

The Square of Opposition and the Four Fundamental Choices

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Abstract. Each predicate of the Aristotelian square of opposition includes the word “is”. Through a twofold interpretation of this word the square includes both classical logic and non-classical logic. All theses embodied by the square of opposition are preserved by the new interpretation, except for contradictions, which are substituted by incommensurabilities. Indeed, the new interpretation of the square of opposition concerns the relationships among entire theories, each represented by means of a characteristic predicate. A generalization of the square of opposition is achieved by not adjoining, according to two Leibniz’ suggestions about human mind, one more choice about the kind of infinity; i.e., a choice which was unknown by Greek’s culture, but which played a decisive role for the birth and then the development of modern science. This essential innovation of modern scientific culture explains why in modern times the Aristotelian square of opposition was disregarded.

Mathematics Subject Classification (2000). Primary 01A20; Secondary 03A05, 03B20.

Keywords. Square of opposition, non-classical logic, incommensurabilities, Leibniz, infinity, fundamental choices.

1. Introduction

I am a historian of the sciences. My frame of reference is logical thinking, not that of natural language, but the one found in the several scientific theories that have been put forward over time. In the following I will therefore suggest a change from a sentence-oriented to a theory-oriented viewpoint.

My previous work on past scientific theories proposed an interpretation of the foundations of science as constituted by four choices, resulting from two basic options, one of which concerns the kind of logic. My personal reason for studying the old square of opposition is to compare this logical structure with the four

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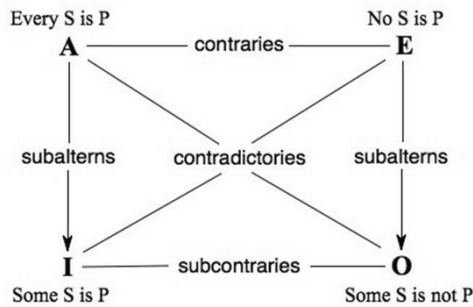
fundamental choices, in order to discover what relationship exists between the old theory of arguing and the modern way of arguing in scientific theories.

2. The modern interpretation of the square of opposition

It is well-known that all inference schemata in traditional logic, either in the theory of immediate inference or in the theory of categorical syllogism sentences, involve only the sentence structures presented by the Aristotle's square of opposition of the four categorical sentences, each one including a quantifier. The theses of these inference schemata concern the logical relations among four logical forms:

| NAME | FORM | TITLE |
|----------|------------------------|------------------------|
| A | <i>Every S is P</i> | Universal Affirmative |
| E | <i>No S is P</i> | Universal Negative |
| I | <i>Some S is P</i> | Particular Affirmative |
| O | <i>Some S is not P</i> | Particular Negative |

The diagram for the traditional square of opposition is:



The theses embodied in this diagram are the following ones:

- 'Every S is P' and 'Some S is not P' are contradictories.
- 'No S is P' and 'Some S is P' are contradictories.
- 'Every S is P' and 'No S is P' are contraries.
- 'Some S is P' and 'Some S is not P' are subcontraries.
- 'Some S is P' is a subaltern of 'Every S is P'.
- 'Some S is not P' is a subaltern of 'No S is P'.

These theses are supplemented, according to Parsons, [23] with the following explanations:

- Two propositions are contradictory iff they cannot both be true and cannot both be false.
- Two propositions are contraries if they cannot both be true but can both be false.

- Two propositions are subcontraries if they cannot both be false but can both be true.
- A proposition is a subaltern of another if it must be true if its superaltern is true, and the superaltern must be false if the subaltern is false.

But Parsons admits that “Probably nobody until the twentieth century ever held exactly these views without holding certain linked ones as well”. In particular, both form E and form I convert simply; instead both A and O do not. Hence, we may consider Aristotle’s square of opposition as an approximation only of what the ancient Greek meant.

3. Interpretation of the square of opposition by means of non-classical logic

In modern times the square of opposition was interpreted by substituting the word “is” with an implication. I object that the connective ‘implication’ is not invariant to a change of the kind of logic; therefore, this substitution implies that we consider “deviant” all kinds of non-classical logic. [16] I, on the other hand, believe that nothing prevents us to mean Aristotle’s effort for formalizing logic by means of the square of opposition, as implicitly including both classical logic and non-classical logic.¹ I admit that my interpretation is a daring interpretation, but I ask that it be evaluated according to its results.

