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TWO REFORMULATIONS OF THE VERIFICATIONIST THESIS IN EPISTEMIC TEMPORAL LOGIC THAT AVOID FITCH'S PARADOX

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Abstract: 1) We will begin by offering a short introduction to Epistemic Logic and presenting Fitch's paradox in an epistemic-modal logic. (2) Then, we will proceed to presenting three Epistemic Temporal logical frameworks created by Hoshi (2009): TPAL (Temporal Public Announcement Logic), TAPAL (Temporal Arbitrary Public Announcement Logic) and TPAL+P! (Temporal Public Announcement Logic with Labeled Past Operators). We will show how Hoshi stated the Verificationist Thesis in the language of TAPAL and analyze his argument on why this version of it is immune from paradox. (3) Edgington (1985) offered an interpretation of the Verificationist Thesis that blocks Fitch's paradox and we will propose a way to formulate it in a TAPAL-based language. The language we will use is a combination of TAPAL and TPAL+P! with an Indefinite (Unlabeled) Past Operator (TAPAL+P!+P). Using indexed satisfiability relations (as introduced in (Wang 2010; 2011)) we will offer a prospective semantics for this language. We will investigate whether the tentative reformulation of Edgington's Verificationist Thesis in TAPAL+P!+P is free from paradox and adequate to Edgington's ideas on how „all truths are knowable" should be interpreted.

Keywords: Fitch's paradox, knowability, dynamic epistemic logic, epistemic logic, epistemic temporal logic, protocols

I. INTRODUCTION

This paper is concerned with two reformulations of the Verificationist Thesis that avoid Fitch's paradox: (1) we will present Hoshi's (2009) interpretation of the Verificationist Thesis in the language of TAPAL (Temporal

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Arbitrary Public Announcement Logic[1]) and his argument that in the logical framework of TAPAL it will not lead to paradox, and (2) we will consider Edgington's (1985) interpretation of the Verificationist Thesis, offer a formalization of this interpretation in a prospective logical framework that combines Hoshi's TAPAL and TPAL+P! (TPAL with Labeled Past Operators) and discuss the result. We will begin with short introductions to Epistemic Logic, Fitch's paradox and Hoshi's TPAL, TAPAL and TPAL+P! logics.

II. EPISTEMIC LOGIC

In this section we will present the syntax and semantics of Epistemic Logic[2] following (Fagin et al. 1995), (van Ditmarsch et al. 2006), (Blackburn et al. 2002).

II.1. Syntax and Axioms

The language of EL, hereafter L_{EL}, is given by the following BNF, for $p \in$ *Atoms* (the set of the atoms of L_{EL}) and $a \in Ag$ (the set of agents):

$$\phi ::= p \mid \bot \mid \top \mid \neg\phi \mid \phi \,\&\, \psi \mid \phi \vee \psi \mid \phi \rightarrow \psi \mid K_a\phi$$

Formulas of type $K_a\phi$ are read *agent a knows that* ϕ, whereas all the other formulas keep their Boolean readings. One will have to distinguish between $\neg K_a\phi$, meaning that agent a does not know that ϕ, and $K_a\neg\phi$, meaning that agent a knows that it is false that ϕ.

In the following we will use the most popular system of epistemic logic, S5. Its axioms are:

(Taut) All propositional tautologies

(Axiom K) $K_a(\phi \rightarrow \psi) \rightarrow (K_a\phi \rightarrow K_a\psi)$

(Axiom T or *Veridicity*) $K_a\phi \rightarrow \phi$

(Axiom 4 or the *Positive Introspection*) $K_a\phi \rightarrow K_aK_a\phi$

(Axiom 5 or the *Negative Introspection*) $\neg K_a\phi \rightarrow K_a\neg K_a\phi$

Rules of deduction:

1. Modus Ponens: if $\vdash \phi$ and $\vdash \phi \rightarrow \psi$, then $\vdash \psi$
2. If $\vdash \phi$, then $\vdash K_a\phi$

II.2. Semantics

The semantics of Epistemic Logic is traditionally offered in terms of Kripke models. A Kripke model is a structure $M=(W, \{R_a\}_{a\in Ag}, V)$, W being a

[1] Developed in (Hoshi 2009).

[2] Hereafter: „EL".

set of possible worlds, $\{R_a\}_{a \in Ag}$ a set of equivalence accessibility relations for all agents in group Ag: $R_a \subseteq W \times W$, and V a function that assigns sets of possible worlds to atoms: $V: Atoms \rightarrow 2^W$. An atom p will be true in a possible world w iff function V assigned a set containing w to p, and the Boolean connectives will keep their usual meanings, but relative to some possible world of W. The epistemic operator K_a is defined as follows: an agent a knows that ϕ iff ϕ is true in all epistemic alternatives to w:

$M, w \models p$ iff $w \in V(p)$

$M, w \models \neg\phi$ iff $M, w \not\models \phi$

$M, w \models \phi \,\&\, \psi$ iff $M, w \models \phi$ and $M, w \models \psi$

$M, w \models K_a\phi$ iff: $\forall u$: if $wR_a u$ then $M, u \models \phi$

So we have a definition of what does it mean for a formula to be true at a possible world in a model. We call ϕ *true in a model M* if ϕ is true in all the possible worlds of M's domain, and we call ϕ *valid* if ϕ is true in all models.

II.3. Fitch's Paradox of Knowability

In this subsection we will present a proof of Fitch's paradox as it is represented in a modal-epistemic logical framework. The content of this subsection will be based on the proofs presented in (Edgington 1985), (van Benthem 2004), (van Ditmarsch et al. 2012) and (Holliday forthcoming).

Fitch's argument proves a non-intuitive formula, one that states that all truths are (already) known (OT), from an intuitive one that states that all truths can be known, a statement known as the Verificationist Thesis (VT), wherefrom the paradox. (VT)'s most popular formalization is done in an epistemic-modal language: a language that contains both epistemic and modal operators, whose BNF is:

$$\phi ::= p \mid \bot \mid \top \mid \neg\phi \mid \phi \,\&\, \psi \mid \phi \vee \psi \mid \phi \rightarrow \psi \mid K_a\phi \mid \Diamond\phi$$

Formulas of type $\Diamond\phi$ will be read *it is possible that* φ and their meaning is that there is a possible world in which φ is true. Formally, their semantics is usually offered in terms of Kripke models $M = (W, \{R_a\}_{a \in Ag}, \sim, V)$: $M, w \models \Diamond\phi$ iff: $\exists u$: $w \sim u$ and $M, u \models \phi$ (in model M, at w it is true that $\Diamond\phi$ iff there is a world u in M, accessible from w by relation $\sim \subseteq W \times W$, in which it is true that ϕ). Since Fitch's proof does not use the axioms of Modal Logic, we will not present them[3], but we will have to observe that $\Diamond\bot$ is false in all possible worlds and all models.

