Revisiting Normativity with Deleuze

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The Question of Deleuze's Neo-Leibnizianism

Simon Duffy

Deleuze's texts are replete with examples of mathematical problems drawn from different historical periods. These engagements with mathematics rely upon the extraction of mathematical problems or problematics from the history of mathematics and the development of alternative lineages in the history of mathematics in order to reconfigure particular philosophical problems and to develop new concepts in response to them. The example that will be explored in this chapter is the problem of continuity as encountered by Leibniz's mathematical approach to natural philosophy, which draws upon the law of continuity as reflected in the calculus of infinite series and the infinitesimal calculus. Deleuze traces alternative lineages in the history of mathematics based on non-canonical research and solutions that have subsequently been offered to these problems. The relation between the canonical history of mathematics and the alternative lineages that Deleuze extracts from it are most clearly exemplified in the difference between what can be described as the axiomatized set-theoretical explications of mathematics and those developments or research programmes in mathematics that fall outside of the parameters of such an axiomatics; for example, algebraic topology, functional analysis and differential geometry, to name but a few. This difference can be understood to be characteristic of the relation between what Deleuze and Guattari in *A Thousand Plateaus* (1987) refer to as Royal or major science and nomadic or minor science. Royal or major science refers to those practices that fall within the scientific norms and methodological conventions of the time, whereas nomad or minor science refers to those practices that fall outside of such disciplinary habits and resist attempts to be reduced to them. Scientific normativity can therefore be understood to operate as a set of principles according to which respectable research in mathematics is conducted, despite the fact that developments continue to be made that undermine such constraints and, by a process of destabilization and regeneration, lead to the development of alternative systems for structuring normative frameworks. The aim of this chapter is to provide an account of the mathematical engagement that Deleuze undertakes with Leibniz, which he draws upon to structure the alternative normative framework that is developed in his philosophy. An understanding of any of the mathematical engagements that Deleuze undertakes throughout his work requires a clear explication of the history of mathematics from which the specific mathematical problematic has been extracted and of the alternative lineage in the history of mathematics that is generated in relation to it. These mathematical problematics extracted from the history of mathematics are directly redeployed by Deleuze as philosophical problematics in relation to the history of philosophy. This is achieved by mapping the alternative lineages in the history of mathematics onto corresponding alternative lineages in the history of philosophy, that is, by isolating those points of convergence between the mathematical and philosophical problematics extracted from their respective histories. The redeployment of mathematical problematics as philosophical problematics is one of the strategies that Deleuze employs in his engagement with the history of philosophy.[1]

Deleuze has gained a lot of respect among historians of philosophy for the rigour and historical integrity of his engagements with figures in the history of philosophy. Particularly in those texts that engage with the intricacies of seventeenth century metaphysics and the mathematical developments that contributed to its diversity.[2] One of the aims of these engagements is not only to explicate the detail of the thinker's thought but also to recast aspects of their philosophy as developments that contribute to his broader project of constructing a philosophy of difference. Each of these engagements therefore provides as much insight into the developments of Deleuze's own thought as it does into the detail of the thought of the figure under examination. For the purposes of this chapter, Deleuze's engagement with Leibniz is singled out for closer scrutiny. Much has been made of Deleuze's Neo-Leibnizianism,[3] however not very much detailed work has been done on the specific nature of Deleuze's critique of Leibniz that positions his work within the broader framework of Deleuze's own philosophical project. The present chapter undertakes to redress this oversight by providing an account of the reconstruction of Leibniz's metaphysics that Deleuze undertakes in *The Fold*. Deleuze provides a systematic account of the structure of Leibniz's metaphysics in terms of its mathematical underpinnings. However, in doing so, Deleuze draws upon not only the mathematics developed by Leibniz – including the law of continuity as reflected in the calculus of infinite series and the infinitesimal calculus – but also the developments in mathematics made by a number of Leibniz's contemporaries – including Newton's method of fluxions – and a number of subsequent developments in mathematics, the rudiments of which can be more or less located in Leibniz's own work – including the theory of functions and singularities, the theory of continuity and Poincaré's theory of automorphic functions. Deleuze then retrospectively maps these developments back onto the structure of Leibniz's metaphysics. While the theory of continuity serves to clarify Leibniz's work, Poincaré's theory of automorphic functions offers a solution to overcome and extend the limits that Deleuze identifies in Leibniz's metaphysics. Deleuze brings this elaborate conjunction of material together in order to set up a mathematical idealization of the system that he considers to be implicit in Leibniz's work. The result is a thoroughly mathematical explication of the structure of Leibniz's metaphysics. What is provided in this chapter is an exposition of the very mathematical underpinnings of this Deleuzian account of the structure of Leibniz's metaphysics, which, I maintain, subtends the entire text of *The Fold*.

