What Is a Mathematical Concept?

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Deleuze and the Conceptualisable Character of Mathematical Theories

SIMON B. DUFFY

In this chapter I present an account of what a mathematical concept is, as developed in the work of the French philosopher Gilles Deleuze. In characterising his understanding of what a mathematical concept is, I draw upon the way that he taps into the mathematical philosophy of Albert Lautman and one of Lautman's contemporaries, Jean Cavaillès. To make sense of what Deleuze understands by a mathematical concept requires unpacking what he considers to be the conceptualisable character of a mathematical theory. However, his account of mathematical theories, indeed his account of mathematics, is much broader than what is usually understood by these terms. Mathematics is more than just the sum of its theories, and mathematical theories are determined by more than just the proofs upon which the theories are based. Deleuze identifies a problem-solution complex in operation at the base of every theory.

By problem-solution complex Deleuze does not mean a 'problem set' and the written 'solutions' to them that are encountered in pedagogical settings, but rather the mathematical problems that emerge during, and indeed characterize, the historical development of mathematics, to which mathematicians have responded by developing more or less rigorous solutions in the form of mathematical concepts derived from: conjectures, for which proofs have yet to be developed; proof, either of conjectures or of other mathematical problems in general; formal theories, based on the proofs; and informal, or semi-formal, theories, for which no rigorous proof has developed, or at least not for all aspects of the proof. Informal theories rely rather on heuristic proofs or inductive approximations derived intuitively. For Deleuze, the mathematical problems, for which a theory is a solution, retain their relevance to the theories not only as the conditions that govern their development but also insofar as they can contribute to determining the conceptualisable character of those theories. Deleuze presents two examples of mathematical problems that operate in this way. These two are the problem of the solvability of quintics and the problem of the diagrammatic representation of essential singularities.

The first is formal: Galois' group-theoretic proof of the solvability of higher-degree polynomial equations. Galois proved that, despite some specific quintics actually having solutions, it was impossible to construct an algebraic formula that solved all quintics. And the second is informal: Poincaré's qualitative theory of differential equations which develops the concept of an essential singularity. The different kinds of essential singularity are determined by virtue of the observed trajectories of variables across a potential function, rather than by a formal mathematical proof. The diagrammatic representation of essential singularities, which are determined in relation to the problem of the representation of meromorphic functions, remains only intuitive. The question of the solvability of quintics and the diagrammatic representation of essential singularities are each examples of mathematical problems, the conditions of solvability of which are immanent to the problems themselves. Deleuze considers these mathematical problems, and the mathematical concepts derived from them, to be characteristic of a more general theory of mathematical problems.

This distinction between formal and informal characterizations of the mathematical expression of problems is important for determining how Deleuze's approach to the relation between mathematics and philosophy differs from those committed to foundationalist approaches to the philosophy of mathematics. Deleuze traces the development of what he characterizes as a more general theory of mathematical problems through the history of mathematics drawing upon the work of Karl Weierstrass, Henri Poincaré, Bernhard Riemann, and Hermann Weyl, and the historical insights of Lautman and Cavaillès. An account of what a mathematical concept is for Deleuze will be developed by tracing the contours of this history and of its broader significance to his work, chief amongst which is showing how such a theory of mathematical problems can be deployed as a way of studying problems in other discourses, or fields and contexts.

Deleuze develops an account of mathematical theories, and thus of the concepts derived from them, that presents them as responses to problems, the immanent conditions of which not only govern the very development of the concepts but continue to be implicated in the concepts as the conditions of their development (Deleuze, 1994, p. 160). The implications are that mathematical concepts are not solely derived from theories that resolve mathematical problems (Deleuze, 1994, p. 158), but rather mathematical concepts are themselves *problematic* insofar as the mathematical problems that govern the development of the theories continue to be implicated in the concepts, as the conditions of their very development. Deleuze wants to challenge any presumption that mathematical problems disappear as problems, or can be abandoned, with the discovery of their solution, and therefore also what this presumption falsely entails, that it is the truth of the solution that retrospectively governs the development of the problems.