As a first step of my interpretation, in both form A (*Every S is P*) and form E (*No S is P*) the word “is” is intended as an equality, rather than an implication.² The second step of my interpretation essentially involves non-classical logic by interpreting the word “is” in both form I (*Some S is P*) and form O (*Some S is not P*) as a more comprehensive word than the implication of classical logic. It is intended as a notion which, although it is in natural language a slightly different notion from equality, in logic is a very different notion, i.e., “equivalence”; just as physics textbooks state that in thermodynamics “Heat is equivalent to work”, and in special relativity all inertial reference systems are said to be “mutually equivalent”, that does not mean “the same”.³

The word “equivalent” (or “similar”, or, in ancient Greek, *analògos*) may be formalized by means of a double negated statement (“It is not true that it is

¹However, my interpretation is supported by the long interpretative work by G. Calogero [2–4] that gave evidence for an essential dualism inside ancient logic, between dianethic logic and noethic logic; they respectively correspond to classical logic and non-classical logic since the ancient times logic was severed in two kinds, i.e., the classical logic and the dialectical logic. In particular, he stressed the basic difference in Aristotle’s writings on logic between *ei esti* and *ti esti*. Moreover, non-classical logic was followed by ancient schools of both philosophy and theology; usually these schools are called “negative”; actually, their thinking relies upon DNS whose second negative words are often covert or implicit. See my paper [6].

²Within classical logic an equality is stronger than an implication, because it adds an implication in inverse direction.

³Notice that in Aristotle the O form is given by the words: ‘Not every S is P’ [1]; these words are more suitable to be translated in a DNS: ‘Not every S is equivalent to P’.

not. . .”),⁴ which is not equivalent to the affirmative corresponding statement (“It is. . .”) for lack of evidence. For instance: “It is not true that heat is not work” cannot be changed into “Heat is equal to work”, because there is no means of converting the entire quantity of heat into work; hence, only under some constraints or modality are heat and work equal, i.e., under the constraints exerted by an engine, which cannot achieve the complete conversion of the heat into work. This kind of sentence will be called a DNS. According to several studies of mathematical logic of the last century, the failure of the double negation law qualifies a DNS as belonging to a non-classical logic, intuitionist logic as first. [13, 27]⁵

In my interpretation the form I becomes: ‘It is not true that some *S* is not *P*’; and the form O becomes: ‘It is not true that some *S* is not not *P*’.⁶

This interpretation of the square of opposition is corroborated by the following remarks:

- i) In the common use of all natural languages there exists a slight difference between equality and equivalence (or similarity); a careful investigation of the ancient texts may provide evidence for my interpretation.
- ii) The ancient interpretation characterized the couple (A,E) and the couple (I,O) as being structurally equivalent, but qualitatively very different: 1) It attributed to each couple the same characteristic two words, but for each couple it emphasized a different letter; indeed, the four predicates are characterized by the following words: “Affirmo, nEgo, affIrmo, negO”. 2) It attributed a logical quantifier to all forms, but the total one to the former couple and the existential quantifier to the latter couple. These characteristic features may correspond to the respective higher and lower places of the two couple of forms within the graphic representation of the square of opposition. But they may correspond also to emphasize the difference between the two main formalizations of natural logic, i.e., the classical one and the non-classical one, as my interpretation does. In other words, the two ancient characterizations of the couple (A,E) and (I,O) may be considered to be hints of the ancient logicians’ way of emphasizing the qualitative difference between classical logic and non-classical logic.
- iii) The ancient square of opposition gives a relationship from a superaltern to a subaltern, but not the inverse relationship. Hence the set of relationships among the four forms is not complete. A strange “square” results; i.e., a partially oriented square. Moreover, the qualification (“opposition”) of the square also has a partial meaning. In the past, to my knowledge, there was

⁴Here and in the following the underlined words manifest the negated words inside a DNS.

⁵One may suggest as an alternative interpretation, considering E and O as DNSs, by merely adding a negation to each form. But it is difficult to believe that ancient logicians confused one negation with a double negation.