Deleuze's project in the *Fold* is predominantly oriented by Leibniz's insistence on the metaphysical importance of mathematical speculation. What this suggests is that mathematics functioned as an important heuristic in the development of Leibniz's metaphysical theories. Deleuze puts this insistence to good use by bringing together the different aspects of Leibniz's metaphysics with the variety of mathematical themes that run throughout his work. Those aspects of Leibniz's metaphysics that Deleuze undertakes to clarify in this way, and upon which this chapter will focus, include: the definition of the monad, the conception of matter and motion and the representation of the continuum.

1 The concept of matter, motion and the representation of the continuum

Leibniz considered nature to be infinitely divisible such that 'the smallest particle should be considered as a world full of an infinity of creatures'.[4] However, his interpretation of infinitesimals as useful fictions, which he arrived at as early as 1676, means that they are without status as actual parts of the continuum.[5] This syncategorematic interpretation of the continuum, which means not only that there is no actually infinitely small but rather for any assignable finite quantity there is always another that is smaller but also that there is no number of all numbers, or actually infinite number but only numbers greater than others without bound. The fictional status of the infinite and the infinitely small has significant implications for Leibniz's mathematical approach to natural philosophy and its metaphysical foundations, particularly his understanding of what is perceived in perceptual experience as continuous motion and the problem of how matter and the objects we perceive in perceptual experience as bodies are grounded.

It is in the *Pacidius Philalethi*, 1676 (Leibniz, 2001), that Leibniz first makes a detailed attempt to work out a theory of motion that is in harmony with his syncategorematic interpretation of the continuum. Indeed, in the *Pacidius*, Leibniz develops an analysis of matter and continuity that prefigures his later metaphysical views.[6] Implicit in Leibniz's reasoning is the assumption of a direct correspondence between a curve as a mathematical object and a curve understood as the trajectory of a physical body. The trajectory of a body that traces or maps directly onto a continuous curve would be both continuous and uniform, that is it would be both uninterrupted and moving with constant acceleration, respectively. Uniformly accelerated motion is represented mathematically by the curve of a function that pairs as bodies change in position with respect to time.

The problem with this picture is that Leibniz actually denies the uniformity of motion, and instead considers the contrary hypothesis of non-uniform motion, which he maintains 'is also consistent with reason, for there is no body which is not acted upon by those around it at every single moment' (Leibniz, 2001: 208; Levy, 2003: 384). Leibniz, following Huygens,[7] subscribed to an impulse account of the acceleration of a body, according to which the motion of a body was 'due to a series of instantaneous finite impulses punctuating tiny subintervals of uniform motion so that in each successive subinterval the moving body has a fixed higher (or lower) velocity than it had in the preceding one' (Levy, 2003: 385). Such accelerated motion is more accurately represented by a polygonal curve that only approximates the 'smooth' character of a curve.

Leibniz's work on the infinitangular polygon, which actually approaches the smooth character of the curve, comes to the fore here, as it was only by representing motion as a smooth curve that the seventeenth century resources of algebra and geometry were able to be deployed to calculate the velocity and acceleration of a body at any time.

However, the Leibnizian model of the structure of matter satisfies the premises of the syncategorematic idea of infinite division, such that any finite portion of matter is able to be infinitely divided into progressively smaller finite parts, each of which is also infinitely divisible. The infinity of infinitely divisible parts of matter forms a plenum. The continuously curved trajectory of a body is the mathematical representation of a fictitious limit of the trajectory followed by the body which is constantly subject to the impact of other bodies from all directions in the plenum. So when Leibniz denies the uniformity of motion, he is denying not only the uniformity of acceleration but also the kind of directionality represented by a polygonal curve.

