**Platonism by the Numbers**

**Reflections on the Act of Counting**

Mathematical Platonism or *realism about mathematical objects* contends that numbers are real, abstract objects existing independently of the physical world and of our awareness of them, so that they are neither physical objects nor mere ideas existing in the mind. Numbers are necessary beings. They exist in all possible worlds and are related to each other in the same manner in every world in which they exist. For this reason, all mathematical truths about numbers are necessary truths whose denial involves a demonstrable contradiction. Further, since there is no highest number (for any number we can conceive of we can always form a higher number by adding 1 to it) there are an infinite amount of numbers. At the same time, all these numbers are accessible to the intellect as objects of thought/contemplation, and are knowable *a priori*, independently of any justification derived from sense-experience. We apprehend mathematical truths in such acts of contemplation through the power of rational intuition by means of which all formal truths are both known by us and known to be necessarily true with intrinsic certainty. Although there is an infinite amount of mathematical truths, any such truth considered individually is ultimately accessible to us, even though there are many such truths that no one has ever contemplated and no one will in fact ever actually contemplate.

Hardly anyone these days is a straightforward Platonist about mathematics. Like substance dualism in the philosophy of mind, Platonism in mathematics is shunned like the plague, or perhaps better, a cast-off lover. Most current philosophers of mathematics embrace some sort of mathematical nominalism or anti-realism concerning numbers and other mathematical objects considered as elements of our ontology. For reasons I shall broach later in this paper, it is not surprising that this should be the case in the current intellectual atmosphere. However, I will suggest that none of the indicated motivations for the current anti-Platonic *animus* in the philosophy of mathematics are intellectually compelling. Further, just as I (and others) have argued in the case of substance dualism, I shall argue that Platonism is the default position in the philosophy of mathematics and thus that there is a natural presumption in favor of it that requires either the discovery of deep theoretical problems within Platonism or irresistible gains in simplicity and explanatory power on the part of some other theory to overcome. As we shall see, however, it is arguable that neither of these is the case. My not very sanguine hope is that, after reading this paper, disinterested readers will confidently endorse Platonism and reject anti-Platonist accounts of mathematics, at least in the case of numbers, which will be my sole concern here. I will leave it to others more knowledgeable than myself to extend its results to algebra, geometry, and the more advanced branches of mathematics and the mathematical objects over which such theories, treated realistically, must “quantify” in order for its statements to be true in the correspondence sense. Those expecting lots of equations or discussions of esoteric mathematical or physical concepts will be disappointed, I’m afraid. My defense of Platonism will be homey and mathematically unsophisticated but, I am hoping, all the better for that fact. Sometimes an overly-sophisticated approach to these questions can cause us to forget the home truths that we need to keep firmly before us.

**The Platonic Presumption**

Let’s begin with a common, low-grade mathematical statement:

2 + 2 = 4

This statement is true. Further, it is a statement that I know to be true simply through my *understanding* it, i.e. my knowing what the terms of this statement (“2,” “4,” “+” and “=”) mean, both considered individually and in the context of this statement as they constitute it. To know that some statement is true simply by understanding it, without the need for any sort of empirical justification, is to know that it is true *a priori*. More than this, I not only know that this statement is true, but I also know that it is true in a particular way, one not shared by all true statements: 2+2=4 is a *necessary* truth. It is not the case that the statement 2+2=4 just happens to be true, though it might have been false. Quite the contrary, given my understanding of what the component terms of the statement mean, it is simply and literally *inconceivable* that this statement could be false under any imaginable circumstances (or in any “possible world.”). It is precisely for this reason that I do not need empirical evidence in order “confirm” its truth. Indeed, my clear and distinct perception of the necessity of 2+2=4 confers *intrinsic certainty* on that statement received as an object of belief.[[1]](#footnote-1) The evidence for that belief provided by my clear and distinct perception of its certainty warrants my acceptance of that belief as something that I know with greater certainty than *any* empirical claim, including those of empirical science. 2+2=4, then, possesses *transparently* the properties we attribute to mathematical claims generally regardless of whether they transparently possess them: truth, necessary truth, knowability *a priori*, and intrinsic certainty.

In addition, all of these claims have the status of *facts* about the statement 2+2=4.[[2]](#footnote-2) These claims are not merely conceptual or conjectural, they are not mere definitional consequences of some esoteric set of axioms, or theory-laden in any obvious way. Indeed, these claims appear to be *directly* and *immediately* apprehended by us. So much is this the case that statements such as 2+2=4 appear to be and have been received by many, both in the past and currently not merely as obviously true, but as paradigmatic examples of knowledge. So much is this the case that many philosophers have held that mathematical knowledge sets the standard for epistemic justification to which all truly scientific knowledge should aspire and to which substantive empirical knowledge-claims inevitably fall short. Be that as it may, there is a clear and solid basis in experience (which here obviously means more than mere sense experience) for affirming each of the foregoing claims as factually true about the statement 2+2=4, facts accessible to any schoolboy clever enough to understand that statement and receive it as true. In other words, they are *common facts*, accessible to and admitted by all, unless they have been misled by the dogmatism associated with what Descartes called “preconceived ideas.” As such, any philosophy of mathematics that implies that these claims are not true, or only true in some *ersatz* sense seems just obviously wrong, at least *prima facie*.

