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Maimon's Theory Of Differentials As The Elements Of Intuitions

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Abstract

Maimon's theory of the differential has proved to be a rather enigmatic aspect of his philosophy. By drawing upon mathematical developments that had occurred earlier in the century and that, by virtue of the arguments presented in the *Essay* and comments elsewhere in his writing, I suggest Maimon would have been aware of, what I propose to offer in this paper is a study of the differential and the role that it plays in the *Essay on Transcendental Philosophy* (1790). In order to do so, this paper focuses upon Maimon's criticism of the role played by mathematics in Kant's philosophy, to which Maimon offers a Leibnizian solution based on the infinitesimal calculus. The main difficulties that Maimon has with Kant's system, the second of which will be the focus of this paper, include the presumption of the existence of synthetic a priori judgments, i.e. the question quid facti, and the question of whether the fact of our use of a priori concepts in experience is justified, i.e. the question quid juris. Maimon deploys mathematics, specifically arithmetic, against Kant to show how it is possible to understand objects as having been constituted by the very relations between them, and he proposes an alternative solution to the question quid juris, which relies on the concept of the differential. However, despite these arguments, Maimon remains sceptical with respect to the question quid facti.

Keywords: Maimon; Kant; differential; mathematics; concepts; intuitions

Maimon's critique of Kant concerns the central thesis of the first *Critique* that purports to resolve the problem of the relation between pure concepts of the understanding and empirical intuitions by means of the schematism. The question quid juris for Maimon is a query concerning the objective use of the concept, i.e. whether this use is legitimate, and if it is, what exactly is the nature of this legitimacy (Maimon, 2010: p. 51)?[1] The solution that Kant provides is in the chapter on the Schematism (A137/B176); however, Maimon does not accept Kant's response to the question. He considers Kant to have presupposed that concepts and intuitions necessarily unite in cognition. It is not the necessity of this relation that Maimon disputes but the presumption, because he does not think that Kant can justify the presumption. The alternative solution that Maimon proposes draws upon 'the Leibnizian-Wolffian system' (2010: p. 64).

Maimon maintains that as long as sensibility is regarded as an independent source of cognition to the understanding, the possibility of applying concepts to sensible intuition cannot be comprehended. The connection between the two can only be explained by demonstrating that they both derive from the same cognitive source. While Kant only asked the quid juris question about the relation between the pure concepts and a posteriori intuitions, Maimon extends this to include a priori intuitions as well, and it is in relation to mathematics that Maimon demonstrates the primacy of this question. The purpose of this paper is to provide an account of the 'hypothesis' that Maimon adopts in order to render this connection 'comprehensible' (Maimon, 2010: p. 364).

Maimon's starting point is to distinguish between two types of a priori cognition, that which is pure and a priori, and that which is merely a priori (Maimon, 2010: p. 56). Cognition that is both a priori and pure does not refer to sensibility in any way, neither to the a posteriori, i.e. to specific sensations, nor to that which constitutes a condition for the sensation of objects, namely, space and time. This type of a priori is completely conceptual. The other type of a priori, which is not pure, also doesn't refer to specific sensations, but does involve space and time and therefore the forms of sensation. The range and philosophical significance of Maimon's two types of a priori cognition differ from that of the types of cognition discussed by Kant. While for both, pure cognition involves the categories,[2] Kant also refers to mathematical concepts as pure sensible concepts. Maimon on the other hand claims that while mathematical concepts are indeed a priori, not all of them are pure. What this means for Maimon is that there is a distinction between mathematical concepts that are pure, and about which we can only think, and those that are not pure and of which we are only conscious because of their representation in a priori intuition. The difference between Kant and Maimon on this issue comes down to the difference in the nature of the representation of mathematical concepts in a priori intuition. If the concepts of the numbers are taken as a preliminary example of this difference, for Kant, the concept of a number, 5 for example, is constructed in pure intuition by means of the representation of discrete strokes, for example | | | | | (Kant, 1998: A240/B299). Whereas Maimon considers the concepts of the numbers to be 'merely relations' that

> do not presuppose real objects because these relations are the objects themselves. For example, the number 2 expresses a ratio of 2:1 at the same time as it expresses the object of this relation, and if the latter is necessary for its consciousness, it is certainly not necessary for its reality. All mathematical truths have their reality prior to our consciousness of them.[3] (Maimon, 2010: p. 190)

Maimon considers it to be 'an error to believe that things (real objects) must be prior to their relations' (Maimon, 2010: p. 190). The difference between these two accounts is that, for Kant, the a priori intuitions are supplements to and given independently of the concepts of number that are applied to them. Whereas for Maimon, the a priori intuition is merely 'an image or distinguishing mark' (Maimon, 2010: p. 69) of the relational concept of the magnitude itself, which results from what Maimon characterizes as our limited knowledge of it, and is therefore not so heterogeneous with it.

Maimon's difference in understanding of arithmetic is deployed in relation to Kant's account of synthetic a priori judgments in the first *Critique*. The distinction between their different approaches to arithmetic allows Maimon's question quid juris to be formulated specifically in relation to mathematical cognition. For Maimon, the question regarding the connection between the categories of the understanding and the forms of sensibility is generalized into a demand to understand the connection between mere a priori cognition, which draws on intuition, and pure a priori cognition which doesn't. The question that Maimon poses is how the possibility of such a connection can be accounted for, i.e. the possibility of applying a pure relational concept to an intuition that is a priori but not pure? The example that Maimon gives of this connection is the proposition that 'the straight line is the shortest between two points' (2010: p. 65), which is also one of Kant's examples of a synthetic a priori judgment in the first *Critique* (1998: B16). On Kant's analysis, the judgment that a straight line is the shortest between two points adds a further property, i.e. the intuited property of the line being straight, to the conceptual property of being the shortest distance between two points. Maimon understands this example quite differently. According to him, the intuition in question is not a supplement to the concept, but rather 'an image' of that concept, i.e. it represents the concept on which it is founded. What is represented as a straight line, i.e. a line with a single, fixed direction, is in fact an image of the shortest distance between two points. Maimon acknowledges that there is a synthesis between the two components of the proposition. On the one hand, there is the straight line, which, as far as Maimon is concerned, is an a priori cognition which appears in intuition and is therefore impure. On the other hand, there is the property of being the shortest distance between two points, which refers solely to the magnitude of the distance, which is a category and therefore belongs to pure cognition. The two are synthesized in the proposition. It therefore remains a synthetic a priori proposition for Maimon; however, the nature of the synthesis is different.[4]