Now, note that *all truths are known* is easily represented by an epistemic formula (a formula that contains only Boolean and epistemic operators):

[3] See, for example, (Blackburn et al. 2002).

(OT) $\phi \rightarrow K_a \phi$

The reading of (OT) is: *if something is a truth, then agent a knows it*. But the Verificationist Thesis does not state something about the actual knowledge of an agent, but something about what an agent can, in principle, acquire as knowledge. In other words, it expresses the potential or possible state of knowledge of an agent. But modal formulas like $\Diamond\phi$ are read *it is possible for ϕ to be true*, so, if we agree that statements about potential knowledge are statements about what is possible for an agent to know, we arrive at the following formal translation of *ϕ is knowable*: $\Diamond K_a\phi$. Now, remember that the Verificationist Thesis states that all truths are knowable, so we can translate this as: *if ϕ is true, then $\Diamond K_a\phi$ is true*:

(VT) $\phi \rightarrow \Diamond K_a\phi$

Now we can see how, using propositional, modal and epistemic reasoning, we can arrive at (OT) from (VT)[4]. Note that the following is an instance of (VT):

(1) $(p \,\&\, \neg K_a p) \rightarrow \Diamond K_a (p \,\&\, \neg K_a p)$

Using the distribution of knowledge over conjunction, we have

(2) $(p \,\&\, \neg K_a p) \rightarrow \Diamond(K_a p \,\&\, K_a \neg K_a p)$

But knowledge implies truth, so:

(3) $(p \,\&\, \neg K_a p) \rightarrow \Diamond(K_a p \,\&\, \neg K_a p)$

The consequent of (3) is a contradiction:

(4) $(p \,\&\, \neg K_a p) \rightarrow \Diamond\bot$

But it is impossible for a contradiction to be true, $\neg\Diamond\bot$, so, by propositional reasoning:

(5) $\neg(p \,\&\, \neg K_a p)$

Which is equivalent to:

(6) $p \rightarrow K_a p$

And because the choice of p was arbitrary:

(OT) $\phi \rightarrow K_a \phi$

The argument is based on deriving a contradiction in the consequent of (VT) by substituting $p \,\&\, \neg K_a p$ for ϕ, and then falsifying the antecedent and deriving omniscience from it.

[4] Cf. (Edgington 1985), (van Benthem 2004), (van Ditmarsch et al. 2012) and (Holliday forthcoming).

III. PUBLIC ANNOUNCEMENT LOGIC

In this section we will present, based on (van Ditmarsch et al. 2006) and (Hoshi 2009), Public Announcement Logic[5], a logical framework developed by Jan Plaza (1989) and Jelle Gerbrandy (1997). Public Announcement Logic was created in order to describe the evolution of a group of agents' knowledge under learning truths about the world or about other agents' knowledge. Consider the following example. Bob wants to visit a museum but he does not know whether it is still open. But so it happens that he runs into Anne who just returned from visiting the museum and tells him that they just closed until tomorrow. Using Kripke models it's easy to model the situation of Bob's not knowing whether the museum is still open. Let p denote the proposition *the museum is closed*, w, the actual world, in which it is true that p, and u a possible world in which it is false that p. Now, not knowing that the museum is closed is representable by letting the accessibility relation R_{Bob} connect w and u. Then, if Bob „sees" a possible world in which it is true that the museum is still open, $\neg p$, then it is false that $K_{Bob}\, p$: $\neg K_{Bob}\, p$. But once Anne informs him that the museum is closed, he comes to know that p: $K_{Bob} p$. The fact that Anne revealed him the truth about the museum leads Bob not to consider anymore the possibility that $\neg p$, therefore not considering u as a possible candidate to the actual world, since u satisfies $\neg p$. Now, because the accessibility relation R_{Bob} only connects w with itself, being an equivalence accessibility relation, it is true in the actual world, w, that $K_{Bob}(p)$. Anne's informing Bob that p corresponds to a public announcement in the group $\{Bob\}$. Therefore, the semantic effect of a public announcement of p will consist in restricting the domain of possible worlds to only the p-satisfying worlds of the initial Kripke model representing the epistemic state of all agents in a group.

The language of PAL, L_{PAL}, extends the language of EL with binary operators $[!\, \cdot]\, \cdot$ and $<!\, \cdot>\, \cdot$. The two operators connect the announced formula with the result of its announcing. L_{PAL} is given by the following BNF (for $p \in Atoms$ and $\psi \in L_{EL}$):

$$\phi ::= p \mid \perp \mid \top \mid \neg\phi \mid \phi \,\&\, \psi \mid \phi \vee \psi \mid \phi \rightarrow \psi \mid K_a\, \phi \mid <!\, \psi>\phi \mid [!\, \psi]\phi$$

Formulas of type $<!\, \phi>\psi$ will be read ϕ *can be announced, after which* ψ *is true*, whereas formulas of type $[!\, \phi]\psi$ will have the meaning: *after* ϕ *is announced,* ψ *is true*[6]. The semantics of these operators follows the intuition presented above: after a public announcement is made at world w, the domain of the Kripke model will be refined into another Kripke model, and the effect of the announcement, expressed in terms of a formula, will be evaluated in the new model, in the world where the announcement was made:

(D1) $M, w \vDash [!\, \phi]\psi$ iff: if $M, w \vDash \phi$, then $M!\phi, w \vDash \psi$,

[5] Hereafter: „PAL".
[6] Cf. (Hoshi 2009, 24, 27).

(D2) $M, w \vDash <!\ \phi>\psi$ iff: $M, w \vDash \phi$ and $M!\phi, w \vDash \psi$,

Where model $M!\phi = (W', R', V')$ is defined so as the new domain, W', will contain only worlds that satisfy the formula announced and both the accessibility relations and the valuation function will be restricted to the new domain: $W'=\{w \in W \mid M\ \ w \vDash \phi\}$ $R_a'=R_a \cap (W' \times W'), \forall a \in Ag$, and $V'=V \mid W'$. The two public announcement operators are dual: $\neg[!\ \phi]\ \psi \leftrightarrow <!\ \phi>\neg\psi$.

