

# BIMODAL QUANTUM THEORY<sup>\*</sup>

In process

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## Abstract

Some variants of quantum theory theorize dogmatic “unimodal” states-of-being, and are based on hodge-podge classical-quantum language. They are based on *ontic* syntax, but *pragmatic* semantics. This error was termed semantic inconsistency [1]. Measurement seems to be central problem of these theories, and widely discussed in their interpretation. Copenhagen theory deviates from this prescription, which is modeled on experience. A *complete* quantum experiment is “*bimodal*”. An experimenter *creates* the system-under-study in *initial* mode of experiment, and *annihilates* it in the *final*. The experimental *intervention* lies beyond the theory. I theorize most rudimentary bimodal quantum experiments studied by Finkelstein [2], and deduce “bimodal probability density”  $\pi = |\psi_{\text{IN}}\rangle \otimes \langle \phi_{\text{FIN}}|$  to represent *complete* quantum experiments. It resembles core insights of the Copenhagen theory.

**keywords.** Pragmatism; Bimodal Logic; Probability.

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## 1 Introduction

The name “Quantum Mechanics” carries the germ of mechanistic philosophy. Many people endure to fit quantum theory into this name. They seem to work on *causal* or *ontic* interpretations of quantum theory. Practical quantum theory is not mechanistic. It resembles Bohm’s *anti-Bohmian* proposals [3]. We can not reduce an individual quantum system-under-study to *finite* and *coherent* set of *quantitative* entities, that define its *qualitative* infinity [3]. A mechanistic theory suffices this requisite. Classical mechanics is a mechanistic theory, where coherent quantitative set  $(q, p)$  represents qualitative diversity of the system-under-study. Quantum theory has no such representatives. The doctrine with name “Quantum Mechanics” tacitly entails mechanistic prescription, that deceptively raises paradoxes from experiments that are incompatible with mechanistic prescription. We change the prescription to avoid paradoxes, which seems neater way to reduce ambiguity. Our prescription is *pragmatic*, *bimodal* and *non-mechanistic*, which obviates paradoxes. I call a practical quantum theory “Pragmatic Relativity”, in that its syntax is pragmatic, and it relativizes absolute “states-of-being” [4]. Pragmatic relativity is not a variant of Copenhagen theory.

Mechanistic theories are modeled on states-of-being. The “state-of-being” is unimodal dogmatic entity. An experimenter measures state-of-being of the system-under-study solely in *initial* mode of the experiment. A *final* mode seems redundant, in that it measures the same state. Classical experiments are unimodal *idealistic* measurements, that envisage states-of-being of the system-under-study. States-of-being objectivize the system-under-study, and entail its ontology.

Copenhagen theory deviates from this prescription. It has its roots in the philosophy of Niels Bohr. It appeared to him when he thought “What does it mean to *know*

the atom, while we see it only when it *changes*". He brought Heraclitean tendency into physics.

I recapitulate the Copenhagen theory, theorizing most rudimentary bimodal quantum experiments, and deduce "bimodal probability density"  $\pi = |\psi_{\text{IN}}\rangle \otimes \langle \phi_{\text{FIN}}|$  to represent *complete* quantum experiments.

## 1.1 The System Interface

Quantum theories *model* experiments on a miniscule part of Cosmos — a system-under-study  $\mathfrak{S}$  — a microcosm. An *isolated* system-under-study is oxymora; we see a system while we interact with it. This prescription respects temporal locality; a quantum system is "local" *immediate connection* with the experimenter. There is no "global" isolated system. Quantum experimenter  $\mathfrak{E}$  (who lives in exosystem,  $\mathfrak{E} \subset \mathfrak{X}\mathfrak{S}$ ) divides Cosmos into the dichotomy; endosystem  $\mathfrak{S}$  and exosystem  $\mathfrak{X}\mathfrak{S}$ , creating the *system interface* or *experimental channel*  $\mathfrak{S} | \mathfrak{X}\mathfrak{S}$ . Experiment destroys this interface<sup>1</sup>, being sole *process* that *connects*  $\mathfrak{S}$  with  $\mathfrak{X}\mathfrak{S}$ .

Experiment *entangles* system  $\mathfrak{S}$  with exosystem  $\mathfrak{X}\mathfrak{S}$ .

