:

If *a* is a general proposition of the form “any element of set K possesses the property A”, and if in addition the set K is infinite, then the negation of [a proposition] *a* “*a* is false” does not satisfy the above principle. In order to avoid this situation Brouwer gives a new definition of negation: “*a* is false” should mean ”*a* leads to a contradiction.” Thus [the problem of] the negation of *a* is transformed into [the problem of] an *existential proposition:* There exists a chain of logical inferences that, under the assumption of the correctness of *a*, leads to a contradiction. (Kolmogorov 1932, p. 332)

He then discussed how formalize a negation. Yet, in my opinion, his arguing is disputable, and in fact, it was forgotten. After a century, the long debate on this question is still unresolved (Sundholm 1994; Raatikainen 2004; Raatikainen 2013). The latter author distinguished three attitudes; *i*) strict actualism, i.e., to assume the existence of the contradiction proof in all cases; *ii*) possibilism, i.e. the proof is no more than possible and *iii*) an intermediated case of liberalized actualism. He concluded as follows:

We have examined the three basic choices there are for the intuitionist theory of truth, the strict actualism, the liberalized actualism and possibilism, and found all them wanting. (Raatikainen 2004, p. 143)

I suggest that the question is still unresolved because in the past it was scrutinized with reference to only the AO, without taking in account the PO. Let us consider under this new light the difference between classical logic and intuitionist logic about a negation. The former kind of logic considers a negation as implying in all cases absurdity, i.e. as suggesting the existence of a proof of its absurdity no matter of the idealistic nature of this existence; whereas, after Kolmogorov, the latter kind of logic is formalized according to the theoretical development of a PO theory, which in general suggests no more than hypotheses; hence, about a negation it suggests a mere hypothesis of the existence of a proof of the absurdity of negation; that is, in a PO theory not always this proof actually exists. So we have two methods obtaining a same notion, negation, but attributing to it two different meanings. According to the first method the proof is idealistically actual; in the second case the proof is no more than possible, because intuitionist logic is related to the use of the double negation law and PSR. In this latter case one achieves an affirmative conclusion by changing the kind of logic by means of an appeal to the human rationality (PSR). The same occurs about the existence of the proof related to a negation. Before the organization of the theory is changed by PSR, this proof is merely possible; and in order to the negation be affirmed as absurdity, it requires a constructive proof; instead, after the change, when the theory is governed by classical logic, the proof is always assumed as actual, albeit also an idealistic one.

In conclusion, the nature of the definition of negation is of a structural kind, rather than of an objective kind, or worst a subjective kind.

**Conclusions**

All in the above proves that the traditional view confining the DNPs of the original texts of scientific theories to only a rhetorical use obstructed past scholars to discover what Beth looked for: an alternative kind of organization of a theory, and in particular a better foundation of intuitionist logic than the foundation given by the intuitionist Heyting who had manifestly deviated from Brouwer’s program.

All in the above does not represents an enlargement of classical logic, but a logical divide, caused by the failure or not of the double negation law, of the general, universal body of logic into at least two kinds of equally relevant kinds of logic. This divide implies a deep division between the kind of single propositions as well as the highest logical and philosophical principles of logic. This situation represents an essential pluralism which is supported by the past four centuries of universally shared, objective reasoning on the most formal subjects of the scientific enterprise, i.e. scientific theories.

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1. As Norwood Hanson (Hanson 1963, p. 103) put it: “… the First Law of Motion - the "Law of Inertia" - this has everything a logician of science could look for. Understanding the complexities and perplexities of this fundamental mechanical statement is in itself to gain insight into what theoretical physics in general really is.” [↑](#footnote-ref-1)
2. Already Gottfried Leibniz stated that theoretical physics is based not on “necessary propositions”, but “contingent propositions”, i.e. on those whose “the contrary ones do not imply contradiction”, i.e. a DNP. His methodological priciple was “the impossibility of a perpetual (= without an end) motion”. His version of the inertia principle was “the in-difference of bodies to rest or motion”. His goal was to find out the “in-variants”. (McDonough 2019, sect. 2.3 (1)) Unfortunately, he did not achieve a completed theory of mechanics because he lacked of the mathematical formula of the principle of virtual works (which verbally is a DNP: “The work of virtual [= non real] works is not positive”]; it was stated one year after his death (1717). A century after Leibniz, this principle was the basic one of Lazare Carnot’s mechanics, which completed Leibniz’ program in mecahnics. [↑](#footnote-ref-2)
3. [↑](#footnote-ref-3)
4. Unfortunately, Immanuel Kant believed that classical logic was the only possible logic. He formulated eight ontological proofs, all *ad absurdum* proofs. He thought to have obtained contradictions between their conclusions; actually, he uniformely applied the double negation law to their conclusions of double negation of classical logic; instead the application of this law is appropriate to the metaphysical thinking, but not to the empirist thinking, whose AAA are the weak ones of non-classical logic. Instead of rejecting the conclusions of the above proofs as mutually contradictory, he rather could evidentiate a logical difference between two kinds of logic, and hence he coud accept the essential pluralism of two different kinds of logic. Let us consider another case of this belòief. Hilbert mantained that there exists two version of the principle of mathemaical inducton, an *inhaltiche* and a *formale*; but he was unable to clarify the definition of the former one. (van Hijenoort 1967, p. 481) Actually, he could refer it to non-classical logic, which yet he believed does not exist.

 In fact, both Henri Poincaré (Poincarè 1903, chp.VII) and Albert Einstein (Miller 1981, pp. 123-142) suggested that past physical theories presented two different kinds of organisation. But these suggestions have been ignored. Let us also recall that in philosophy of mathematics Wlliam Beth stressed that present science is biased by a unique model for the organization of a scientific theory, i.e. the deductive one, theorized by Aristotle and in modern times axiomatized by Hilbert. However, he did not present an alternative model of organization.(Beth 1959) The above analysis obtains in a formal way Beth’s goal. [↑](#footnote-ref-4)
5. The book (de Stigt 1990, pp. 238-270) offers a more detailed analysis of the subject. [↑](#footnote-ref-5)