Routledge
Taylor & Francis Group

ANGELAKI
journal of the theoretical humanities
volume 9 number 3 december 2004

simon duffy

SCHIZO-MATH[1]
the logic of different/ciation and the philosophy of difference

In the paper "Math Anxiety," Aden Evens explores the manner by means of which concepts are implicated in the problematic Idea according to the philosophy of Gilles Deleuze. The example that Evens draws from *Difference and Repetition* in order to demonstrate this relation is a mathematics problem, the elements of which are the differentials of the differential calculus. What I would like to offer in the present paper is an historical account of the mathematical problematic that Deleuze deploys in his philosophy, and an introduction to the role that this problematic plays in the development of his philosophy of difference. One of the points of departure that I will take from the Evens paper is the theme of "power series."[2] This will involve a detailed elaboration of the mechanism by means of which power series operate in the differential calculus deployed by Deleuze in *Difference and Repetition.* Deleuze actually constructs an alternative history of mathematics that establishes an historical continuity between the differential point of view of the infinitesimal calculus and modern theories of the differential calculus. It is in relation to the differential point of view of the infinitesimal calculus that Deleuze determines a differential logic which he deploys, in the form of a logic of differen*t*/*c*iation, in the development of his project of constructing a philosophy of difference.

the differential point of view of the infinitesimal calculus

The concept of the differential was introduced by developments in the infinitesimal calculus during the latter part of the seventeenth century. Carl Boyer, in *The History of the Calculus and its Conceptual Development*, describes the early stages of this development as being "bound up with concepts of geometry [...] and with explanations of [...] the infinitely small."[3] Boyer describes the infinitesimal calculus as dealing with "the infinite sequences [...] obtained by continuing [...] to diminish ad infinitum the intervals between the values of the independent variable. [...] By means of [these] successive subdivisions [...] the smallest possible intervals or differentials" are obtained (CB 12). The differential can therefore be understood to be the infinitesimal difference between consecutive values of a continuously diminishing quantity. Boyer refers to this early form of the infinitesimal calculus as the infinitesimal calculus from "the differential point of view" (CB 12). From the differential point of view of the infinitesimal calculus, Boyer

ISSN 0969-725X print/ISSN 1469-2899 online/04/030199–17
DOI: 10.1080/0969725042000307727

argues that "the derivative would [...] be defined as the quotient of two such differentials, and the integral would then be the sum of a number (perhaps finite, perhaps infinite) of such differentials" (CB 12).

The infinitesimal calculus consists of two branches which are inverse operations: differential calculus, which is concerned with calculating derivatives, or differential relations; and integral calculus, which is concerned with integration, or the calculation of the infinite sum of the differentials. The derivative, from the differential point of view of the infinitesimal calculus, is the quotient of two differentials, that is, a differential relation, of the type dy/dx. The differential, dy, is an infinitely small quantity, or what Deleuze describes as "a vanishing quantity":[4] a quantity smaller than any given or givable quantity. Therefore, as a vanishing quantity, dy, in relation to y, is, strictly speaking, equal to zero. In the same way dx, in relation to x, is, strictly speaking, equal to zero; that is, dx is the vanishing quantity of x. Given that y is a quantity of the abscissa, and that x is a quantity of the ordinate, $dy = 0$ in relation to the abscissa, and $dx = 0$ in relation to the ordinate. The differential relation can therefore be written as $dy/dx = 0/0$. However, although dy is nothing in relation to y, and dx is nothing in relation to x, dy over dx does not cancel out, that is, dy/dx is not equal to zero. When the differentials are represented as being equal to zero, the relation can no longer be said to exist since the relation between two zeros is zero, that is $0/0 = 0$; there is no relation between two things which do not exist. However, the differentials do actually exist. They exist as vanishing quantities in so far as they continue to vanish as quantities rather than having already vanished as quantities. Therefore, despite the fact that, strictly speaking, they equal zero, they are still not yet, or not quite equal to, zero. The relation between these two differentials, dy/dx, therefore does not equal zero, $dy/dx \neq 0$, despite the fact that $dy/dx = 0/0$. Instead, the differential relation itself, dy/dx, subsists as a relation. "What subsists when dy and dx cancel out under the form of vanishing quantities is the relation dy/dx itself" (DSS). Despite the fact that its terms vanish, the relation itself is nonetheless real. It is here that Deleuze considers seventeenth-century logic to have made "a fundamental leap," by determining "a logic of relations" (DSS). He argues that "under this form of infinitesimal calculus is discovered a domain where the relations no longer depend on their terms" (DSS). The concept of the infinitely small as vanishing quantities allows the determination of relations independently of their terms. "The differential relation presents itself as the subsistence of the relation when the terms vanish" (DSS). According to Deleuze, "the terms between which the relation establishes itself are neither determined, nor determinable. Only the relation between its terms is determined" (DSS). This is the logic of relations that Deleuze locates in the infinitesimal calculus of the seventeenth century.

The differential relation, which Deleuze characterises as a "pure relation" (DSS) because it is independent of its terms, and which subsists in so far as $dy/dx \neq 0$, has a perfectly expressible finite quantity designated by a third term, z, such that dy/dx equals z. Deleuze argues that "when you have a [differential] relation derived from a circle, this relation doesn't involve the circle at all but refers [rather] to what is called a tangent" (DSS). A tangent is a line that touches a circle or curve at one point. The gradient of a tangent indicates the rate of change of the curve at that point, that is, the rate at which the curve changes on the y-axis relative to the x-axis. The differential relation therefore serves in the determination of this third term, z, the value of which is the gradient of the tangent to the circle or curve.

When referring to the geometrical study of curves in his early mathematical manuscripts, Leibniz writes that "the differential calculus could be employed with diagrams in an even more wonderfully simple manner than it was with numbers."[5] Leibniz presents one such diagram in a paper entitled "Justification of the Infinitesimal Calculus by that of Ordinary Algebra," when he offers an example of what had already been established of the infinitesimal calculus in relation to particular problems before the greater generality of its methods was devel-

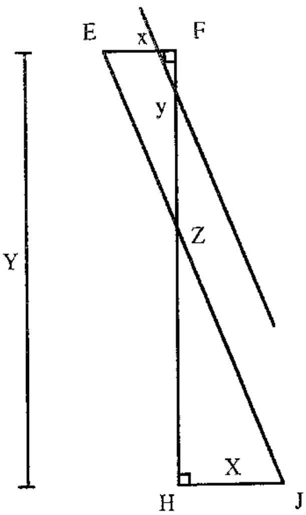

Fig. 1.

oped.[6] An outline of the example that Leibniz gives is shown in Fig. 1.