Quantum experiment is system-epi-system entanglement  $\mathfrak{S} - \mathfrak{X}\mathfrak{S}$ . An experimenter *creates* the system-under-study in *initial* mode of experiment, and *annihilates* it in the *final*. We do not theorize the system-under-study apart from these two pragmatic events. Quantum systems *jump* from *initial* mode to *final* mode of experiment, without carrying a dogmatic entity that defines its reality after the experiment. The *initial* system does not *evolve* into the *final* in *intervention* of the experiment. In classical continuous experiments, an auxiliary "system interface" separates the system from being effected by the experimenter; the system-under-study carries dogmatic state-of-being that we seem to predict *causally* after the experiment. Classical experimental channel is *unidirectional interface*; system-under-study acts on the experimenter, but not conversely. These experiments are often termed "ideal experiments", and deceptively preempted by ontologists in the discussion of quantum theories. We see classical systems as they are. There is no *interface* in quantum experiments, and actions respect reciprocity. Classical systems are dogmatic unimodal objects. Quantum systems are bimodal pragmatic events. In classical experiments, the system-under-study is an *absolute* object. In quantum experiments, the experimenter is an *absolute* subject. A theory based on *relative* experimenter lies beyond Copenhagen theory, and being developed elsewhere [4].

## 2 Quantum Disconnections and Acausality

Causality is succession of events: how an event descends from the preceding one. An event is an act or happening, like *initial* and *final* modes of an experiment. Einstein

<sup>1</sup>This interface was *arbitrary* in Heisenberg's version of Copenhagen theory, but Bohr's insistence eliminates it [1]. Heisenberg refrained from eliminating this *split* between observer and the system-under-study, in order to preclude the dilemma of considering whole universe as system-under-study [5, Ch. VII.1]. Heisenberg retained *interface* in order to save the *observer*. Rendering a theory of whole cosmos non practical, I retain Bohr's proposals.

called collision of two bodies an “event”. He ascribed “space-time” address to his event; a dogmatic event. In our prescription, an event is pragmatic; the happening itself (verb), than its address (noun).

Von Neumann’s theory [6], though rendered a variant of Copenhagen theory [2], is unimodal. It theorizes *initial* mode vector  $\psi$  (or *ket*  $|\psi\rangle$  in Dirac’s terminology). Inferences from past experiments endow the system-under-study with states-of-being, while predictions for future experiments are statistical. This theory seems to have temporal asymmetry; past experience represents the system-under-study causally, while future predictions do not. Dirac’s theory [7] is also unimodal, that ascribes the system-under-study an state-of-being  $|\psi\rangle$ . The *error* lies in their *modal structure* that we cure next.

## 2.1 Bimodal Prescription and Vertex Model

Quantum theory<sup>2</sup> advances modal logic. Classical experiments were unimodal; a classical experimenter *prepared* and *registered* the system-under-study simultaneously. This simultaneity ascribes to the system its unimodal “states-of-being”. Classical experimenter measures state-of-being of the system-under-study.

Quantum theory renounces this *simultaneity* and *unimodality*. Quantum experiments are bimodal; a quantum experimenter *injects*, *prepares* or *creates* the system-under-study in its *initial* mode, and *extracts*, *registers* or *annihilates* it in the *final* mode of experiment. Quantum experimenter specifies bimodal *external* acts, not unimodal *internal* states.

Quantum experiment is succession of external acts on the system-under-study. We can only define it by its *initial* and *final* modes. Initial mode *creates* the system-under-study by sending a *probe*, and interacting with it. The system-under-study undergoes a drastic *irreversible* change; we call it “intervention” of the experiment. The experimenter is not supposed to know of the intervention; an attempt to know it ends the experiment, that we call its *final* mode. Final mode *annihilates* system-under-study.

We know (and can know) the system-under-study only while interacting with it, or acting upon it; in *initial* and *final* modes the experiment. Quantum knowledge is both *bimodal* and *pragmatic*.

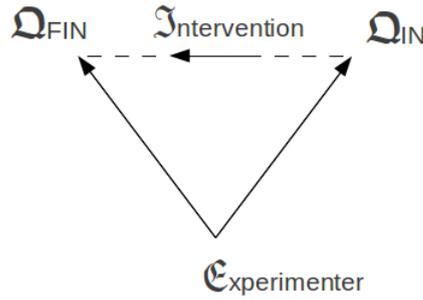
A quantum experiment can be described by at least two modes; *initial* and *final*, with no causal connection.

### 2.1.1 Quantum Topology

Quantum topology is theory of quantum connections; *how* quantum events do *connect*. Processes such as *creation* and *annihilation* are quantum events.