Such, of course, is not the case where Platonism is concerned. Platonism takes these facts at face value and reads off its philosophy of mathematics from them. In cases of clear and distinct rational intuition, we apprehend mathematical objects and their properties in a manner analogous to the way we apprehend perceptible things using the senses.[[3]](#footnote-3) As a result, we find ourselves in possession of spontaneous judgments, such as that 2+2=4, which we can confirm to be true on the basis of the internal evidence provided by our mere understanding of the meaning of that claim and its component terms. In reflection on that act of understanding, we are able to self-consciously appropriate the content of that understanding in the form of distinct concepts of mathematical objects, such as numbers, their essential properties, their internal relations to each other, and the facts about complexes of numbers resulting from those properties and relations between those numbers, all of which are recognized to be true, necessarily true, known *a priori*, and when clearly and distinctly perceived, intrinsically certain for us. Platonism, then, “quantifies” over numbers, their properties, relations between numbers, and the resulting facts about numbers related to each other in particular, stateable ways. According to Platonism, mathematical truth is truth in the correspondence sense, objective truth about real, albeit abstract, entities existing independently of our awareness of them. Our knowledge of these truths is a consequence of our having apprehended these entities by means of rational intuition and the resultant understanding that arises from the operation of that faculty, taking occurrent and articulate form in spontaneous mathematical judgments, which when reflected upon acquaint us with their truth, necessity, and intrinsic certainty as objects of *a priori* knowledge. On this view, then, the scientific status of mathematics depends on the existence of mathematical objects such as numbers, without which it has no content, no subject matter, and nothing to tell us about anything. The foregoing thus models mathematics as a branch of substantive theoretical inquiry by specifying its content and subject-matter, the specific department of the real to which it stakes its claim and over which it claims special expertise and authority, except that in this case we have a formal rather than an empirical science.[[4]](#footnote-4) As such, it is numbers, understood as real, abstract objects, that alone makes this possible for any branch of mathematics that employs numbers or represents them with variables in its axioms, equations, etc. to be true. Platonism secures this status by interpreting that subject matter realistically.

**Mathematics without numbers – Why?**

Since the mid-1970’s, it seems that the mainstream view, and subsequent focus of research, has been on eliminating numbers, and other mathematical objects, from our ontology. I shall say something about the history of this chapter in the philosophy of mathematics shortly. To begin with, however, I want to consider the question of what the motives for such a project might be. I bring this up because there do not seem to be any reasons internal to mathematics or natural science as done by actual mathematicians and scientists (such as mathematical physicists) that suggest even so much as the propriety, let alone the urgency, of accomplishing such a project.[[5]](#footnote-5) Whether such deflationary projects succeed or fail, it seems, everything remains the same in these fields and their truth confers no benefit on either. So, since there is nothing to be gained through such a reduction in ontology for mathematics or science, what reasons could we have for expending such much energy and cleverness in the attempt to accomplish this?

In his *A Structural Account of Mathematics*, Charles Chihara suggests a number of reasons for being interested in such a project.[[6]](#footnote-6) To my mind, these reduce to three basic concerns. First, Chihara thinks that there are no strong arguments for Platonism, understood as a theory that postulates mathematical objects in order to explain our knowledge of mathematical truth. Second, the accounts given by Platonists to account for our knowledge of mathematical truth are vague and “unscientific,” since they appeal to rational intuition and the understanding as the source of mathematical knowledge. Third, Platonism is inconsistent with physicalism, a view to which many philosophers have a foundational commitment, so we have a strong motivation to attempt to provide an account of mathematics consistent with physicalism. Let me consider each of these criticisms in turn.

First, as I have already argued, there is an experiential presumption in favor of Platonism. As such, the playing field is not level and Platonism does not compete on all fours with its nominalist rivals. Platonism, I have claimed, can simply be read off of our ordinary, everyday mathematical judgments such as 2+2=4 as we reflectively apprehend them in daily life. Platonism is the simplest, most natural, and therefore *prima facie* best explanation for the facts I noted earlier about such judgments. It is thus in need of no other, positive argument in order to solicit our rational acceptance and allegiance. In particular, it does not require that the Quine/Putnam indispensability argument (which Chihara critiques at great length) be sound, so that refuting such an argument does not thereby refute mathematical Platonism.[[7]](#footnote-7) By contrast, competing nominalist theories about mathematics are precisely that, and thus require positive argument for their acceptance. Of course, if there were reasons to be drawn from mathematics or the proven results of natural science (as opposed to merely philosophical interpretations of natural science) for doubting the truth of Platonism, these might serve as defeaters for this presumption. Since none are apparently extant, it is useless to speculate further about this possibility here.[[8]](#footnote-8)

With regard to the second objection, it seems sufficient to note that there is a distinction between the *fact* of our knowing and *possessing an explanation* of that fact – between *knowing* and *showing*. My knowledge that 2+2=4 is true, necessary, *a priori* and intrinsically certain is, I have claimed, something that I possess directly and immediately, simply as a result of reflection on what I understand when I consider the meaning of that statement. The fact that I have no ready explanation of this fact does not in any way cast any doubt *at all* on either the truth or certainty of any of these claims. Quite the contrary, given the nature of what I know in this instance, my certainty trumps all but the most hyperbolical doubts concerning my claim to know, doubts which if admitted would call all of my knowledge, including my scientific knowledge, into question. In this case, then, our knowledge of simple mathematical claims becomes *data* to which any plausible account of mathematical knowledge must accommodate itself, even if it turns out to be incompatible with currently fashionable assumptions about the way the world is.[[9]](#footnote-9)

