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The Role of Mathematics in Deleuze's Critical Engagement with Hegel

Simon Duffy [a]
[a] University of Sydney, Australia

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Simon Duffy

Abstract

The role of mathematics in the development of Gilles Deleuze's (1925–95) philosophy of difference as an alternative to the dialectical philosophy determined by the Hegelian dialectic logic is demonstrated in this paper by differentiating Deleuze's interpretation of the problem of the infinitesimal in *Difference and Repetition* from that which G. W. F Hegel (1770–1831) presents in the *Science of Logic*. Each deploys the operation of integration as conceived at different stages in the development of the infinitesimal calculus in his treatment of the problem of the infinitesimal. Against the role that Hegel assigns to integration as the inverse transformation of differentiation in the development of his dialectical logic, Deleuze strategically redeploys Leibniz's account of integration as a method of summation in the form of a series in the development of his philosophy of difference. By demonstrating the relation between the differential point of view of the Leibnizian infinitesimal calculus and the differential calculus of contemporary mathematics, I argue that Deleuze effectively bypasses the methods of the differential calculus which Hegel uses to support the development of the dialectical logic, and by doing so, sets up the critical perspective from which to construct an alternative logic of relations characteristic of a philosophy of difference. The mode of operation of this logic is then demonstrated by drawing upon the mathematical philosophy of Albert Lautman (1908–44), which plays a significant role in Deleuze's project of constructing a philosophy of difference. Indeed, the logic of relations that Deleuze constructs is dialectical in the Lautmanian sense.

Keywords: Deleuze; mathematics; Hegel

1 Deleuze and the History of Mathematics

Deleuze's engagements with mathematics can be characterized in a general and schematic way as consisting of three different components, all of which are mutually implicated with one another. The explication of each of these components, and of its relations of implication, is required in order to develop an adequate understanding of these kinds of engagements.

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1. The first component can be characterized as the history of mathematics relevant to each of the programmes or mathematical disciplines with which Deleuze engages, and the alternative lineages in the history of mathematics that are determinable in relation to them. An explication of how these 'histories' function as alternative lineages to the accepted retrospectively constructed history of mathematics that dominates the general understanding of the discipline at various moments of its history involves determining each of these alternative lineages as being determined in relation to a mathematical problem or problematic that challenges the self-imposed limits of the discipline at that time. The tensions that Deleuze characterizes between the history of mathematics and the mathematical problematics that are extracted from it can be understood to be characteristic of the relation between what Deleuze and Guattari in *A Thousand Plateaus* deem to be Royal or major science and nomadic or minor science.

An understanding of each of the mathematical engagements which Deleuze undertakes throughout his work therefore requires a clear explication of the history of mathematics from which the specific mathematical problematic has been extracted, and of the alternative lineage in the history of mathematics that is generated in relation to it. The example that is used in section 2 of this paper involves the relation between the problem of the infinitesimal in the history of the differential calculus and the development of the Hegelian dialectical logic.[1]

2. The second component of each of Deleuze's engagements with mathematics can be characterized as the explication of the manner by means of which these interventions in the history of mathematics are redeployed by Deleuze in relation to the history of philosophy. The mathematical problematics extracted from the history of mathematics are directly redeployed by Deleuze as philosophical problematics in relation to the history of philosophy. This is achieved by mapping the alternative lineages in the history of mathematics onto corresponding alternative lineages in the history of philosophy, that is, by isolating those points of convergence between the mathematical and philosophical problematics extracted from their respective histories. The redeployment of mathematical problematics as philosophical problematics is one of the strategies that Deleuze employs in his engagement with the history of philosophy. Deleuze actually extracts philosophical problematics from the history of philosophy, and then redeploys them either in relation to one another, or in relation to mathematical problematics, or in relation to problematics extracted from other discourses, to create new concepts, which, according to Deleuze and Guattari in *What is Philosophy?*, is the task of philosophy.

3. The creation of new concepts by bringing together mathematical and philosophical problematics constitutes the third component of these Deleuzian engagements in mathematics. The mutual implication of these three components constitutes the mechanism by means of which Deleuze's

interventions in the history of mathematics serve in his project of constructing a philosophy of difference.

Deleuze is therefore interested in particular kinds of mathematical problematics that can be extracted from the history of mathematics, and in the relationship that these problematics have to the discourse of philosophy. He can therefore be understood to redeploy not only the actual mathematical problematics that are extracted from the history of mathematics in relation to the history of philosophy, but also the logic of the generation of mathematical problematics in relation to the history of philosophy, in order to generate the philosophical problematics which are then redeployed in his project of constructing a philosophy of difference. The logic of the generation of mathematical problematics is characterized as the calculus of problems in section 3 of this paper by drawing upon the work of Albert Lautman. It is in relation to the history of philosophy that Deleuze then determines the logic of the generation of philosophical problematics as the logic characteristic of a philosophy of difference.

This logic, the logic of the calculus of problems, is not simply characteristic of the relative difference between Royal and nomadic science, or between the history of mathematics and its related mathematical problematics. It is rather characteristic of the very logic of the generation of each mathematical problematic itself. It is this logic that Deleuze redeploys in relation to the history of philosophy as a logic of difference in order to generate the philosophical problematics that he then uses to construct a philosophy of difference. Developing an understanding of the nature of this logic is the key to understanding Deleuze's engagement with the history of mathematics and his use of mathematical problematics throughout his work.