Maimon agrees with Kant on the point that the Wolffian definition of the straight line as the 'identity of direction of its parts' is 'useless', as Maimon puts it, since it presupposes that the parts have already arisen and, 'because the similarity of the parts to the whole can only be in direction', it also 'already presupposes lines' (Maimon, 2010: p. 70).[5] However, he disagrees with Kant, who Maimon argues makes 'a concept of reflection', that is, the shortest distance between two points, 'into the rule for the production of an object', i.e. of the straight line as a real object of mathematics, by claiming that it is constructed by being represented in intuition. Maimon on the contrary argues that 'a concept of reflection should really be thought between already given objects' (Maimon, 2010: p. 68), i.e. between real objects of mathematics which are pure a priori concepts of the understanding. Maimon is thinking here of the phrase 'the shortest distance between two points', which he argues the understanding thinks as a rule in order to produce the straight line as an object. Maimon considers this rule to be a concept of reflection, 'a relation of difference with respect to magnitude' (2010: p. 68), i.e. thought between two already given real objects of mathematics or pure a priori concepts of the understanding, i.e. the two points between which a judgment of magnitude is made, both of which can be defined independently of the intuitions. This is achieved according to Euclid's definition 1.1, 'A point is that which has no part' (Euclid, 1956: p. 153); and from Maimon's argument presented above about numbers being 'merely relations' that 'do not presuppose real objects because these relations are the objects themselves' (Maimon, 2010: p. 190). Maimon argues that the two points referred to in this rule of the understanding are 'pure magnitudes prior to their application to intuition', and that this 'cannot be supposed otherwise, because it is only by means of such relations that the magnitudes become objects in the first place' (2010: p. 69). So, contrary to Kant, Maimon distinguishes arithmetic from geometry in this respect insofar as in arithmetic 'without the thought of a relation there is indeed no object of magnitude', whereas geometry 'does provide us with objects prior to their subsumption under the category of magnitude, namely figures that are already determined through their position' (Maimon, 2010: p. 69). In arithmetic, 'the inner (the thing in itself) does not precede the outer (the relation to other things) as is the case with other objects, but rather the reverse' (p. 69).

In the next step of his argument, Maimon provides an analytic proof 'that one line (between two points) must be shorter than several lines (between the same points)' (2010: p. 65). He does this by initially comparing two lines between the points with one line between the same points. These three lines can be understood to constitute a triangle, and therefore allow the use of Euclidian proposition I.20, which concerns the relations between the sides of a triangle. Proposition, 20 states that 'In any triangle two sides taken together in any manner are greater than the remaining one' (Euclid, 1956: p. 293). Maimon then claims that this proof can be extended to 'several lines that lie … between the same points'. The reason being that 'a rectilinear figure will always arise that can be resolved into triangles' (2010: p. 66).

What this means for Maimon is that, just as an intuited number 2 is necessary for consciousness of the magnitude, but is not necessary for the reality of the object 2 in the understanding, because the relation 2:1 is the object itself, so too can the rule, 'the shortest distance between two points', be thought by the understanding independently of the intuition, even though it can only be brought to consciousness as an object by means of the intuition. What is brought to consciousness is 'the straight line', which, in keeping with Maimon's solution to the quid juris question, that sensibility and understanding flow from one and the same cognitive source, is 'an image [*Bild*] or the distinguishing mark [*Merkmal*] of this relational concept' (Maimon, 2010: p. 69). Maimon acknowledges that we can and do 'already have cognition of this proposition by means of intuition alone prior to its proof'; however, he maintains that this perception of the 'distinguishing mark or image in intuition … can only be made clear, not distinct' (Maimon, 2010: p. 70). This provides a good example of how to account for Maimon's claim that the sensible is an 'image' of the intellectual and that 'sensibility and understanding … flow from one and the same cognitive source' (Maimon, 2010: pp. 69, 64). Rather than there being a sensible intuition belonging to the faculty of the imagination that represents the concept in a different faculty, i.e. in the faculty of the understanding, and which is necessary for its construction; for Maimon, the straight line is an image in intuition; however, intuition, as an image or mark of the concept, is itself conceptual, although only a limited version of the conceptual. The relational concept, 'the shortest distance between two points', is thought as a mathematical rule of the understanding in order to produce the straight line as an object of the understanding independently of the intuition. So for Maimon, the synthesis is between different conceptual components of the proposition, rather than between a concept and an intuition. For Kant the representation in intuition results in the construction of the mathematical concept, whereas for Maimon it merely brings the concept to consciousness. This understanding is consistent with Maimon's broader logical principles and is the key to his system as a whole.

Maimon's point about the relation between a priori cognition and pure cognition, i.e. the idea that intuition as the image of the concept is always already conceptual though limited, is also applicable to the empirical realm. In fact, this relation functions as a model for the way empirical intuitions are understood to relate to the objects of which they are the images. Just as the a priori intuition of the straight line is an image of the concept of the shortest distance between two points, which serves as the rule of the understanding by means of which it is defined, by the same procedure, an empirical intuition can be understood to be the image or representation of the concept of a sensible object. However, sensible objects are not the same kind of object as the real objects produced by the understanding according to rules that are determined by mathematics. Instead, what is immediately striking about Maimon's account of the concept of the sensible object is the fact that for Maimon there are no objects outside consciousness. Maimon gives new meaning to the Kantian idea of the 'thing in itself' by conceiving 'the thing itself' and phenomena solely as functions of knowledge.

Maimon's solution to the quid juris question, which involves the claim that intuitions are images of concepts, supplants the role proposed by Kant for the thing in itself as what produces the affections of sensibility, because, unlike Kant, Maimonic intuition has a ground which is not extra-cognitive (see Maimon, 2010: p. 30). However, while Maimon's solution renders the thing in itself redundant, sensible objects of the intuitions are still represented to the understanding as being extra-cognitive. Maimon's explanation of this 'illusion' is that sensible objects of intuition are represented to the understanding from 'outside of us' as a consequence of being represented from the point of view of our limited understanding, i.e. the cognized sensible object is restricted to the finite point of view of human consciousness.