Since the two right triangles, ZFE and ZHJ, that meet at their apex, point Z, are similar, it follows that the ratio y/x is equal to $(Y - y)/X$. As the straight line EJ approaches point F, maintaining the same angle at the variable point Z, the lengths of the straight lines FZ and FE, or y and x, steadily diminish, yet the ratio of y to x remains constant. When the straight line EJ passes through F, the points E and Z coincide with F, and the straight lines, y and x, vanish. Yet y and x will not be absolutely nothing since they preserve the ratio of ZH to HJ, represented by the proportion $(Y - y)/X$, which in this case reduces to Y/X, and obviously does not equal zero. The relation y/x continues to exist even though the terms have vanished since the relation is determinable as equal to Y/X. In this algebraic calculus, the vanished lines x and y are not taken for zeros since they still have an algebraic relation to each other. "And so," Leibniz argues, "they are treated as infinitesimals, exactly as one of the elements which [...] differential calculus recognises in the ordinates of curves for momentary increments and decrements" (PPL 545). That is, the vanished lines x and y are determinable in relation to each other only in so far as they can be replaced by the infinitesimals dy and dx, by making the supposition that the ratio y/x is equal to the ratio of the infinitesimals, dy/dx. In the first published account of the calculus, Leibniz defines the ratio of infinitesimals as the quotient of first-order differentials, or the differential relation. He says that "the differential dx of the abscissa x is an arbitrary quantity, and that the differential dy of the ordinate y is defined as the quantity which is to dx as the ratio of the ordinate to the subtangent" (CB 210). Leibniz considers differentials to be the fundamental concepts of the infinitesimal calculus, the differential relation being defined in terms of these differentials.

a new theory of relations

Leibniz recognised integration to be a process not only of summation but also of the inverse transformation of differentiation, so the integral is not only the sum of differentials but also the inverse of the differential relation. In the early nineteenth century, the process of integration as a summation was overlooked by most mathematicians in favour of determining integration, instead, as the inverse transformation of differentiation. The main reason for this was that by extending sums to an infinite number of terms, problems began to emerge if the series did not converge. The value or sum of an infinite series is determinable only if the series converges. Divergent series have no sum. It was considered that reckoning with divergent series would therefore lead to false results. The problem of integration as a process of summation from the differential point of view of the infinitesimal calculus did, however, continue to be explored. It was Augustin Cauchy (1789–1857) who first introduced specific tests for the convergence of series, so that divergent series could henceforth be excluded from being used to try to solve problems of integration because of their propensity to lead to false results (CB 287).

The object of the process of integration in general is to determine from the coefficients of the given function of the differential relation the original function from which they were derived. Put simply, given a relation between two differentials, dy/dx, the problem of integration is how to find a relation between the quantities themselves, y and x. This problem corresponds to the geometrical method of finding the function of a curve characterised by a given property of its tangent. The differential relation is thought of as another function which describes, at each point on an original function, the gradient of the line tangent to the curve at that point. The value of this "gradient" indicates a specific quality of the original function; its rate of change at that point. The differential relation therefore indicates the specific qualitative nature of the original function at the different points of the curve.

The inverse process of this method is differentiation, which, in geometrical terms, determines the differential relation as the function of the line tangent to a given curve. Put simply, to determine the tangent of a curve at a specified point, a second point that satisfies the function of the curve is selected, and the gradient of the line that runs through both of these points is calculated. As the second point approaches the point of tangency, the gradient of the line between the two points approaches the gradient of the tangent. The gradient of the tangent is, therefore, the limit of the gradient of the line between the two points.

It was Newton who first came up with this concept of a limit. He conceptualised the tangent geometrically, as the limit of a sequence of lines between two points on a curve, which he called a secant. As the distance between the points approached zero, the secants became progressively smaller; however, they always retained "a real length." The secant therefore approached the tangent without reaching it. When this distance "got arbitrarily small (but remained a real number),"[7] it was considered insignificant for practical purposes, and was ignored. What is different in Lebniz's method is that he "hypothesized infinitely small numbers – infinitesimals – to designate the size of infinitely small intervals" (LN 224). For Newton, on the contrary, these intervals remained only small, and therefore real. When performing calculations, however, both approaches yielded the same results. But they differed ontologically, because Leibniz had hypothesised a new kind of number, a number Newton did not need, since "his secants always had a real length, while Leibniz's had an infinitesimal length" (LN 224).

For the next two hundred years, various attempts were made to find a rigorous arithmetic foundation for the calculus. One that did not rely on either the mathematical intuition of geometry, with its tangents and secants, which was perceived as imprecise because its conception of limits was not properly understood; or on the vagaries of the infinitesimal, which made many mathematicians wary, so much so that they refused the hypothesis outright, despite the fact that Leibniz "could do calculus using arithmetic without geometry – by using infinitesimal numbers" (LN 224–25). It was not until the late nineteenth century that an adequate solution to this problem of rigour was posed. It was Karl Weierstrass (1815–97) who "developed a pure nongeometric arithmetization for Newtonian calculus" (LN 230), which provided the rigour that had been lacking. "Weierstrass's theory was an updated version of Cauchy's earlier account" (LN 309), which had also had problems conceptualising limits. Cauchy actually begs the question of the concept of limit in his proof.[8] In order to overcome this problem of conceptualising limits, Weierstrass "sought to eliminate all geometry from the study of [...] derivatives and integrals in calculus" (LN 309). In order to characterise calculus purely in terms of arithmetic, it was necessary for the idea of a function, as a curve in the Cartesian plane defined in terms of the motion of a point, to be completely replaced with the idea of a function that is, rather, a set of ordered pairs of real numbers. The geometric idea of "approaching a limit" had to be replaced by an arithmetised concept of limit that relied on static logical constraints on numbers alone. This approach is commonly referred to as the epsilon-delta method. Deleuze argues that:

> It is Weierstrass who bypasses all the interpretations of the differential calculus from Leibniz to Lagrange, by saying that it has nothing to do with a process [...] Weierstrass gives an interpretation of the differential and infinitesimal calculus which he himself calls

static, where there is no longer fluctuation towards a limit, nor any idea of threshold.[9]

The calculus was thereby reformulated without either geometric secants and tangents or infinitesimals; only the real numbers were used.

Because there is no reference to infinitesimals in this Weierstrassian definition of the calculus, the designation "the infinitesimal calculus" was considered to be "inappropriate" (CB 287). Weierstrass's work not only effectively removed any remnants of geometry from what was now referred to as the differential calculus but it also eliminated the use of Leibniz-inspired infinitesimal arithmetic in doing the calculus for over half a century. It was not until the late 1960s, with the development of the controversial axioms of non-standard analysis by Abraham Robinson (1918–74), that the infinitesimal was given a rigorous foundation,[10] and a formal theory of the infinitesimal calculus was constructed, thus allowing Leibniz's ideas to be "fully vindicated,"[11] as Newtown's had been thanks to Weierstrass.