The episode in the history of mathematics from which the mathematical problematic that is used in this paper is extracted is the problem of the infinitesimal in the history of the calculus and its various (alternative) lines (or lineages) of development, which were only put on a rigorous algebraic foundation towards the end of the nineteenth century, by the Weierstrassian epsilon-delta method (see Potter, p. 85).

Because there is no reference to infinitesimals in the Weierstrassian definition of the calculus, the designation 'the infinitesimal calculus' was considered to be 'inappropriate' (Boyer, p. 287). Weierstrass's work effectively eliminated the use of the Leibnizian-inspired infinitesimals in doing what was now referred to as the differential calculus for over half a century. It was not until the late 1960s, with the development of the controversial axioms of non-standard analysis by Abraham Robinson (1918–74), that the infinitesimal was given a rigorous foundation (see Bell, 1998), thus allowing the inconsistencies to be removed from the Leibnizian infinitesimal calculus without removing the infinitesimals themselves.[2] Leibniz's ideas about the role of the infinitesimal in the calculus have therefore been 'fully vindicated' (Robinson, p. 2), as Newton's had been thanks to Weierstrass.[3]

In response to the protracted history of these developments,[4] Deleuze brings renewed scrutiny to the relationship between the developments in the history of mathematics and the metaphysics associated with these developments, which were marginalized as a result of efforts to determine the rigorous algebraic foundations of the calculus. This is a part of Deleuze's broader project of constructing an alternative lineage in the history of philosophy that tracks the development of a series of metaphysical schemes that respond to and attempt to deploy the concept of the infinitesimal. The aim of the project is to construct a philosophy of difference as an alternative philosophical logic that subverts a number of the commitments of the Hegelian dialectical logic which supported the elimination of the infinitesimal in favour of the inverse operation of differentiation as reflected in the operation of negation, the procedure of which postulates the synthesis of a series of contradictions in the determination of the concept. The operation of this was then taken for granted and redeployed by subsequent generations of scholars, the Hegelian dialectical logic becoming one of the most influential and entrenched philosophical logics that guided the development of philosophy for the centuries that followed.

2 Deleuze and Hegel on the Problem of the Infinitesimal

The differential calculus consists of two branches which are inverse operations: differential calculus, which is concerned with calculating derivatives, or, in Leibnizian terms, differential relations or quotients; and integral calculus, which is concerned with integration, or the calculation of the infinite sum of the differentials in the form of series. The differential point of view of the infinitesimal calculus approaches integration as a process of summation by considering the problem of finding the area under the graph of a function. This problem is dealt with by dividing up the area under the curve into a large number of rectangles. The area under the curve is the sum of the infinitely small and infinitely numerous rectangles. The difference between this sum and the actual area is considered small enough to be neglected. The integral is therefore the finite magnitude of the area. In the development of the infinitesimal calculus Leibniz recognized integration to be a process not only of summation, but also of the inverse transformation of differentiation, so the integral is not only the sum of differentials, but also the inverse of the differential relation. Leibniz called this inverse relationship between differentiation and integration the fundamental theorem of the calculus.

In the early nineteenth century, the process of integration as a summation was overlooked by most mathematicians in favour of determining integration, instead, as the inverse transformation of differentiation, according to which integrals are computed by finding anti-derivatives, otherwise known as primitive functions. This operation of inversion is effected by means of a large number of rules, or algorithms. Hegel continued this tendency by defining the integral solely as the inverse of the differential relation. According to Hegel 'the integral calculus has been simplified and more correctly determined merely by the fact that it is no longer taken to be a *method of summation* in which it appeared essentially connected with the *form of a series*'.[5] However, the problem of integration as a process of summation from the differential point of view of the infinitesimal calculus did continue to be explored. This method was later reformulated by Augustin Cauchy (1789–1857) and Georg Riemann (1826–66), and by later I mean that it was a development that post-dates Hegel. In contrast to this move by Hegel, Deleuze appeals to the interpretations of the differential calculus according to which integration is a method of summation in the form of a series.

The object of the process of integration in general is to determine from the coefficients of the given function of the differential relation the original function from which they were derived. Put simply, given a relation between two differentials, dy/dx, the problem of integration is how to find a relation between the quantities themselves, y and x. This problem corresponds to the method of finding the function of a curve characterized by a given property of its tangent. The differential relation is thought of as another function which describes, at each point on an original function, the gradient of the line tangent to the curve at that point. The value of this 'gradient' indicates a specific quality of the original function: its rate of change at that point. The differential relation therefore indicates the specific qualitative nature of the original function, that is, the shape and behaviour of the graph of the function or curve, at the different points of the curve.

The inverse process of this method is differentiation, which in general determines the differential relation as the function of the line tangent to a given curve. To put it simply, to determine the tangent of a curve at a specified point, a second point that satisfies the function of the curve is selected, and the gradient of the line that runs through both of these points is calculated. As the second point approaches the point of tangency, the gradient of the line between the two points approaches the gradient of the tangent. The gradient of the tangent is, therefore, the limit of the gradient of the line between the two points.