Unlike Kant, who treats sensibility and understanding as two different faculties, for Maimon 'sensibility is incomplete understanding' (Maimon 2004: p. 183). He argues that consciousness is limited insofar as it remains oblivious to the cause and the mode of production of what is given in sensibility as an empirical intuition. If it is not extra cognitive objects that we are conscious of, then what is it that we are conscious of in sensibility? What is it that constitutes an empirical intuition? First of all, empirical intuitions are distinct from a priori intuitions. An example of a concept the consciousness of which can be attained is the mathematical concept of the straight line as the shortest distance between two points. The rule of the understanding must be attached to the a priori intuition in order to achieve consciousness of a straight line as a mathematical concept rather than just as an empirical intuition of something like an extended stroke. However, even with mathematical concepts, the question of what exactly we are conscious of in an a priori intuition, or what is its content, is yet to be answered. So, before addressing the question of the content of empirical intuitions, another mathematical example will be presented in order to determine the content of a priori intuitions, which will assist in setting up the discussion of the contents of empirical intuitions.

One of the other paradigm examples of a mathematical concept that Maimon discusses is the concept of the circle. To define the circle, 'the understanding prescribes for itself this rule or condition: that an infinite number of equal lines are to be drawn from a given point, so that by joining their endpoints together the concept of the circle is produced' (Maimon, 2010: p. 75). Maimon maintains that 'the possibility of this rule, and hence of the concept itself, can be shown in intuition' in the image of a circle, which is constructed by 'rotating a line around the given point' (2010: p. 75). In the example of the circle, the rule of the understanding of the circle cannot be given in intuition because 'only a finite number of equal lines can be drawn' in intuition, whereas the rule of the understanding calls for an infinite number of lines. What is provided in conscious intuition is described by Maimon as the 'unity of the manifold', which he refers to as 'an idea of the understanding', whereas the pure a priori concept of a circle is 'an idea of reason' (2010: p. 80). The idea of the understanding of the circle should be understood as 'a limit concept' (p. 76), because it can only be approached, like an asymptote. Maimon describes an asymptote as 'complete according to their rule, but in their presentation they are always incomplete. We grasp how their construction must be completed without being able to construct them completely' (2010: p. 79). What is brought to consciousness by the image of a circle is therefore the a priori intuition of the circle as an idea of the understanding, and not the concept of the circle as an idea of reason.

This distinction also holds in the example of the straight line. Maimon maintains that 'the principle that a straight line is the shortest between two points is all the more correctly applied to a given line, the more straight parts can be identified in it' (2010: p. 80). The a priori intuition or image of the straight line is therefore an idea of the understanding, rather than an idea of reason, because identifying the straight parts of a line by distinguishing them from curved parts as an intuitive exercise would remain incomplete on the understanding that a line is divisible into an infinite number of parts. The straight line and the circle are therefore examples of concepts that involve infinity and that can nevertheless be brought to consciousness as ideas of the understanding by means of them being attached to their respective a priori intuitions, or images. Maimon (2010: p. 78) maintains that the distinction between ideas of reason and ideas of the understanding is 'indispensible for extending the use of the understanding' in his account of cognition.

When it comes to empirical intuitions and the concepts of the sensible objects of which they are intuitions, Maimon maintains that we only come across these concepts 'themselves as well as their relations to one another incompletely and in a temporal sequence according to the laws of sensibility' (2010: p. 182). In his discussion of the role of sensation in intuition, sensation and intuition being the two constituents of sensibility, Maimon argues that

> sensation is a modification of the cognitive faculty that is actualized within that faculty only passively (without spontaneity); but this is only an idea that we can approach by means of ever diminishing consciousness, but can never reach because the complete absence of consciousness = 0 and so cannot be a modification of the cognitive faculty. (Maimon, 2010: p. 168)

When it comes to sensation, we can only ever have an idea of it, and here Maimon means an idea of an empirical intuition, because we are not talking about an a priori intuition. However, the way that we understand sensation involves applying an a priori concept to it in intuition. For Maimon, the idea of sensation[6] is the lowest degree of consciousness that can be accounted for by the ever diminishing series of degrees that distinguishes clearly determined consciousness from the privation of consciousness, which would result if this exercise were carried out to its limit, i.e. to zero. The limit can therefore only be approached, without ever being reached. Maimon argues that what we understand to be characteristic of the idea that we have of sensation, insofar as it approaches this limit, is the 'differential' (2010: p. 33), the idea of which is drawn from the differential calculus.[7] When thought in relation to mathematics, the differential as an idea of the understanding is understood solely as a limit concept. Maimon maintains that 'with differentials we do not think them in intuition, but merely have cognition of them' (2010: p. 290). However, when thought in relation to an empirical intuition as an idea of sensation, a differential is brought to consciousness as the intuitive idea of that of which it is predicated.

This characterization of the idea of sensation as a differential is the key to Maimon's solution to the quid juris question. While this is only one aspect of Maimon's account of the characteristics of our experience in intuition when faced with a manifold of sensation, it is crucial for developing an understanding of how the integral calculus is deployed in Maimon's account of cognition. The characterization of an idea of sensation as a differential is an example of the application of an a priori rule of the understanding, i.e. a mathematical concept, to an empirical intuition. The differential is the pure a priori concept that is applied to sensation in order to characterize its constituents, i.e. to represent them in imagination, of which we can then have an idea. Maimon distinguishes between

> two kinds of infinitely small namely a symbolic and an intuitive infinitely small. The first signifies a state that a quantum approaches ever closer to, but that it could never reach without ceasing to be what it is, so we can view it as in this state merely symbolically. On the other hand, the second kind signifies every state in general that a quantum can reach; here the infinitely small does not so much fail to be a quantum at all as it fails to be a determined quantum. (Maimon, 2010: p. 352)