This leads directly to Chihara’s third concern, that mathematical Platonism is inconsistent with physicalism. Guilty as charged. However, given the certainty attaching to our everyday mathematical knowledge, a certainty far greater than we can rest in a merely philosophical doctrine such as physicalism, if we must choose between these two the obvious, rational choice is to reject physicalism on the ground that it conflicts with something that we know with the penultimate degree of certainty to be necessarily true, e.g. mathematical statements like 2+2=4. For the same reason, we ought to rationally affirm the truth of Platonism and alter our metaphysical commitments accordingly. From this point of view, the entire nominalist project appears to be merely motivated by a dogmatic commitment to what Descartes would have called “preconceived ideas,” i.e. irrevisable theoretical commitments held independently of evidence.[[10]](#footnote-10)

This can be confirmed, I think, by a brief review of course of events that has lead us to the present situation in the philosophy of mathematics. Quine was well known for his commitment to physicalism, understood as the metaphysical thesis that nothing ultimately exists except for the entities “quantified over” by physics. This thesis, in turn, depended on his commitment to scientific realism, the claim that natural science (especially physics) describes the world the way it actually is. However, given the ingression of mathematics into modern physics, with the result that modern physical theories are essentially mathematical constructs in which terms that formerly were attributed a substantive meaning (like “electron”) now function mostly as parameters in mathematical laws, it does not seem that such theories can be true unless mathematics is true. This led Quine to his influential “indispensability” argument for mathematical Platonism. If it really is the case that physics cannot be true unless the mathematics essential to it is true, then the truth of physics crucially depends on the truth of mathematics. In turn, mathematics cannot be true unless there are mathematical objects about which those mathematical statements are true. Thus, in order to save scientific realism, we need to embrace Platonism in mathematics. Quine thus affirmed mathematical Platonism for this reason, and passed this conviction on to many of his students, including (e.g.) Hilary Putnam.

It is evident that if Platonism is indeed, as Quine thought it was, the consequence of his scientific realism, then it is likewise inconsistent with his Physicalism, since mathematical objects are abstract, non-physical entities and thus if Platonism is true ineliminable from our ontology. It seems, then, that Quine’s philosophical position is ultimately incoherent. In order to justify his commitment to physicalism, he needs scientific realism to be true. However, given that modern physics cannot be true unless the mathematics that is essential to it is true as well, scientific realism seems to require the truth of Platonism, and thus that we are forced to quantify over abstract, non-physical mathematical entities. In that case, physicalism shows itself to be incompatible with scientific realism and must be sacrificed in order to preserve it. In that case, however, Quine’s general philosophical position is self-refuting. It is not surprising, then, that philosophers of mathematics after Quine who find themselves committed to physicalism have attempted to remove this evident incoherence in Quine’s position by getting somehow getting rid of mathematical objects.

Many years ago I was in the philosophy library at the UW looking at the new acquisitions shelf. A fellow graduate student, Brad Rind, who was one of the few students in my cohort with the interest and background to read philosophy of mathematics to profit, happened to be going by. I pointed to a book and said, “Hey, Brad, look! Here’s a new book: *Science without Numbers*. You gonna read it?” “No,” answered Brad, “They always end up having something just as bad.” Although I still have not read *Science without Numbers*, from what I have read in the field I am strongly inclined to believe that Brad’s comment was correct.[[11]](#footnote-11) Like the speculative cosmologists whose book cover promises to show how the universe created itself out of nothing, when one examines the contents one discovers that one has been the victim of an intellectual bait-and-switch. No one can show that the universe created itself from nothing, since this is self-evidently impossible. Instead, we are offered some sort of world-generating mechanism, wave function, “quantum vacuum” or multiverse which pre-exists the universe we live in and somehow acts as the ultimate productive cause of its existence. In the same way, philosophies of mathematics promising to eliminate or get rid of mathematical objects such as numbers always end up replacing one set of abstract objects (e.g., the familiar numbers that we work with every day) with some more exotic, invented, and merely posited sort of abstract object (sets, structures, Chihara’s constructability quantifiers and open-sentence functions) into which numbers-talk has been translated.[[12]](#footnote-12) Numbers are then dismissed with a wave of the hand in the same way that a stage magician makes his assistant disappear in a puff of smoke.[[13]](#footnote-13) Of course, we know that the magician’s assistant has not really vanished into thin air. In the same way, we also know in the back of our minds that nothing along this line is ever going to succeed in eliminating numbers without any residual commitment to abstract objects of some sort – we will find them if only we are willing to look. As such, this strategy, still-born from the start, cannot remove the necessity of positing abstract objects and so cannot rescue physicalism from the threat represented by mathematical realism. It is very likely for this reason that a more natural progression in one’s intellectual commitments follows the track represented by Hilary Putnam. After he abandoned his Quinean mathematical realism around 1990 and became an anti-realist about abstract objects, he quickly progressed to scientific anti-realism as well, a position from which a commitment to physicalism can amount to nothing more than the expression of an aesthetic preference. In that case, to each his own, and I can only say that such a preference is not mine.[[14]](#footnote-14)