Given that the method of integration provides a way of working back from the differential relation, the problem of integration is, therefore, how to reverse this process of differentiation. This can be solved by determining the inverse of the given differential relation according to the inverse transformation of differentiation. Or, a solution can be determined from the differential point of view of the infinitesimal calculus by considering integration as a process of summation in the form of a series, according to which, given the specific qualitative nature of a tangent at a point, the problem becomes that of finding, not just one other point determinative of the differential relation, but a sequence of points, all of which together satisfy, or generate, a curve and therefore a function in the neighbourhood of the given point of tangency; which therefore functions as the limit of the function.[6]

Deleuze considers this to be the base of the infinitesimal calculus as understood or interpreted in the seventeenth century. The formula for the problem of the infinite that Deleuze extracts from this seventeenth-century understanding of the infinitesimal calculus is that 'something finite consists of an infinity under a certain relation'. Deleuze considers this formula to mark 'an equilibrium point, for seventeenth-century thought, between the infinite and the finite, by means of a new theory of relations'.[7]

In his account of the differential calculus from the point of view of the dialectical logic, Hegel argues that 'the infinitely small which presents itself in the differential calculus as *dx* and *dy*, does not have merely the negative, empty meaning of a non-finite, non-given magnitude' (*SL* 268). On the contrary, the infinitely small has 'the specific meaning of the qualitative nature of what is quantitative, of a moment of a ratio as such' (268).

Hegel defines the quantitative difference of a differential as constituting its qualitative character, and the differential relation functions therefore as the qualitative determination of quantity. He defines the qualitative character of what is quantitative in the differential by arguing that, in the differential, 'a quantitative difference, the definition of which is that it not only *can*, but *shall* be smaller than any given difference, is no longer a quantitative difference, this is self-evident, as self-evident as anything can be in mathematics' (268). The quantitative difference of a differential thus constitutes its qualitative character. Hegel argues that 'the demonstrated *qualitative character as such* of the form of magnitude here under discussion in what is called the infinitesimal, is found most directly in the category of *limit of the ratio*' (266). The limit of the ratio is determined by Hegel as the differential relation. However, for Hegel, 'the idea of limit […] impl[ies] the true category of the *qualitatively* determined relation of variable magnitudes' (266). The differential relation functions therefore as the qualitative determination of quantity. Hegel can therefore argue that 'the so-called infinitesimals express the vanishing of the sides of the ratio as quanta, and that what remains is their quantitative relation solely as qualitatively determined; far from this resulting in the loss of the qualitative relation the fact is that it is just this relation which results from the conversion of finite into infinite magnitudes' (269). The finite magnitudes that Hegel is referring to are *x* and *y*; and the infinite magnitudes into which these finite quanta are converted are the differentials *dx* and *dy*, that is, the resulting qualitative relation of each of the finite quanta.

According to Hegel, 'x and y as such are still supposed to signify quanta; now this significance is altogether and completely lost in the so-called *infinitesimal differences*' (253). What Hegel contends is that the infinitesimal calculus failed to consider adequately the x of dx and the y of dy as quanta; instead it simply posited the infinite value of dx and dy themselves, without accounting for the conversion or transition from x to dx or from y to dy. Hegel argues that 'it is this concept which has been the target for all the attacks made on the fundamental determination of the mathematics of this infinite, that is, of the differential and integral calculus' (253), and he contends that it is the inability of mathematics 'to justify the object as *Concept* [*Begriff*][8] which is mainly responsible for these attacks' (254). He argues that 'the originators of the definitions did not establish the thought as Concept and found it necessary in the application to resort again to expedients which conflict with their better cause' (255). According to Hegel, the object, which is determined by the limit of the ratio, is justified as the Concept by means of the dialectical logic. The product of the primary negation of the finite quantum, x, is the infinite qualitative relation, or differential, dx. The absolute negation of dx is effected in the differential relation, dy/dx, according to which the finite quanta are realized in the finite determinateness of the limit of the ratio.

What Hegel dismisses as 'the so-called infinitesimal' is for him the relation of primary negation of the finite quanta, x and y, in dx and dy. He argues that 'the infinitesimal signifies, strictly, the negation of quantum as quantum, that is, of a so-called finite expression, of the completed determinateness possessed by quantum as such' (299). The absolute negation of the infinitesimal, in the differential relation, he describes as 'the vanishing of the sides of the ratio'. He argues that 'the specific nature of the notion of the so-called infinitesimal is the *qualitative* nature of determinations of quantity which are related to each other primarily as quanta' (275). What remains of the finite quantum, x and y, in the differential relation, he argues, is their quantitative relation solely as qualitatively determined, that is, the limit of the ratio solely as determined by dx and dy as dy/dx.

The operation of differentiation 'does not confine itself to the *finite* determinateness of its object', that is, determining the differential relation as the limit of the ratio. 'On the contrary', Hegel argues, 'it converts it into an identity with its opposite, for example converting a curved line into a straight line, the circle into a polygon, etc.' (254). Differentiation determines the limit of the ratio as the tangent to the circle or curve in what Hegel refers to as 'the moment of quantitative transition'. As with differentiation, the operation of integration converts the differential relation into an identity with its opposite; however, it effects this conversion in the opposite direction, from the limit of the ratio as the tangent, that is, from dy/dx, to the curve itself, as a function of x and y. Integration expresses 'the transition of straight lines which are infinitely small, into curved lines, and their relation in their infinity as a relation of curves' (271). A straight line is defined as 'the *shortest* distance between two points, its difference from the curved line is [therefore] based on the determination [...] of a *quantum*' (271). Hegel argues that 'this determination vanishes in the line when it is taken as an intensive magnitude, as an infinite moment' (271). The finite quantum, *x*, is determined as having vanished, or as having been negated, in the differential *dx*, when *dx* is taken as an intensive magnitude or infinite moment. Insofar as the differential is taken as an intensive magnitude, 'the straight line and arc no longer retain any quantitative relation nor consequently [...] any qualitative difference from each other either; on the contrary, the former passes into the latter' (271), in what Hegel refers to as 'the moment of qualitative transition' (304). Integration is therefore the determination, from the differential, *dx*, as an intensive magnitude, of the finite quantum on the curve of the original function.