The reading of the two definitions are as following: (D1) tells that after an announcement of ϕ it is true that ψ at w iff: if ϕ is true at w, then, in the restricted model, ψ is true in w, and (D2) tells that ϕ can be announced, after which ψ is true iff: ϕ is true at w, and ψ is true in the restricted model, in the world where the announcement was made.

Note that public announcements have the property of truthfulness: one cannot announce a falsity, or only true formulas can be announced, a property embedded in definition (D1) as the antecedent of the conditional or as the first conjunct of the (D2) definition. The notion of protocol, to be presented, will be used to add a new constraint to what formulas can be publicly announced. After all, it seems intuitive that social conventions may not allow one to publicly inform anyone of everything.

The axioms of PAL are reduction axioms, equations that translate the formulas of L_{PAL} into formulas of L_{EL}. This type of axiomatics reduces the problem of PAL's completeness to the completeness of EL, meaning S5, a system proven to be complete.

Axioms of PAL[7]:

$<!\ \phi>p \leftrightarrow \phi\ \&\ p\ (p \in Atoms)$

$<!\ \phi>\neg\psi \leftrightarrow \phi\ \&\ \neg<!\ \phi>\ \psi$

$<!\ \phi>\ (\psi\ \&\ \chi) \leftrightarrow\ <!\ \phi>\psi\ \&\ <!\ \phi>\ \chi$

$<!\ \phi>K_a\ \psi \leftrightarrow \phi\ \&\ K_a(\phi \rightarrow\ <!\ \phi>\psi)$

The complete proofs for the reduction of PAL to EL can be read in (Kooi 2007) and (van Ditmarsch et al. 2006).

IV. THREE EPISTEMIC TEMPORAL LOGIC SYSTEMS

In this section we will present three Epistemic Temporal Logic systems developed by T. Hoshi: Temporal Public Announcement Logic (TPAL)[8], Temporal Arbitrary Public Announcement Logic (TAPAL)[9] and Temporal Public Announcement Logic with Labeled Past Operators (TPAL+P!)[10].

[7] As offered in (Hoshi 2009, 51).
[8] See (Hoshi 2009, 44 – 66).
[9] See (Hoshi 2009, 78 – 80).
[10] See (Hoshi 2009, 90 – 97).

IV.1. Temporal Public Announcement Logic (TPAL)

We have seen that in order for a formula to be announced it has to be true. But the truthfulness of the announced formula should not be the only restriction imposed on the set of announceable formulas. Some formulas, though true, cannot be announced due to, say, social conventions. This intuition was modeled in the general framework of dynamic epistemic logics in, among others, (Hoshi 2009) and (Wang 2010; 2011). Following (Hoshi 2009, 28, 44), from a syntactic point of view, a protocol is a set of sequences of formulas of L_{EL} closed under finite prefix (so every finite subsequence is also part of the protocol: if $\phi\psi \in \pi$, then $\phi \in \pi$, for π a protocol). But how is the protocol incorporated in a model? From a semantic point of view, the protocol describes all the possible transformations of a model under announcing the formulas of the protocol π and in the order specified by π (Hoshi 2009). Therefore, as Hoshi (2009) argues, the model fit for incorporating and modeling the notion of a protocol would not be a Kripke model, but a *Kripke forest*, a set of Kripke models, each of them representing the result of announcing a formula in the protocol, in the order specified. For example, if the protocol is $\pi = \{pq, r\}$, and the initial model is M, then the Kripke forest generated by M and π will contain four models: (1) the initial model, M, (2) the model M updated by p, $M!p$, (3) the model $M!p$ updated by q, $M!p!q$, and (4) the initial model M updated by r: $M!r$. In order to interpret formulas with epistemic and public announcement operators, Hoshi (2009) proposes using Epistemic Temporal models[11] generated by the Kripke forests. Let us get through Hoshi's entire method of constructing the Kripke forest generated by a protocol.

The language of TPAL, L_{TPAL}, is inductively constructed as is presented by the following BNF, for $p \in Atoms$ and $\psi \in L_{EL}$:

$$\phi ::= p \mid \bot \mid \top \mid \neg\phi \mid \phi \,\&\, \psi \mid \phi \vee \psi \mid \phi \rightarrow \psi \mid K_a\phi \mid <!\, \psi > \phi$$

Now, the Kripke forest generated by $M = (W, R, V)$ and protocol π, $M^{\sigma,\pi} = (W^{\sigma,\pi}, R^{\sigma,\pi}, V^{\sigma,\pi})$, is constructed by induction on the length of σ, a sequence in π, following the rules (cf. (Hoshi 2009, 45)):

1) $W^{\sigma 0,\pi} = W$, $R_a^{\sigma 0,\pi} = R_a$ (for $a \in Ag$), $V^{\sigma 0,\pi} = V$

2) $w\sigma_{n+1} \in W^{\sigma n+1,\pi}$ iff (a) $w \in W$, (b) $M^{\sigma n,\pi}, w\sigma_n \vDash \sigma_{n+1}$, (c) $\sigma_{n+1} \in \pi$

3) $\forall w\sigma_{n+1}, u\sigma_{n+1} \in W^{\sigma n+1,\pi}: (w\sigma_{n+1}, u\sigma_{n+1}) \in R_a^{\sigma n+1,\pi}$ iff $(w, u) \in R_a$,

4) $\forall p \in Atoms$: $V^{\sigma n+1,\pi}(p) = \{ w\sigma_{n+1} \in W^{\sigma n+1,\pi} \mid w \in V(p)\}$

In the following, $w!p$ will denote a possible world of a model $M!p$, the model obtained from M after announcing p in it. The fact that $w!p$ is part of the domain of $M!p$ implies that p is true in w and that announcing p was allowed by the protocol. The same, $w!p!q$ is a possible world in $M!p!q$.

[11] Hereafter: „ETL models".

An ETL model is a structure $(\sum, H, R_a, V)$, where $\sum$ is a set of events and $H \subseteq \sum^*$ ($\sum^*$ being the set of all finite sequences of events in $\sum$) is closed under finite prefix and $\varepsilon \notin H$ (ε being the empty sequence), relations R_a are equivalence accessibility relations between the elements of H and $V: Atoms \rightarrow 2^W$ (cf. (Hoshi 2009, 25)). The finite sequences of events of H are called *histories*, but alternatively we will call them *possible worlds*, in analogy with the name of the elements of the domain of a Kripke model.