While Deleuze would acknowledge that most mathematical concepts do seem to fit with this model, i.e. of dissolving the problem and retrospectively governing the development of the problem for which they are the solution, he would maintain that this seeming fit is misleading and at best a convenient shorthand. It is misleading insofar as it sets up the expectation that all problems will have solutions, or rather that all adequately determined problems will have solutions. When the expectation of a solution is thwarted, the fault is considered to lie with the particular characterisation of the problem, rather than with the expectation of a solution. Deleuze considers this to be a poor model for a theory of mathematical problems, as it fails to account for those problems that are able to be characterised adequately, i.e. with generally acceptable mathematical rigour, and yet whose expectation of solution is thwarted, i.e. the associated mathematical proof disproves a solution or proves that there is no solution. The example that Deleuze uses is Galois' group-theoretic proof of the solvability of higher-degree polynomial equations. The conceptualisable character that Deleuze extracts from these mathematical concepts is presented as a theory of mathematical problems, and would be something like this: Well-defined problems don't necessarily have rigorous solutions, indeed informal diagrammatic representations of results do not provide solutions, but rather characterise the conditions of the problem itself. Deleuze argues that these kinds of concepts are responses to the particular conditions of the problem itself and are governed by those conditions rather than by an actual solution whose discovery is expected, since there is no such solution. Deleuze considers this example to function as an effective counterexample to what I have characterised as the poor model, and wants to generalise from this particular case to a general theory of mathematical problems.

Deleuze proposes the account of mathematical concepts that he develops as a more inclusive alternative. The implication is that mathematical concepts are not solely derived from theories that are the expected solutions to mathematical problems, but rather mathematical concepts are themselves problematic insofar as the mathematical problems that govern the development of the theories from which the mathematical concepts are derived continue to be implicated in the concepts as the conditions of their very development. So problems are not forgettable, or able to be discarded or abandoned, relative to their solutions. Nor are they simply useful for pedagogical reasons, i.e. to educate budding mathematicians about solutions, which are then discarded or seamlessly displaced by the solution itself. Mathematical problems are rather intrinsic features of the enduring problem-solution complex.

Deleuze then uses this theory of mathematical problems as a model to develop a general theory of problems, and specifically philosophical problems, with the aid of tracing an alternative lineage in the history of philosophy that tracks these developments in mathematics. For example, (1) Maimon's account of intuitions as differentials allows Deleuze to incorporate a critique of representation within the structure of his philosophy, to which I return at the end of this chapter; and (2) Riemann's work allows Deleuze to critique and reconfigure Kant's pure intuitions of space and time along Bergsonian lines, although arguably modelled more effectively on the mathematics that inspired Bergson.[1]

THE THEORY OF MATHEMATICAL PROBLEMS AND THE HISTORY OF MATHEMATICS

Deleuze traces the development of his alternative account of a theory of mathematical problems through the history of mathematics drawing upon the work of Weierstrass, Poincaré, Riemann, and Weyl. This history actually begins with the Leibnizian method of approximation (1684) using successive orders of the differential relation, which was developed into a theorem about power series expansions by Brooke Taylor in 1715, for which Lagrange attempted to provide an algebraic proof in 1772 (1797). "Lagrange tried to make the calculus rigorous by reducing it to the algebra of infinite series" (Grabiner, 1983, p. 203). The foundation of algebra was generally thought to be sound in the eighteenth century. Lagrange thought he had proved that every differentiable function was the sum of a Taylor series. He coined the expression 'derived function' and the notation for it f'(x), and recursively f''(x), etc., which is the origin of the term 'derivative.' The concept of function helped 'free the concept of derivative from the earlier ill-defined notions' – Leibniz's differential quotient, a ratio with vanishing quantities; and, Newton's fluxion – since the derivative, as a function, was "the same sort of object as the original function" (Grabiner, 1983, p. 203).

[1] See Duffy, 2013, pp. 89–116.

However, the problem with Lagrange's method was that it assumed that "the algebra of finite quantities' could 'automatically be extended to infinite processes" (Grabiner, 1983, p. 203), which turned out not to be the case. The subsequent development of the epsilon-delta method by Weierstrass, which reformulated the calculus without either fluxions or vanishing quantities using only real numbers, allows Lagrange's method to be formalised in the calculus by Weierstrass' theory of analytic continuity (1872), which is a theory of integration as the approximation of functions from differential relations according to a process of summation in the form of series, using Taylor series or power series expansions.