'Consequently', Hegel argues, 'the operations which [the mathematics of the infinite] allows itself to perform in the differential and integral calculus are in complete contradiction with the nature of merely finite determinations and their relations and would therefore have to be justified solely by the *Concept*' (254). The relation between the finite determinations, *x* and *y*, and their relations, for example *y/x*, is in complete contradiction with the operations dealing with the differentials, *dy* and *dx*, and their relation, *dy/dx*. Insofar as both differentiation, as the transformation of *x* and *y* to *dy/dx*, and integration, as the inverse transformation of *dy/dx* to *x* and *y*, deal with the relation between finite determinations, and their relations, and differentials, and their relation, they are contradictory operations, which are, furthermore, in contradiction with each other. The reference to the mechanism of the principle of contradiction in relation to the inverse processes of differentiation and integration belies Hegel's consideration of the relation between differentiation, as the moment of quantitative transition, and integration, as the moment of qualitative transition, as being justified solely by the Concept, that is, the transformation from one to the other is determined solely according to the dialectical logic. So, according to the dialectical logic, the moment of quantitative transition, which determines the limit of the ratio, is negated in the moment of qualitative transition, which determines the differential as an intensive magnitude, the absolute negation of which determines the finite determination of the differential relation, that is, its quantitative determinateness, as actually infinite; in other words, the differential relation realizes itself as actually infinite. It is 'in this concept of the infinite', Hegel argues, that 'the quantum is genuinely completed into a qualitative reality; it is posited as actually infinite; it is sublated [*aufgehoben*] not merely as this or that quantum but as quantum generally' (253). In Chapter 3 of the *Science of Logic*, Hegel defines the differential relation, which he refers to as 'the quantitative relation or quantitative ratio', from the point of view of the dialectical logic, when he declares that 'the infinity of quantum has been determined to the stage where it is the negative beyond of quantum, which beyond, however, is contained within the quantum itself. This beyond is the qualitative moment as such. The infinite quantum as the unity of both moments, of the quantitative and qualitative determinateness, is in the first instance a *ratio*' (314).

According to Hegel, the infinitesimal interpretation of the calculus lacks an adequate expression of the differential as the qualitative moment of the quantitative relation, and therefore as an infinite quantum. Hegel argues that 'the infinite which is associated with infinite series, the indeterminate expression of the negative of quantum in general, has nothing in common with the affirmative determination belonging to the infinite of this calculus' (301). Hegel considers the operation of the differential calculus according to the dialectical logic to provide the logical basis for such an adequate expression of the differential.[9]

Deleuze recognizes that Hegel grasped what is at stake in *dx* when he writes that 'Hegel seems to recognise the presence of a genuine infinite in the differential calculus, the infinity of "relation".'[10] Indeed, it is difficult to grasp a discordance between each of their declarations on the differential, each considering the differential as that moment where the terms of the relation, as vanishing quantities, are only determinable according to their relation. In *Difference and Repetition*, Deleuze writes:

> In relation to *x*, *dx* is completely undetermined, as *dy* is to *y*, but they are perfectly determinable in relation to one another. For this reason, a principle of determinability corresponds to the undetermined as such. The universal is not a nothing since there are, in Bordas's expression, 'relations of the universal'. [...] The relation *dy/dx* is not like a fraction which is established between particular quanta in intuition, but neither is it a general relation between variable algebraic magnitudes or quantities. Each term exists absolutely only in its relation to the other.
>
> (*DR* 172)

And in the *Science of Logic* Hegel writes:

> *Dx*, *dy*, are no longer quanta, nor are they supposed to signify quanta; it is solely in their relation to each other that they have any meaning, *a meaning merely as moments*. They are no longer *something* (something taken as a quantum), not finite differences; but neither are they *nothing*; not empty nullities. Apart from their relation they are pure nullities, but they are intended to be taken only as moments of the relation, as *determinations* of the differential co-efficient *dx/dy*.
>
> (*SL* 253)[11]

However, Deleuze affirms the existence of a discordance with the suggestion that Hegel only 'seems' to recognize the presence of a genuine infinite in the differential calculus. Deleuze argues that 'for Hegel, infinite representation cannot be reduced to a mathematical structure: there is a non-mathematical or supra-mathematical architectonic element in continuity and in the differential calculus' (*DR* 310 n9). As far as Deleuze is concerned, the Hegelian concept of the genuine infinite, rather than being determined by the differential calculus, is determined, on the contrary, by the implication of the differential calculus in the dialectical logic. Indeed, far from explaining the nature of the differential calculus, Hegel's presentation presupposes it in the form of the dialectical logic.