The ETL model based on the Kripke forest $M^{\sigma,\pi} = (W^{\sigma,\pi}, R^{\sigma,\pi}, V^{\sigma,\pi})$, will be $H=(H, R', V')$ and constructed as below (see Hoshi 2009, p. 45):

1) $H=\{h \mid h \in W^{\sigma,\pi}$, for some $\sigma \in \pi\}$
2) $(h, h') \in R'_a$ iff $(h, h') \in R_a^{\sigma,\pi}$, for $\sigma \in \pi$ and every $h, h' \in H$ such that $h=w\sigma$ and $h'=u\sigma$
3) $h \in V'(p)$ iff $h \in V^{\sigma,\pi}$, *for* $p \in Atoms, \sigma \in \pi, h=w\sigma$.

Now, the epistemic and public announcement operators can be given semantic definitions in terms of the ETL model $\mathbb{H} = (H, R', V')$ generated by $M^{\sigma,\pi}=(W^{\sigma,\pi}, R^{\sigma,\pi}, V^{\sigma,\pi})$. The epistemic operator, K_a, will keep its EL reading and meaning, i.e. truth in all epistemic alternatives:

$\mathbb{H}, h \vDash K_a\phi$ iff: $\forall h' \in H($ if $hR_a'h'$ then $\mathbb{H}, h' \vDash \phi)$

However, the public announcement operator will have a slightly different definition, one that will account for the fact that only formulas in the protocol will be allowed to be announced:

(D3) $\mathbb{H}, h \vDash \langle ! \phi \rangle \psi$ iff $h!\phi \in H$ and $\mathbb{H}, h!\phi \vDash \psi$

Definition (D3) tells that φ can be announced, after which ψ holds, if and only if $h!\phi$ is part of the domain of the ETL model $\mathbb{H}$, meaning that announcing ϕ was allowed by the protocol, and ψ is true at $h!\phi$.

As opposed to PAL, TPAL is not reducible to a complete logic like S5. However, Hoshi (2009, 52-56) proved its completeness with respect to the following axioms and rules of deduction:

Axioms of TPAL:

$K_a(\phi \rightarrow \psi) \rightarrow (K_a \phi \rightarrow K_a \psi)$

$[! \phi](\psi \rightarrow \chi) \rightarrow ([! \phi] \psi \rightarrow [! \phi] \chi)$

$\langle ! \phi \rangle p \leftrightarrow \langle ! \phi \rangle \top \,\&\, p$ ($p \in Atoms$)

$\langle ! \phi \rangle \neg\psi \leftrightarrow \langle ! \phi \rangle \top \,\&\, \neg\langle ! \phi \rangle \psi$

$\langle ! \phi \rangle(\psi \,\&\, \chi) \leftrightarrow \langle ! \phi \rangle \psi \,\&\, \langle ! \phi \rangle \chi$

$\langle ! \phi \rangle K_a \psi \leftrightarrow \langle ! \phi \rangle \top \,\&\, K_a(\langle ! \phi \rangle \top \rightarrow \langle ! \phi \rangle \psi)$

$\langle ! \phi \rangle \top \rightarrow \phi$

Rules of deduction: Modus Ponens, Necessitation for K and Necessitation for Public Announcements.

IV.2. Temporal Arbitrary Public Announcement Logic (TAPAL)

In this subsection we will present Hoshi's TAPAL logic[12]. As an extension of TPAL, this framework also allows for operators that quantify over public announcements. First, the language of TAPAL, L_{TAPAL}, includes an operator $\Diamond$, and formulas of type $\Diamond\phi$ will be read *there is an announcement after which it is true that* ϕ. L_{TAPAL} is defined by the following BNF (for $p \in Atoms$ and $\psi \in L_{EL}$):

$$\phi ::= p \mid \bot \mid \top \mid \neg\phi \mid \phi \,\&\, \psi \mid \phi \vee \psi \mid \phi \rightarrow \psi \mid K_a\phi \mid <!\,\psi>\phi \mid \Diamond\phi$$

To the set of semantic rules of TPAL are added the semantic rules for operators that quantify over announcements:

(D4) $\mathbb{H}, h \vDash \Diamond\phi$ iff $\exists\, \psi \in L_{EL}\, (\, h!\psi \in H$ and $\mathbb{H}, h!\psi \vDash \phi)$

$\mathbb{H}, h \vDash \Box\phi$ iff $\forall \psi \in L_{EL}\, ($ if $h!\psi \in H$, then $\mathbb{H}, h!\psi \vDash \phi)$

(D4) tells that $\Diamond\phi$ is true at h iff there is a formula ψ such that after announcing it, it becomes true that ϕ. Operators $\Box$ and $\Diamond$ are dual.

TAPAL's axioms[13]**:**

TPAL's axioms

$<!\,\phi>\psi \rightarrow \Diamond\psi$

The rules of inference: TPAL's rules and one that governs the behavior of the quantifying operator $\Box$: if $\vdash \phi \rightarrow [\sigma][!p]\psi$ then $\vdash \phi \rightarrow [\sigma]\Box\psi$, for $p \in$ *Atoms* and $!p$ is not in ϕ[14].

Hoshi's soundness and completeness proofs for TAPAL can be read in (Hoshi 2009, 84 – 90).

IV. 3. Temporal Public Announcement Logic with a Labeled Past Operator (TPAL+P!)

In this subsection we will present Hoshi's TPAL+P! logic[15]. The language TPAL+P!, $L_{TPAL+P!}$, contains all the formulas of TPAL and formulas of type $P_{!\psi}\, \phi$, for ψ in L_{EL} and ϕ in $L_{TPAL+P!}$. Formulas of type $P_{!\psi}\, \phi$ will be read *before announcing* ψ *it was true that* ϕ. $L_{TPAL+P!}$ is given by the following BNF (for $p \in$ *Atoms* and $\psi \in L_{EL}$):

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[12] See (Hoshi 2009, 78 – 90).
[13] See (Hoshi 2009, 83).
[14] Idem.
[15] See (Hoshi 2009, 90 – 97).

$$\phi ::= p \mid \bot \mid \top \mid \neg\phi \mid \phi \,\&\, \psi \mid \phi \vee \psi \mid \phi \rightarrow \psi \mid K_a\phi \mid <!\,\psi>\phi \mid P_{!\,\psi}\,\phi$$

The semantic rule for $P_{!\,\psi}\,\phi$-formulas is:

(D5) $\mathbb{H}, h \vDash P_{!\,\psi}\,\phi$ iff $\exists g(\, h{=}g!\psi$ and $\mathbb{H}, g \vDash \phi)$

TPAL+P! is sound and complete if axiomatized as following (Hoshi 2009, 93 - 97):

Axioms of TPAL+P!:

The axioms of TPAL

$\neg P_{!\,\chi}\,\neg(\phi \rightarrow \psi) \rightarrow (\neg P_{!\chi}\,\neg\phi \rightarrow \neg P_{!\chi}\,\neg\psi)$

$<!\,\phi>P_{!\,\phi}\,\psi \leftrightarrow <!\,\phi>\top \,\&\, \psi$

$<!\,\chi>P_{!\psi}\,\phi \rightarrow \bot$ if $\chi \neq \psi$

Rules of inference: See (Hoshi 2009, 93).