**Counting**

The marriage of mathematics with physics, which began in the seventeenth century and has continued down to the present time, has proved as incredibly fruitful as it is remarkably unlikely. Indeed, the twentieth century could almost be called the Pythagorean century, since in that century we came closer to the concrete realization of the Pythagorean conviction that “all things are numbers” than at any previous time in human history. Nevertheless, many people have questioned the very applicability of mathematics to real things on the supposition that Platonism is true. This is especially so with respect to physics, despite that fact that it is precisely in physics and cosmology that the ingression of mathematics into natural science has achieved its most impressive results. On the one hand, the applicability of abstract, non-physical entities like numbers to concrete, material nature in such a way as to be capable of facilitating the discovery of substantive truths about the physical world, let alone in such a way as to be essential to our current physical theories, is both unexpected and mysterious.[[15]](#footnote-15) On the other hand they clearly are, so how can this be? Since I don’t have the detailed expertise necessary to address *that* question, let me consider a similar question in a slightly different context. Let’s examine the applicability of mathematics to real objects in a much less complex employment, i.e. the simple act of counting.

Suppose a child creates a line of pebbles in the sand by beginning with a single pebble, then places another pebble to the right of the original pebble, then another, then another, each time adding a single pebble to the series, until he has constructed an impressive series of pebbles all in a line. How many pebbles are there? If there are a sufficient number, we can’t tell “just by looking.” Instead, we have to *count* them – and counting (as we shall see) requires numbers, in particular, the natural numbers or positive integers. In order to count the pebbles, I take each individual pebble in the line as a discrete countable *unit* to which a different number is applied. So, I begin with the first pebble and say “one,” move to the next and say “two,” and so on until I get to the last pebble at the end of the line. The last number I count, corresponding to the last pebble in the line, also corresponds to the total number of pebbles in the line.

Although I use the numbers to count the pebbles, I have not finished counting them unless by the time that I reach the last pebble in that line I can answer the question “How many pebbles are there in this line?” In turn, I can only do this if I have arrived at the *total number* of pebbles there are in the line taken as a whole for the purpose of arriving at precisely that numerical result. Suppose that there are forty-two pebbles in the line. When I reach the last pebble and say “forty-two,” the number forty-two serves two functions: first, it counts the last (individual) pebble in the series and assigns a number to it and second, it gives us the total number of the pebbles in the line. Further, it does the second as a consequence of doing the first. In this way, “42” corresponds to the total number of pebbles in the line and corresponds to the number I assign to the last unit in the group that I count, having counted each such unit a single time.[[16]](#footnote-16)

In the same way, each number I count both numbers the individual to which it is assigned and also tells us the total number of objects counted to that point. In this case, for example, “twelve” both refers to a particular pebble in the series of pebbles I just counted and to the number of pebbles I have counted up to that point on the way to counting them all. Even though it is a contingent, empirical fact about the world that there are 42 pebbles in this line, we cannot know this fact without using numbers. While I count individual units of the non-mathematical aggregate (in this case, the line of pebbles), it is only by arriving at the total number of pebbles in the line that I can answer the “How many?” question that motivated my act of counting them in the first place. Thus, while I count individual pebbles in the line of pebbles constructed by the child, I do this simply to arrive at the total number of units in the line treated as an *aggregate*, a loose collection of merely externally related things, which we may call the *number of* the aggregate, treated as a whole solely for purposes of determining the total number of units composing it. We can do this, I think, even if or even despite the fact that what we regard as the line of pebbles is nothing over and the above the externally-related pebbles composing that aggregate.

The natural numbers can accomplish this task because they constitute an ordered series the members each of which is *internally* related to all the others in the series in a particular way, i.e. such that for any member of the series, n, the value of n always exceeds that of the preceding number, m, by a single unit of whatever is being counted or could be counted using that number.[[17]](#footnote-17) When we count, we assign a number n to each such unit, i.e. discrete, non-mathematical member of the aggregate to be counted, e.g. a particular pebble in the line. Further, this assignment is not made randomly, but rather in a particular order determined by the manner in which those numbers are internally related to one another in order of position in the number-series, an order which in turn endows each number with a unique “size” in relation to all other members of that series. In this case, “42” is a unique number which nevertheless contains all of the previous members of the series and by so doing *outranks* all the numbers previously used in counting the line of pebbles. 42 thus outranks or is a larger, bigger, or higher number than all the other numbers in the series used to count the number of pebbles in the line, as would be clear if we were to mark each number separately with an appropriate series of strokes: I, II, III, IIII, etc.[[18]](#footnote-18) More than this, 42 outranks each of its discrete to the distance in units counted from 42 to each of those predecessors, e.g. 7, 19, 23, 31, 40, and so on, which distances of units constitute countable series in their own right, to which a number corresponding to the number of units in the distance between the two numbers can be assigned: 35, 23, 19, 11, and 2.[[19]](#footnote-19) In the same way, 42 will be outranked by each of its successors in the series of integers in the same way that 42 outranks each of its predecessors in the number series. For example, 52 will outrank 42 by a distance of 10 (in this case imaginary) units and thus be related to each of its successors as well, with a unique result for every number. Each number thus occupies a distinct position in the series that relates it to every other number in the series and which constitutes a unique set of such relations for each number, thus essentially individuating it from every other. So, just as I can take a series of unitary objects, such as pebbles, and form an aggregate of them, so too can I count each member of the series using a unique number, despite the fact that the individual members of the aggregate that I am counting are merely externally related to one another and could, e.g. have been placed in any order whatsoever and thus actually counted by any discrete number. No matter how I shuffle or shift the pebbles around, so long as I do so without introducing additional units to the aggregate without removing any of the present units or removing present units from the aggregate without replacing them with additional units, my act of counting will always end with “42.” As such, there is nothing intrinsic to the individual members composing an aggregate of things counted that grounds the act of counting or determines what the number of the aggregate will be. With this in mind, it may be instructive at this point to consider what some contemporary philosophers of mathematics have to say about counting.