In order to differentiate his thought from that of Hegel, Deleuze writes that 'just as we oppose difference in itself to negativity, so we oppose *dx* to not-A, the symbol of difference [*Differenzphilosophie*] to that of contradiction' (170). Deleuze refers to the Hegelian concept of the differential as 'not-A', rather than '*dx*', to indicate the implication of the differential as a moment of the dialectical logic, which results from the primary negation of a finite quantum, 'A'. It is the tradition which adheres to the realism of the infinitely small that motivates Hegel's reticence in regard to the infinitesimal. The differential, from the point of view of this tradition, is represented in an unlimited series, or what Hegel refers to as a bad infinite. It is thereby understood by him to be condemned to the unsatisfactory status of justified approximation or negligible error.[12] What is at stake in the debate on the legitimacy of the infinitesimal is 'the integration of the infinitesimal into the register of quantity' (Salanskis 71), that is, of the infinite in the finite, which comes down to the alternative between infinite and finite representations. This is precisely what is at issue in what Deleuze describes as 'the "metaphysics" of the calculus' (DR 176). Throughout the eighteenth century, there was disagreement as to the particular kind of 'metaphysics' by which 'to rescue the procedures of the calculus' from the vagaries of the infinitesimal.

The Deleuzian solution to the debate on the legitimacy of the infinitesimal distinguishes itself from the Hegelian solution insofar as it is not resolved according to the dialectical logic. Rather than being involved in what Deleuze describes as the 'circulation of opposing representations which would make their coincidence in the identity of a concept' (178), Deleuze argues that the alternative between infinite and finite representations, and therefore the metaphysics of the calculus, is 'strictly immanent to the techniques of the calculus itself' (176). Deleuze's thesis is that the differential, *dx* or *dy*, which he considers to feature in the discourse of the pioneers, cannot be classified within the dialectical logic, which rather asserts the opposition of the finite and the infinite.

In accord with Robinson's 'vindication' of the infinitesimal, Deleuze finds in the example of the method of integration as a process of summation not simply the primitive expression of the differential calculus, but rather the logic of the differential from the differential point of view of the infinitesimal calculus.[13] By implicating Leibniz's now vindicated understanding of the early form of the infinitesimal calculus in his account of the history of the development of the calculus, Deleuze demonstrates how the process of integration as a summation eludes the grasp of the dialectical progression of the history of philosophy, and in doing so nominates Leibniz as one of the figures with whom he engages in his project of renewing the history of philosophy by constructing an alternative lineage in the history of philosophy.

Deleuze considers seventeenth-century thought to have developed a new theory of relations by means of the infinitesimal calculus, one which is determined according to the logic of the differential. The relation between the two differentials of a differential, *dy/dx*, does not equal zero, or is not undefined, despite the fact that *dy/dx = 0/0*. Instead, the differential relation itself, *dy/dx*, subsists as a relation. 'What subsists when *dy* and *dx* cancel out under the form of vanishing quantities is the relation *dy/dx* itself' (Deleuze, 17 Feb.). Despite the fact that its terms vanish, the relation itself is nonetheless real. It is here that Deleuze considers seventeenth-century logic to have made 'a fundamental leap', by determining 'a logic of relations'.[14] He argues that 'under this form of infinitesimal calculus is discovered a domain where the relations no longer depend on their terms' (Deleuze 10 Mar.). It is this logic of relations that provides a starting point for the investigation into the logic that Deleuze deploys throughout his work, and which can be traced through *Difference and Repetition* as a part of his project of constructing a philosophy of difference; a logic which functions as an alternative to the Hegelian dialectical logic.

Having located the logic of the differential from the differential point of view of the infinitesimal calculus in the work of Leibniz, Deleuze then tracks the subsequent developments that this logic undergoes in relation to the work of some of the key figures in the history of this branch of the infinitesimal calculus. In particular: Cauchy and Riemann, whose work on integration as a process of summation has already been alluded to; Weierstrass and Poincaré, specifically Weierstrass's theory of analytic continuity and Poincaré's theory of automorphic functions;[15] Albert Lautman, whose work is the focus of the next section; and Abraham Robinson's proof of the infinitesimal. These figures are implicated in an alternative lineage in the history of mathematics by means of which the differential point of view of the infinitesimal calculus is aligned with the differential calculus of contemporary mathematics. The logic of the differential from the differential point of view of the infinitesimal calculus is then implicated in the development of Deleuze's project of constructing a philosophy of difference.

3 Deleuze and Albert Lautman's Mathematical Philosophy

Lautman's views on mathematical reality and on the philosophy of mathematics parted with the dominant tendencies of mathematical epistemology of the time. Lautman considered the role of philosophy, and of the philosopher, in relation to mathematics to be quite specific. He writes that: 'in the development of mathematics, a reality is affirmed that mathematical philosophy has as its function to recognize and to describe'.[16] He goes on to characterize this reality as an 'ideal reality' that 'governs' the development of mathematics. He maintains that 'what mathematics leaves for the philosopher to hope for, is a truth which would appear in the harmony of its edifices, and in this field as in all others, the search for the primitive concepts must yield place to a synthetic study of the whole' (AL 24).

One of the tasks, indeed the challenges, that Lautman sets himself, but never carried through because of his early and tragic demise – Lautman was captured by the Nazis in 1944 and shot for being an active member of the resistance – was the task of deploying the mathematical philosophy that he had developed in other domains. The commentator who shows the most assiduity in his engagement with Lautman by taking up this challenge is Gilles Deleuze. The mathematical work that is drawn upon and that plays a significant role in Deleuze's philosophical project is that of Lautman. Indeed, the philosophical logic that Deleuze constructs as a part of his project of constructing a philosophy of difference is dialectical in the Lautmanian sense. The aim of this part of the paper is to give an account of this Lautmanian dialectic, of how it operates in Lautman's work, and to characterize what Deleuze does to Lautman's dialectic when it is incorporated into his project of constructing a philosophy of difference.