The history that Deleuze traces incorporates Poincaré's qualitative theory of differential equations (1881–1882), which provides a diagrammatic response to problems with the representation of meromorphic functions, or divergent series, in Weierstrassian analytic continuity, and further extends to Riemann's concept of qualitative multiplicity, or Riemann space (1854). The work of Weyl on Riemann surfaces (1913)[2] is instrumental to the development of the mathematical model that Deleuze develops.[3] Weyl makes Riemann's intuitive representation of Riemann space more explicit by using a generalisation of Weierstrass' analytic continuity – effectively demonstrating that Riemann surfaces are the surfaces of Weierstrassian power series expansions – to show that Riemann space is composed of Riemann surfaces, and therefore of Weierstrassian power series expansions.[4] It is by tracing the history of the philosophical engagements with and responses to these mathematical developments, and by drawing upon the work of Lautman and Cavaillès, that Deleuze develops a theory of mathematical problems, which he then uses as a model for a theory of problems in general as they arise in other discourses.

It is important to note that Deleuze eschews characterising his engagement with mathematics as simply analogical or metaphorical. He is careful to distinguish between those mathematical concepts that are quantitative and exact in nature, which he considers it to be 'quite wrong' to use metaphorically 'because they belong to exact science' (Deleuze, 1995, p. 29), and those mathematical problems that are 'essentially inexact yet completely rigorous' (Deleuze, 1995, p. 29) and which have led to important

[2] See Weyl, 1964.

[3] Deleuze is aware of Weyl's work on Riemann via Lautman's commentary on Weyl, which Deleuze cites in Deleuze and Guattari, 1987, pp. 485. See Lautman, 2011, pp. 133–137 and Duffy, 2013, pp. 103–115. The importance of Lautman's work to Deleuze's engagement with mathematics in Deleuze, 1994 is explored in Duffy, 2013, pp. 117–136.

[4] Deleuze extends this model with Poincaré's qualitative theory of differential equations to characterise the relations between discontinuous (Riemann) surfaces of Riemann space.

developments not only in mathematics and science in general but also in other nonscientific areas such as philosophy and the arts. Deleuze argues that this sort of concept is "not unspecific because something's missing but because of its nature and content" (Deleuze, 1995, p. 29). An example of an inexact and yet rigorous concept is Poincaré's qualitative theory of differential equations which develops the concept of an essential singularity. The different kinds of essential singularity are determined by virtue of the observed trajectories of variables across a potential function, rather than by a specific formal mathematical proof. Another example is a Riemann Space. Bernhard Riemann generalises Gauss' work on the differential geometry of surfaces – namely, that the curvature of a surface embedded in three-dimensional space may be understood intrinsically to that surface, i.e. independently of the three dimensional space in which it is embedded – into higher dimensions. While Euclidean 'finite' geometry holds for three-dimensional linear point-configurations, curved three-dimensional spaces are not necessarily flat. However, these spaces still resemble Euclidean space in the infinitesimal neighbourhood of each point. By considering the infinitesimal neighbourhood around each point as a small bit of Euclidean space, the entire space can then be constructed with the step-by-step juxtaposition, or accumulation, of these infinitesimal neighbourhoods. The resulting Riemannian space can be defined as an assemblage of local spaces, each of which can be mapped onto a flat Euclidean space, without this determining the structure of the manifold or multiplicity as a whole.

While Deleuze recognises that citing mathematical concepts of the exact kind outside of their particular sphere would rightly expose one to the criticism of "arbitrary metaphor or of forced application" (Deleuze, 1989, p. 129), he defends the use he makes of mathematical concepts of the inexact kind. He does so on the grounds that by taking from these mathematical concepts "a particular conceptualizable character which itself refers to non-scientific areas" (Deleuze, 1989, p. 129), the redeployment of this conceptualisable character in a nonscientific area is justified. What this means is that the other nonscientific area "converges with science without applying it or making it a metaphor" (Deleuze, 1989, p. 129). A useful way of characterising the relation between the conceptualisable character of the inexact mathematical concept and this conceptualisable character as redeployed in other nonscientific areas, insofar as the latter converges with the former, is to refer to it as a modelling relation. That is, the conceptualisable character which is redeployed in a nonscientific area is modelled on the conceptualisable character of the inexact mathematical concept. What distinguishes a modelling relation from a relation of analogy or metaphor is that there are "correspondences without resemblance" (Deleuze, 1994, p. 184) between them. That is, there is a correspondence between the conceptualisable character in each instance; however, there is no *resemblance* between the mathematical elements of the mathematical problem and the non-mathematical elements of the discourse in which this conceptualisable character has been redeployed. It is the *conceptualisable character* of the two examples above, or at least of how the former is implicated in the latter, that Deleuze redeploys in his philosophy. He is interested in the conditions of the discontinuity between the two discontinuous analytic functions in Poincaré's qualitative theory of differential equations, which, post Weyl, would characterise the relations between discontinuous (Riemann) surfaces of a Riemann space.