**Definition 2.1.** A quantum system  $\mathfrak{Q}$  is represented by two *bimodal pragmatic vertices*  $\mathfrak{Q}_{\text{IN}}$  and  $\mathfrak{Q}_{\text{FIN}}$ , connecting system-under-study with the experimenter  $\mathfrak{E}$ . The initial vertex  $\mathfrak{Q}_{\text{IN}}$  represents *initial* mode of experiment, and final vertex  $\mathfrak{Q}_{\text{FIN}}$  represents the *final* mode.  $\mathfrak{Q}_{\text{IN}}$  represents *creation* of the system-under-study and  $\mathfrak{Q}_{\text{FIN}}$  its *annihilation*. The archetypal action diagram of a “complete” quantum experiment is give by

<sup>2</sup>By quantum theory, I tacitly mean Copenhagen theory, unless otherwise explicated.



Arrows  $\mathcal{E}\Omega_{\text{IN}}$  and  $\mathcal{E}\Omega_{\text{FIN}}$  represent *initial* and *final* actions  $|\psi_{\text{IN}}\rangle$  and  $\langle\phi_{\text{FIN}}|$  of episystem  $\mathcal{E}$  on the system  $\Omega$ . The *intervention*  $\mathcal{I}$  is a disconnected event, that lies outside the theory. The system-under-study *jumps* from  $\Omega_{\text{IN}}$  to  $\Omega_{\text{FIN}}$ , at least for the experimenter  $\mathcal{E}$ . Actions *connecting* pragmatic vertices of  $\Omega$  with  $\mathcal{E}$  constitute experimenter’s frame  $\mathfrak{F}_{\mathcal{E}}\{\psi_i, \phi_f; \tau\}$ ; where  $\psi$  and  $\phi$  represent *initial* and *final* external actions of his choices,  $i$  and  $f$  represent their indices, and  $\tau$  represents “proper time” of experimenter’s clock.  $\psi$  or *ket*  $| \ \rangle$  represents *creator*, and  $\phi$  or *bra*  $\langle \ |$  represents *annihilator*. Experimental frame  $\mathfrak{F}_{\mathcal{E}}\{\psi_i, \phi_f; \tau\}$  belongs to episystemic composite frame space  $\text{IN} \otimes \text{FIN} \otimes \text{TIME}$ ;  $|\psi_{\text{IN}}\rangle \in \text{IN}$ ,  $\langle\phi_{\text{FIN}}| \in \text{FIN}$ ,  $\tau \in \text{TIME}$ .

## 2.2 Quantum Jumps and Acausality

Quantum systems “jump” from  $\Omega_{\text{IN}}$  to  $\Omega_{\text{FIN}}$  with no causal connection. A complete bimodal quantum experiment connects  $\Omega_{\text{IN}}$  and  $\Omega_{\text{FIN}}$  to experimenter  $\mathcal{E}$ . The experimenter does not know how  $\Omega_{\text{IN}}$  evolved into  $\Omega_{\text{FIN}}$ . His knowledge is restricted merely to the mode vectors of his choices *randomly* drawn from his frame  $\mathfrak{F}_{\mathcal{E}}\{\psi_i, \phi_f; \tau\}$ .

Quantum events  $\Omega_{\text{IN}}$  and  $\Omega_{\text{FIN}}$  are disconnected, and we can not entail causality to assimilate their evolution. This discontinuity was often called *quantum jump* and endured by ontologists to refute. It appears that quantum jumps are indispensable, and can not be renounced while defining reality. We are eventually led to probabilistic prescription; talk about what is possible, than real.

Many people blame quantum theory for not being *causal*. Some disqualify it, and work on its causal or ontic variants, that lack modal semantics; this compendium is abundant. Heuristically, I entail acausality a symptom of more elaborate prescription for theorization. Unimodal dogmatic theories, having mechanistic syntax, are *causal*; bimodal pragmatic theories are not.

## 3 Quantum Probability

Quantum probability was erroneously called charge density, when probability was confused with (electron) density. Born later resolved this problem in his probabilistic interpretation of quantum theory [8], and generalized charge density  $e|\psi|^2$  of electrons in atoms to more general quantum systems. Contemporary quantum theories are modeled on density operator  $\rho = |\psi\rangle \otimes \langle\psi|$  (a variant of Pauli’s “probability amplitude”). However, density operator  $\rho = |\psi\rangle \otimes \langle\psi|$  (often called density matrix) is not Schrödinger probability operator; it does not fulfill criterion of Schrödinger operators, and does not correspond to his eigenvalue equation. It was erroneously

called a quantum operator because it was represented by matrices. It seems a pseudo Schrödinger operator, in that  $P = \text{Tr}(\rho)$ . Nevertheless, I retain terms *density operator* or *statistical operator* for it.