What is quite clear in Lautman's work is that he was not concerned with specific foundational questions in mathematics, with those relating to its origins, to its relationship to logic or to the problem of foundations. What he is interested in rather is shifting the ground of this very problematic by presenting an account of the nature of mathematical problematics in general. Lautman was inspired by the work of Hilbert on the axiomatic concept of mathematics to deploy the potential of an axiomatic-structuralism in mathematics. The essential point that motivated this move was Lautman's conviction 'that a mathematical theory is predominantly occupied with the relations between the objects that it considers, more so than with the nature of those objects' (Dieudonné, AL 16). The introduction of the axiomatic method into mathematics means that there is an 'essential dependence between the properties of a mathematical object and the axiomatic field to which it belongs' (AL 146). It is precisely in the meta-mathematical work of Herbrand and Gödel that Lautman considers a new theory of the mathematical real to have been affirmed. It is one that is 'as different from the logicism of the formalist as from the constructivism of the intuitionist' (89). Indeed, Lautman maintains that 'logic is not a priori compared to mathematics, but that for logic one needs a mathematics to exist' (48). Lautman sets himself the task of disengaging from this new mathematical real 'a philosophy of mathematical genesis, whose range goes far beyond the field of logic' (89). What we have with this conception of the mathematical real is 'the statement of a logical problem without at all having the mathematical means of resolving it' (28). Lautman proposes to characterize the problematic 'distinction between the position of a logical problem and its mathematical solution' (28) by means of an 'exposé' of what he calls 'the metaphysics of logic' (87). This takes the form of 'an introduction to a general theory of the connections which unite the structural considerations' of the axiomatic conception of mathematics with the 'affirmations of existence' of a particular dynamic conception (87). The particular dynamic conception of mathematics that Lautman deploys is further characterized as 'the ideal reality which is solely capable of giving its sense and value to the mathematical experience' (Cavaillès and Lautman, p. 39). This ideal reality is constituted by what he refers to as 'abstract Ideas'. Lautman proposes to call the relation between the development of mathematical theories and the Ideas of this ideal reality 'dialectical' (AL 28), and he refers to these Ideas as 'dialectical ideas'. Lautman's principal thesis is that mathematics participates in a dialectic that governs [*domines*] it in an abstract way. He argues that the Ideas 'which appear to govern the movement of certain mathematical theories', and which are conceivable as independent of mathematics, 'are not however susceptible of direct study' (29). He goes on to claim that it is these dialectical Ideas that 'confer on mathematics its eminent philosophical value' (29). This is why Lautman considers mathematics to tell, in addition to the constructions in which the mathematician is interested, 'another more hidden story [that is] made for the philosopher' (28). The gist of the story is that there is a 'dialectical action [that] is constantly at play in the background and it is towards its clarification' that Lautman directs his research (28).

What Lautman wants to do is restore to Ideas what he considers to be 'the true Platonic meaning of the term', that is, the understanding of these abstract dialectical ideas as 'the structural schemata according to which effective theories are organized' (AL 204).[17] He characterizes these structural schemata as establishing specific connections between contrary concepts such as: local–global; intrinsic–extrinsic; essence–existence; continuous–discontinuous; and finite–infinite. Lautman provides many examples of these contrary concepts, including the introduction of analysis into arithmetic; of topology into the theory of functions; and the effect of the penetration of the structural and finitist methods of algebra into the field of analysis and the debates about the continuum (see Chevalley, p. 60). The nature of mathematical reality for Lautman is therefore such that 'mathematical theories … give body to a dialectical *ideal*' (AL 253). This dialectic is constituted 'by couples of opposites', and the Ideas or structural schemata of this dialectic are presented in each case 'as the problem of establishing connections between opposing concepts' (253). Lautman makes a firm distinction between concepts and dialectical Ideas: the Ideas 'consider possible relations between dialectical concepts' (210), or conceptual couples,[18] and 'these connections are only determined within the fields where the dialectic is incarnated' (253). What Lautman is proposing is a philosophical logic that considerably broadens the field and range of the meta-mathematics that he adopts from Hilbert. While meta-mathematics examines mathematical theories from the point of view of the concepts of non-contradiction and completeness, Lautman argues that there are 'other logical concepts, also likely to be connected eventually to one another within a mathematical theory' (28). These other logical concepts are the conceptual couples of the structural schemata.[19]

So, for Lautman, Ideas constitute, along with mathematical facts, objects and theories, a fourth point of view of the mathematical real that 'are naturally integrated with one another: the facts consist in the discovery of new objects, these objects organize themselves in theories and the movement of these theories incarnates the schema of connections of certain Ideas' (135). For this reason, the mathematical real depends not only on the base of mathematical facts but also on dialectical ideas that govern the mathematical theories in which they are actualized. Lautman thus reconsiders meta-mathematics in metaphysical terms, and postulates the metaphysical regulation of mathematics. However, he is not suggesting the application of metaphysics to mathematics. Mathematical philosophy such as Lautman conceives it 'does not consist … in finding a logical problem of traditional metaphysics within a mathematical theory' (142). Rather it is from the mathematical constitution of problems that it is necessary to turn to the metaphysical, that is, to the dialectic, in order to give an account of the ideas which govern the mathematical theories. Lautman maintains that the philosophical meaning of mathematical thought appears in the incorporation of a metaphysics (or dialectic), of which mathematics is the necessary consequence. 'We would like to have shown', he argues, 'that this bringing together of metaphysics and mathematics is not contingent but necessary' (203). Lautman doesn't consider this to be 'a diminution for mathematics, on the contrary it confers on it an exemplary role' (10).[20]