It is specifically in relation to these developments that Deleuze contends that, when understood from the differential point of view of that infinitesimal calculus, the value of z, which was determined by Leibniz in relation to the differential relation, dy/dx, as the gradient of the tangent, functions as a limit. When the relation establishes itself between infinitely small terms it does not cancel itself out with its terms but rather tends towards a limit. In other words, when the terms of the differential relation vanish, the relation subsists because it tends towards a limit, z. Since the differential relation approaches closer to its limit as the differentials decrease in size, or approach zero, the limit of the relation is represented by the relation between the infinitely small. Of course, despite the geometrical nature of the idea of a variable and a limit, where variables "decrease in size" or "approach zero," and the differential relation "approaches" or "tends towards" a limit, they are not essentially dynamic, but involve purely static considerations, that is, they are rather "to be taken automatically as a kind of shorthand for the corresponding developments of the epsilon-delta approach" (LN 277). It is in this sense that the differential relation between the infinitely small refers to something finite. Or, as Deleuze suggests, it is in the finite itself that there is the "mutual immanence" (DSS) of the relation and the infinitely small.

Given that the method of integration provides a way of working back from the differential relation, the problem of integration is, therefore, how to reverse this process of differentiation. This can be solved by determining the inverse of the given differential relation according to the inverse transformation of differentiation. Or, a solution can be determined from the differential point of view of the infinitesimal calculus by considering integration as a process of summation in the form of a series, according to which, given the specific qualitative nature of a tangent at a point, the problem becomes that of finding not just one other point determinative of the differential relation but a sequence of points, all of which together satisfy, or generate, a curve and therefore a function in the neighbourhood of the given point of tangency, which therefore functions as the limit of the function.

Deleuze considers this to be the base of the infinitesimal calculus as understood or interpreted in the seventeenth century. The formula for the problem of the infinite that Deleuze extracts from this seventeenth-century understanding of the infinitesimal calculus is that "something finite consists of an infinity under a certain relation" (DSS). Deleuze considers this formula to mark "an equilibrium point, for seventeenth-century thought, between the infinite and the finite, by means of a new theory of relations" (DSS). It is the logic of this theory of relations that provides a starting point for the investigation into the logic that Deleuze deploys in *Difference and Repetition* as a part of his project of constructing a philosophy of difference.

the logic of the differential

Having located the logic of the differential from the differential point of view of the infinitesimal calculus in the work of Leibniz, the subsequent developments that this logic undergoes will now be examined in relation to the work of some of the key figures in the history of this branch of the infinitesimal calculus. These figures are implicated in an alternative lineage in the history of mathematics by means of which the differential point of view of the infinitesimal calculus is aligned with the differential calculus of contemporary mathematics. The logic of the differential from the differential point of view of the infinitesimal calculus is then implicated in the development of Deleuze's project of constructing a philosophy of difference. The manner by means of which the figures in the history of the differential point of view of the infinitesimal calculus are implicated in an alternative lineage in the history of mathematics will now be examined.

Ironically, one of the mathematicians who contributed to the development of the differential point of view of the infinitesimal calculus is Karl Weierstrass, who considers the differential relation to be logically prior to the function in the process of determination associated with the infinitesimal calculus; that is, rather than determining the differential relation from a given function, the kinds of mathematical problems that Weierstrass dealt with involved investigating how to generate a function from a given differential relation. Weierstrass develops a theory of integration as the approximation of functions from differential relations according to a process of summation in the form of series. Despite Weierstrass having eliminated both geometry and the infinitesimal from the calculus, Deleuze recovers this theory in order to restore the Leibnizian perspective of the differential, as the genetic force of the differential relation, to the differential point of view of the infinitesimal calculus, by means of the infinitesimal axioms of non-standard analysis.

According to Deleuze's reading of the infinitesimal calculus from the differential point of view, a function does not precede the differential relation, but rather is determined by the differential relation. The differential relation is used to determine the overall shape of the curve of a function primarily by determining the number and distribution of its distinctive points, which are points of articulation where the nature of the curve changes or the function alters its behaviour. For example, in geometrical terms, when the differential relation is zero, the gradient of the tangent at that point is horizontal, indicating that the curve peaks or dips, therefore determining a maximum or minimum at that point. These distinctive points are known as stationary or turning points. The differential relation characterises or qualifies not only the distinctive points which it determines but also the nature of the regular points in the immediate neighbourhood of these points, that is, the shape of the branches of the curve between each distinctive point. Where the differential relation gives the value of the gradient at the distinctive point, the value of the derivative of the differential relation, that is, the second derivative, indicates the rate at which the gradient is changing at that point, which allows a more accurate approximation of the nature of the function in the neighbourhood of that point. The value of the third derivative indicates the rate at which the second derivative is changing at that point. In fact, the more successive derivatives that can be evaluated at the distinctive point, the more accurate will be the approximation of the function in the immediate neighbourhood of that point.

This method of approximation using successive derivatives is formalised in the calculus according to Weierstrass's theory by a Taylor series or power series expansion. A power series expansion can be written as a polynomial, the coefficients of each of its terms being the successive derivatives evaluated at the distinctive point. The sum of such a series represents the expanded function provided that any remainder approaches zero as the number of terms becomes infinite; the polynomial then becomes an infinite series which converges with the function in the neighbourhood of the distinctive point.[12] This criterion of convergence repeats Cauchy's earlier exclusion of divergent series from the calculus. A power series operates at each distinctive point by successively determining the specific qualitative nature of the function at that point. The power series determines not only the specific qualitative nature of the function at the distinctive point in question but also the specific qualitative nature of all of the regular points in the neighbourhood of that distinctive point, such that the specific qualitative nature of a function in the neighbourhood of a distinctive point insists in that one point. By examining the relation between the differently distributed distinctive points determined by the differential relation, the regular points which are continuous between the distinctive points, that is, in geometrical terms, the branches of the curve, can be determined. In general, the power series converges with a function by generating a continuous branch of a curve in the neighbourhood of a distinctive point. To the extent that all of the regular points are continuous across all of the different branches generated by the power series of the other distinctive points, the entire complex curve or the whole analytic function is generated.

So, according to Deleuze's reading of the infinitesimal calculus, the differential relation is generated by differentials and the power series are generated in a process involving the repeated differentiation of the differential relation. It is due to these processes that a function is generated to begin with. The mathematical elements of this interpretation are most clearly developed by Weierstrassian analysis, according to the theorem on the approximation of analytic functions. An analytic function, being secondary to the differential relation, is differentiable, and therefore continuous, at each point of its domain. According to Weierstrass, for any continuous function on a given interval, or domain, there exists a power series expansion which uniformly converges to this function on the given domain. Given that a power series approximates a function in such a restricted domain, the task is then to determine other power series expansions that approximate the same function in other domains. An analytic function is differentiable at each point of its domain, and is essentially defined for Weierstrass from the neighbourhood of a distinctive point by a power series expansion which is convergent with a "circle of convergence" around that point. A power series expansion that is convergent in such a circle represents a function that is analytic at each point in the circle. By taking a point interior to the first circle as a new centre, and by determining the values of the coefficients of this new series using the function generated by the first series, a new series and a new centre of convergence is obtained, whose circle of convergence overlaps the first. The new series is continuous with the first if the values of the function coincide in the common part of the two circles. This method of "analytic continuity" allows the gradual construction of a whole domain over which the generated function is continuous. At the points of the new circle of convergence which are exterior to or extend outside of the first, the function represented by the second series is then the analytic continuation of the function defined by the first series – what Weierstrass defines as the analytic continuation of a power series expansion outside its circle of convergence. The domain of the function is extended by the successive adjunction of more and more circles of convergence. Each series expansion which determines a circle of convergence is called an element of the function.[13] In this way, given an element of an analytic function, by analytic continuation one can obtain the entire analytic function over an extended domain. The analytic continuation of power series expansions can be continued in this way in all directions up to the points in the immediate neighbourhood exterior to the circles of convergence where the series obtained diverge.