V. FITCH'S PARADOX IN THE TAPAL FRAMEWORK

In this section we will present and discuss Hoshi's (2009) reformulation of the Verificationist Thesis as a theorem in TAPAL, and his solution to Fitch's paradox. In the second subsection of this section we will show how Edgington's reformulation of the Verificationist Thesis can be restated in the language of an extension of TAPAL and discuss the result.[16]

V.1. Hoshi's Reformulation of the Verificationist Thesis in TAPAL

Hoshi (2009) argues that the Verificationist Thesis (all truths are knowable) can be stated in the logical framework of TAPAL and the paradoxical derivation of omniscience (all truths are known) can be avoided. In order to reformulate the Verificationist Thesis and use TAPAL's deductive power and semantics, Hoshi devises a new set of readings for L_{TAPAL} formulas. Note that the change in the readings of L_{TAPAL} formulas will not have any effect on their semantics. Hoshi's new „reading rules" for L_{TAPAL} formulas are (cf. (Hoshi 2009, 131)):

(R1) Publicly announcing a formula in TAPAL represents the execution of the verifying procedure for that formula.

(R2) Formula $<!\,\phi>\top$ is read: the procedure that verifies ϕ can be successfully executed (Hoshi 2009, 131).

[16] TAPAL with Labeled Past Operators, see (Hoshi 2009, 77 – 97), and an Indefinite Past Operator.

(R3) Formula $<! \phi>\psi$ is read: the procedure that verifies ϕ can be successfully executed, and after its execution it is true that ψ (Hoshi 2009, 131).

(R4) Formula $[! \phi]\psi$ means that after successfully executing the procedure that verifies whether ϕ, it is true that ψ (Hoshi 2009, 131).

(R5) Formula $\Diamond\psi$ means that there is a successfully executable verifying procedure such that after its execution it is true that ψ: „$\Diamond\phi$: 'the verification procedure of some statement can be successfully executed, after which ϕ is true'" (Hoshi 2009, 131).

(R6) As a consequence, formula $\Diamond K_a\phi$ will be read: there is a successfully executable verifying procedure such that after its execution agent *a* knows that ϕ: „Some successful execution can be made after which ϕ is true" (Hoshi 2009, 131). In other words, the successful executability of a verifying procedure will lead to knowledge of ϕ.

After establishing the new rules for reading the TAPAL formulas, Hoshi (2009, 134) proposes restating the Verificationist Thesis as the following (HVT):

(HVT) If $\vdash [! \phi]\, \phi$, then $\vdash <! \phi> \top \rightarrow \Diamond K_a\phi$

In the light of Hoshi's reading rules presented above, (HVT) is to be read as: if after the successful execution of the verifying procedure of ϕ it is true that ϕ, then, if the verifying procedure of ϕ can be successfully executed, ϕ is knowable.

Does (HVT) catch the meaning of „all truths are knowable"? The consequent of (HVT), $<! \phi> \top \rightarrow \Diamond K_a\phi$, can be understood as saying that as a result of the successful execution of the verification procedure of ϕ one may come to know that ϕ. It does not express actual knowledge of ϕ, as it should not, but knowledge conditional on verifying whether ϕ holds: „when we learn some true statements, we learn them by checking in one way or another whether they are true or not. That is, in our terms, we learn them by successfully executing their verification procedures" (Hoshi 2009, p. 131). What about the antecedent, $[! \phi]\, \phi$? It does not state that ϕ is true, as the antecedent of (VT) does. Hoshi assumes that all statements ϕ have a canonic verifying procedure such that the result of executing the verification procedure of ϕ is „determined by the way the world is when the procedure is performed" (Hoshi 2009, 128). Moreover, an execution of a verifying procedure „*yields* the value *success* whenever the corresponding statements are true" (Hoshi 2009, 128). So the truth of ϕ determines a successful execution of the verification procedure of ϕ. Now, recall the truth condition of $[! \phi]\, \phi$:

$\mathbb{H}, h \vDash [! \phi]\, \phi$, iff:

if $h!\phi \in H$, then $\mathbb{H}, h!\phi \vDash \phi$, iff:

after the verification procedure of ϕ is successfully executed[17], it is true that ϕ.

Note that the truth of ϕ is a consequence of successfully executing its verification procedure. So not only that the truth of ϕ determines successful executability, but also successful executability implies the truth of ϕ. Therefore, equating ϕ with $[!\ \phi]\phi$ is justifiable in Hoshi's philosophical and logical framework.

Now, let us follow Hoshi's argument[18] that (HVT) is immune to Fitch's paradox. Note that (HVT) is not a formula in a formal language, as the modal-epistemic formula $\phi \rightarrow \Diamond K_a\phi$ is; instead, (HVT) is a meta-theorem of TAPAL, stating the conditional derivation of the knowability of ϕ. The proof of (HVT) is offered in (Hoshi 2009, 135). However, following Hoshi's argument, $[!\phi]\ \phi$ is not valid in TAPAL: $\nvdash [!\phi]\ \phi$, for $\phi := p\ \&\ \neg K_a p$. This is easily checked by constructing a counter-model[19]: an ETL model with a world that does not satisfy $[!\phi]\ \phi$. Let $H = (H, R', V')$ be an ETL model constructed over the initial Kripke model $M=(W, R, V)$ and the protocol $\pi=\{!(p\ \&\ \neg Kp)\}$. M is specified as follows:

(1) $W=\{w, u\}$,

(2) $R=\{(w,w), (w, u), (u, w), (u,u)\}$ (R connects worlds w and u), and

(3) $w \in V(p)$, while $u \notin V(p)$, so p is true at w but not at u.

The protocol specifies that there is only one permitted announcement: $p\ \&\ \neg Kp$, so $H=(H, R', V')$ will be:

(1) $H=\{w, u, w!(p\ \&\ \neg Kp)\}$,

(2) $wR'u$, $uR'w$, $wR'w$, $uR'u$, $w!(p\ \&\ \neg Kp)R'w!(p\ \&\ \neg Kp)$, and

(3) $w \in V'(p)$, $u \notin V'(p)$, $w!(p\ \&\ \neg Kp) \in V'(p)$.