Not much, as it turns out. Chihara attempts to reduce counting simply to the application of a “rule learned in childhood for constructing and ordering the Arabic numerals.”[[20]](#footnote-20) For his part, Charles Parsons appears to think that counting is the act of creating a one-to-one correspondence between non-mathematical entities and the (Arabic) numerals used as counters, which at least highlights the idea of correspondence between counters and things counted.[[21]](#footnote-21) However, as should now be clear I cannot use the Arabic numerals *as such* to count. These are mere marks or sounds – *flatus vocis*. A parrot that has been taught to say 1, 2, 3, 4, 5…10” is not counting and cannot count simply through reciting that series of sounds, even if it is also taught to wait to emit the next sound until a new object has been presented in its visual field. Further, Arabic numerals considered in themselves are merely a series of externally related marks or sounds; they thus lack the essential feature of the number series, namely that the members of that theory are internally related in a unique way, one that allows us to use numbers to count. Simply to utter “1, 2, 3…” and thereby make a one-to-one correspondence between those sounds and another aggregate of things like pebbles is no more to count than it would be to point and say “pebble, pebble, pebble…” or intone “one, one, one…” as I focus on each pebble in the series. In the same way, I could have used “3” to designate the number we in fact designate using the numeral “1,” and so on. Because numerals are merely externally related to each other, our use of any particular numeral to designate any particular number is merely conventional. There is thus no essential connection between numerals and numbers and thus no prospect of reducing the latter to the former.

In the same way, it is not enough in order for me to count that I be able to create a correlation between concrete things and the numbers in accordance with a rule or algorithm that generates a unique “output” given a unique set of “inputs,” even if the output is the one that we would regard as correct. Suppose I am taught to apply the Arabic numerals in the standard way to the line of pebbles in our example by writing the appropriate numeral under each pebble, thus creating a one-to-one correlation between pebbles and numerals. Then I am told that, in response to the question “How many?” when asked with respect to pebbles I should look at the numeral I wrote under the last pebble in the line and utter that numeral aloud in response. Thus, when asked how many pebbles there are in the line, I look at the last numeral I wrote and say, “42.” Did I count the pebbles and by so doing come to know that there are 42 pebbles in the line of pebbles in the example? If I am a behaviorist or a Wittgensteinian my answer will probably be “Yes,” since I meet the applicable, publically observable criteria for determining whether someone knows how to count. Further, my answer to the question asked is correct and might well be correct on any occasion that I was challenged to enact this protocol. Nevertheless, to the extent that my performance is simply the result of my generating the numeral-series and answering that question in accordance with a rule that I am following, my apparently flawless performance is perfectly consistent with my having no idea what I am doing, why I am doing it, or even with what “42” means. I could do all of this even if I was the equivalent of the man in John Searle’s Chinese Room and had no notion of what it is to count, what I am doing when I count, or what counting is for and was instead just mechanically following a rule I had been taught as part of a “language game.” What’s missing in nominalist accounts of numbers as used in counting is precisely the recognition that counting involves the *enumeration* of the member of an aggregate of countable objects with the end of determining how many of them constitute that aggregate, i.e. the *number* of the members of that aggregate, something that is not given to me by the numeral just as such or established by creating a one-to-one correspondence between pebbles and numerals. In order to count in a meaningful way, then, I have to understand numerals as the conventional *names* for numbers, objects of thought that do not consist in any aggregate of merely externally related non-mathematical/physical objects, which is all that numerals considered as such could ever be in any case.[[22]](#footnote-22) Indeed, no series of merely physical things possesses has the kind of internal (if you will, “structural”) relation between its members that subsists between the members of the number-series in virtue of which we can use numbers to count and so cannot be used to replace the number-series in the activity of counting. This, as we have seen, is true as well of numerals, considered simply as a series of inscriptions or utterances, since these too constitute merely a conventionally ordered series of externally related objects, i.e. a mere aggregate.