⁶It is commonly stated that the law of triple negation also holds true in intuitionist predicate logic; if this law holds true, the new version of form O is equivalent at the same time to ‘It is not true that some *S* is *P*’ (ancient form E) and ‘Some *S* is not *P*’ (ancient form O); hence, my interpretation unites into form O both classical forms E and O.

no explanation of this approximate qualification. In fact, the inverse translation is not allowed by my interpretation, since both subalterns represent the double negation translations of the respective superalterns: $I = \neg\neg A$ and $O = \neg\neg E$, which in non-classical logic do not admit inverse translations. [28]

- iv) A further suggestion for non-classical logic comes from the relationships “contraries” and “subcontraries”; they have different names, although each of them concerns the same logical operation, i.e., negation: $E = \neg A$ and $O = \neg I$; this difference in name may suggest that the negation in E is not the same in O ; in my interpretation the former negation is a classical negation and the latter one is a non-classical negation; hence, they require similar, but different, names.

4. Contradiction and incommensurability

Let us remark that, as previous interpretations, the theses, embodied in the diagram, of both contraries and subcontraries are preserved by my interpretation. Moreover, the conversion laws apply to both form E (*‘No S is P’* is equivalent to *‘No P is S’*) and form I (*‘Some S is P’* is equivalent to *‘Some P is S’*), and do not apply to both forms A and form O .

On the other hand, the crucial thesis of contradiction is not preserved by my interpretation. Let us see why. The square of opposition is composed of four statements which are logical predicates. It is well-known that in a scientific theory we find a predicate when we deal with its laws and principles.

There are usually all kinds of scientific principles contained in the word “principles”. One has, however, to differentiate two kinds of principles, i.e., “axiomatic principles” and “methodological principles”. The former belong to a deductive theory, where they play the role of starting points for an infinite chain of deductions, which are governed by classical logic, while the latter initiate an inductive search for finding out a new method capable of solving a general problem. These latter principles, by representing orientations instead of certainties, are expressed by means of DNSs; hence, they do not belong to classical logic.

Instances of methodological principles are the following: “It is impossible a motion without an end”,⁷ which in past centuries gave rise to a substantial part of theoretical mechanics as well as the whole of thermodynamics; the previously mentioned sentence: “It is not true that heat is not work” was one more methodological principle in thermodynamics. The same holds true for the inertia principle in L. Carnot’s version: “Once a body... is in motion, it cannot change its speed

⁷This version only is adequate in a scientific context, the usual word “perpetual” being without meaning in science. Let us remark that the corresponding positive statement “All motion has an end” lacks scientific content, since we can state neither the location where the motion will end, nor the time at which it will end; because the friction-function along a path is known only after the end of the path has been reached. The second DNS is not equivalent to “Heat is equal to work”, because only one part of a quantity of heat can be converted into work.

and direction”. [5]⁸ In other words, the difference between the two kinds of principle relies upon the two different kinds of logic, i.e., the classical in the former case, the non-classical in the latter cases.

Let us now compare in a general way two principles belonging to two scientific theories. If both are axioms-principles, they are in a mutual relationship of either independence or dependence; in the latter case the relationship may be of contradiction. On the other hand, when one axiom-principle is compared with a methodological principle which is a DNS, these two principles represent two mutually incompatible kinds of logic; in this case there is no way to obtain a result from their comparison; rather, they can be considered in a relationship of incommensurability – a notion that in recent times Feyerabend [15] and, independently, Kuhn [20] introduced in order to compare entire theories with one another.

In conclusion, in agreement with previous explanations of the propositions of the square of opposition (Section 2), I add the following one:

- “Two propositions, each one being true within a different respective theory, are incommensurable iff they cannot be compared in logical terms; in such a case they belong to two incommensurable theories.”⁹

Incidentally, let us remark that only by relating each single sentence to an entire theory can we control the meaning of our arguing; otherwise, the relation of this sentence with the other sentences may imperceptibly shift from contradiction to incommensurability. Notice that Feyerabend and Kuhn generalized to a couple of theories the ancient Greek notion of an incommensurable couple of numbers. Ancient Greeks did not generalize this notion, because they ostracized its corresponding situation as forbidden; moreover, because they had at their disposal one theory only, Euclid’s *Elements*. Hence, they made use of the notion of contradiction only; but in modern times, by disposing of numerous scientific theories, we are led to consider more general problems and relations than those considered by the ancient Greeks.

In summary, my interpretation is not mined by the lack of the thesis of the contradictories, since this thesis is substituted by a more general thesis:

⁸This DNS is not equivalent to “. . . perseveres in both speed and direction”, because the animistic word “perseveres”, used by Newton in his version of the inertia principle, lacks scientific evidence.