While Deleuze does refer to his project as developing a "*mathesis universalis*" (Deleuze, 1994, p. 181), he does not consider there to be a definite system of mathematical laws at the base of nature. Mathematics is not privileged in this way over other discourses. There is, however, a peculiarity about the discourse of mathematics that distinguishes it from other discourses, and that is the very general nature of the relation between the objects of the discourse and the ideas that we have of those objects as expressed within the discourse. Mathematics is peculiar insofar as all of its objects are actually constructed by the discourse itself. By this I just mean very generally that they are the product of the discursive practice of mathematics by mathematicians; they are not discovered empirically. I take it as uncontroversial that there are mathematical objects, and that these objects are abstract. The ideas that we have of the objects of mathematics are therefore directly and unproblematically related to the objects themselves. And this is regardless of subsequent questions about the status of those objects, from the point of view of the philosophy of mathematics: questions about the independence of those objects, whether we are talking about objects or structures, or even about competing constructions in mathematics itself. It is for this reason that mathematics is figured as providing a model for our understanding of the nature of this relation, between the objects of a discourse and the ideas of those objects as expressed within the discourse, in discourses other than mathematics, where this relation is far from straightforward.

LAUTMAN, CAVAILLÈS AND THE MATHEMATICAL REAL

What is important about mathematics, for Lautman and Deleuze, is its seeming *a priority*, which allows the structure of problematic ideas, or the theory of mathematical problems, to be recognised as a component of the mathematical real in a way that is not directly accessible in other discourses because of the reasons mentioned in the paragraph above. Deleuze takes Lautman's concept of the mathematical real, which includes the sum of all mathematical theories and the structure of the problematic ideas that govern them, as the basis for his reflections on the theory of problems, and casts it as a model for our understanding of the nature of the relation between the objects of any one discourse and the structure of the problematic ideas that govern them within that discourse. Insofar as he claims that all discourses can be modelled in this way, Deleuze argues that there is a "*mathesis universalis*" (Deleuze, 1994, p. 181). Deleuze is not positing a positive mathematical order to the universe, but he is rather nominating the Lautmanian mathematical real, and the theory of problems that he characterises by means of it, as a model for our understanding of the structure of other discourses.

What does Deleuze mean by the structure of the problematic ideas that govern the development of mathematical theories, which is included in Lautman's concept of the mathematical real and, that I argue, forms the basis of Deleuze's theory of problems? Lautman subscribed to a Platonic understanding of the structure of the problematic idea; however, he understands these to be abstract dialectical ideas, i.e. not universal Forms, but Archetypes or Ideals, which are the touchstones for the selective and organizational function of the Dialectic,[5] and which remain revisable in the face of the demands of that organization. The abstract dialectical ideas to which Lautman is referring include the following: local-global; intrinsic-extrinsic; essence-existence; continuous-discontinuous; and finite-infinite.

It is important to note that Lautman's references to a dialectic of ideas should not be understood as being references to a general dialectic that exists independently of the mathematics. Lautman is quite explicit in claiming that the dialectic of ideas is the fourth point of view of the mathematical real (Lautman, 2011, p. 183). For Lautman, Ideas constitute, along with mathematical facts, objects and theories, a fourth point of view of the mathematical real. "Far from being opposed these four conceptions fit naturally together: the facts consist in the discovery of new entities, these entities are organized in theories, and the movement of these theories incarnates the schema of connections of certain Ideas" (Lautman, 2011, p. 183). For this reason, the mathematical real depends not only on the base of mathematical facts but also on dialectical ideas that govern the mathematical theories

[5] According to Lautman, they are "the structural schemas according to which effective theories are organized" (Lautman, 2011, p. 199).

in which they are actualized. The mathematical real is not just the sum of all mathematical theories. The former should therefore not be collapsed into the latter. To do so would lead to the mistaken thesis that mathematics provides evidence of an external and more general dialectic that is equally accessible by means of some kind of analysis performed in regard to or from within other discourses. What seems to be clear in Lautman's work is that he considers himself to be working within the constraints of the discourse of mathematics, and the structure of the dialectic that he presents is determined as operating within the expanded concept of mathematics that he makes claim to: the mathematical real. The dialectic of ideas is independent of the mathematical theories, or the mathematics per se, but not of the expanded understanding of the mathematical real.