One of the examples that Maimon gives of the first kind is the angle between parallel lines, which arises by moving the meeting point of the lines enclosing a given angle to infinity, 'the angle becomes infinitely small, but it altogether ceases to be an angle' (Maimon, 2010: p. 252; see also p. 289). As such, it is a limit concept, 'i.e. a merely symbolic infinitely small' (p. 352). The second kind of infinitely small, i.e. the intuitive infinitely small, is referred to as intuitive because there is a procedure by means of which the concept is applied to sensation, rather than because it can itself be intuited. The example that Maimon gives of it is 'the differential of a magnitude', which 'does not signify the state where the magnitude ceases to be what it is, but each state that it can reach, without distinction, i.e. a determinable but undetermined state' (2010: p. 352). The mathematical example that Maimon uses here is the differential of a differential ratio, $dx{:}dy = a{:}b$. In this example, dx is a differential of magnitude x, and Maimon argues that 'we can take x to be as small or as large as we want (as long as it has some magnitude)' (2010: p. 352). Maimon defines magnitude as

> something such that something else larger than it or something else smaller than it can be thought; consequently what is *omni dabili majus* (greater than any given magnitude) as well as what is *omni dabili minus*

> (less than any given magnitude) i.e. the infinitely large and the infinitely small, is a magnitude. (Maimon, 2010: p. 352)

It therefore follows from the ratio *x:y*, if *x* is smaller than any given magnitude, that *dx:dy*. One explanation for how this works is to draw upon the Leibnizian syncategorematic definition of the infinitesimal in the example of the calculus of infinite series, which defines the differential as the infinitesimal difference between consecutive values of a continuously diminishing quantity.[8] If the limit of the series is zero, as it is in Maimon's example of 'consciousness = 0', then the differential is defined as the difference between the consecutive values of the continuously diminishing quantity as it approaches zero. This would be the a priori rule of the understanding that is applied to sensation in order to define the idea of sensation as a differential.

The differential itself as a mathematical concept is an idea of the understanding because as a magnitude less than any given magnitude it is not a concept to which an object corresponds. However, because the concept of the differential is less than any given magnitude, it is only ever approached without being reached, and is therefore understood as a limit concept. What distinguishes differentials from the other mathematical concepts dealt with so far is that with differentials there is no corresponding empirical intuition, they therefore cannot be constructed in intuition like lines, circles or numbers. Nevertheless, the differential can be applied to intuition as the predicate of sensation. This is how differentials can be represented in intuition, i.e. not as differentials per se, but as the intuitive ideas of that of which they are predicated. When predicated of sensation, i.e. singling out the differential and applying it to sensation to determine it as an idea of sensation, the differential is represented by the imagination as the intuitive idea of that of which it is predicated.[9]

While Maimon describes the symbolic infinitely small as 'merely the invention of mathematicians that lends generality to their claims' (2010: p. 352), he maintains that the intuitive infinitely small or differential can be understood to be real, and 'can itself be thought as an object (and not merely as the predicate of an intuition) despite the fact that it is itself a mere form that cannot be constructed as an object, i.e. presented in intuition' (2010: p. 353). When considered in relation to sensible representation, Maimon argues that 'a magnitude (*quantum*) is not treated as a large quantity, but rather as a quality abstracted from quantity' (2010: p. 261n1). Maimon describes quality 'abstracted from all quantity' as an intensive magnitude and as the 'differential of an extensive quantity' (2010: p. 395). It is therefore as the intensive magnitude of a sensible representation that the differential can be thought of, and is represented by the imagination, as an object. The infinitely small can legitimately be predicated of the quality of a sensible representation because the a priori rule of the understanding that determines the differential in mathematical cognition can be applied to our understanding of the relation between quality and quantity in sensible representation. Maimon argues that, 'considered in itself as a quality, every sensible representation must be abstracted from all quantity' (2010: p. 26), i.e. as the differential of an extensive quantity. The differential can therefore be thought of, and is represented by the imagination, as both the idea of sensation and as the corresponding object of this idea.

Maimon's explanation of the intuitive infinitely small as able to be thought of as an object is characteristic of his account of the metaphysically infinitely small as real. Maimon claims that: 'The metaphysically infinitely small is real because quality can certainly be considered in itself abstracted from all quantity' (2010: p. 354). The example given by Maimon in which the metaphysically infinitely small is predicated of the quality of a sensible representation is his account of the representation of the color red. Maimon argues that the representation of the color 'must be thought without any finite extension, although not as a mathematical but rather as a physical point, or as the differential of an extension' (2010: p. 27).

The idea of the differential as a physical point, which, as outlined above, is the idea of the corresponding object of the differential as an idea of sensation, must 'be thought without any finite degree of quality, but still as the differential of a finite degree', that is, every sensible representation considered as a quality 'must be abstracted from all quantity' (Maimon, 2010: p. 27) and yet still be understood as a differential of that quantity. Insofar as the differential is predicated of a quality, and is therefore understood to be a real physical point although abstracted from all quantity, each differential is understood to function as a 'determinate unit' of sensation such that when they 'are added to themselves successively, an arbitrary finite magnitude then arises' (Maimon, 2010: p. 29n2). So as physical points of intuition, the differentials of one sensation can be added to one another successively to determine an arbitrary finite magnitude or a manifold of sensation.

In order to be able to distinguish one manifold of sensation from another, Maimon maintains that 'we must assume that these units are different in different objects' (2010: p. 29n2). So, the determinate units of different manifolds of sensation are qualitatively different differentials. This can be accounted for by Maimon's definition of the intuitive infinitely small as the undetermined quantum of 'every state in general that a quantum can reach' (Maimon, 2010: p. 352). The Leibnizian example of the calculus of infinite series provides an explanation for the qualitative difference between different differentials, or the different undetermined quantums of the different states that a quantum can reach. According to Leibniz, the differential varies with the different consecutive values of the continuously diminishing quantity, i.e. there's a differential for each quantity that the series reaches and each of these differentials can be considered to be different, and each can be predicated of a different quality as an intuitive infinitely small, differential of extension or physical point. So, for Maimon, the representations of different manifolds of sensation are qualitatively different 'according to the difference of their differentials' (Maimon, 2010: p. 29).