Power series expansions diverge at specific "singular points" or "singularities" that may arise in the process of analytic continuity. A singular point or singularity of an analytic function is any point which is not a regular or ordinary point of the function. They are points which exhibit distinctive properties and thereby have a dominating and exceptional role in the determination of the characteristics of the function.[14] The distinctive points of a function, which include the turning points, where $dy/dx = 0$, and points of inflection, where $d^2y/dx^2 = 0$, are "removable singular points," since the power series at these points converge with the function. A removable singular point is uniformly determined by the function and therefore redefinable as a distinctive point of the function, such that the function is analytic or continuous at that point. The specific singularities of an analytic function where the series obtained diverge are called "poles." Singularities of this kind are those points where the function no longer satisfies the conditions of regularity which assure its local continuity, such that the rule of analytic continuity breaks down. They are therefore points of discontinuity. A singularity is called a pole of a function when the values of the differential relation, that is, the gradients of the tangents to the points of the function, approach infinity as the function approaches the pole. The function is said to be asymptotic to the pole; it is therefore no longer differentiable at that point but rather remains undefined, or vanishes. A pole is therefore the limit point of a function, and is referred to as an accumulation point or point of condensation. A pole can also be referred to as a jump discontinuity in relation to a finite discontinuous interval both within the same function, for example periodic functions, and between neighbouring analytic functions. Deleuze writes that "a singularity is the point of departure for a series which extends over all the ordinary points of the system, as far as the region of another singularity which itself gives rise to another series which may either converge or diverge from the first" (DR 278). The singularities whose series converge are removable singular points, and those whose series diverge are poles.

The singularities, or poles, that arise in the process of analytic continuity necessarily lie on the boundaries of the circles of convergence of power series. In the neighbourhood of a pole, a circle of convergence extends as far as the pole in order to avoid including it, and the poles of any neighbouring functions, within its domain. The effective domain of an analytic function determined by the process of the analytic continuation of power series expansions is therefore limited to that between its poles. With this method the domain is not circumscribed in advance but results rather from the succession of local operations.

Power series can be used in this way to solve differential relations by determining the analytic function into which they can be expanded. Weierstrass developed his theory alongside the integral conception of Cauchy, which further developed the inverse relation between the differential and the integral calculus as the fundamental theorem of the calculus. The fundamental theorem maintains that differentiation and integration are inverse operations, such that integrals are computed by finding anti-derivatives, which are otherwise known as primitive functions. There are a number of rules, or algorithms, according to which this reversal is effected.

Deleuze presents Weierstrass's theorem of approximation as an effective method for determining the characteristics of a function from the differential point of view of the infinitesimal calculus. The mathematician Albert Lautman (1908–44) refers to this process as integration from "the local point of view," or simply as "local integration."[15] This form of integration does not involve the determination of the primitive function, which is generated by exercising the inverse operation of integration. The development of a local point of view, rather, requires the analysis of the characteristics of a function at its singular points. The passage from the analytic function defined in the neighbourhood of a singular point to the analytic function defined in each ordinary point is made according to the ideas of Weierstrass, by analytic continuity. This method was eventually deduced from the Cauchy point of view, such that the Weierstrassian approach was no longer emphasised. The unification of both of these points of view, however, was achieved at the beginning of the twentieth century when the rigour of Cauchy's ideas, which were then fused with those of Georg Riemann (1826–66), the other major contributor to the development of the theory of functions, was improved. Deleuze is therefore able to cite the contribution of Weierstrass's theorem of approximation in the development of the differential point of view of the infinitesimal calculus as an alternative point of view of the differential calculus to that developed by Cauchy, and thereby establish an historical continuity between Leibniz's differential point of view of the infinitesimal calculus and the differential calculus of contemporary mathematics, thanks to the axioms of non-standard analysis which allow the inclusion of the infinitesimal in its arithmetisation.

the development of a differential philosophy

While Deleuze draws inspiration and guidance from Salomon Maïmon (1753–1800), who "sought to ground post-Kantianism upon a Leibnizian reinterpretation of the calculus" (DR 170), and "who proposes a fundamental reformation of the *Critique* and an overcoming of the Kantian duality between concept and intuition" (DR 173), it is in the work of Höené Wronski (1778–1853) that Deleuze finds the established expression of the first principle of the differential philosophy. Wronski was "an eager devotee of the differential method of Leibniz and of the transcendental philosophy of Kant" (CB 261). Wronski made a transcendental distinction between the finite and the infinitesimal, determined by the two heterogeneous functions of knowledge, namely understanding and reason. He argued that:

> finite quantities bear upon the objects of our knowledge, and infinitesimal quantities on the very generation of this knowledge; such that each of these two classes of knowledge must have laws proper [to them], and it is in the distinction between these laws that the major thesis of the metaphysics of infinitesimal quantities is to be found.[16]

It is imperative not to confuse "the objective laws of finite quantities with the purely subjective laws of infinitesimal quantities" (HW 36, 158). He claims that it is this

> confusion that is the source of the inexactitude that is felt to be attached to the infinitesimal Calculus [...] This is also [why] geometers, especially those of the present day, consider the infinitesimal Calculus, which nonetheless they concede always gives true results, to be only an indirect or artificial procedure. (HW 36, 159)

Wronski is referring here to the work of Joseph-Louis Lagrange (1736–1813) and Lazarre Carnot (1753–1823), two of the major figures in the history of the differential calculus, whose attempts to provide a rigorous foundation for the differential calculus involved the elimination of the infinitesimal from all calculations, or, as Wronski argued, involved confusing objective and subjective laws in favour of finite quantities (see HW 36, 159). Both of these figures count as precursors to the work of Cauchy and Weierstrass. Wronski argued that the differential calculus constituted "a *primitive algorithm* governing the *generation* of quantities, rather than the laws of quantities *already formed*" (CB 262). According to Wronski, the differential should be interpreted "as having an a priori metaphysical reality associated with the generation of magnitude" (CB 262). The differential is therefore expressed as a pure element of quantitability, in so far as it prepares for the determination of quantity. The work of Wronski represents an extreme example of the differential point of view of the infinitesimal calculus which recurs throughout the nineteenth century.