Lautman does claim that the structure of the dialectic is not the sole privy of the mathematical real, and that it can therefore also 'be found' in other discourses. However, he does not claim that this is the case because the dialectic is able to be generalised, or insofar as it is transcendent with respect to the mathematical real. While Lautman makes strong claims to the unity of mathematics, which was controversial at the time and remains so today, he does not make any claim whatsoever as to the unity of all discourses. What Lautman argues rather is that this is the case because of the way the structure of the dialectic operates in the mathematical real functions as a model for recognizing how it can be understood to operate in other discourses.

Lautman maintains that we are able to recognise the logic of relations structured by the dialectic in other discourses solely by virtue of the mathematical theories in which these relations are incarnated; as he argues, "the effectuation of these connections is immediately mathematical theory" (Lautman, 2011, p. 28). That is to say that it is the way in which the mathematical logic is deployed in other discourses that allows such a discourse to be understood to operate according to the dialectic. This is perhaps straightforward in those discourses that deal extensively with mathematics, such as the sciences and social sciences, even philosophy; but not so straighforward in those that do not, such as those under the umbrella of the humanities. But even a simple concept like representation, or the problem of the relation between an object and the idea that we have of that object, referred to above, can be understood to be modelled on mathematics in the requisite way being referred to here by Lautman, and therefore operate according to the dialectic. A more thorough treatment of the problem of representation will be returned to at the end of the chapter. By dialectic Lautman means here the dialectic of the mathematical real. So mathematics is not privileged over other discourses according to Lautman because, on the one hand, he does not consider there to be a definite system of mathematical laws at the base of nature, and, on the other hand, he does consider it to be intimately involved in our understanding of the very dialectical structure of those discourses. What this amounts to is that mathematical theories are not the sole privy of mathematics, or the mathematical real; they also provide the ground for understanding how the dialectic operates in other discourses. So when Lautman argues that "mathematical logic does not enjoy in this respect any special privilege. It is only one theory among others and the problems that it raises or that it solves are found almost identically elsewhere" (Lautman, 2011, p. 28) – by privileged we should also understand exclusive to the mathematical real.

While Lautman subscribed to a Platonic understanding of the structure of the problematic idea, Deleuze also draws upon the comments of Cavaillès, who is critical of this aspect of Lautman's work, to characterise problematic ideas as being immanent to the problems themselves.[6] The example from mathematics that Deleuze uses to characterise the immanent nature of problematic ideas, and thus of a more general theory of mathematical problems, is the question of the solvability of polynomial equations.

GALOIS (FORMAL) OR POINCARÉ (INFORMAL) ON THE MATHEMATICAL EXPRESSION OF PROBLEMS?

Lagrange provided a unified understanding of the general formulas for determining the solutions to polynomial equations of degree less than or equal to four. However, Lagrange was unable to do the same for quintics, and suggested that they might not be solvable in this way. Abel provided the first conclusive proof of this conjecture. He proved that, despite some specific quintics actually having solutions, it was impossible to construct an algebraic formula that solved all quintics. So, the question of the solvability of quintics provides an example of a mathematical problem, the conditions of which are immanent to the problem itself, and which is characteristic of a more general theory of mathematical problems.

Galois developed a more complete theory on the solvability of higher-degree polynomial equations, by showing that a simpler proof of Abel's

[6] 'Cavaillès: "Personally I am reluctant to posit something else that would govern the actual thinking of the mathematician, I see the exigency in the problems themselves. Perhaps this is what he calls the Dialectic that governs; if not I think that, by this Dialectic, one would only arrive at very general relations … The future will show which of us is right." (Lautman, 2011, p. 224).

result could be found along purely group-theoretic lines. Deleuze argues that Galois' theory is not simply another example or expression of the theory of mathematical problems, but rather a formal restatement of the theory of problems as such, in purely group-theoretic terms. Galois' theory can be understood to unite in one formal theory those aspects of the theory of problems that Deleuze draws upon in Poincaré's qualitative theory of differential equations. Galois' theory allows the formal presentation of a feature that remains only intuitive in Poincaré's qualitative theory of differential equations, namely the leap of the variable across the cut of the potential function in the diagrammatic representation of essential singularities, which are determined in relation to the problem of the representation of meromorphic functions, where more formal solutions remain elusive.