⁹I offered a general definition of this notion in [7]. In the history of science the comparison of two incommensurable principles puzzled the scientific community when Einstein’s celebrated paper of 1905 introduced special relativity [14]. He argued by comparing two predicates, i.e., his methodological principle of the constancy of the speed of light and the axiom-principle of Galilean relativity; the former is the representative of the theory of electromagnetism and the latter one is the representative of Newtonian mechanics. The same level of generality is implied by Poincaré in St. Louis lecture, where he mutually compared all the principles coming from all different physical theories. [24] Actually, already fifty years before both Poincaré and Einstein, Kelvin and Clausius pondered for one year in order to mutually compare the affirmative principle of conservation of energy with S. Carnot’s methodological principle, concerning the bound on the efficiency in heat conversions to work.

- v) Both predicates in each couple (A,O) and (E,I), previously considered as contradictories, in my interpretation are two mutually incompatible propositions because they represent a comparison between two incommensurable kinds of logic.

Let us remark that the supplemented theses suggested by Parsons (Section 2) are all preserved by my interpretation, except for the first one, because both couples (A,O) and (E,I), previously considered contradictory predicates, in my interpretation represent two incommensurable predicates, which suggests that my interpretation of the ancient square of opposition generalizes it, in agreement with the most recent advances in the philosophy of science, at the level of relationships between two entire theories.

5. The new square of opposition as a general scheme in general logic

To sum up, we established that the scheme of square of opposition, when interpreted as in the above, enjoys the maximum of generality, because it concerns the relationships between two theories; i.e., first of all, between two formal theories of logic.

By representing in terms of predicate calculus the basic relationships between classical logic and non-classical logic, the square of opposition is much more general than any scheme that can be included by classical logic. In other words, the square of opposition is meant here as Aristotle's maximum effort to grasp all ways of arguing in formal logic irrespective of the several formalizations of natural logic; Kolmogoroff would say that the square of opposition belongs to "general logic", the logic that includes all kinds of logic. [18]

In this light one can re-visit the modern polemic about the validity of the square of opposition. This polemic focused the attention upon the case of the empty case of the predicate S. My interpretation offers two meanings to the empty case; either S is an already formalized predicate, then the empty case is formally the negative of the totality case, \forall ; or S is a not yet formalized predicate, in which case the emptiness of S means no case at all. This ambiguous role played by the empty case shows that the real problem is rather whether the square of opposition has to be considered an already formalized scheme within a particular formal logic (say, the classical one, as modern logic does), or alternatively as an attempt to argue by means of a tool still requiring a formalization, because the level of discourse, involving several kinds of logic, as my interpretation does, pertains to an informal realm which stands before any formalization of natural logic within a specific kind of logic.

Let us also consider the debate according to the result obtained by the modern interpretation of the square of opposition. This interpretation cuts away from this square the relationship of subalternity, which in my interpretation is obtained by doubly negating the predicates of the superalterns; moreover it drops out the

relationships of both contraries and subcontraries, which in my interpretation refer to two different negations; the former negation is a classical one, while the latter is non-classical. To sum up, modern interpretation cuts away almost all the relationships which in my interpretation involve the difference between classical logic and non-classical logic. Moreover, the modern interpretation preserves the relationship of contradictories only. Instead, my interpretation generalized this relationship to the relationship of incommensurability between predicates, representing two incommensurable kinds of logic.

By ignoring the incommensurable phenomena, the modern interpretation is not adequate to recent views on the relationships among the different kinds of logic. It is the result of the reduction of general logic to a mere skeleton.

6. Leibniz's two labyrinths of human reason and the four fundamental choices

In the following I will exploit Leibniz's two clever suggestions, which in previous papers I interpreted as referring to the foundations of science and, more in general, of knowledge. [8] The link that I will suggest with Aristotle's square of opposition will simply be supported by plausible arguments. Why more cogent arguments are not possible will be clear in next section.

In 17th Century Leibniz suggested a "labyrinth of the human reason", i.e., "either law or free will". According to my studies on past scientific theories, this labyrinth expresses in merely subjective terms a dilemma between two ways of organizing a theory, i.e., the deductive way, where all statements are strict consequences, according to the *laws* of classical logic, from few principles; and an inductive way, where one *freely* searches for a new method, capable of solving a universal problem.¹⁰ Let us call them respectively apodictic organization, AO, and problem-based organization, PO.