Another significant figure in this alternative history of mathematics that is constructed by Deleuze is Jean-Baptiste Bordas-Demoulin (1798–1859), who also champions the infinitesimal against those who consider that infinitesimals had to be eliminated in favour of finite quantities. Bordas-Demoulin does not absolve the differential calculus of the accusation of error but rather considers the differential calculus to have this error as its principle. According to Bordas-Demoulin, the minimal error of the infinitesimal "finds itself compensated by reference to an error active in the contrary sense. [...] It is in all necessity that the errors are mutually compensated."[17] The consequence of this mutual compensation "is that one differential is only exact after having been combined with another" (BD 414). Deleuze repeats these arguments of Wronski and Bordas-Demoulin when he maintains that it is in the differential relation that the differential is realised as a pure element of quantitability. Each term of the relation, that is, each differential, each pure element of quantitability, therefore "exists absolutely only in its relation to the other" (DR 172), that is, only in so far as it is reciprocally determined in relation to another.

The question for Deleuze then becomes: "in what form is the differential relation determinable?" (DR 172). He argues that it is determinable primarily in qualitative form, in so far as it is the reciprocal relation between differentials; and then secondarily, in so far as it is the function of a tangent whose values give the gradient of the line tangent to a curve, or the specific qualitative nature of this curve, at a point. As the function of a tangent, the differential relation "expresses a function which differs in kind from the so-called primitive function" (DR 172). Whereas the primitive function, when differentiated, expresses the whole curve directly,[18] the differential relation, when differentiated, expresses rather the further qualification of the nature of the function at, or in the immediate neighbourhood of, a specific point. The primitive function is the integral of the function determined by the inverse transformation of differentiation, according to the differential calculus. From the differential point of view of the infinitesimal calculus, the differential relation, as the function of the tangent, determines the existence and distribution of the distinctive points of a function, thus preparing for its further qualification. Unlike the primitive function, the differential relation remains tied to the specific qualitative nature of the function at those distinctive points, and, as the function of the tangent, it "is therefore differentiable in turn" (DR 172). When the differential relation is differentiated repeatedly at a distinctive point generating a power series expansion, what is increasingly specified is the qualitative nature of the function in the immediate neighbourhood of that point. Deleuze argues that this convergence of a power series with an analytic function, in its immediate neighbourhood, satisfies "the minimal conditions of an integral" (DR 174), and characterises what is for Deleuze the process of "differen*t*iation" (DR 209).

The differential relation expresses the qualitative relation between not only curves and straight lines but also between linear dimensions and their functions, and plane or surface dimensions and their functions. The domain of the successive adjunction of circles of convergence, as determined by analytic continuity, actually has the structure of a surface. This surface is constituted by the points of the domain and the direction attached to each point in the domain, that is, the tangents to the curve at each point and the direction in which the curve goes at that point. Such a surface can be described as a field of directions or a field of vectors. A vector is a quantity having both magnitude and direction. It is the surface of such a vector field that provides the structure for the local genesis of functions. It is within this context that the example of a jump discontinuity in relation to a finite discontinuous interval between neighbouring analytic or local functions is developed by Deleuze, in order to characterise the generation of another function which extends beyond the points of discontinuity which determine the limits of these local functions. Such a function would characterise the relation between the different domains of different local functions. The genesis of such a function from the local point of view is determined initially by taking any two points on the surface of a vector field, such that each point is a pole of a local function determined independently by the point-wise operations of Weierstrassian analysis. The so determined local functions, which have no common distinctive points or poles in the domain, are discontinuous with each other; each pole being a point of discontinuity, or limit point, for its respective local function. Rather than simply being considered as the unchanging limits of local functions generated by analytic continuity, the limit points of each local function can be considered in relation to each other, within the context of the generation of a new function which encompasses the limit points of each local function and the discontinuity that extends between them. Such a function can be understood initially to be a potential function, which is determined as a line of discontinuity between the poles of the two local functions on the surface of the vector field. The potential function admits these two points as the poles of its domain. However, the domain of the potential function is on a scalar field, which is distinct from the vector field in so far as it is composed of points (scalars) which are non-directional; scalar points are the points onto which a vector field is mapped. The potential function can be defined by the succession of points (scalars) which stretch between the two poles. The scalar field of the potential function is distinct from the vector field of the local functions in so far as, mathematically speaking, it is "cut" from the surface of the vector field. Deleuze argues that "the limit must be conceived not as the limit of a [local] function but as a genuine cut [*coupure*], a border between the changeable and the unchangeable within the function itself [...] the limit no longer presupposes the ideas of a continuous variable and infinite approximation. On the contrary, the concept of limit grounds a new, static and purely ideal definition" (DR 172), that of the potential function. To cut the surface from one of these poles to the next is to generate such a potential function. The poles of the potential function determine the limits of the discontinuous domain, or scalar field, which is cut from the surface of the vector field. The "cut" of the surface in this theory renders the structure of the potential function "apt to a creation" (ALI 8). The precise moment of production, or genesis, resides in the act by which the cut renders the variables of certain functional expressions able to "jump" from pole to pole across the cut. When the variable jumps across this cut, the domain of the potential function is no longer uniformly discontinuous. With each "jump," the poles which determine the domain of discontinuity, represented by the potential function sustained across the cut, seem to have been removed. The more the cut does not separate the potential function on the scalar field from the surface of the vector field, the more the poles seem to have been removed, and the more the potential function seems to be continuous with the local functions across the whole surface of the vectorial field. It is only in so far as this interpretation is conferred on the structure of the potential function that a new function can be understood to have been generated on the surface. A potential function is generated only when there is potential for the creation of a new function between the poles of two local functions. The potential function is therefore always apt to the creation of a new function. This new function, which encompasses the limit points of each local function and the discontinuity that extends between them, is continuous across this structure of the potential function; it completes the structure of the potential function, in what can be referred to as a "composite function." The connection between the structural completion of the potential function and the generation of the corresponding composite function is the act by which the variable jumps from pole to pole. When the variable jumps across the cut, the value of the composite function sustains a determined increase. Although the increase seems to be sustained by the potential function, it is this increase which actually registers the generation or complete determination of the composite function.