Since every finite interval of motion is infinitely divisible into increasingly small finite and distinct moments, the moving body suffers the impacts of infinitely many distinct forces during each and every interval of motion, however small. The resulting motion is not accelerated continuously by a force that acts throughout the interval, as we now understand accelerative force to act, but rather each impact adds a distinct and instantaneous change to the motion of the body (see Levy, 2003: 386). According to this impulsive account of acceleration, the non-uniformity of motion is maintained throughout every subinterval, however small.

In the *Pacidius*, Leibniz advances an analysis of the structure of the interval of motion, according to which, at any moment, the moving body is at a new point, and the transition of the moving body from the end of one interval to the beginning of the next occurs by a single step, which Leibniz characterizes as a 'leap' (2001: 79), from an assigned endpoint to what Leibniz describes as the '*locus proximus*' (2001: 168–69), the indistant but distinct beginning point of the next interval. The conclusion that Leibniz comes to in the *Pacidius* is 'that motion is not continuous, but happens by a leap; that is to say, that a body, staying for a while in one place, may immediately afterwards be transplanted to another; i.e. that matter is extinguished here, and reproduced elsewhere' (2001: 79). In *Numeri infiniti* (1676), Leibniz further characterizes motion '*per saltus*', or through a leap, as 'transcreation' (2001: 92–3), where the body is 'annihilated in the earlier state, and resuscitated in the later one' (2001: 194–5).

The endpoints of each subinterval of motion remain nothing but bounds, the ends or beginnings of the subintervals of motion into which a whole subinterval is divided by the actions of impulse forces on the apparently moving body.

The example that Leibniz uses in the *Pacidius* to characterize the continuum, of which the interval of motion that has non-uniform acceleration is an instance, is the folded tunic:

> Accordingly the division of the continuum must not be considered to be like the division of sand into grains, but like that of a sheet of paper or tunic into folds. . . . It is just as if we suppose a tunic to be scored with folds multiplied to infinity in such a way that there is no fold so small that it is not subdivided by a new fold. (2001: 185)

The image of the tunic 'scored with folds multiplied to infinity' is a heuristic for the structure of the continuum (Levy, 2003: 392), and insofar as each moment in the continuum is an endpoint of motion, it is also a heuristic for the structure of the interval of motion.

The interval of motion and the folded tunic therefore display similar structure, and this structure, as Leibniz describes it, displays the very properties that fractal mathematics was later developed to study (Levy, 2003: 393). The fractal curve that best represents the structure of 'folds within folds' that is suggested in the image of the folded tunic in the *Pacidius* is the Koch curve, demonstrated by Helge von Koch in 1904 (Deleuze, 1993, p. 16). The method of constructing the Koch curve is to take an equilateral triangle and trisect each of its sides. On the external side of each middle segment, construct equilateral triangles and delete the abovementioned middle segment. This first iteration resembles a Star of David composed of six small triangles. Repeat the previous process on the two outer sides of each small triangle. This basic construction is then iterated indefinitely. The Koch curve is an example of a non-differentiable curve, that is, a continuous curve that does not have a tangent at any of its points. More generalized Koch or fractal curves can be obtained by replacing the equilateral triangle with a regular n-gon, and/or the 'trisection' of each side with other equipartitioning schemes.[8] In this example, the line effectively and continuously defers inflection by means of the method of construction of the folds of its sides.

Fractal curves typically are not differentiable, that is, there are no points on the curve at which tangents can be drawn, no matter what the scale of magnification. Instead, the intervals display only 'corners' which are singularities, where the nature of the curve changes. Leibniz's account of accelerated motion, as depicted in the image of the folded tunic, displays fractal structure. The action of impulses at every single moment ensures that the interval of motion of the moving body includes infinitely many singularities in every subinterval of the motion. The fractal curve of the motion, like the Koch curve, is therefore not differentiable.

According to Leibniz, each fold or vertex of the fractal curve, which is a singularity, is a boundary of not one but two intervals of motion, each of which is actually subdivided into smaller subintervals. Each vertex or singularity is in fact an aggregate pair of 'indistant points': the end point of one subinterval and the beginning point of the next. A body in motion makes a 'leap' from the end of one subinterval to the beginning of the next, and every leap, which occurs at the boundary between the distinct subintervals of motions, marks a change in the motion of the moving body, both of its direction and velocity. Because these subintervals are infinitely divisible, the divisions of a subinterval of motion are distributed across an indefinitely descending hierarchy of distinct scales, of which, according to Leibniz's sycategorematic account of the infinitely small, there is always a subinterval at a scale smaller than the smallest given scale. Any motion across an interval therefore contains a multiplicity of singularities, vertices or boundaries of intervals of motion, that is, a multiplicity of unextended leaps between the indistant ends and beginnings of its various subintervals of motion, with increasing scales of resolution.