More than this, there appears to be *no* structural feature, whether formal or physical, of aggregates of countable objects that has to correspond to or be isomorphic with the number series and for the objects of which that aggregate of non-mathematical objects is composed, in order to be constituted as countable. Aggregates of countable things are just heaps, or other loose collections of discrete, externally-related things that need share nothing in common other than having been designated as an aggregate by someone for some purpose. Because of this, I can use numbers to count aggregates of just anything at all, no matter how diverse they are in kind or arbitrarily collected the members of that aggregate may be.[[23]](#footnote-23) Suppose I decide to count the number of objects in my room. Of course, the aggregate designated by “the objects in my room” is not a natural kind and there is no strict criterion for the individuation for these objects. The decision concerning what counts as an object for the purposes of counting the objects in my room is for the most part arbitrary, so that if I draw the lines differently on different occasions, the number of objects in my room will be different as well. No matter how I draw the lines, even if I am inconsistent, the resulting aggregate will be countable just so long as it consists of a collection of individual units designated as members of that aggregate for the purposes of counting.

Further, since the relevant structural features of the number-series are constituted by internal relations between the numbers making up that series considered as bearers of a unique relational properties, each in relation to each the others, there is no prospect of treating that series as though it were nothing but a “pure structure” without anything being structured by that structure, or as somehow “multiply realizable” by other collections or series of objects. In the first place, for example, we note actually existing structures are structures of *things*, i.e. structural properties of groups of things organized in and related to each other in some particular way and so parasitic on those things and the relations among them for their very existence. Things “inhere” in those structured collections in the way that real accidents (qualitative properties) inhere in primary substances as traditionally understood in Aristotelian metaphysics. Although there can be abstract concepts associated with structures considered independently of their concrete realization as the structures in or of things, what is instantiated in each case is not a concept but a global or regional property of those things constituting the intentional content of that concept, i.e. *what we think about*, *know*, or *contemplate* by means of that concept when we understand it: in short, a *form*, regardless of how we conceive of this as something existing extramentally.

So, then, for the reasons I have given it seems clear that these structural features or properties of the number-series cannot be instantiated by aggregates of non-mathematical, “physical” things and thus cannot be structures associated with, inherent in, or physically realized by such aggregates. It appears, then, that nothing other than numbers *could* exemplify the structure of the number-series in virtue of which we can use numbers to count, precisely because the internal relations between numbers that constitute the structure in and of that series are unique to those entities, and thus not “multiply realizable” in the way that something like the I formation in football can be realized in different times or places by different football teams, by an arrangement of salt shakers and tableware on a table-top, or chalk marks on a blackboard. Of necessity, then, it seems that *only* the number-series exemplifies (or could exemplify) the structure that makes numbers capable of serving as counters for purposes of counting. Nothing else will do.

We must conclude, then, that it is solely in virtue of the special, structural feature of the number series *itself* that I can use numbers to count. As such, given the unique nature of the number-series, I cannot count with anything other than numbers, in particular with anything physical, such as numerals regarded as mere utterances or inscriptions, paint chips (which may be internally related to one another by hue), potatoes, or marks in the sand just as such since nothing except numbers have this special internal relation to one another that makes them capable of answering the question “How many?” in the case of aggregates of non-numerical objects. When I count, I use numerals to represent numbers, either publically or simply to myself *in mente*, but I do not count with numerals except insofar as I recognize numerals as conventional names for numbers, entities quite differ in nature from any sort of non-mathematical physical thing, including numerals themselves considered as mere inscription-tokens or *flatus vocis*.

Without numbers, then, counting is impossible. Thus, if I can count at all, numbers must exist and exist as something independent of and irreducible to any sort of physical object.[[24]](#footnote-24) Moreover, given that numbers exist and are knowable by me, I can contemplate numbers as objects of inquiry in their own right and note that they have certain properties – immateriality, immutability, sempiternity, and so on – and also explore and discover various ways in which numbers are related to each other independently of their use as “counters” in everyday contexts, i.e. as not just what we use to count units but as uniquely determined individual entities in their own right. In this, then, we have already laid the foundation for the discovery of all the standard arithmetic operations (addition, subtraction, multiplication, division) through reflection on the properties of numbers in the context of the series they form in virtue of the way in which they are internally related to each other. Once again, this constitutes a set of facts that we directly and immediately apprehend through rational intuition. In turn, using rules of inference whose validity is apprehended directly and immediately by us using rational intuition, and thus known *a priori* with intrinsic certainty to be necessarily true, we are able to deductively arrive at many further mathematical truths as well. Since these rules are truth-preserving, these subsequent results have the same necessity and certainty that possessed by the axioms and rules from which they are derived. They are thus known to be true in such a way that empirical evidence is neither necessary for nor even relevant to that knowledge. At the same time, we lay the basis for mathematics as a science, with an actual, albeit abstract and non-physical, subject matter.

**Conclusion**

As a further illustration, let us apply this example to an argument examined by Chihara.[[25]](#footnote-25) Suppose I see that there are seven dimes on this table and four quarters on this table and argue as follows:

There are seven dimes and four quarters on this table.

Every coin on this table is either a dime or a quarter.

No coin on this table is both a dime and a quarter.

7+4=11

Therefore, there are eleven coins on this table.