The complete determination of a composite function by the structural completion of the potential function is not determined by Weierstrass's theory of analytic continuity. A function is able to be determined as continuous by analytic continuity across singular points which are removable, but not across singular points which are non-removable. The poles that determine the parameters of the domain of the potential function are non-removable, thus analytic continuity between the two functions, across the cut, is not able to be established. Weierstrass, however, recognised a means of solving this problem by extending his analysis to meromorphic functions.[19] A function is said to be meromorphic in a domain if it is analytic in the domain determined by the poles of analytic functions. A meromorphic function is determined by the quotient of two arbitrary analytic functions, which have been determined independently on the same surface by the point-wise operations of Weierstrassian analysis. Such a function is defined by the differential relation:

$$\frac{dy}{dx} = \frac{Y}{X},$$

where X and Y are the polynomials, or power series, of the two local functions. The meromorphic function, as the function of a differential relation, is just the kind of function which can be understood to have been generated by the structural completion of the potential function. The meromorphic function is therefore the differential relation of the composite function. The expansion of the power series determined by the repeated differentiation of the meromorphic function should generate a function which converges with a composite function. The graph of a composite function, however, consists of curves with infinite branches, because the series generated by the expansion of the meromorphic function is divergent. The representation of such curves posed a problem for Weierstrass, which he was unable to resolve, because divergent series fall outside the parameters of the differential calculus, as determined by the epsilon-delta approach, since they defy the criterion of convergence.

the qualitative theory of differential equations

Henri Poincaré (1854–1912) took up this problem of the representation of composite functions by extending the Weierstrass theory of meromorphic functions to what was called "the qualitative theory of differential equations" (MK 732). In place of studying the properties of complex functions in the neighbourhood of their singularities, Poincaré was occupied primarily with determining the properties of complex functions in the whole plane, that is, the properties of the entire curve. This qualitative method involved the initial investigation of the geometrical form of the curves of functions with infinite branches – only then was the numerical determination of the values of the function able to be made. While such divergent series do not converge, in the Weierstrassian sense, to a function, they may indeed furnish a useful approximation to a function if they can be said to represent the function asymptotically. When such a series is asymptotic to the function, it can represent an analytic or composite function even though the series is divergent.

When this geometrical interpretation was applied to composite functions, Poincaré found the values of the composite function around the singularity produced by the function to be undetermined and irregular. The singularity of a composite function would be the point at which both the numerator and denominator of the quotient of the meromorphic function determinative of the composite function vanish (or equal zero). The peculiarity of the meromorphic function is that the numerator and denominator do not vanish at the same point on the surface of the domain. The points at which the two local functions of the quotient vanish are at their respective poles. The determination of a composite function therefore requires the determination of a new singularity in relation to the poles of the local functions of which it is composed. Poincaré called this new kind of singularity an essential singularity. Observing that the values of a composite function very close to an essential singularity fluctuate through a range of different possibilities without stabilising, Poincaré distinguished four types of essential singularity, which he classified according to the behaviour of the function and the geometrical appearance of the solution curves in the neighbourhood of these points. The first type of singularity is the saddle point or dip (*col*), through which only two solution curves pass, acting as asymptotes for neighbouring curves. A saddle point is neither a maximum nor a minimum, since it either increases or decreases depending on the direction taken away from it. The second kind of singularity is the node (*nœud*), which is a point through which an infinite number of curves pass. The third type of singularity is the point of focus (*foyer*), around which the solution curves turn and towards which they approach in the same way as logarithmic spirals. And the fourth, called a *centre*, is a point around which the curves are closed, enveloping one another and the centre (see Fig. 2).

The type of essential singularity is determined by the form of the curves constitutive of the meromorphic function. Whereas the potential function remains discontinuous with the other functions on the surface from which it is cut, thereby representing a discontinuous group of functions, the composite function, on the contrary, overcomes this discontinuity in so far as it is continuous in the domain which extends across the whole surface of the discontinuous group of functions. The existence of such a continuous function, however, does not express

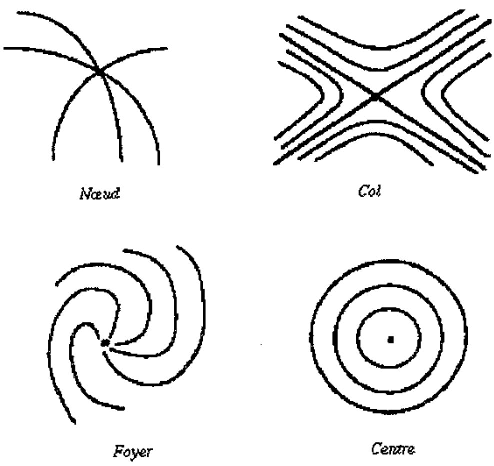

Fig. 2. [20]

any less the properties of the domain of discontinuity which serves to define it. The discontinuous group of local functions and the continuous composite function attached to this group exist alongside each other, the transformation from one to the other being determined by the process of the generation and expansion of the meromorphic function. The potential function is actualised in the composite function when the variable jumps from one pole to the other. Its trajectory, in the form of a solution curve, is determined by the type of essential singularity created by the meromorphic function. The essential singularity determines the behaviour of the composite function, or the appearance of the solution curve, in its immediate neighbourhood by acting as an *attractor* for the trajectory of the variable across its domain. It is the value of this function which sustains a determined increase with each jump of the variable. In so far as the trajectory of each variable is attracted to the same final state represented by each of the different essential singularities, these essential singularities can be understood to represent what Manuel DeLanda describes as the "inherent or intrinsic *long-term tendencies* of a system, the states which the system will spontaneously tend to adopt in the long run as long as it is not constrained by other forces."[21]

Deleuze distinguishes this differential point of view of the infinitesimal calculus from the Weierstrassian theory of approximation when he writes that:

> No doubt the specification of the singular points (for example, dips, nodes, focal points, centers) is undertaken by means of the form of integral curves, which refer back to the solutions for the differential equations. There is nevertheless a complete determination with regard to the existence and distribution of these points which depends upon a completely different instance – namely, the field of vectors defined by the equation itself. The complementarity of these two aspects does not obscure their difference in kind – on the contrary. (DR 177)

The equation to which Deleuze refers is the meromorphic function, which is a differential equation or function of a differential relation determined according to the Weierstrassian approach, from which the essential singularity and therefore the composite function are determined according to Poincaré's qualitative approach. This form of integration is again characterised from the local point of view, and is characterised by what Deleuze describes as "an original process of differenciation" (DR 209). Differenciation is the complete determination of the composite function from the reciprocally determined local functions or the structural completion of the potential function. It is the process whereby a potential function is actualised as a composite function.

Deleuze states that "actualisation or differenciation is always a genuine creation," and that to be actualised is "to create divergent lines" (DR 212). The expanded power series of a meromorphic function is actualised in the composite function in so far as it converges with, or creates, the divergent lines of the composite function. Differenciation, therefore, creates an essential singularity, whose divergent lines actualise the specific qualitative nature of the poles of the group of discontinuous local functions, represented by a potential function, in the form of a composite function. These complex functions can be understood to be what Poincaré called "Fuschian functions," which, as Georges Valiron points out, "are more often called automorphic functions."[22] The discontinuous group of local functions can therefore also be understood to be Fuschian groups. Poincaré's pioneering work in this area eventually led to the definitive founding of the geometric theory of analytic functions, the study of which "has not yet been completely carried out" (GV 173) but continues to be developed with the assistance of computers.