The impulses at the root of motion, that is, the leaps between indistant points, are neither intervals nor endpoints of motion. They remain unextended and are rather effected by divine intervention. The body is transcreated by God from one moment to the next. The changes in motion, that is, the actions of accelerative forces, which Leibniz characterizes as 'primary active force', are not the effects of moving bodies upon one another, which he characterizes as 'derivative forces', they are rather ascribed to God (Leibniz, 1965: 468–70; 1969: 432–3). For Leibniz, motion is not a real property in bodies, but 'merely a positional phenomenon that results from God's creative activity' (Levy, 2003: 406). In Leibniz's later metaphysics, he explains that whatever new states a body will possess have been predetermined by virtue of God's selection of the best of possible worlds and the pre-established harmony that that entails.

According to Leibniz's theory of motion, the properties of motion are divided into (1) those that apply to the *phenomenon* of motion across an interval of space, that is, motion as it appears in perceptual experience and is determined by derivative forces and (2) the conception of motion as a multiplicity of unextended leaps between indistant *loci proximi*, which is reserved for the metaphysical reality that subtends that phenomenon, and which is determined by primary active force. In perceptual experience, motion appears to consist in extended intervals that can be resolved into subintervals, *ad infinitum*. However, metaphysically, motion consists in a multiplicity of unextended leaps. Those leaps that are manifest in experience are the 'singularities' at which motion is perceived to be accelerated, but neither all leaps nor subintervals of motion are perceived consciously. In the sense perception of finite minds, the corporeal world always appears immediately as only finitely complex and piecewise continuous, though upon closer scrutiny it is determined as indefinitely complex and fractal in its structure.

One of the problems with Leibniz's account of the divisibility of matter in the *Pacidius* (1676) that was not resolved until the later development of his metaphysics of monads is that the problem of how matter and the objects we perceive in perceptual experience as bodies are grounded.

Any particular part of matter is infinitely divisible into progressively smaller finite parts without ever reaching or being resolved into a smallest part which could serve as its ground. The division doesn't terminate in atoms or material indivisibles. The problem is that there must be something in virtue of which the bodies, as the objects of our perceptual experience in the corporeal world, are true unities despite their indefinite subdivision into parts. There must be foundations for matter, but those foundations cannot be parts of matter. The grounding of bodies that are the objects of our perceptual experience issues from something immaterial in the foundations of matter whose unity is not subject to the same indefinite, and therefore, problematic division. The indivisible unities, whose reality provides a metaphysical foundation for matter while residing outside of the indefinite regress of parts within parts, are immaterial substances that Leibniz calls *monads* (*Monadology*, 1714). It is by means of the monad that the multiplicity of parts of matter that make up a body can be considered as a unity. The monad is prior to the multiplicity that constitutes the body, and the monad exists phenomenally only through the body it constitutes.

The constructivism of the syncategorematic infinite explains the content of our experience of reality; however, it has no place in the account of metaphysical reality.

What is real metaphysically, as far as Leibniz is concerned, are simple substances or monads and aggregates of them. Bodies, as the objects of perceptual experience that are composed of a multiplicity of parts of matter, are the 'well-founded phenomena' that are grounded by monads. In fact, the consensus in Leibniz studies is beginning to swing from an understanding of Leibniz's mature metaphysics as idealist in regards to matter – according to which the bodies perceived in our perceptual experience are mere phenomena, solely the products of our limited understanding – towards an understanding of the actual existence of corporeal substances as constituted by aggregates of monads, or of Leibniz as a realist in regards to matter, although it is not clear that Leibniz himself solved this problem satisfactorily once and for all (Garber, 2009, 557). These aggregates of monads are then determined as the bodies perceived in our perceptual experience by the dominant monad that unites them. That is, one dominant monad unites each aggregate of monads which manifests phenomenally as an identifiable body.