Now, we have that $\mathbb{H}, w!(p\ \&\ \neg K_a p) \vDash p$ and, because $w!(p\ \&\ \neg K_a p)$ only sees itself, we have that: $\mathbb{H}, w!(p\ \&\ \neg K_a p) \vDash K_a p$. So $\mathbb{H}, w!(p\ \&\ \neg K_a p) \vDash \neg(p\ \&\ \neg K_a p)$. Because $w!(p\ \&\ \neg K_a p) \in H$ and $\mathbb{H}, w!(p\ \&\ \neg K_a p) \vDash \neg(p\ \&\ \neg K_a p)$, we have that $\mathbb{H}, w \vDash \langle !(p\ \&\ \neg K_a p)\rangle \neg(p\ \&\ \neg K_a p)$, which, due to the duality of the announcement operators[20], is equivalent to $\mathbb{H}, w \vDash \neg[!(p\ \&\ \neg K_a p)]\ (p\ \&\ \neg K_a p)$ and, finally, $\mathbb{H}, w \nvDash [!(p\ \&\ \neg K_a p)]\ (p\ \&\ \neg K_a p)$ what was needed to prove that $[!(p\ \&\ \neg K_a p)]\ (p\ \&\ \neg K_a p)$ is not derivable in TAPAL.

[17] Because $h!\phi \in H$ iff $\mathbb{H}, h \vDash \langle !\ \phi\rangle \top$.
[18] See (Hoshi 2009, 135).
[19] Along the lines of Hoshi's indications, see (Hoshi 2009, 135).
[20] $\langle !\phi\rangle \neg\psi \leftrightarrow \neg[!\phi]\psi$.

Hoshi concludes the argument of (HVT)'s immunity to Fitch's paradox[21] by saying that substituting $p \ \& \ \neg K_a p$ for ϕ in (HVT) will make it vacuously true since its antecendent will be false: $\nvdash [!\ (p \ \& \ \neg K_a p)]\ p \ \& \ \neg K_a p$.

In the following subsections we will consider a different formulation of the Verificationist Thesis and offer an Epistemic Temporal semantics for a translation of it in a language that mixes two of the languages already presented: L_{TAPAL} and $L_{TPAL+P!}$.

V.2. Edgington's Formulation of the Verificationist Thesis

Edgington (1985) argued that the Verificationist Thesis should be understood in the following manner: if ϕ is true in the actual world, then an agent can come to know that ϕ is true in the actual world. Now, recall stating the Verificationist Thesis as (VT) $\phi \rightarrow \Diamond K_a\phi$, and note that the formula is true if the agent a comes to know that ϕ is true in a possible world u. But, following Edgington's intuition, the thesis should be represented as saying that agent a comes to know something about the actual world and not something about a possible world.[22]

In order to formally represent this intuition, Edgington (1985) introduced an actuality operator that imposes evaluating its argument in the actual world, so: $A\phi$ is true iff ϕ is true in the actual world. Therefore, the formal translation of the Verification Thesis should be:

(EVT1) $A\phi \rightarrow \Diamond K_a A\phi$.

But what makes this reformulation immune from paradox? Edgington argues that nothing contradictory follows from saying that the agent will know something about how things are in the actual world. Contradiction only follows from saying that in a possible world u the agent a will know that in u it is true that $p \ \& \ \neg K_a p$, since $K_a(p \ \& \ \neg K_a p)$ is invalid, therefore false in every possible world. However, nothing contradictory follows from $K_a A(p \ \& \ \neg K_a p)$. Another way to understand why it does not imply a contradiction is by pointing out that the A operator does not commute with the knowledge operator, therefore the K operator will not be distributed over the conjunction $p \ \& \ \neg K_a p$ and from this derive $K_a p \ \& \ \neg K_a p$.

Based on the same intuition on how the Verificationist Thesis should be interpreted, Edgington (1985) also proposes a stronger version of the Thesis: „If 'p' is true at t, then (∃ t')(someone knows at t' that 'p' is true at t)"[23] (Edgington

[21] See (Hoshi 2009, 135).

[22] In Edgington's words: „Either way, (6), the statement that all truths are some time known, does not imply that the time of the knowledge is the same as the time of truth" (Edgington 1985, 560).

[23] As opposed to „ If 'p' is true at t, then (∃ t') someone knows at t' that 'p' is true at t' „ (Edgington 1985, 560)

1985, 560). Hoshi resumes it using the following formula, in which the relation *In(x, y)* is read *x is true in y* (Hoshi 2009, 125):[24]

$(EVT2)\ (\forall s)(In(\phi, s)) \rightarrow (\exists t)(In(K_a(In(\phi, s), t)))$

Its reading is: in all states *s* it is true that: if ϕ is true in *s*, then there is a state *t* such that in *t* it is true that *a* knows that in *s* it is true that ϕ.[25] (EVT2) eludes Fitch's paradox for reasons similar to (EVT): agent *a* will not come to know at *t* that $p \,\&\, \neg K_a p$, which would lead to the contradictory $K_a(p \,\&\, \neg K_a p)$, but, at *t*, agent *a* will acquire knowledge that $p \,\&\, \neg K_a p$ is true in *s*, a non-contradictory claim.

V.3. Edgington's Verificationist Thesis in an Epistemic Temporal Framework

In order to interpret Edgington's (EVT2) in an Epistemic Temporal framework, we will choose TAPAL+P! augmented with an Indefinite Past Operator. Recall that Hoshi introduced a Labeled Past Operator in the logic TPAL+P!. But combining the ◊-operator and the binary *P!* is problematic: the ◊-operator picks an arbitrary formula from the language of EL and announces it and the *P!*-operator relates two formulas, the announcement made and the formula to be evaluated before the announcement was made. But it is not the case that we can always know beforehand what announcement the ◊-operator will pick, so we cannot use the binary Labeled Past Operator, *P!*, to go back to the situation before the announcement selected by ◊. In order to solve this problem, we will use a unary Indefinite Past Operator *P* that only takes as argument the formula to be evaluated. So the language of TAPAL+P!+P will contain formulas like: $\phi \rightarrow \Diamond KP\phi$, expressing the fact that if ϕ is true, then some formula can be announced, after which the agent will know that before the announcement was made it was true that ϕ. Observe again that using the binary *P!* would have been impossible: we don't know what announcement would make the argument of ◊ true.