While this explains that the different manifolds of sensation are different and distinct representations, it does not explain how each of these representations is brought to consciousness. Maimon outlines the next stage of this process by which the differential, as both an idea and unit of sensation, is brought to consciousness as follows:

> Sensibility thus provides the differentials to a determined consciousness; out of them, the imagination produces a finite (determined) object of intuition; out of the relations of these different differentials, which are its objects, the understanding produces the relation of the sensible objects arising from them. (Maimon, 2010: p. 32)

Sensibility provides the differentials as ideas of sensation, and the imagination produces a finite (determined) object of intuition from the manifold of sensation that results from the 'addition' (Maimon, 2010: p. 29n2) or sum of the differentials as determinate units of sensation.[10] Before explicating how this takes place, a more detailed account of Maimon's understanding of intuition is required.

For Maimon, intuition, like sensation, is also 'a modification of the cognitive faculty', however it is 'actualized within that faculty in part passively and in part actively' (Maimon, 2010: p. 168). The passive part is termed its matter, and is supplied by sensation. The active part is its form, which is supplied by the a priori intuitions of space and time. What has been accounted for so far in this explication is only the passive part of intuition. As regards the active part, Maimon maintains that

> consciousness first arises when the imagination takes together several homogeneous sensible representations, orders them according to its forms (succession in time and space), and forms an individual intuition out of them. (Maimon, 2010: p. 30)

Each homogeneous sensible representation that Maimon is referring to is the product of having taken together, or having added together successively, the differentials as ideas or objects of a particular sensation to form a manifold of sensation. This correlates with the successive addition of the differentials as determinate units of sensation that determines an 'arbitrary finite magnitude' (Maimon, 2010: p. 29n2), or 'finite (determined) object of intuition' (p. 32). However, it is only when manifolds of sensation are ordered according to the a priori intuitions of space and time that an arbitrary finite magnitude, or finite (determined) object of intuition is formed and brought to consciousness as an individual empirical intuition.

The example that Maimon gives of the way that two different homogeneous sensible representations, or manifolds of sensation, are ordered in space and time to form distinct individual empirical intuitions is the way a distinction is made between the perception, or passive intuition, of a red and a green manifold of sensation.

> When a perception, for example red, is given to me, I do not yet have any consciousness of it; when another, for example green, is given to me, I do not yet have any consciousness of it in itself either. But if I relate them to one another (by means of the unity of difference), then I notice that red is different from green, and so I attain consciousness of each of the perceptions in itself. If I constantly had the representation red, for example, without having any other representation, then I could never attain consciousness of it. (Maimon, 2010: pp. 131–2)

It is therefore only insofar as individual empirical intuitions are related to one another that they are brought to consciousness, and it is by means of what Maimon refers to as the 'unity of difference' that they are able to be related to one another. In the case of the representation of red and green, Maimon refers to this unity of difference as a relation between differentials:

> For example, if I say that red is different from green, then the pure concept of the understanding of the difference is not treated as a relation between the sensible qualities (for then the Kantian question *quid juris?* remains unanswered), but rather either (according to the Kantian theory) as the relation of their spaces as *a priori* forms, or (according to my theory) as the relation of their differentials, which are *a priori* ideas of reason. (Maimon, 2010: p. 33)

In the 'Notes & Clarifications' to the *Essay*, Maimon provides an account of how individual intuitions are brought to consciousness by means of the relations between their differentials, which he refers to in this passage initially as 'elements':

> the pure concepts of the understanding or categories are never directly related to intuitions, but only to their elements, and these are ideas of reason concerning the way these intuitions arise; it is through the mediation of these ideas that the categories are related to the intuitions themselves. Just as in higher mathematics we produce the relations of different magnitudes themselves from their differentials, so the understanding (admittedly in an obscure way) produces the real relations of qualities themselves from the real relations of their differentials. So, if we judge that fire melts wax, then this judgment does not relate to fire and wax as objects of intuition, but to their elements, which the understanding thinks in the relation of cause and effect to one another. (Maimon, 2010: pp. 355–6)

By 'higher mathematics', Maimon is referring to the operations of the calculus, where the ratios or relations of different magnitudes, for example *x:y*, can be produced from the ratio of their differentials, *dx:dy*. So too can the understanding apply this a priori rule to the elements of sensation, or differentials, to produce, 'admittedly in an obscure way', the real relations of qualities themselves from the real relations of their differentials.

The specific mathematical operation being referred to is integration. The mathematical concept of integration can be understood both as the inverse operation of differentiation and also as a method of summation in the form of series. The method of integration in general provides a way of working back from the differential relation to the construction of the curve whose tangent it represents. The problem of integration is therefore that of reversing the process of differentiation. That is, given a relation between two differentials, *dy/dx*, the problem of integration is to find a relation between the quantities themselves, *y* and *x*. What distinguishes the inverse operation of integration from integration as a method of summation in the form of series is that the operation of the former is dependent upon the brute application of different algorithms that have been proven to supply the correct solutions, whereas the latter is distinct insofar as it involves a process of summation, where successive coefficients are added together recursively to form an infinite series, the graphical representation of which approaches the solution curve. It is this difference that singles out the latter as being a more appropriate explanation of the process that Maimon has in mind. Given that the elements of sensation Maimon is working with are modeled on differentials, and that, as determinate units of sensation, they are characterized as being 'added to themselves successively' to determine an arbitrary finite magnitude or manifold of sensation, the method of integration that Maimon applies as a rule of the understanding to the elements of sensation should be understood implicitly to be the method of summation.

The application of the mathematical rule of the understanding, which is the operation of integration as a method of summation in the form of series, to the elements of sensation, which are modeled on differentials, brings the manifolds of sensation to consciousness as sensible objects of intuition. In the first step of the process, two different manifolds of sensation characterized by different differentials are brought into consciousness by virtue of the application of integration as a rule of the understanding to the elements of sensation that models the real relation between the two qualities themselves, as sensible objects, on the real relation between their differentials. This happens as follows. Each manifold of sensation is brought to consciousness as a sensible object by virtue of the relation between their respective differentials, *dy/dx*, and the application of the operation of integration to this relation. In integration, the differential relation *dy/dx* gives the slope of the tangent to the graph of a function, or curve, where the tangent is a straight line that touches a curve at only one point. Let's call this point *b*. It is important to note that at this stage of the operation there is no curve, the only information available is that consciousness

= 0 at a point, let's call this point *a*, and that a point *b* can be determined as a potential point of tangency by virtue of the contingent nature of the relation between two differentials of sensation.