In the sense perception of finite minds, the corporeal world always appears immediately as only finitely complex and piecewise continuous, though upon closer scrutiny it is determined as being indefinitely complex and fractal in its structure. Matter 'only appears to be continuous' because our imperfect perceptual apparatus obscures the divisions which actually separate the parts of bodies. Leibniz's postulate of the best of possible worlds, chosen by God, can be characterized as an actual infinite, in which all the divisions of matter, and the relations of motion that are exhibited between them in perceptual experience, are actually assigned and the resolution into singularities or leaps, that are more or less perceived in perceptual experience, is complete, independently of the limited capacity of the mind to represent only a temporal section of this in consciousness.

Before discussing Deleuze's response to this material, I'd first like to give two brief outlines of some of the material that I will draw upon in the argument that follows.

(1) The first brief outline is of Deleuze's Leibnizian account of the theory of compossibility. A crucial test for Deleuze's mathematical reconstruction of Leibniz's metaphysics is how to deal with Leibniz's subject-predicate logic. Deleuze maintains that Leibniz's mathematical account of continuity is reconcilable with the relation between the concept of a subject and its predicates. What Deleuze proposes involves demonstrating that the continuity characteristic of the infinitesimal calculus is isomorphic to the infinite series of predicates contained in the concept of a subject that express the infinite series of states of the world, although – and I will say more about this later – each particular subject in fact only expresses clearly a small finite portion of it from a certain point of view.

Deleuze offers a 'Leibnizian' interpretation of the difference between compossibility and incompossibility 'based only on divergence or convergence of series' (1993: 150), that is, the series of predicates contained in the concept of a subject. He proposes the hypothesis that there is compossibility between two singularities – where a singularity is a distinctive point on a curve, for example where the shape of the curve changes, whether a maxima, minima, or point of inflection – 'when series of ordinaries converge', that is, when the values of the series of regular points that derive from two singularities coincide, 'otherwise there is discontinuity. In one case, you have the definition of compossibility, in the other case, the definition of incompossibility'.[9] If the series of ordinary or regular points that derive from singularities diverge, then you have a discontinuity. When the series diverge, when you can no longer compose the continuity of this world with the continuity of this other world, then it can no longer belong to the same world. There are therefore as many worlds as divergences. All worlds are possible, but they are incompossibles with each other.[10] God conceives an infinity of possible worlds that are not compossible with each other, from which He chooses the best of possible worlds, which happens to be the world in which, for example, Adam sinned, which is incompossible, and therefore diverges with, the world in which Adam doesn't sin. A world is therefore defined by its continuity. What separates two incompossible worlds is the fact that there is a discontinuity between the two worlds.[11]

(2) The second brief outline is of the solution, in certain circumstances, to the problem of the discontinuity between the divergent poles of two otherwise continuous local functions represented by Poincaré's theory of automorphic functions. Local functions can be generated from a differential relation at any point and extended in either direction through the length of the curve up to the point where the curve diverges at a pole; what is meant by divergence here is that a point is reached where the function is no longer defined. The solution curve, which results from the 'jump' of the variable across the domain of discontinuity between the poles of two local functions, is a composite function determined by the quotient of the two divergent local functions, which have been determined independently on the same surface.

The graph of the composite function consists of curves with infinite branches or that are divergent. The representation of such curves however posed a problem because divergent series fall outside the parameters of what was understood of the differential calculus at the time, since they defy the criterion of convergence. It was considered that reckoning with divergent series, which have no sum, would therefore lead to false results.

The representation of the divergent curves of composite functions remained a problem until Poincaré (b.1854–1912) proposed 'the qualitative theory of differential equations' or theory of automorphic functions. While such divergent series do not converge to a function, they may indeed furnish a useful approximation to a function if they can be said to represent the function asymptotically. When such a series is asymptotic to the function, it can represent a composite function even though the series is divergent. However, the representation of a composite function requires the determination of a new singularity in relation to the poles of the local functions of which it is composed. Poincaré called this new kind of singularity an essential singularity. Poincaré distinguished four types of essential singularity, which he classified according to the behaviour of the function and the geometrical appearance of the solution curves in the neighbourhood of these points: the saddle point (col); the node (nœud); the focus (foyer) and the centre. Singularities develop increasingly complex relations with the increasing complexity of the curves.[12]

The construction of new essential singularities is the problem that Deleuze draws upon to offer a solution to overcome and extend the limits of Leibniz's account of compossibility.