But, though we can include in our language formulas talking about what someone will know to have been the truth before an arbitrary announcement, the Indefinite Past Operator will not tell us where in the past we should evaluate its argument. What will help us evaluate the past truth of ϕ? Translating the *P*-prefixed formula into a *P!*-formula, for which we already have a semantics. But will this not take us one step back? How will *P!* pick out its first argument? We will need a way to introduce in the semantics a „memory" of what were the announcements picked by the possibility operators. An instant of such a „memory" can be easily represented as a

[24] Also see (Williamson 1987, 257).

[25] Note that if we replace the In relation to that of satisfiability relative to a possible world, we obtain a (possibly) more intuitive representation of the underlying idea behind (EVT2), though not a formula in any language: $(\forall s)\ (s \vDash \phi \rightarrow (\exists t)(t \vDash K_a(s \vDash \phi)))$.

regular expression σ and we can introduce it in the semantics by indexing the satisfiability relation $\models$ by σ. This method of endowing the semantics with a memory was introduced by Yanjing Wang in (Wang 2010) and (Wang 2011). If the possibility operator will pick an announcement ψ from the protocol, we will evaluate the argument of $\Diamond$ with respect to the relation $\models_{\sigma\cdot\psi}$, where $\sigma\cdot\psi$ is the result of concatenating ψ with sequence σ. In this way, the $\models$ relation will „remember" the announcements selected and pass them to the *P!* operators. Now, say we have to evaluate the truth value of $P\phi$ with respect to $\models_{\sigma\cdot\psi}$. If we pop the last component of $\sigma.\psi$ out, meaning ψ, we can settle to evaluate the formula $P_{!\psi}\,\phi$, for which we already have a semantics. This way, the *P*-operator will be just an intermediary between $\Diamond$ and *P!*.

The semantic rules of TAPAL+P!+P

The satisfiability relation will be indexed by a regular expression, so, first, we have to set a regular language. Its expressions will be given by the BNF $\sigma ::= \mathbf{0}, \mathbf{1}, \phi, \sigma\cdot\sigma$, where ϕ is in L_{EL}, $\mathbf{1}$ will denote the empty word, $\mathbf{0}$ will denote the empty language and $\cdot$ will be the concatenation operation. The models we will use are ETL-models $H = (H, R', V')$ based on a Kripke model and a protocol. The semantic rules for TAPAL+P!+P will be:

(TP1) $\mathbb{H}, h \models_\sigma p$ iff $h \in V'(p)$, for $p \in Atoms$

(TP2) $\mathbb{H}, h \models_1 \phi$ iff $\mathbb{H}, h \models \phi$

(TP3) $\mathbb{H}, h \models_\sigma \neg\phi$ iff $\mathbb{H}, h \not\models_\sigma \phi$

(TP4) $\mathbb{H}, h \models_\sigma \phi \,\&\, \psi$ iff $\mathbb{H}, h \models_\sigma \phi$ and $\mathbb{H}, h \models_\sigma \psi$

(TP5) $\mathbb{H}, h \models_\sigma K_a\phi$ iff: if $\forall h' \in H\,(hR_a{'}h'$ then $\mathbb{H}, h' \models_\sigma \phi)$

(TP6) $\mathbb{H}, h \models_\sigma \langle !\,\psi\rangle\phi$ iff $h!\psi \in H$ and $\mathbb{H}, h!\psi \models_{\sigma\cdot\psi} \phi$

(TP7) $\mathbb{H}, h \models_\sigma \Diamond\phi$ iff $\exists\, \boldsymbol{\psi} \in L_{EL}\,(h!\psi \in H$ and $\mathbb{H}, h!\psi \models_{\sigma\cdot\boldsymbol{\psi}} \phi)$

(TP8) or $\sigma\cdot\psi \neq \mathbf{1}$: $\mathbb{H}, h \models_{\sigma\cdot\psi} P\,\phi$ iff $\mathbb{H}, h \models_\sigma P_{!\psi}\,\phi$,

For $\sigma\cdot\psi = \mathbf{1}$: $\mathbb{H}, h \models_{\sigma\cdot\psi} P\,\phi$ iff **never**,

(TP9) $\mathbb{H}, h \models_\sigma P_{!\psi}\,\phi$ iff $\exists g(h{=}g!\psi$ and $\mathbb{H}, g \models_\sigma \phi)$

Observe that only the semantic rules for $\Diamond$, announcements $\langle ! \cdot \rangle$ and *P* change the sequence that indexes the consequence relation: the possibility and announcements operators by pushing a formula at the end of the string and the Indefinite Past Operator by taking out the last component of the sequence that indexes $\models$. Also, the semantic rule for *P* accomplishes the transition to the *P!*-operator by providing both arguments: the first one is the literal popping out of the sequence and the second is the formula that *P* prefixes. The rules will make formulas like $P\phi$ come out as false when the memory is empty (meaning that $\models$ is indexed by the empty word, **1**: $H, h \models_1 P\,\phi$ will never hold by (TP8)).

Now, we will propose a tentative translation schema **t** from TAPAL+P!+P to TAPAL+P!:

Translation schema from TAPAL+P to TAPAL+P! :

$\mathbf{t}_1(\phi) = \mathbf{t}(\phi)$

$\mathbf{t}_\sigma(p) = p$

$\mathbf{t}_\sigma(\neg\phi) = \neg\mathbf{t}_\sigma(\phi)$

$\mathbf{t}_\sigma(\phi \,\&\, \psi) = \mathbf{t}_\sigma(\phi) \,\&\, \mathbf{t}_\sigma(\psi)$

$\mathbf{t}_\sigma(<!\, \psi>\phi) = <!\, \psi>\mathbf{t}_{\sigma\cdot\psi}(\phi)$

$\mathbf{t}_\sigma(\Diamond\phi) = \bigvee_{!\psi \in EL}(<!\, \psi> \top \,\&\, <!\, \psi>\mathbf{t}_{\sigma\cdot\psi}(\phi))$

$\mathbf{t}_{\sigma\cdot\psi}(P\phi) = \mathrm{P}_{!\psi}\mathbf{t}_\sigma(\phi)$

$\mathbf{t}_\sigma(P_{!\psi}\,\phi) = P_{!\psi}\,\mathbf{t}_\sigma(\phi)$

Edgington's Verification Thesis in TAPAL+P!+P

Recall that Hoshi defined new readings for formulas of TAPAL. We will add to the list (R1)-(R6) the „verificationist" readings for $P_{!\psi}\phi$ and $P\phi$:

(R7) „$P_{!\psi}\phi$" will be read: before the successful execution of the verifying procedure of ψ it was true that ϕ.