By substantiating the logical ideas of the two quantifiers by means of objective ideas, I interpret the total quantifier as the logical capability to organize a totality of objects as a system, and the existential quantifier as a heuristic effort to search for a specific object. In other words, as corresponding to respectively AO and PO.

Now let us remark that in Aristotle's square of opposition each of both the top forms, A and E, includes a universal quantifier, whereas each of both the bottom forms, I and O, includes an existential quantifier. Leibniz' notion of "law" may be interpreted as either form A or form E; i.e., as the idea of the systematic organization; whereas Leibniz's "free will" as either form I or form O; i.e., as the idea of a search.

To sum up, I suggest that Leibniz' dilemma relates the two general ways of organizing the same theory to the square of opposition; i.e., law and free will to respectively the superalterns to AO and the subalterns to PO. Let us recall

¹⁰Already D'Alembert supported the idea that there exist two organizations of a scientific theory. [21] Both Poincaré [25] and Einstein [22] recognized two kinds of organizations within the list of past physical theories.

TABLE 1. Relationships among the four forms, the two quantifiers, the two kinds of organization, and the two kinds of infinity.

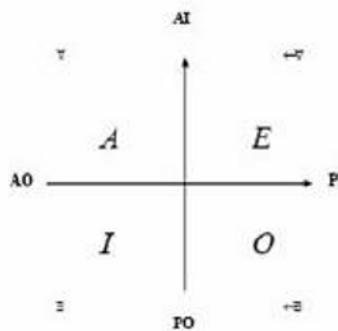
| Name | Form | Title | Quantifiers | Organization | Infinity |
|------|---|------------------------|---------------|---------------------|----------|
| A | <i>Every S is P</i> | Universal Affirmative | \forall | AO ("Law") | AI |
| E | <i>No S is P</i> | Universal Negative | $\neg\forall$ | AO ("Law") | PI |
| I | <i>Some S is \cong P</i> | Particular Affirmative | \exists | PO ("Free will") | AI |
| O | <i>Some S is not \cong P</i> | Particular Negative | $\neg\exists$ | PO ("Free will") | PI |

that Leibniz rightly, in my interpretation, qualifies the dilemma as a “labyrinth”, because it actually concerns an incommensurability phenomenon which cannot be solved by human reason.

Actually, Leibniz suggested one more labyrinth of the human mind; i.e., the labyrinth given by the dilemma between actual infinity and potential infinity; it is independent of logic. In the square of opposition this dilemma may be represented by the couple on the right-hand and the couple on the left-hand;¹¹ indeed, both forms A and I of *affirmo* can be interpreted as the full capability of managing actual infinity, so that either they collect together all objects down to the last object, or they single out exactly some object; whereas both forms E and O of *nego* can be interpreted as the results of an investigation performed constructively step by step in order to reach an unbounded term of investigation; in other terms, as a search relying upon a finite algorithm of investigation.

We obtain Table 1.

One more illustration of the same relationships is obtained by means of a “windrose” graph.



¹¹An alternative interpretation attributes actual infinity to both form A and form E and potential infinity to both form I and form O. But my interpretation agrees with the ancient names of the relationships among them: “contraries”, instead of the relationships of “subalterns”.

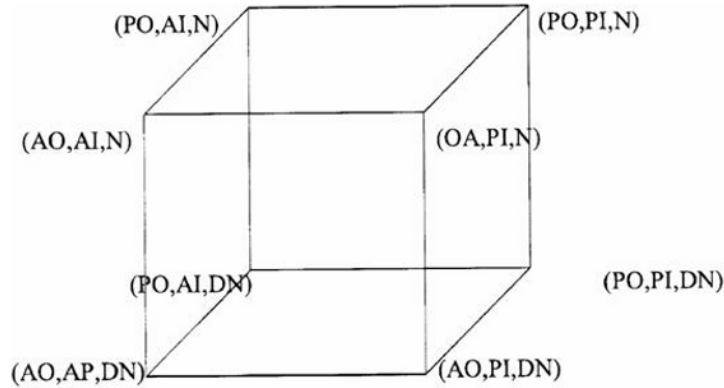


FIGURE 1. AI = O Actual Infinity; AO: Apodictic Organization; DN = Double negation of non-classical logic; N = Negation of classical logic; PI = Potential Infinity; PO = Problem-based Organization.