The method of integration as a process of summation in the form of series that I suggest Maimon deploys is the method of approximation of a differentiable function around a given point provided by a Taylor series or power series expansion.[11] Buzaglo claims that 'Maimon was aware' of 'the mathematical developments of his time', and that 'he exhibited a profound understanding of their implications, which he eventually expressed in his critique of Kant' (Buzaglo, 2002: p. 67). We know that Maimon was aware of the work of Leonard Euler, Alexis Clairaut and Abraham Kästner, as he mentions them and their work in his autobiography (Maimon 1888) and in his *Philosophisches Wörterbuch* (1791). In the former, Maimon claims to have taught Euler's *Algebra* (1771) to one of his students (Maimon 1888: p. 274), and in the latter Maimon compares the different approaches to mathematics taken by Clairaut and Kästner in their texts on algebra (Maimon 1791: p. 154). Clairaut was well aware of Taylor's developments because in his 1734 paper, which developed what is now known as 'Clairaut's differential equations', he presents a number of geometric problems that require the determination of a curve in terms of the properties of its tangents that are 'common to all points of the curve' (Rozov, 2001). This work uses the very developments made by Taylor in 1715. In Clairaut's text, the *Elémens d'Algebre* (1746), which we know Maimon was aware of because of his comments about it in his autobiography, Clairaut introduces a new demonstration of the binomial formula, developed by Newton, and, drawing upon his previous work on Taylor, shows 'the different uses that can be drawn from this formula in order to find all sort of quantities by approximation, which could prepare the novice for infinite analysis' (Clairaut 1746: p. xiii). While these demonstrations are limited to simple approximations that are strictly algebraic, Clairaut does signal that more complex examples are more easily dealt with 'when aided by geometry'. However, he defers dealing with these more complex problems as they are beyond the purview of a textbook on algebra. This would not however have prevented those interested in seeking out examples of these more complex problems elsewhere in the mathematical literature of the time from doing so. Given Maimon's own demonstrated grasp of these kinds of problems, it is my claim that it is likely that he did in fact do this. In his autobiography, Maimon compares Clairaut's text, which starts by presenting and solving problems in such a way that 'the explanations and the theorems of this science are explained and proven in their natural order' (Maimon, 1791: p. 154), to that of Kästner (1768–9), which starts with abstract principles and theorems and only then derives the problems and examples. The advantage of Clairaut's text is that it is well suited for students, whereas Maimon finds Kästner's text preferable because it is more conducive to advanced study as it provides 'an overview of science and all its applications with the help of a few explanations and theorems' (Maimon, 1791: p. 154). This doesn't prevent Maimon from criticizing the presentation of some problems in Kästner, and in the work of other mathematicians (see Freudenthall, 2006: pp. 76–9), it does however signal Maimon's interest in the broader implications of mathematical problems, and demonstrates his aptitude to seek out and engage with the more complicated texts of his day. While Lagrange, a contemporary of Maimon's, did attempt to provide an algebraic proof of Taylor's theorem as early as 1772, the work in which it was published did not appear until 1797 (Lagrange, 1797), after Maimon had written the *Essay* (2010) in 1790.

The method of integration as a process of summation in the form of series is appropriate for Maimon because the coefficients of the function depend solely on the relations between the differentials at that point. The power series expansion can be written as a polynomial, the coefficients of each of its terms being the successive differential relations evaluated at the given point. The sum of such a series represents the differentiable function provided that any remainder approaches zero as the number of terms becomes infinite; the polynomial then becomes an infinite series which converges with the function around the given point. Given the differential relation, dy/dx, what can be determined at this point is the power series expansion of this differential relation. As the number of terms of the power series expansion approaches infinity, the polynomial of the power series converges with the function, which is therefore its limit.

For Maimon, the differentiable function would therefore be the idea of the understanding of the polynomial of the power series expansion of this particular differential relation, which we can only understand as a limit concept. Therefore the operation of integration that Maimon has in mind, and which is the rule of the understanding for how sensible objects are brought to consciousness, is the process of determining the polynomial of the power series expansion of the differential relation at the given point b to an arbitrary finite number of terms. It is common practice to use a finite number of terms of the series to approximate the function in the immediate neighborhood of the given point.[12] The finite polynomial of the power series expansion would be the a priori intuition of this particular relation. At the given point b of tangency to the curve, y can be approximated as a function of x in the immediate neighborhood of the given point by expanding the polynomial of the power series expansion to an arbitrary finite number of terms. x can therefore also be approximated as a function of y in the immediate neighborhood of the given point. x and y then function as the empirical correlates of the concept of the differentiable function as a limit concept. The result of attaching this limit concept to the a priori intuition of the arbitrarily finite polynomial brings the x and y to consciousness as intuitions, and x and y are then represented to consciousness as sensible objects.[13]

The clearest statement in the *Essay* of how these components of Maimon's system displace those of Kant's first *Critique* is as follows:

> These differentials of objects are the so-called *noumena*; but the objects themselves arising from them are the *phenomena*. With respect to intuition = 0, the differential of any such object in itself is $dx = 0$, $dy = 0$ etc.; however their relations are not = 0, but can rather be given determinately in the intuitions arising from them. These *noumena* are ideas of reason serving as principles to explain how objects arise according to certain rules of the understanding. (Maimon, 2010: p. 32)

Maimon is referring to the relation between *dy* and *dx* in the differential relation *dy/dx*, which despite the terms equaling zero, does not itself equal zero. In mathematics, while the terms between which the relation is established are neither determined nor determinable, the relation between the terms is determined,[14] and is the basis for determining the real relation between the qualities themselves by means of the operation of integration as a method of summation. The Kantian *noumena* is displaced by the metaphysical infinitely small, which is predicated of the quality of a sensible representation by virtue of the application of a rule of the understanding to sensation. And Kantian *phenomena* is displaced by the sensible objects produced by the synthetic unity which is determined by the operation of integration on these infinitely small elements.