A key point for Lautman is that dialectical ideas 'only exist insofar as incarnated mathematically' (203). Lautman insists on this point. He argues that 'the reality inherent in mathematical theories comes to it from the fact that it takes part in an ideal reality which is governing of the mathematics, *but which is only recognizable through it*' (290). This is what distinguishes Lautman's conception from 'a naive subjective idealism' (Petitot, p. 86). Mathematical theories are constituted in an effort to bring a response to the problem posed by dialectical Ideas; however, the conceptual couples of the logical schemata '*are not anterior to their realization within a theory*' (AL

142). The fundamental consequence is that the constitution of new logical schemata and problematic Ideas '*depend on the progress of mathematics itself*' (142). Mathematical philosophy such as Lautman conceives it consists in 'apprehending the structure of [a mathematical] theory globally in order to extract the logical problem which is both defined and resolved by the very existence of this theory' (143). For Lautman, although the dialectic is anterior to mathematics, it 'does not form part of mathematics, and its concepts are without relationship to the primitive concepts of a theory' (210). Lautman defines the 'anteriority of the dialectic' as that of the 'question' in relation to the 'response': 'it is of the nature of the response to be an answer to a question already posed … even if the idea of the question comes to mind only after having seen the answer' (210). The dialectic therefore functions by extracting logical problems from mathematical theories. The apprehension of the conceptual couple, that is, the logical schema of the problematic Idea, only comes after having extracted the logical problem from the mathematical theory. And, it is the logical problem itself, rather than the problematic Idea, that directly drives the development of mathematics. The problematic idea governs the extraction process that deploys the logical problem in the further development of new mathematical theories. So for Lautman, 'The philosopher has neither to extract the laws, nor to envisage a future evolution, his role only consists in becoming aware of the logical drama which is played out within the theories' (142). This effort on the part of the philosopher to 'comprehend dialectical Ideas adequately' is itself 'creative of the system of more concrete concepts where the connections between the [concepts] are defined' (205).

The method that Lautman uses in his mathematical philosophy is 'descriptive analysis'. The particular mathematical theories that he deploys throughout his work constitute for him 'a given' in which he endeavours 'to extract the ideal reality in which this material participates' (40). That is, Lautman starts with mathematical theories that are already in circulation. In relation to these mathematical theories Lautman argues that while

> it is necessary that mathematics exists as an example where the ideal structures of the dialectic can be realized, it is not necessary that the examples which correspond to a particular dialectical structure are of a particular kind; what generally happens on the contrary is that the organizing power of the same structure is affirmed in different theories; they present affinities of mathematical structure which testify to the common dialectical structure in which they take part.
>
> (AL 213)

One of the examples that is developed by Lautman is the operation of the local–global conceptual couple in the theory of the approximate representation of functions.[21] The same conceptual couple is illustrated in geometry (40–4); distinct mathematical theories can therefore be structured by the same conceptual couple.[22]

Lautman sees in the local–global conceptual couple the source of a dialectical movement in mathematics that produces new theories. According to Lautman, the problematic nature of the connections between conceptual couples 'can arise apart from any mathematics, but the effectuation of these connections is immediately mathematical theory' (288). As a consequence, he maintains that 'Mathematics thus plays with respect to the other domains of incarnation, physical reality, social reality, human reality, the role of model where the way that things come into existence is observed' (209). This is an important point for Deleuze which shapes his strategy of engagement with a range of discourses throughout his work. Lautman's final word on mathematical logic is that it 'does not enjoy in this respect any special privilege; it is only one theory among others and the problems which it raises or which it solves are found almost identically elsewhere' (288).

Deleuze, following Lautman, considers the concept of genesis in mathematics to 'play the role of model … with respect to all other domains of incarnation' (AL 209). While Lautman explicated the philosophical logic of the actualization of ideas within the framework of mathematics, Deleuze (along with Guattari) follows Lautman's suggestion and explicates the operation of this logic within the framework of a multiplicity of domains, including, for example, philosophy, science and art in *What is Philosophy?*, and the variety of domains which characterize the plateaus in *A Thousand Plateaus*. While, for Lautman, a mathematical problem is resolved by the development of a new mathematical theory, for Deleuze, it is the construction of a concept that offers a solution to a philosophical problem, even if this newly constructed concept is characteristic of or modelled on the new mathematical theory.

One of the differences between Lautman and Deleuze is that while Lautman locates the ideas in a specifically Platonic and idealist perspective, the ideas that Deleuze refers to are not Platonic,[23] and Lautman's idealism is displaced in Deleuze's work by an understanding of the Lautmanian idea as 'purely' problematic. There is no ideal reality associated with ideas in Deleuze; rather ideas are constituted by the purely problematic relation between conceptual couples. Deleuze defines the 'Idea' as 'a structure. A structure or an Idea is … a system of multiple, non-localisable connections between differential elements which is incarnated in real relations and actual terms' (*DR* 183). For Deleuze, it is the problematic nature of the relations between conceptual couples that incarnate problematic ideas and which govern the kinds of solutions that can be offered to them.