Notice that the ancient Greeks excluded actual infinity from the foundations of knowledge and of science. Hence, through Leibniz' suggestions, we are exploring general logic in a more general framework than Aristotle's one; this new framework is suggested by modern science, which is born by including in its foundations precisely both actual infinity (through infinitesimals) and potential infinity (through classical chemistry and thermodynamics).¹²

In such a way the square of opposition is linked directly to what I meant as the accomplishment of Leibniz' *Scientia Generalis*, i.e., an entire philosophy of science.

7. The cube of opposition

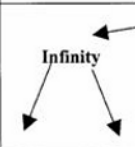
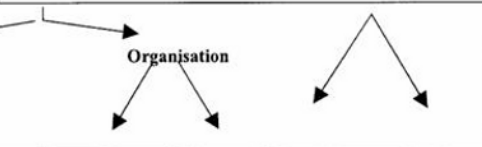


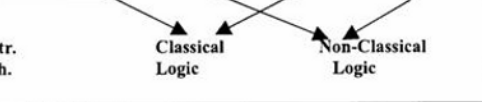
Whereas in the ancient square of opposition we have predicates built by means of **two** basic variables, i.e., *quantifiers* (either \forall or \exists) and *negation* (either double or not), after Leibniz's suggestions we have **three** basic variables, i.e., the *organization* of a theory (either AO or PO), the *infinity* (either AI or PI) and the negation (either double or not). The absolute novelty with respect to Aristotle's square of opposition is the introduction of the kind of infinity, a notion which the Greek exorcized from the foundations of both science and knowledge.

The above three dichotomic variables give a cube as in Figure 1.

By identifying, as in the above, the quantifiers with the two kinds of organization, we remark that Aristotle's square of opposition does not correspond to any one face in the cube; but to the rectangle joining the top edges of the front

¹²This thesis is supported by the magnificent work performed by Koyré. [19]

TABLE 2. The historical development of the square of opposition.

| | | |
|--|--|---|
| Ancient Greeks' Logic and Medieval Logic | SQUARE OF OPPOSITION | $\left\{ \begin{array}{l} \text{Total} \\ \text{quantifier} \end{array} \right\} \times \left\{ \begin{array}{l} \text{Pos.} \\ \text{Neg.} \end{array} \right\} S \text{ is } P$ $\left\{ \begin{array}{l} \text{Existential} \\ \text{quantifier} \end{array} \right\}$ |
| Modern Science |  |  |
| Leibniz' two labyrinths | AI PI | AO PO = Equal ≈ Equivalent |
| Intuitionism, Poincaré-Einstein, Bishop |   |  |
| Modern Foundations of Science | CUBE OF OPPOSITION | $\left\{ \begin{array}{l} \text{AO - Total} \\ \text{quantifier} \end{array} \right\} \times \left\{ \begin{array}{l} \text{Pos.} \\ \text{Neg.} \end{array} \right\} \times \left\{ \begin{array}{l} \text{AI} \\ \text{PI} \end{array} \right\} S \text{ is } P$ $\left\{ \begin{array}{l} \text{PO - Existential} \\ \text{quantifier} \end{array} \right\}$ |

face with the bottom edges of the back face. In other words, according to Leibniz's suggestions, Aristotle's square of opposition is only a mixture of predicates, which fortunately correspond to the most usual ones; but they are not so well connected, since they collapse the several possibilities of the cube (six faces) into a further quadric scheme.

I summarize my entire interpretation in Table 2, where the square of opposition is expressed by means of a synthetic formula relying upon dichotomic variables.

8. Two verifications: The logical-mathematical principles and two versions of inertia principle

From this point on, the square of opposition is no longer considered an ancient tool, but rather a hint for exploring the foundations there are scientific predicates mutually connected according to Aristotle's square of opposition.

A square of opposition is obtained for the logical-mathematical principles that compare a sentence S with the predicate "to be a truth". They may be characterized by the following four mottos (where the double negated statements of the intuitionist logic have been translated in modal logic *via* S4 model):

A = 'Every S is a truth'; for instance, Pythagoras's 'All is number'; where the "number" is, according to Pythagoras, the symbol of a perfect truth.

E = ‘No [isolated] S is a truth’ (e.g., a negated sentence is not a certain truth, owing to the several meanings of a negation); this statement was proved true by Sophists; only a well-organized system of propositions assures the truth of a proposition.

I = ‘Some S are Provable’, as, after Goedel’s theorem, modern proof theory states.