With Galois we obtain a formal proof of a theory of mathematical problems. However, the differences in kind between differential calculus and group theory, namely the formalisation of the latter versus the informal intuitive results of the former, are 'merely secondary' in Deleuze's reckoning. What is important is that they are each characterisations of the mathematical expression of problems as such. So even though Galois' theory is a formal restatement of the theory of problems in purely group-theoretic terms, throughout his work Deleuze prefers and continues to draw upon the informal model that brings together Weierstrass, Poincaré and Riemann, and remains quiet, or is epistemically modest, in regards to mathematical foundations.

This sets up the mathematical real, and the structure of the theory of problems as it operates in the mathematical real, to function as a model for the structure of other discourses, and for how we can understand these other discourses to operate. It is the conceptualisable character of the theory of mathematical problems as a theory of problems that allows other discourses to be understood by the non-mathematician to operate according to a theory of problems, or to be structured by a theory of problems modelled on the theory of problems in mathematics.

What distinguishes Deleuze from those committed to foundational approaches to the philosophy of mathematics is that Deleuze's claims are not dependent on the success of one or other of the foundational approaches to provide foundations for mathematics. Deleuze's claims are rather epistemically modest. Rather than being drawn into making the claim that the model he is proposing is *the* mathematical model, which is defendable by virtue of it being underpinned by a particular foundational approach to mathematics, for Deleuze, the proposed informal model is simply *a* mathematical model that he considers to be more useful than other potential models.

The work of Deleuze does more than merely provide a descriptive account of a theory of problems as the foundations for our understanding of the operations of all discourses. Rather, his work develops an argument for a particular kind of theory of problems that can be understood to operate in relation to other discourses by virtue of the way that it operates as a theory of problems in mathematics, here borrowing the expanded account of mathematics derived from Lautman's concept of the mathematical real. The detail of the structure of the theory of problems can only be offered in mathematical terms; however, the structure of this theory of problems can be used to model the structure and mode of operation of problems in other discourses.

For Deleuze, the way that a mathematical theory, and the mathematical concept derived from it, is implicated immanently in the conditions of the problem that determines it serves as a model for the way that a philosophical concept is implicated in the philosophical conditions of the problem which determines it. There is, therefore, a correspondence between the structure of the theory of problems in mathematics and the structure of the theory of problems that is deployed in Deleuze's philosophy, insofar as the latter is modelled on the former. There are "correspondences without resemblance" (Deleuze, 1994, p. 184) between them, insofar as the structure of the theory of problems conditions each discourse, but without there being a resemblance between the respective problems that do the conditioning.

The philosophical implications of this modelling relation are developed by Deleuze[7] in his critique of representation, or of the presumption of a connection between an idea and something in the world that it represents, presented in *Difference and Repetition*. For Deleuze, the 'idea' is not bound to the representation of an object or a concept, nor is it the property of individual consciousness. Difference, in Deleuze's sense of the term, is also not tied to representation, thus it does not involve a comparison of one thing or concept to another. Deleuze insists that "[d]ifference is not and cannot be thought in itself, so long as it is subject to the requirements of representation" (Deleuze, 1994, p. 262). One aspect of the way that mathematics models the theory of problems for Deleuze is that the mathematics that he draws upon to develop this model also

[7] In *Expressionism in Philosophy* (Deleuze, 1990) in relation to his reading of Spinoza's theory of relations in the *Ethics* (see Duffy, 2004; 2006), and in *Bergsonism* (Deleuze, 1991), and *Cinema 1* and *2* (Deleuze, 1986; 1989) in relation to his understanding of Bergson's intention "to give multiplicities the metaphysics which their scientific treatment demands" (Deleuze, 1991, p. 112). But I want to focus here on how this modelling relation operates in *Difference and Repetition* (Deleuze, 1994).