The argument is a deductive argument and presumably valid. As such, given the standard account of success for a deductive argument, the foregoing argument can be sound and thus the conclusion true only if all premises are true, and one of the premises, 7+4=11, is a purely mathematical one. So, it can be true that there are eleven coins on the table only if 7+4=11. Mathematical nominalists like Chihara go to great lengths to show how, using various complex and exotic translation schemes, we can use 7+4=11 as a premise in such arguments without regarding it as true, at least about numbers. However, the natural riposte to such efforts is simply to point out that 7+4=11 is *obviously* true, necessarily so, known to be so *a priori* with intrinsic certainty and most naturally understood as being about numbers.[[26]](#footnote-26) At the very least, much more powerful reasons for doubting these claims need to be supplied than Chihara, or anyone else, has offered to this point. A dogmatic preference for physicalism is simply not enough to recommend such efforts to our credence, however interesting, clever, and otherwise useful the results may be.

Nevertheless, the question still remains concerning how it is that the fact that 7+4=11, being true about numbers, has any relevance at all to the contingent, empirical fact that there are eleven coins on this table. Given the foregoing, however, the answer is not far to seek. It is simply impossible for it to be the case that there are eleven coins on this desk without it being the case that total number of those coins arrived at by correctly counting them is also eleven. Indeed, if this were not the case, we could have no confidence that we could arrive at the number of coins on the desk by correctly counting the individual coins on the desk one-by-one. But we clearly can do this and I know of no one who denies this – this would be a move of desperation indeed .Thus, while it may be a contingent fact that there are eleven coins on this table, that the *number of coins* on this table (in this case, the number of the aggregate being counted) is eleven *given that fact* is not and thus that the number of coins on this table is eleven is *also a fact*, albeit a mathematical rather than an empirical fact. As such, there are eleven coins on this table if and only if the number of the coins on this table is eleven. Thus, my correctly counting the coins on the table and arriving at eleven as the number of those coins (something that lies within the limits even of my extremely limited mathematical competence) deductively entails that there are eleven coins on the table in the conditions specified in the original problem.

A further consequence of this fact is that if there are eleven coins on this table then the sum of any two numbers that add up to eleven will also correspond to the number of coins on the table. Thus, if there are eleven coins on this table, it will also be the case that there are 7+4 coins on this table and, by the same token, if there are 7+4 coins on this table, there are also eleven coins on this table. So, if I correctly counted the dimes and quarters on the table separately and correctly added those sums rather than counting each coin separately, it nevertheless follows that the number of coins on the table is eleven, and that therefore it is true that, and an empirical fact that, there are eleven coins on the table. Where is the mystery in that?