I theorize a probability  $\omega$  to account for system's ontology, and call it *mode probability*. It represents modes-of-being of the system-under-study. When we set out observation on the system-under-study, we may or may not observe all its characteristic (or systemic) variables or coordinates proper to the experimenter  $\mathfrak{E}$  or his frame  $\mathfrak{F}_{\mathfrak{E}}\{\psi_i, \phi_f; \tau\}$ . The system-under-study might fall in one of these modes-of-being:

- **Apparent mode** - The system-under-study may be rendered in *apparent mode of being* when all its variables are specifiable at one instance (in experimenter's frame), and system-under-study could be specified (or objectivized) uniquely by means of these systemic variables. [Theories modeled on apparent mode perception are relic of ontology.]
- **Partial mode** - The system-under-study may be rendered in *partial mode of being* when some of its variables are specifiable at one instance (in experimenter's frame), and system-under-study could not be specified with ultimate precision by means of these variables. Systems in partial mode of being are *maximal-informative-systems* (in language of Von Neumann) or simply quantum systems. [Practical quantum theories are usually modeled on partial mode perception, which are pragmatic in usage; deceptively ontic in form.]
- **Hidden mode** - The system-under-study may be rendered in *hidden mode of being* when none of its variables are specifiable at one instance (in experimenter's frame), and system-under-study could not be specified by any means.

These modes can alternatively be called *Full*, *Possible* and *Null* modes of being. Quantum mode is possible mode, not full (or apparent) being. Quantum mode is maximal informative mode.

Systemic variables are often *complementary*. Two complementary variables, say  $(q, p)$  entail phase or state space of classical systems. In quantum usage, knowledge of one precludes that of another. Classical systems or *objects* are systems in apparent-mode-of-being; with specification to *all* their systemic variables. Quantum systems are maximal-informative-systems, in partial-mode-of-being; with specification to *few* of their systemic (complementary) variables, or *all* within minimal uncertainty set out by Heisenberg [9].

Some endure to supplement quantum systems with some "hidden" noncomplementary variables, that render it in apparent mode-of-being, and anticipate that we would eventually recede to classical epistemology. These hidden-variables could not be subjected to perception, as their name implies, and these endeavors lead nowhere.

We (can) specify the system-under-study (only) by means of its mode-of-being. I entail some systems in practice:

- **Objects** - If the system-under-study is in apparent-mode-of-being, and an experimenter  $\mathfrak{E}$  learns (or measures) all its systemic variables, we call the system-under-study an *object*. [It is not in that all experimenters agree upon its appearance. They change it when then attempt to perceive it. They independently,

and rather separately render it an object. Our object differs from ontological “absolute” objects (like Moon). See Section 8.1]

- **Maximal-informative-systems** - If the system-under-study is in partial-mode-of-being, and an experimenter  $\mathfrak{E}$  learns (or measures) only *few* of its systemic variables, or *all* within minimal uncertainty implied by uncertainty relations [9], we call the system-under-study a *maximal-informative-system* or *quantum system*  $\mathfrak{Q}$ .
- **Forbidden-systems** - If the system-under-study is in forbidden-mode-of-being, and an experimenter  $\mathfrak{E}$  does not (or can not) learn (or measure) any of its systemic variables, we call the system-under-study a *forbidden-system*. [Vacuum is a forbidden system.]

In quantum usage, we confront with maximal-informative-systems. Experiments perceiving maximal-informative-systems many be called *maximal informative experiments* or simply quantum experiments.

## 4 Probability Density Operator

Despite its rudimentary form  $S = K \ln \psi$  [10], Schrödinger’s wavefunction is often written nowadays in polar form [11]

$$\psi(R, S) := R \exp\left(\frac{i}{\hbar} S\right). \tag{4.1}$$

Its partial differentiation w.r.t action S yields

$$\psi(R, S) + i\hbar \frac{\partial \psi(R, S)}{\partial S} = 0. \tag{4.2}$$

**Definition 4.1.** A hypothetical “toy” identity operator  $\mathcal{I}$  (without physical meaning), in analogy to identity matrix  $\mathbb{I}$ , corresponds to Schrödinger eigenvalue equation (SE)  $\widehat{\mathcal{I}}|\psi\rangle = \mathcal{I}|\psi\rangle = |\psi\rangle$  with  $\mathcal{I} \equiv \mathbb{I} \equiv 1$ ,