To return to the example of the judgment that fire melts wax that Maimon gives in the *Essay* (2010: p. 356), two steps are required to make this judgment. The first involves the application of the mathematical rule of the understanding, which is the concept of integration as a method of summation, to the elements of sensation, i.e. differentials, which brings the manifolds of sensation to consciousness as sensible objects of intuition that are then ordered in space and time. The second is the judgment that involves the application of the pure concept of cause to the differentials, which appear as sensible objects. This is because pure concepts of the understanding, whether mathematical or categorical, 'never relate to intuitions, but only to their elements' (Maimon, 2010: p. 355). It is the relation between these elements that gives rise to the sensible intuitions in the first place. Maimon describes a similar judgment in relation to the elements of heat and presumably frozen water as follows: 'there is a necessity connected with the actual perception of fluidity following heat … from which I judge that heat makes the water fluid (is the cause)' (Maimon, 2010: p. 129). The judgment in the case of the wax applies the pure concept of cause to the elements of the intuited relation between fire and wax, i.e. to their differentials as qualities of magnitudes. The judgment that fire (as the cause) melts wax is then made in accord with the 'necessity connected with the actual perception of fluidity following heat'. The application of mathematical rules of the understanding to sensation is what determines the objects of sensation as objects and makes them available to be ordered in space and time and therefore available as the objects of categorical judgments.

When it comes to regulative ideas, Maimon distinguishes himself from Kant by proposing 'a single Idea (of an infinite understanding)' to displace Kant's three Transcendental Ideas: God, the World and the human Soul. Maimon attributes an

> objective reality to this idea (not, it is true, viewed in itself – for this is contrary to the nature of an idea – but only in so far as it acquires objective reality for us in so many ways by means of objects of intuition). And also the other way around, i.e. intuitions acquire objective reality only because they must eventually resolve into this idea … Now the understanding … insists on absolute totality in these concepts so that this totality belongs as much to the essence of the understanding as concepts in general even if we cannot attain it. (Maimon, 2010: p. 367)

The regulative use of the concept of the infinite understanding does not make Maimon's system theocentric. Nor does Maimon presuppose the infinite understanding as 'the originator of the world of appearances', whose ideas constitute its elements.[15] The infinite understanding for Maimon is only an idea of the understanding that functions as an ultimate limit concept that our understanding continuously approaches without ever reaching. The limit concept is applied to the intuition of a totality of objects, where the thought of the element of each is perceived as conditioned by the thought of all the others. This is a totality, understood as an idea of reason, that approaches the infinite; however, it is not, or is only erroneously considered by the imagination to be, a privileged reality projected as external to us like an object.

The hypothesis that Maimon adopts in order to render the connection between pure a priori concepts and their intuitions comprehensible is that both are modifications of the same cognitive faculty. The question quid juris is thus resolved because the understanding does not subject something given in a different faculty, the faculty of the imagination, to its rules a priori, as is the case with the Kantian schematism. What Maimon proposes instead is that the understanding produces this something as an intuition that conforms to its rules by virtue of being of the same faculty. One of the conditions of the exercise of the understanding in bringing sensible objects of intuition to consciousness is the prior existence of difference (as intrinsic, between intensive magnitudes, and not simply numerical difference). Maimon's idealism therefore accounts for representation without recourse to the thing in itself. Instead he uses the model of the differential in Taylor series as the means of solving the problem of explaining the intellectual character of the content that is given to us.

The production of phenomena according to the model of differentials and their relations is made in the same way that mathematical figures are determined in intuition, i.e. according to the rule that expresses the image of this figure. However, sensations in themselves, of which differentials are predicated, do not yet result in consciousness. What is required to bring sensations to consciousness is the integration of the differentials by means of the method of summation. From the point of view of the finite understanding, the consciousness of sensations is not reflected upon as happening according to the application of a mathematical rule of the understanding, but rather is (erroneously) considered to actually be produced by the differentials themselves.

As for the question quid facti, however, Maimon defers to Hume (see Maimon, 2010: pp. 9, 215, 371).[16] Maimon doesn't doubt that sensation is presented to us, he rather considers the presentation of sensation and the identities inferred in its mode of presentation to be produced in accordance with the mathematical rules of the understanding applied to sensation, i.e. the rule according to which the understanding thinks the object. However, because of our limited understanding we can only assume that the world is a product of reason and of the regulative idea of the infinite understanding, but this cannot be proved. The rational principle that matter flows from the understanding therefore remains merely a *hypothesis*.[17] The gap between the given sensation and the a priori rule is still not bridged. Maimon's hypothesis is therefore tested and found wanting, the result being that skepticism is not fully eradicated from the system.

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Notes

1 According to Maimon, 'what is justified is what is legitimate, and with respect to thought, something is justified if it conforms to the laws of thought or reason' (2010: p. 363). English translations are from Maimon 2010, which also provides page numbers to Maimon 2004.
2 However, unlike Kant, Maimon deduces the formal forms from the categories, rather than the inverse, and Maimon does not recognize the category of quantity. See Bergman 1967: pp. 117–20.
3 For further discussion of Maimon's interpretation of number as a ratio and not as a multitude of units, see Freudenthal, 2011: pp. 89–97.
4 See Freudenthal's (2006) exhaustive analysis of Maimon's prevarications with respect to the status of this proposition about the straight line. While my analysis of the proposition is different, Freudenthal concurs that the proposition is a synthetic a priori judgment for Maimon that is, however, understood differently to Kant, see Freudenthal, 2006: pp. 95, 102, 110–11. While Freudenthal surveys all of Maimon's work for an account of this difference, the focus of this paper is the role of the differential in providing an account of this difference in the *Essay*.
5 See also Freudenthal's discussion of this point: Freudenthal, 2006: p. 29.
6 While Maimon doesn't refer specifically to 'the idea of sensation', in the paragraph cited above he does refer to 'sensation' as an 'idea' that can only be approached (Maimon, 2010: p. 168).
7 Maimon's theory of the differential has proved to be a rather enigmatic aspect of his system. Commentators have argued either that it plays a central role in determining the structure of this system (Kuntze, 1912; Atlas, 1964; Bergman, 1967; Engstler, 1990), or on the contrary that it as too incoherent to do so (Buzaglo, 2002: p. 125). Alternatively they have focused on the importance of other aspects