Benoit Mandelbrot (b. 1924) considers Poincaré, with his concept of essential singularities, to be "the first student of fractal ('strange') attractors," that is, of the kinds of attractors operative in fractals which occur in mathematics, and cites certain theories of Poincaré as having "led [him] to new lines of research," specifically "the theory of automorphic functions," which made Poincaré and Felix Klein (1849–1925) famous.[23]

Deleuze does not consider this process of differen*c*iation to be arrested with the generation of a composite function, but rather to continue, generating those functions which actualise the relations between different composite functions, and those functions which actualise the relations between these functions, and so on. The conception of differen*c*iation is extended in this way when Deleuze states that "there is a differenciation of differenciation which integrates and welds together the differenciated" (DR 217); each differen*c*iation is simultaneously "a local integration," which then connects with others, according to the same logic, in what is characterised as a "global integration" (DR 211).

The logic of the differential, as determined according to both differen*t*iation and differen*c*iation, designates a process of production, or genesis, which has, for Deleuze, the value of introducing a general theory of relations which unites the Weierstrassian structural considerations of the differential calculus to the concept of "the generation of quantities" (DR 175). "In order to designate the integrity or the integrality of the object," when considered as a composite function from the differential point of view of the infinitesimal calculus, Deleuze argues that "we require the complex concept of differen*t*/*c*iation. The *t* and the *c* here are the distinctive feature or the phonological relation of difference in person" (DR 209). Deleuze argues that differen*c*iation is "the second part of difference" (DR 209), the first being expressed by the logic of the differential in differen*t*iation. Where the logic of differen*t*iation characterises a differential philosophy, the complex concept of the logic of differen*t*/*c*iation characterises Deleuze's "philosophy of difference."

The differential point of view of the infinitesimal calculus represents an opening, providing an alternative trajectory for the construction of an alternative history of mathematics; it actually anticipates the return of the infinitesimal in the differential calculus of contemporary mathematics, thanks to the axioms of non-standard analysis. This is the interpretation of the differential calculus to which Deleuze is referring when he appeals to the "barbaric or pre-scientific interpretations of the differential calculus" (DR 171). Deleuze thereby establishes an historical continuity between the differential point of view of the infinitesimal calculus and modern theories of the differential calculus which surpasses the methods of the differential calculus that Weierstrass uses in the epsilon-delta approach to support the development of a rigorous foundation for the calculus. While Weierstrass is interested in making advances in mathematics to secure the development of a rigorous foundation for the differential calculus, Deleuze is interested in using mathematics to problematise the reduction of the differential calculus to set theory, by determining an alternative trajectory in the history of mathematics, one that retrospectively allows the reintroduction of the infinitesimal to an understanding of the operation of the calculus. According to Deleuze, the "finitist interpretations" of the calculus given in modern set-theoretical mathematics – which are congruent with "Cantorian finitism,"[24] that is, "the idea that infinite entities are […] considered to be finite within set theory"[25] (JS 66) – betray the nature of the differential no less than Weierstrass, since they "both fail to capture the extra-propositional or sub-representative source […] from which calculus draws its power" (DR 264).

In constructing this theory of relations characteristic of a philosophy of difference, Deleuze draws significantly from the work of Albert Lautman, who refers to this whole process as "the metaphysics of logic" (ALI 3). It is in *Difference and Repetition* that Deleuze formulates a "metaphysics of logic" that corresponds to the logic of the differential from the differential point of view of the infinitesimal calculus. However, he argues that "we should speak of a dialectics of the calculus rather than a metaphysics" (DR 178), since:

> each engendered domain, in which dialectical Ideas of this or that order are incarnated, possesses its own calculus. [...] It is not mathematics which is applied to other domains but the dialectic which establishes [...] the direct differential calculus corresponding or appropriate to the domain under consideration. (DR 181)

Just as he argued that mathematics:

> does not include only solutions to problems; it also includes the expression of problems relative to the field of solvability which they define. [...] That is why the differential calculus belongs to mathematics, even at the very moment when it finds its sense in the revelation of a dialectic which points beyond mathematics. (DR 179)

It is in the differential point of view of the infinitesimal calculus that Deleuze finds a form of the differential calculus appropriate to the determination of a differential logic. This logic is deployed by Deleuze, in the form of the logic of different*/c*iation, in the development of his project of constructing a philosophy of difference.

The relation between the finite and the infinitesimal is determined according to what Lautman describes as "the logical schemas which preside over the organisation of their edifices."[26] Lautman argues that "it is possible to recover within mathematical theories, logical Ideas incarnated in the same movement of these theories" (ALII 58). The logical Ideas to which Lautman refers include the relations of expression between the finite and the infinitesimal. He argues that these logical Ideas "have no other purpose than to contribute to the illumination of logical schemas within mathematics, which are only knowable through the mathematics themselves" (ALII 58). The project of the present paper has been to locate these "logical Ideas" in the mathematical theory of the infinitesimal calculus from the differential point of view, in order then to determine how Deleuze uses these "logical Ideas" to develop the logical schema of a theory of relations characteristic of a philosophy of difference.

notes

I would like to thank Daniel W. Smith for his generous comments when reviewing an early version of this paper.

1 It is in *Anti-Oedipus* that Deleuze coins the phrase "schizophrenic mathematics" (Gilles Deleuze and Félix Guattari, *Anti-Oedipus: Capitalism and Schizophrenia* 372), which I have borrowed and shortened to "Schizo-Math" in order to expand upon some of the themes introduced in the paper "Math Anxiety" by Aden Evens (*Angelaki* 5:3 (2000): 105).

2 Gilles Deleuze, *Difference and Repetition* 114. Hereafter DR.

3 Carl Benjamin Boyer, *The History of the Calculus and its Conceptual Development* 11. Hereafter CB.

4 Gilles Deleuze, "Sur Spinoza," 17 Feb. 1981. Hereafter DSS.

5 Gottfried Wilhelm Leibniz, *The Early Mathematical Manuscripts* 53.

6 Leibniz, "Letter to Varignon, with a Note on the 'Justification of the Infinitesimal Calculus by that of Ordinary Algebra'" 545. Hereafter PPL.

7 George Lakoff and Rafael E. Nafiez, *Where Mathematics Comes From: How the Embodied Mind Brings Mathematics into Being* 224. Hereafter LN.

8 For a thorough analysis of this problem with limits in Cauchy, see CB 281.

9 Deleuze, "Sur Leibniz," 22 Feb. 1972.

10 See J.L. Bell, *A Primer of Infinitesimal Analysis*.

11 Abraham Robinson, *Non-Standard Analysis* 2.

12 Given a function, $f(x)$, having derivatives of all orders, the Taylor series of the function is given by

$$\sum_{k=0}^{\infty} \frac{f^{(k)}(a)}{k!}(x-a)^k,$$

where $f^{(k)}$ is the kth derivative of f at a. A function is equal to its Taylor series if and only if its error term R_n can be made arbitrarily small, where

$$R_n = \left| f(x) - \sum_{k=0}^{\infty} \frac{f^{(k)}(a)}{k!}(x-a)^k \right|.$$

The Taylor series of a function can be represented in the form of a power series, which is given by

$$\sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n + \cdots,$$

where each a is a distinct constant. It can be shown that any such series either converges at $x = 0$, or for all real x, or for all x with $-R < x < R$ for some positive real R. The interval $(-R, R)$ is called the circle of convergence, or neighbourhood of the distinctive point. This series should be thought of as a function in x for all x in the circle of convergence. Where defined, this function has derivatives of all orders. See H.J. Reinhardt, *Analysis of Approximation Methods for Differential and Integral Equations*.