1. By *belief* I mean “rational assent to the truth of a proposition.” For my distinction between intrinsic and extrinsic certainty, see *The Proof of the External World*, Eugene, OR, Wipf and Stock, 2007, 51-53. [↑](#footnote-ref-1)
2. By *fact* I mean an objectively constituted state-of-affairs capable of serving as the truth-condition for a substantive claim or proposition. Numbers, on my view, are beings, i.e. really existing things with essential properties. [↑](#footnote-ref-2)
3. This apprehension is indirect because mediated by concepts. The acquisition of concepts and their manner of being “stored” by the intellect are pre-conscious processes involving both the agent intellect and the understanding. Intuition is the source of spontaneous judgments, i.e. those evoked by sense-experience and are the first part of this process that is introspectable and of which we are occurrently aware. For more on this, see “Counting, Numerals, and Numbers,” (in preparation). [↑](#footnote-ref-3)
4. For a sustained defense of the existence of abstract objects and the notion of formal science as applied to linguistics, see Jerrold Katz, *Realistic Rationalism*, Cambridge, MA, MIT Press, 2000. A similar case can be made for mathematical objects as well. [↑](#footnote-ref-4)
5. As, e.g., Chihara admits, see *A Structural Account of Mathematics*, Oxford, Clarendon Press, 2007, 160-161 [↑](#footnote-ref-5)
6. Op. cit., 159-161. [↑](#footnote-ref-6)
7. See Chihara, op. cit., especially chapters 5 and 9. Despite Chihara’s criticisms, I am persuaded that some version of this argument must be sound. Showing this, however, I must leave to heads cleverer than mine. [↑](#footnote-ref-7)
8. Not even Chihara suggests anything like this. [↑](#footnote-ref-8)
9. As evinced by Chihara, op. cit., 6; see also 341. However, Chihara gives no proof, at least in this book, that modern science has somehow shown that rational intuition is impossible or irrelevant to the explanation of mathematical knowledge. Having said this, I think that a plausible account of the nature of mathematical knowledge and role that the senses play in its acquisition can be given and confirmed by the analysis of actual cases of mathematical and scientific discovery, based on the work of Bernard Lonergan – see his *Insight*, New York, Darton, Longman, and Todd, 1957, especially 3-33. Lonergan, however, provides only the starting point for such an account, which needless to say assumes a different ontology than the physicalist theory to which Chihara subscribes. [↑](#footnote-ref-9)
10. Indeed, there are many reasons for rejecting physicalism, drawn from various sources, which I have discussed in several papers archived on this website. So mathematical Platonism is just one arrow in the quiver and far from being the last obstacle to the achievement of a complete naturalization of reality. [↑](#footnote-ref-10)
11. The book is by Hartry Field and was published by Princeton University Press in 1980; for an account of the basic theory, see Chihara, op. cit., 108-113. Geoff Hellman’s equally provocative title, *Mathematics without Numbers*, was published by Oxford University Press in 1989; for discussion see Chihara, 113-15 and 131-135. [↑](#footnote-ref-11)
12. This seems to be case with regard to the Structuralist philosophers of mathematics, such as Resnick and Shapiro, whom he classes as “Platonists” precisely for this reason, though they are certainly not Platonists in the sense I am defending here. Notwithstanding, I think that Chihara is open to the same criticism – see footnote 13. [↑](#footnote-ref-12)
13. In Chihara’s case, the preferred objects are open sentences containing a special logical operator, the “constructability quantifier.” Since open sentence-types would too obviously count as abstract objects, Chihara refuses to “quantify” over such entities, claiming that he recognizes only open-sentence tokens in his theory (170-171). However, he is ambivalent about what these tokens are. He does not seem to think that they are merely physical objects, since given that no amount of merely physical analysis of inscriptions or utterances will reveal to us that any such inscription or utterance is in fact even so much as a sentence-token, let alone an open-sentence token. Indeed, Chihara questions why anyone would even think this, so he clearly does not think that his view is committed to anything as radical as this. However, if sentence-tokens are anything more than this, then it seems that the natural suggestion is that open-sentence tokens are tokens of open sentences, i.e. sentence-types expressed or made public at a particular time and place, “physically realized” by an appropriate open sentence-token. These sentence types (repeatable, expressible in various languages, serving as the cognitive content or meaning conveyed by these tokens) bid fair to be abstract objects, just as Chihara fears. [↑](#footnote-ref-13)
14. Putnam has since abandoned the “internal realism” he espoused at that time, but rather than reverting to realism, has in fact moved even closer to full-blown pragmatism about theories generally. Chihara, for his part, continues to espouse scientific realism, contending that our current physical theories are largely true. [↑](#footnote-ref-14)
15. See Eugene Wigner, "The Unreasonable Effectiveness of Mathematics in the Natural Sciences," in *Communications in Pure and Applied Mathematic*s, vol. 13, No. I (February 1960), 1-14. [↑](#footnote-ref-15)
16. This appear to correspond to one sense in which the terms “ordinal” and “cardinal” are used in application to numbers. However, since I don’t endorse set theory, these terms do not function for me as properties of sets. [↑](#footnote-ref-16)
17. If we supplement the natural numbers/positive integers with the negative integers, then the same definition will apply to both zero and one, since -1+1 = 0, and 0+1=1. As such, the overall account of the number series will be self-referentially consistent and there is no need to have, as Peano does, a separate axiom designating 0 as a number. [↑](#footnote-ref-17)
18. For more on this, see my “Rational Intuition and the Truth about Numbers,” in preparation. [↑](#footnote-ref-18)
19. Thus, numbers can sometimes be counted with numbers. However, when numbers are counted in this way, they are treated as members of an aggregate or collection of externally related individuals, rather than as numbers *per se*. In such cases, we prescind from their character as numbers and simply regard them as another loose collection of things to be counted with the intention of arriving at the total number of things composing that aggregate. At the same time, in counting that collection of numbers taken in this way we use the numbers by which we count them *precisely as numbers* (as numbers *per se*) in order to arrive at the *total number* of the numbers in the aggregate being counted. Only by doing this can we answer the question “how many?” of the aggregate being counted, with each number in that series both counting a member of the aggregate and summing the number of members of that series counted to that point. (Whew!) [↑](#footnote-ref-19)
20. See Chihara, op. cit., 173. [↑](#footnote-ref-20)
21. Charles Parsons, *Mathematical Thought and its Objects*, Cambridge, Cambridge University Press, 2008, 191. [↑](#footnote-ref-21)
22. Indeed, I suppose that numerals stand to numbers in the same way that substantive terms stand to concepts; numerals express numbers linguistically while numerals refer to or denote numbers and make them occurrently present to consciousness as intentional objects. [↑](#footnote-ref-22)
23. I can even use numbers to count numbers, as I did above. [↑](#footnote-ref-23)
24. *Pace* Chihara, how can I know that the dimes on the desk satisfy a constructability open sentence (“possess a seven property”) without counting them? *That is in fact how I know that there are seven dimes on the desk*. As such, talk about constructability open sentences being true of the world is ultimately parasitic on counting, which as I have argued is impossible without numbers. Thus, in saying that numbers exist, I am saying that they are real things, existing independently of both the physical universe and the human mind, though knowable (since known) by the intellect. [↑](#footnote-ref-24)
25. Chihara, op. cit., 229-235. [↑](#footnote-ref-25)
26. At one point, Chihara (305) does not claim to know what 2+2=4 means. This is surely disingenuous. Does Chihara wish us to believe that he does not *understand* the statement 2+2=4? If he can count and add, as I am sure he can, then he perfectly well understands the statement 2+2=4 and the terms composing it. Nothing more than this is needed in order to know what that statement means and to know *a priori* with intrinsic certainty that it is necessarily true. As such, I can only assume we ought to understand Chihara’s claim in some other, more technical sense of “means” that is very likely irrelevant to the sense at issue here. [↑](#footnote-ref-26)