The process of the genesis of mathematical theories that are offered as solutions to mathematical problems corresponds to the Deleuzian account of the construction of concepts as solutions to philosophical problems. The mathematical problematics that Deleuze extracts from the history of mathematics, following Lautman's lead, are directly redeployed by Deleuze as philosophical problematics in relation to the history of philosophy. This is achieved by mapping the alternative lineages in the history of mathematics onto corresponding alternative lineages in the history of philosophy, that is, by isolating those points of convergence between the mathematical and philosophical problematics extracted from their respective histories. The redeployment of mathematical problematics as philosophical problematics is one of the strategies that Deleuze employs in his engagement with the history of philosophy. In the example presented in this paper, Deleuze extracts the problem of the infinitesimal from the history of mathematics and redeploys it in relation to the philosophical problematic of the development of the dialectical philosophy determined by the Hegelian dialectic logic. Deleuze effectively bypasses the methods of the differential calculus which Hegel uses to support the development of the dialectical logic by strategically redeploying Leibniz's account of integration as a method of summation in the form of a series in relation to the differential calculus of contemporary mathematics, and he does this in order to construct an alternative logic of relations characteristic of a philosophy of difference. It is the purely problematic relations between conceptual couples that Deleuze develops in relation to Lautman's philosophy of mathematics, and the solutions that can be offered to those problems that characterize the logic of relations that Deleuze develops as an alternative to the dialectical logic.

University of Sydney, Australia

Notes

1 For another example, see Duffy, *The Logic of Expression*, where Deleuze's interpretation of the geometrical example of Spinoza's Letter XII (on the problem of the infinite) is differentiated from that which Hegel presents in the *Science of Logic*.

2 The infinitesimal is now considered to be a hyper-real number that exists in a cloud of other infinitesimals or hyper-reals floating infinitesimally close to each real number on the hyper-real number line (Bell, 2005: p. 262). The development of non-standard analysis has not in fact broken the stranglehold of classical analysis to any significant extent; however, this seems to be more a matter of taste and practical utility than of necessity (Potter, p. 85).

3 Non-standard analysis allows 'interesting reformulations, more elegant proofs and new results in, for instance, differential geometry, topology, calculus of variations, in the theories of functions of a complex variable, of normed linear spaces, and of topological groups' (Bos, p. 81).

4 Robinson's Non-Standard Analysis is the most recent development that Deleuze refers to (in *The Fold*), but this is by no means the end, or indeed the beginning, of this story. The history of these developments predate Robinson, as does Deleuze's engagement with this history. There have also been a number of independent formalizations of the infinitesimal since Robinson (Bell, 1998, Connes),

each of which allows reformulations, more elegant proofs and new results in a range of areas in mathematics.

5 Hegel, *Hegel's Science of Logic*, p. 294. Hereafter *SL*.

6 For a more extensive account of Leibniz's infinitesimal calculus as deployed by Deleuze see Duffy, 'The Differential Point of View of the Infinitesimal Calculus in Spinoza. Leibniz and Deleuze'.

7 Deleuze, *Sur Spinoza*, 17 February 1981. Hereafter Deleuze, 17 Feb.

8 The reasoning of Jean-Michel Salanskis is followed in translating *Begriff* as *Concept*: 'The term for the German *Begriff* in the English translation is *Notion*, whereas I prefer to follow the French usage in translating it as *Concept*.' See Salanskis, p. 79.

9 For a balanced assessment of the mathematics developed by Hegel in the *Science of Logic*, see Pinkard, 1981 and 1988.

10 Deleuze, *Difference and Repetition*, 310 n9. Hereafter *DR*.

11 Juliette Simont juxtaposes the same two citations in order to demonstrate the difficulty in differentiating the respective philosophical logics of Hegel and Deleuze. See Simont, p. 281.

12 It should be noted that Leibniz also characterized the infinitesimal in this way; however, he justified its use on the basis of its effectiveness.

13 According to Deleuze, the 'finitist interpretations' of the calculus given in modern set-theoretical mathematics – which Salanskis considers to be congruent with what Penelope Maddy calls 'Cantorian finitism', 'namely the idea that infinite entities are so to speak seen and considered to be finite within set theory' (Salanskis, p. 66) – betray the nature of the differential no less than Hegel, since they 'both fail to capture the extra-propositional or sub-representative source […] from which calculus draws its power' (*DR* 264). See Maddy.

14 Deleuze, *Sur Spinoza*, 10 March 1981. Hereafter Deleuze, 10 Mar.

15 For a more extensive account of Deleuze's deployment of the work of Weierstrass and Poincaré see Duffy, 'The Mathematics of Deleuze's Differential Logic and Metaphysic'.

16 Lautman, *Essai sur L'Unité des Mathématiques et Divers écrits*, 23. Hereafter AL.

17 See also AL 143–4, 302–4; Barot, p. 7 n2.

18 They are also referred to and operate as 'dualities'. See Alunni, p. 78.

19 He therefore also refers to them as 'logical schemata'. See AL 142.

20 From Lautman's correspondence with Fréchet dated 1 February 1939.

21 AL 32, 45–7. The 'global conception of the analytic function that one finds with Cauchy and Riemann' (32) is posed as a conceptual couple in relation to Weierstrass's approximation theorem, which is a local method of determining an analytic function in the neighbourhood of a complex point by a power series expansion, which, by a series of local operations, converges around this point (45–7). See Duffy, 'The Mathematics of Deleuze's Differential Logic and Metaphysics'.

22 See Barot, p. 10; Chevalley, pp. 63–4.

23 See Widder for an account of Deleuze's reversal of Platonism and its implied idealism.

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