13 See Morris Kline, *Mathematical Thought from Ancient to Modern Times* 643–44. Hereafter MK.

14 Deleuze argues that "It was a great day for philosophy when […] Leibniz proposed […] that there is no reason for you simply to oppose the singular to the universal. It's much more interesting if you listen to what mathematicians say, who for their own reasons think of 'singular' not in relation to 'universal', but in relation to 'ordinary' or 'regular'" (Deleuze, "Sur Leibniz," 29 Mar. 1980).

15 Albert Lautman, *Essai sur les notions de structure et d'existence en mathématiques* 38; my trans. Hereafter ALI.

16 Höené Wronski, *La Philosophie de l'infini: Contenant des contre-refléxions sur la métaphysique du calcul infinitesimal* 35; large sections of this text, translated by M.B. DeBevoise, appear in Michel Blay (ed.), *Reasoning with the Infinite: From the Closed World to the Mathematical Universe* 158. Hereafter HW. Page references will be given to the French and the English translation respectively.

17 Jean-Baptiste Bordas-Demoulin, *Le Cartésianisme ou la véritable rénovation des sciences, suivi de la théorie de la substance et de celle de l'infini* 414; my trans. Hereafter BD.

18 Note: the primitive function $f(x)dx$, expresses the whole curve $f(x)$.

19 It was Charles A.A. Briot (1817–82) and Jean-Claude Bouquet (1819–85) who introduced the term "meromorphic" for a function which possessed just poles in that domain (MK 642).

20 June Barrow-Green, *Poincaré and the Three Body Problem* 32.

21 Manuel DeLanda, *Intensive Science and Virtual Philosophy* 15.

22 Georges Valiron, "The Origin and the Evolution of the Notion of an Analytic Function of One Variable" 171. Hereafter GV.

23 Benoit B. Mandelbrot, *The Fractal Geometry of Nature* 414. Mandelbrot qualifies these statements when he says of Poincaré that "nothing I know of his work makes him even a distant precursor of the fractal geometry of the visible facets of Nature" (ibid. 414).

24 Penelope Maddy, "Believing the Axioms" 488.

25 Jean-Michel Salanskis, "Idea and Destination" 71.

26 Albert Lautman, *Essai sur l'unité des sciences mathématiques dans leur développement actuel* 58; my trans. Hereafter ALII.

bibliography

Barrow-Green, June. *Poincaré and the Three Body Problem*. Rhode Island: American Mathematical Society, 1997.

Bell, J.L. *A Primer of Infinitesimal Analysis*. New York: Cambridge UP, 1998.

Blay, Michel. *Reasoning with the Infinite: From the Closed World to the Mathematical Universe*. Chicago: U of Chicago P, 1998.

Bordas-Demoulin, Jean-Baptiste. *Le Cartésianisme ou la véritable rénovation des sciences, suivi de la théorie de la substance et de celle de l'infini*. Paris: Gauthier-Villars, 1874.

Boyer, Carl Benjamin. *The History of the Calculus and its Conceptual Development*. New York: Dover, 1959.

DeLanda, Manuel. *Intensive Science and Virtual Philosophy. Transversals*. London and New York: Continuum, 2002.

Deleuze, Gilles. *Difference and Repetition*. Trans. Paul Patton. New York: Columbia UP, 1994.

Deleuze, Gilles. *Expressionism in Philosophy. Spinoza*. Trans. Martin Joughin. New York: Zone, 1992.

Deleuze, Gilles. "Seminars." The seminars on Leibniz and on Spinoza, 1971–87. "Sur Leibniz," 22 Feb. 1972, trans. Simon Duffy; 29 Mar. 1980, trans. Charles J. Stivale. "Sur Spinoza," 17 Feb. 1981, trans. Timothy S. Murphy. Available <http://www.webdeleuze.com>.

Deleuze, Gilles and Félix Guattari. *Anti-Oedipus: Capitalism and Schizophrenia*. Trans. Robert Hurley, Mark Seem and Helen R. Lane. Minneapolis: U of Minnesota P, 1983.

Evens, Aden. "Math Anxiety." *Angelaki* 5.3 (2000): 105–15.

Kline, Morris. *Mathematical Thought from Ancient to Modern Times*. Oxford: Oxford UP, 1972.

Lakoff, George and Rafael E. Nafiez. *Where Mathematics Comes From: How the Embodied Mind Brings Mathematics into Being*. New York: Basic Books, 2000.

Lautman, Albert. *Essai sur les notions de structure et d'existence en mathématiques*. Paris: Hermann, 1938.

Lautman, Albert. *Essai sur l'unité des sciences mathématiques dans leur développement actuel*. Paris: Hermann, 1938.

Leibniz, Gottfried Wilhelm. *The Early Mathematical Manuscripts*. Trans. J.M. Childs. Chicago: Open Court, 1920.

Leibniz, Gottfried Wilhelm. "Letter to Varignon, with a Note on the 'Justification of the Infinitesimal Calculus by that of Ordinary Algebra.'" *Philosophical Papers and Letters*. Trans. Leroy E. Loemker. Dordrecht: Reidel, 1969.

Leibniz, Gottfried Wilhelm. *Die Philosophischen Schriften von G. W. Leibniz*. Berlin: Weidmannsche, 1875–90.

Maddy, Penelope. "Believing the Axioms." *Journal of Symbolic Logic* 53.2 (1988): 481–511.

Mandelbrot, Benoit B. *The Fractal Geometry of Nature*. San Francisco: Freeman, 1982.

Reinhardt, H.J. *Analysis of Approximation Methods for Differential and Integral Equations*. New York: Springer, 1985.

Robinson, Abraham. *Non-Standard Analysis*. Revised ed. *Princeton Landmarks in Mathematics and Physics*. Princeton: Princeton UP, 1996.

Salanskis, Jean-Michel. "Idea and Destination." *Deleuze: A Critical Reader*. Ed. Paul Patton. Oxford: Blackwell, 1996.

Valiron, Georges. "The Origin and the Evolution of the Notion of an Analytic Function of One Variable." *Great Currents of Mathematical Thought*. Ed. François Le Lionnais. Vol. I. New York: Dover, 1971.

Wronski, Hoené. *La Philosophie de l'infini: contenant des contre-refléxions sur la métaphysique du calcul infinitesimal*. Paris, 1814.

Simon Duffy
Department of Philosophy
Main Quad A14
University of Sydney
Sydney, NSW 2006
Australia
E-mail: sduffy@usyd.edu.au