2 Overcoming the limits of Leibniz's metaphysics

Poincaré's development of the representation of composite functions means that in certain circumstances a continuity can be established across divergent series. What this means is that the Leibnizian account of compossibility as the unity of convergent series, which relies on the exclusion of divergence, is no longer required by mathematics.[13] The mathematical idealization has therefore exceeded the metaphysics, so, in keeping with Leibniz's insistence on the metaphysical importance of mathematical speculation, the metaphysics requires recalibration.

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Post Poincaré, the infinite series of states of the world is no longer contained in each monad. There is no pre-established harmony. The continuity of the states of the actual world and the discrimination between what is compossible and what is incompossible with this world is no longer pre-determined. The logical possibilities of all incompossible worlds are now real possibilities, all of which have the potential to be actualized by monads as states of the current world, albeit with different potentials. As Deleuze argues 'To the degree that the world is now made up of divergent series (the chaosmos), . . . the monad is now unable to contain the entire world as if in a closed circle that can be modified by projection' (1993: 137). So while the theory of continuity is able to be mapped onto the Leibnizian account of the unity of convergent series, the subsequent developments by Poincaré provide a solution that can be understood to overcome these explicit limits of Leibniz's metaphysics.

When it comes to Leibniz's account of motion, Deleuze endorses the hypothesis of a fractal account of our perception of motion. However, the recalibration of Leibniz's metaphysics that Deleuze undertakes in line with the more recent developments in mathematics explicated above has repercussions for Leibniz's impulse account of accelerated motion. According to Leibniz's later metaphysics, the impulses at the root of motion, that is, the leaps between indistant points that result in changes in motion, are not the effects of moving bodies upon one another, but rather the effects of the actions of accelerative forces, determined by primary active force that are predetermined by virtue of God's selection of the best of possible worlds and the pre-established harmony of the relations between monads – past, present and future – that this entails. However, according to Deleuze, one of the repercussions of Poincaré's theory of automorphic functions is that there is no longer a pre-established harmony of the relations between monads, and the world is no longer understood to have been the subject of a divine selection as the best of the possible worlds. What this means for Leibniz's mature account of accelerated motion is that the impulses at the root of motion can no longer be explained by monads and a pre-established harmony of the relations between them. Instead, a mathematical explanation can be drawn from Poincaré's theory of automorphic functions. What displaces the monad on this Deleuzian account and takes on the role of bringing unity to the multiplicities of parts of matter is the essential singularity. The 'jump' of the variable across the domain of discontinuity between the poles of two local functions, which actualizes the infinite branches of the Poincaréan composite function, corresponds to what Leibniz refers to in his impulse account of accelerated motion as the unextended 'leap' made by a body in motion from the end of one subinterval to the *locus proximus*, the indistant but distinct beginning point of the next interval, which marks a change in the direction and velocity of the moving body.[14] However, rather than marking a change in direction and velocity of the moving body, the essential singularity brings unity to the variables of the composite function, which correspond to the compossible predicates contained in the concept of the subject, insofar as it determines the form of a solution curve in its immediate neighbourhood by acting as an *attractor* for the trajectory of the variables that 'jump' across its domain.

According to this Neo-Leibnizian account, in the sense perception of finite minds, the corporeal world still appears immediately as only finitely complex and piecewise continuous, and matter 'only appears to be continuous' because our imperfect perceptual apparatus, which is differential in nature, obscures the minute perceptions of the divisions which actually separate the parts of bodies. However, the relations of motion that are exhibited between parts of matter that are more or less perceived in perceptual experience are no longer predetermined according to the pre-established harmony nor are they resolved into leaps in relation to the impulses of monads, determined by primary active force. Instead, motion can actually be considered to be the result of the impact of bodies upon one another and is explained by mechanics. And the jumps of variables in relation to essential singularities, which displace the leaps in relation to impulses of monads, no longer determine the forces of motion, but rather determine the transformations of individuals to different levels or degrees of individuation. The concept of individuation that is being used here is that developed by Deleuze in relation to Spinoza. The essential singularities take on the role of the dominant monads as unities. Any particular degrees of individuation appear immediately as only finitely complex and piecewise continuous, though upon closer scrutiny they are determined to be composed of a multiplicity of degrees of individuation and thus to be indefinitely complex and fractal in structure. Rather than motion exhibiting a fractal structure, it is the multiplicity of degrees of individuation that now exhibits fractal structure, that is, the complexity of individuation, which consists of a mapping of essential singularities, exhibits fractal structure. Of course, the resolution of the jumps of variables in relation to essential singularities, or of the compossible propositions in the concept of the individual or monad, because no longer predetermined, is far from complete. It is rather open ended, and the logical possibilities of all incompossible worlds are now real possibilities, all of which have the potential to be actualized by essential singularities, or individuated, as the composite functions characteristic of states of the current world.[15]