actually models the nature of the illusory relation of representation between an idea and that which it represents in discourses other than mathematics. Deleuze draws upon the work of Salomon Maimon on the concept of the differential to develop this aspect of his critique of representation. According to Maimon, the operation of integration functions as a mathematical rule of the understanding that is applied to the elements of sensation, which are modelled on differentials, in order to account for how the manifolds of sensation are brought to consciousness as sensible objects of intuition. Here, the determinate units of different manifolds of sensation are projected to be qualitatively different differentials. What appear to us as external objects are therefore constructed as such by the understanding, and the retrospective explanation of the construction is that it is the result of the application of a mathematical rule of the understanding to the elements of sensation.

In the first step of the process, two different manifolds of sensation characterised by different differentials are brought into consciousness by virtue of the application of integration as a rule of the understanding to the elements of sensation or differentials. The method of integration that Maimon deploys as a rule of the understanding to proceed from differentials to functions, or from projected elements of sensation to the qualities of sensible objects, is the method of approximation of a differentiable function around a given point provided by the process of summation in the form of a Taylor series or power series expansion, what is eventually formalised by Weierstrass as analytic continuity. Deleuze champions the inexact extension of this method in Poincaré's qualitative theory of differential equations. The real relation between the two qualities themselves, as sensible objects, is modelled on the mathematical relation between their differentials. A primary physical judgement is then made about the products of integration which determines them as sensible objects. What this amounts to is that all physical judgements, whatsoever, are predicated on a prior mathematical judgement, which "escapes consciousness" (Gueroult, 1929, p. 64). It is therefore an illusion that sensible or real objects appear as external objects to us, when in fact they are the product of our understanding.

What can be seen in operation here is an example of both how a mathematical theory is deployed in another discourse and of the conceptualisable character of the mathematical theory. In other words, Poincaré's qualitative theory of differential equations, which is that a function can be generated from differentials, corresponds structurally to the problem as articulated in another discourse, which is that objects can be generated from the elements of sensation, without there being any resemblance between the respective problems that do the conditioning, and therefore how such a discourse, and indeed all discourses relying on such a form of representation, can be understood to operate according to the dialectic. It should be clear that the conceptualisable component which is deployed in the non-mathematical discourse is modelled on the conceptualisable component of the inexact mathematical concept.

In summary, a mathematical concept for Deleuze is the conceptualisable character of a mathematical theory. He develops an account of mathematical theories, and thus of the concepts derived from them, that presents them as responses to problems, the immanent conditions of which govern not only the very development of the theories, but continue to be implicated in the theories as the conditions of their development. The implication being that mathematical concepts are not solely derived from theoretical resolutions of mathematical problems, but rather mathematical concepts are themselves problematic insofar as the mathematical problems that govern the development of the theories, from which the mathematical concepts are derived, continue to be implicated in the theories as the conditions of their very development. It is on the basis of this account of mathematical concepts as problematic that Deleuze considers the mathematical problems that he identifies in the developments of Galois, Weierstrass, Poincaré, Riemann, and Weyl, and the mathematical concepts derived from them, to be characteristic of a more general theory of mathematical problems. By tracing the history of the philosophical engagements with and responses to these mathematical developments, and by drawing upon the work of Lautman and Cavaillès, Deleuze develops a theory of mathematical problems, which he then uses as a model for a theory of problems in general as they arise in other discourses.

Deleuze does consider the theory of mathematical problems to have been formalised in Galois' group-theoretic proof of the solvability of higher-degree polynomial equations. However, the mathematical problem of primary interest to Deleuze, and which features prominantly in the model he constructs, is not ultimately reducible to a formal proof. The theory of problems derived from Deleuze's informal model is the same as that furnished by a group-theoretic approach, it just arrives at this theory by other means, albeit informal mathematical means. What Deleuze gains by this selection is a mathematical model that affords him a critical perspective on representation not available in the group-theoretic approach.[8] Deleuze

[8] This is not to rule out that other possible critical perspectives on representation cannot be drawn from the group-theoretic approach – just that Deleuze opts for a model furnished by the informal approach.

can be understood to find in the informal mathematical problem that he selects a way to use the relevance of sense experience in obtaining informal results for mathematical problems, to model the operation of sense experience relevant not only to science but to all discursive practices relying on representation.

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