O = ‘Some S are not Provable’; e.g., Goedel statements; in physics, ‘It is impossible to prove a motion without an end’.

Let us now consider theoretical mechanics. We will take into account the inertia principle, i.e., the principle which characterizes modern science with respect to ancient scientific thinking. I will show that the predicates of its two versions represent respectively the two positive predicates A and I.

I. *Newton*: ‘All bodies at rest or in rectilinear uniform motion [r.u.m.] perseveres in their state of motion unless acted on by a force’.

Here, every S = all bodies, P = either is at rest or it is in r.u.m.

Hence, Newton’s version is an Affirmo: ‘Every S is P’.¹³

B. *Cavalieri*: “I say, moreover, that, if we consider the motion of a bullet, when it is fired in a given direction, if there is no other force acting on it causing it to move in a different direction, it will reach the place towards which the bullet’s motion in a straight line is directed, . . . ; from that straight line it is not reasonable that the mobile body should detach itself, when no other moving virtue it there exists. . . I moreover say that that projectile not only would proceed along a straight line. . . but also that in equal times it would go through equal spaces on the same line”.

Some years after Cavalieri, Torricelli summarized this same version in a shorter passage: “Let a moving body be launched from A with any inclination. It is clear that, without gravity’s attraction, the body would proceed with uniform and rectilinear motion in direction AB”. [10,11]

Both Italians considered a situation which in Nature never occurs, i.e., the limit situation of null force, this situation may be expressed by the sentence “the motion is equivalent (i.e., it is not true it is not equal) to the R.U.M.”.

some S = a bullet, P = if there is no other force. . . causing it to move in a different direction, it is at the limit equivalent to the r.u.m.

Hence, Cavalieri’s version of inertia principle is an Affirmo: ‘Some S is \cong P’.

Actually, there are four versions of the inertia principle. [29] But we have to remember that included essentially in the predicate of the inertia principle is the notion of infinity, which is not included in Aristotle’s square of opposition, except for a modern interpretation of the left predicates by means of actual infinity and the right predicates to potential infinity.

Hence, the two previous versions of the inertia principle suggest that my interpretation of the square of opposition generalizes it in agreement with the most advanced theoretical science; the fact that they are only two versions confirms that

¹³About the idealistic nature of this version see [9,17].

Aristotle's formalization of predicate logic into the square of opposition is not the most general.¹⁴

9. Conclusions

I generalized the square of opposition in two steps. The first step generalized it in a very comprehensive scheme, corresponding to the relationships between two theories; above all, between two formal theories of logic.

This point stresses the increase of generality we obtained, i.e., to argue about relations between two entire theories; in such a way the square of opposition is, rather than a basic tool for arguing within the universe of predicate calculus, a tool for grasping the foundations of logic and, more in general, of science.

A second step manifested the limited nature of the old square of opposition with respect to the birth of modern science. This fact explains why this tool of Aristotle, instead being a key for discovery in modern science, for a long time was dismissed as a logical curiosity. However, we saw that, by generalizing it, it may be connected to the foundations of modern science.

Hence, Aristotle's logical investigation, although in a disguised way, was in fact oriented at the foundations of science. His cultural bound was the ancient Greek one, the exclusion of infinity from the foundations of science.

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¹⁴More in general, according to the two basic options on the kind of the organization and the kind of infinity, one obtains four ways of arguing. [12] These four ways correspond to the four couples of choices on the two above illustrated options. They are recognized in the branch of science that for two millennia has been most linked to lucid argument, i.e., geometry. Actually, the choices AO and AI are the foundations of Euclidean deductive geometry, which includes the points at infinity of parallel lines. The choices PO and PI are represented by Lobachevsky's presentation of hyperbolic geometry. After him, the number of geometrical theories grew enormously. However, one short way to conclude this investigation into which are the most relevant geometries, is to refer to the geometries which are the most often applied by theoretical physics. Elliptic geometry, which is founded on choices PI and AO; whereas the fourth geometry, which is widely used by physicists, i.e., Minkowsky's geometry, is founded upon the choices AI (because it includes in its space the lines at infinity of the light-cone) and PO (because it has to solve the problem of tracing lines in a constrained space). Poincaré recognized just these four geometries as the basic ones and moreover he suggested some predicates which characterize each of them. [26]

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Submitted: 17 September 2007.

Accepted: 25 November 2007.