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The reconstruction of Leibniz's metaphysics that Deleuze provides in *The Fold* draws upon not only the mathematics developed by Leibniz but also upon developments in mathematics made by a number of Leibniz's contemporaries and a number of subsequent developments in mathematics. Deleuze then retrospectively maps these developments back onto the structure of Leibniz's metaphysics in order to bring together the different aspects of Leibniz's metaphysics with the variety of mathematical themes that run throughout his work. The result is a thoroughly mathematical explication of Leibniz's metaphysics, and it is this account that subtends the entire text of the *Fold*. It is these aspects of Deleuze's project in *The Fold* that represent the 'new Baroque and Neo-Leibnizianism' (1993: 136) that Deleuze has explored elsewhere in his body of work and that structures the alternative normative framework developed in his philosophy, notably in Chapters 1 and 4 of *Difference and Repetition* (1994) and in the ninth and the sixteenth series of the *Logic of Sense* (1990b), where Deleuze explicates his Neo-Leibnizian account of the problematic and his account of the genesis of the individual.

Notes

1 Deleuze actually extracts philosophical problematics from the history of philosophy and then redeploys them either in relation to one another or in relation to mathematical problematics, or in relation to problematics extracted from other discourses, to create new concepts, which, according to Deleuze and Guattari in *What Is Philosophy?* (1994), is the task of philosophy.
2 Levy 2003, p. 413. Levy cites Deleuze (1993, p. 16) as one of the commentators to have picked up on the idea of fractal structure to describe the 'folding of matter' in Leibniz's metaphysics.
3 See Hallward 2003, p. 382; Rajchman 1997, p. 116 and Simont 2003, p. 42.
4 Letter to Simon Foucher (1693), Leibniz 1965, I, pp. 415–16.
5 For a discussion of the Leibnizian fictional or syncategorematic definition of the infinitesimal, see Jessup 2008, 215–34.
6 A summary of which appears in Leibniz's *Monadology*, 1714 (Leibniz 1991, pp. 68–81).
7 Huygens in his 1656 study *De Motu corporum ex percussione* ('On the Motion of Bodies by Percussion'), parts of which were published in 1669. Newton also handles accelerated motion in essentially this way in the *Principia* (1687).
8 See Lakhtakia et al. 353.
9 Deleuze, *sur Leibniz*, 29 April.
10 In *Difference and Repetition*, Deleuze argues that 'for eachworld, a series which converges around a distinctive point [singularity] is capable of being continued in all directions in other series converging around other points, while the incompossibility of worlds, by contrast, is defined by the juxtaposition of points which would make the resultant series diverge' (Deleuze, 1994: 48).
11 For further discussion of Deleuze's interpretation of the difference between compossibility and incompossibility, see Duffy, 2010.
12 For further discussion of Deleuze's engagement with Poincaré's theory of automorphic functions, see Duffy, 2006b.
13 See Deleuze 1994, p. 49, where Deleuze characterizes the limitations of the concept of convergence in Leibniz's philosophy.
14 The 'jump' of the variable across the domain of discontinuity also corresponds to the 'leap' that Deleuze refers to in *Expressionism in Philosophy* (1990a) when an adequate idea of the joyful passive affection is formed (283). It characterizes the 'leap' from inadequate to adequate ideas, from joyful passive affections to active joys, from passions to actions. For a further explication of the correspondence between the 'jump' and the 'leap' in Deleuze's engagement with Spinoza, see Duffy 2006a, 158–63, 185–7.
15 Further developments of this framework in the history of mathematics can be traced through the nineteenth century, particularly in the work of Gauss and Riemann, which then feed into the developments of twentieth-century physics.

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