(R8) „$P\phi$" will be read: before some successful execution of a verifying procedure it was true that ϕ.

Now, we propose a reformulation of Edgington's Verificationist Thesis in the language of TAPAL+P+P!:

(EVT3) $[!\, \phi]\, \phi \rightarrow \Diamond K_a P\phi$

Formula (EVT3) seems to capture the meaning of Edgington's (EVT2): „If 'p' is true at t, then (∃ t')(someone knows at t' that 'p' is true at t)" (Edgington 1985, 560). The reading of (EVT3) is: if after the successful execution of the verification procedure of ϕ it is true that ϕ, then ϕ is knowable in the sense that there is a procedure of verification such that after its successful execution the agent comes to know that ϕ was true before successfully executing that verification procedure. The possibility operator will select an announcement from the protocol and the evaluation of its argument, meaning $K_aP\phi$, will be made in a „future" state. Also, the formula to be evaluated in that possible state says that agent *a* knows a truth that holds in the previous state and not in the state in which $K_aP\phi$ is to be evaluated. This follows Edgington's intuition on how the Verificationist Thesis should be interpreted and, following Edgington's argument, no contradiction should follow from this. By taking ϕ to be $p \,\&\, \neg K_a p$, the antecedent will come out as false, therefore making (EVT3) vacuously true. Also, the choice of $[!\, \phi]\, \phi$ for the antecedent is not arbitrary. As Hoshi argued (an argument we have presented in the previous subsection), because the truth of ϕ determines the success of the execution of ϕ's verification procedure and the successful execution of ϕ's verifying procedure implies ϕ's truth, ϕ can be equated with $[!\phi]\ \phi$. So, (EVT3) seems to be a good candidate for a Verificationist Thesis that follows Edgington's ideas.

But problems arise if we delve into the truth conditions of (EVT3) in TAPAL+P!+P. Let us focus on the consequent, for $\phi := p \ \& \ \neg K_a p$:

$\mathbb{H}\ h \vDash_1 \Diamond K_a P(p \ \& \ \neg K_a p)$ iff,

$\exists\psi \in L_{EL}$ ($h!\psi \in H$ and $\mathbb{H}, h!\psi \vDash_{1\cdot\psi} K_a P(p \ \& \ \neg K_a p)$), iff

$\exists\psi \in L_{EL}$ ($h!\psi \in H$ and $(\forall y)($ if $h!\psi R_a y$ then $\mathbb{H}, y \vDash_{1\cdot\psi} P(p \ \& \neg K\ p)))$ iff

$\exists\psi \in L_{EL}$ ($h!\psi \in H$ and $(\forall y!\psi)($ if $h!\psi R_a y!\psi$ then $\mathbb{H}, y!\psi \vDash_{1\cdot\psi} P(p \ \& \ \neg K_a p)))$[26], iff

$\exists\psi \in L_{EL}$ ($h!\psi \in H$ and $(\forall y!\psi)($ if $h!\psi R_a y!\psi$ then $\mathbb{H}, y!\psi \vDash_1 P_{!\psi}(p \ \& \ \neg K_a p)))$

The second conjunct is satisfied if all the alternative histories that $h!\psi$ can see satisfy $p \ \& \ \neg K_a p$ before the execution of ψ[27]. For exemplification, take M with two worlds, w and u, both satisfying q and only w satisfying p, and a protocol that only permits announcing q. Then, in the ETL model constructed from M we will have worlds $w!q$ and $u!q$, and $P_{!q}(p \ \& \ \neg K_a p)$ will fail to be true in $u!q$ because p is not true in u. Therefore, $\Diamond K_a P(p \ \& \ \neg K_a p)$ will be false in w, but not because $p \ \& \ \neg K_a p$ is false at w (it is not) but because it is false in u. The fact that the knowability of $p \ \& \ \neg K_a p$ is decided taking into consideration the truth of $p \ \& \ \neg K_a p$ in more than one world and, worse, not only in the actual world is not an intended consequence of Edgington's Verificationist Thesis.

However, there are cases in which knowability of ϕ is decided only by taking into consideration how things are in the actual world[28]. Take the same model M as above, but protocol $\pi = \{!(p \ \& \ \neg K_a p)\}$. Because $p \ \& \ \neg K_a p$ is the only announceable formula, the ETL model constructed out of M and π will contain worlds w, u, and $w!(p \ \& \ \neg K_a p)$. In this case, $\Diamond K_a P(p \ \& \ \neg K_a p)$ will be true at w because in $w!(p \ \& \ \neg K_a p)$ it is true that $K_a P(p \ \& \ \neg K_a p)$: $w!(p \ \& \ \neg K_a p)$ only sees itself, and in $w!(p \ \& \ \neg K_a p)$ it is true that $P_{!(p \ \& \ \neg Kp)}(p \ \& \ \neg K_a p)$, because $p \ \& \ \neg K_a p$ is true in w. The moral of this case is that under certain conditions, the semantic behaviour of $\Diamond K_a P\phi$ does not conflict with Edgington's interpretation of the Verificationist Thesis. A solution to the problem raised might rest in finding what conditions should the protocols respect in order not to obtain the case of deciding knowability of a formula by checking the formula's truth in other worlds than the actual one.

VI. CONCLUSIONS

We have presented two ways of expressing the Verificationist Thesis in an Epistemic Temporal framework: Hoshi's, in TAPAL, and Edgington's, in

[26] By the construction rules of an ETL model, y has he form $y!\psi$.

[27] In a temporal reading, before the event ψ happened.

[28] The world in which we intend to decide whether someone can come to know whether φ.

a prospective TAPAL+P+P!. We have presented how Hoshi's reformulation avoids Fitch's paradox and argued that the reformulation of Edgington's Verification Thesis is also safe from paradox. We have argued that the formulation of Edgington's Verification Thesis in the language of TAPAL+P!+P respects Edgington's ideas on how should „all truths are knowable" be interpreted. But the semantics devised in order to evaluate it revealed a conflict with Edgington's ideas. We have also constructed a case in which the conflict does not appear, and proposed that finding what constraints should be put on the protocols that generate the Epistemic Temporal models will make this conflict avoidable. However, the framework that we have devised in order to formalize and evaluate Edgington's Verificationist Thesis is not fully worked out: (1) it needs a proof that the **t** schema translates the language of TAPAL+P+P! into the language of TAPAL+P! (or a fragment of it) and (2) completeness and soundness results.

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