$$SE : \mathcal{I} \longrightarrow \widehat{\mathcal{I}}. \tag{4.3}$$

In view to **Def. 4.1**, (4.2) yields Schrödinger identity operator

$$\widehat{\mathcal{I}} = -i\hbar \frac{\partial}{\partial S}. \tag{4.4}$$

I theorize, phenomenologically, most rudimentary bimodal quantum experiments studied by Finkelstein (1996) [2]; Malus-Born experiment. Our system-under-study is a beam of “photons” traversing optical arrangement of Malus experiment. The archetype of the experiment is

absorb  $\longleftarrow$  analyze  $\longleftarrow$  polarize  $\longleftarrow$  emit .

The experimenter has a frame of experiment  $\mathfrak{F}_{\mathfrak{E}}\{\psi_i, \phi_f; \tau\}$ , that constitutes of his choices for *initial* and *final* mode vectors (or polarization vectors)  $\psi_{\text{IN}}$  and  $\phi_{\text{FIN}}$ ;

where  $i$  and  $f$  represent indices of *initial* and *final* mode vectors. [**Caution!**  $\phi_{\text{FIN}}$  is distinguished from  $\psi_{\text{IN}}$ , and both represent physically distinguished actions (i.e.,  $\psi_{\text{IN}} \neq \phi_{\text{FIN}}$ ).  $\psi_{\text{IN}}$  and  $\phi_{\text{FIN}}$  belong to different mathematical spaces called *initial space* and *final space*. Finkelstein studies their operational symmetry [2]. Here,  $\psi$  (and *ket*  $| \rangle$ ) is preempted for *initial* and  $\phi$  (and *bra*  $\langle |$ ) for *final* mode vectors. *bra*  $\langle |$  represents *modal dual* (MD) of *ket*  $| \rangle$  for arbitrary mode vectors,  $\text{MD} : | \rangle \longrightarrow \langle |$ , and  $\phi_{\text{FIN}}$  may not be confused as *modal dual* of  $\psi_{\text{IN}}$ , except for  $\psi \equiv \phi$ .] Extreme classical cases are with  $\phi_{\text{FIN}} \parallel \psi_{\text{IN}}$  and  $\phi_{\text{FIN}} \perp \psi_{\text{IN}}$ ; former *passes* the photon, while later *blocks* it. Quantum phenomena occurs with *oblique* polarizers  $\phi_{\text{FIN}} \angle \psi_{\text{IN}}$ . For oblique polarizers, Malus calculated the fraction of photons transmitted, that he found to be

$$P = \cos^2 \Delta\theta, \quad (4.5)$$

where  $\Delta\theta$  is angle difference between *polarizer*  $|\psi_{\text{IN}}\rangle$  and *analyzer*  $\langle\phi_{\text{FIN}}|$ . His fraction  $P$  was rediscovered by Born, that he called transition probability [2]. The generalized form of Malus-Born transition probability is

$$P = |\langle\phi_{\text{FIN}}|\psi_{\text{IN}}\rangle|^2. \quad (4.6)$$

For Malus experiment,  $\psi_{\text{IN}}$  and  $\phi_{\text{FIN}}$  are both normalized:  $\langle\psi_{\text{FIN}}|\psi_{\text{IN}}\rangle = \langle\phi_{\text{FIN}}|\phi_{\text{IN}}\rangle = 1$ , where  $\psi_{\text{FIN}}$  is *modal dual* of  $\psi_{\text{IN}}$  and  $\phi_{\text{FIN}}$  is *modal dual* of  $\phi_{\text{IN}}$ . Modal dual of *polarizer* is *analyzer* with same polarization angle  $\theta$ . The *analyzer*  $\langle\phi_{\text{FIN}}|$  with same polarization angle  $\theta$  (i.e.,  $\psi \parallel \phi$ ) as *polarizer*  $|\psi_{\text{IN}}\rangle$  *transmits* the photon.  $\psi_{\text{IN}}$  and  $\phi_{\text{FIN}}$  are not orthogonal necessarily, except for  $\phi \perp \psi$ ,  $\langle\phi_{\text{FIN}}|\psi_{\text{IN}}\rangle = 0$ ; orthogonal *polarizer* and *analyzer* “block” the photon. In either case,  $\psi_{\text{IN}}$  and  $\phi_{\text{FIN}}$  are not *normalized* or *orthogonal* simultaneously; they do not form *orthonormal* bases for Malus experiment.