of Maimon's work because it is too ambiguous to play such a central role (Beiser, 1987; Bransen, 1991; Franks, 2005). Thielke provides a more balanced approach to the concept of the differential as it operates in Maimon, without however taking his analysis as far as it could, and arguably should, be taken (Thielke, 2003: pp. 115–19). In a recent article, Ehrensperger remarks on this enigmatic status by noting that 'Despite its prominence, an in-depth study of the differential in Maimon is still a desideratum' (Ehrensperger, 2010: p. 2). What is offered in the remainder of this paper is a study of the differential and the role that it plays in the *Essay* by drawing upon mathematical developments that had occurred earlier in the century.

8 For further discussion of the Leibnizian syncategorematic or fictional definition of the infinitesimal, see Jesseph, 2008: pp. 215–34.

9 This account runs counter to those claims by commentators, such as Bergman (1967: p. 68) and Engstler (1990: pp. 23, 143), that differentials should themselves somehow be understood to be real.

10 Maimon does not mean a simple sum of addition, because infinitesimals, being infinitesimal, cannot be added to one another to produce a finite magnitude. He is rather referring to a particular method of integration that is a method of summation in the form of series, which is discussed below.

11 Taylor, 1715. Taylor actually adopts the Newtonian methodology of 'fluxions' in his account of power series expansions. While the fluent and the differential were in general used interchangeably by mathematicians at the time, it has also been argued that Maimon uses them interchangeably (Kuntze, 1912: p. 329).

12 Note that the greater the number of terms, the greater the degree of approximation.

13 If the differential of a third quantity has a relation with either of the other two, then it in turn can also be determined as a sensible object by means of the same procedure.

14 Maimon is drawing here upon Leibniz's account of the differential and the differential relation.

15 This is the thesis proposed by Engstler, 1990: p. 24.

16 For further discussion of the question quid facti and of the reasons for Maimon's acceptance of Hume's objection, see Freudenthal, 2006: pp. 39–45.

17 i.e. a hypothesis of a regulative idea.

References

Atlas, Samuel H. (1964) *From Critical to Speculative Idealism: The Philosophy of Solomon Maimon*, The Hague: Nijhoff.

Beiser, Frederick C. (1987) *The Fate of Reason: German Philosophy from Kant to Fichte*, Cambridge: Harvard University Press.

Bergman, Samuel Hugo (1967) *The Philosophy of Solomon Maimon*, Jerusalem: Magnes Press.

Bransen, Jan (1991) *The Antinomy of Thought: Maimonian Skepticism and the Relation between Thoughts and Objects*, Boston: Kluwer.

Buzaglo, Meir (2002) *Solomon Maimon: Monism, Skepticism, and Mathematics*, Pittsburgh: University of Pittsburgh Press.

Clairaut Alexis (1746) *Elémens d'Algebre*. Paris: Guérin, David, Durand; German trans. C. Mylius (1752) *Anfangsgründe der Algebra*. Berlin: Nicolai. 2nd edn (1778) Leipzig: Kummer.

Ehrensperger Florian (2010) 'The Philosophical Significance of Maimon's Essay on Transcendental Philosophy', *Journal for Jewish Thought*, University of Toronto.

Engstler, Achim (1990) *Untersuchungen zum Idealismus Salomon Maimons*, Stuttgart-Bad Cannstatt: Frommann-Holzboog.

Euclid (1956) *The Thirteen Books of Euclid's Elements*, 2nd edn, trans. T. Heath, New York: Dover.

Euler Leonard (1771) *Anleitung zur Algebra*, St Petersburg: Royal Academy of Sciences.

Franks, Paul (2005) *All or Nothing: Systematicity, Transcendental Arguments, and Skepticism in German Idealism*, Cambridge: Harvard University Press.

Freudenthal, Gideon (2006) *Definition and Construction: Salomon Maimon's Philosophy of Geometry*, Preprint 317, Berlin: Max-Planck-Institute.

——— (2011) 'Maimon's Philosophical Program: Understanding versus Intuition', *International Yearbook of German Idealism* 8 (2010): 83–105.

Jesseph, Douglas (2008) 'Truth in Fiction: Origins and Consequences of Leibniz's Doctrine of Infinitesimal Magnitudes', in U. Goldenbaum & D. Jesseph (eds) *Infinitesimal Differences: Controversies between Leibniz and his Contemporaries*, New York: de Gruyter.

Kant, Immanuel (1998) *Critique of Pure Reason*, trans. P. Guyer and A. Wood, New York: Cambridge University Press.

Kästner Abraham (1768–9) *Der mathematischen Anfangsgründe*, Vols 1–4, Göttingen.

Kuntze, Friedrich (1912) *Die Philosophie Salomon Maimon*, Heidelberg: Winter.

Lagrange Joseph Louis (1797) *Théorie des fonctions analytiques*, Paris: République.

Maimon Salomon (1791) *Philosophisches Wörterbuch*, Berlin: Unger.

——— (1888) *Solomon Maimon: An Autobiography*, trans. J. Clark Murray, London: Gardner.

——— (2004) *Versuch über die Transzendentalphilosophie* (1790), ed. F. Ehrensperger, Hamburg: Meiner.

——— (2010) *Essay on Transcendental Philosophy* (1790), trans. N. Midgley et al., London: Continuum.

Rozov Kh., N. Kh. (2001) 'Clairaut Equation', *Encyclopedia of Mathematics*, ed. M. Hazewinkel, Berlin: Springer.

Taylor Brook (1715) *Methodus Incrementorum Directa et Inversa*, London: Innys.

Thielke, Peter (2003) 'Intuition and Diversity: Kant and Maimon on Space and Time', in G. Freudenthal (ed.) *Salomon Maimon: Rational Dogmatist, Empirical Skeptic, Critical Assessments*, Dordrecht: Kluwer.