*Trace* ( $\text{Tr}$ ) of Schrödinger “toy” identity operator (4.4),

$$\text{Tr}(\widehat{\mathcal{I}}) = \langle\phi_{\text{FIN}}|\widehat{\mathcal{I}}|\psi_{\text{IN}}\rangle = \langle\phi_{\text{FIN}}|\psi_{\text{IN}}\rangle, \quad (4.7)$$

returns *amplitude* of Malus-Born experiment.

**Definition 4.2.** *Bimodal probability density*  $\pi = |\psi_{\text{IN}}\rangle \otimes \langle\phi_{\text{FIN}}|$  represents a binary composite action, creation  $\oplus$  annihilation, of experimenter  $\mathfrak{E}$  on the system-under-study. It is mere notational, and order of  $\psi_{\text{IN}}$  and  $\phi_{\text{FIN}}$  is irrelevant. It represents a *complete* quantum experiment proper to experimenter’s frame  $\mathfrak{F}_{\mathfrak{E}}\{\psi_i, \phi_f; \tau\}$ .

**Caution!** It may not be confused with unimodal density matrix  $\rho$ , which is  $\rho_{\text{IN}} = |\psi_{\text{IN}}\rangle\langle\psi_{\text{IN}}|$  for *initial* mode, and  $\rho_{\text{FIN}} = |\phi_{\text{FIN}}\rangle\langle\phi_{\text{FIN}}|$  for *final* mode of experiment. Unimodal quantum theories are modeled on “initial density matrix” or “initial statistical operator”  $\rho_{\text{IN}}$ , not *final*  $\rho_{\text{FIN}}$ .

*Trace* ( $\text{Tr}$ ) of bimodal probability density  $\pi$ ,

$$\text{Tr}(\pi) = \langle\phi_{\text{FIN}}|\pi|\psi_{\text{IN}}\rangle = |\langle\phi_{\text{FIN}}|\psi_{\text{IN}}\rangle|^2, \quad (4.8)$$

returns transition probability (4.6) of Malus-Born experiment. [**Caution!** For unimodal density operators,  $\text{Tr}(\rho_{\text{IN}}; \phi) = \langle\phi_{\text{FIN}}|\rho_{\text{IN}}|\phi_{\text{IN}}\rangle = |\langle\phi_{\text{FIN}}|\psi_{\text{IN}}\rangle|^2$  and  $\text{Tr}(\rho_{\text{FIN}}; \psi) = \langle\psi_{\text{FIN}}|\rho_{\text{FIN}}|\psi_{\text{IN}}\rangle = |\langle\phi_{\text{FIN}}|\psi_{\text{IN}}\rangle|^2$ . It owes *semantic error*, and makes no experimental sense; we preempt *bimodal density operator*  $\pi = |\psi_{\text{IN}}\rangle\langle\phi_{\text{FIN}}|$  and *bimodal trace*  $\text{Tr}(\pi; \psi_{\text{IN}}, \phi_{\text{FIN}}) = \langle\phi_{\text{FIN}}|\pi|\psi_{\text{IN}}\rangle$ , instead.]

**Definition 4.3.**  $\omega$  is probability of mode-of-being of the system-under-study, or simply *mode probability*. For a system-under-study in apparent-mode-of-being,  $\omega = 1$ . For a system-under-study in hidden-mode-of-being,  $\omega = 0$ . For quantum systems in partial-mode-of-being,  $\omega \in (0, 1)$ .

For  $\Delta\theta = \pi/4$ , Malus calculated  $P = 1/2$ . This seems paradoxical for beam consisting of one photon. It makes an absurd assertion “half of the photon passes through analyzer”. For one photon,  $\omega \in (0, 1)$ ; an individual quantum system is maximal-informative-system with mode probability  $\omega \in (0, 1)$ . For extreme classical cases,  $\Delta\theta = 0$  or  $\pi/2$ , we know whether a photon passes or not e.g.,  $\omega = 1$  or  $0$ . For  $\Delta\theta = 0$  and  $\omega = 1$ , a photon is apparent (or objective); experimenter knows that it passes the analyzer. For  $\Delta\theta = \pi/2$  and  $\omega = 0$ , it is forbidden; experimenter knows that it does not pass the analyzer. For  $\Delta\theta \in (0, \pi/2)$  and  $\omega \in (0, 1)$ , experimenter does not know the system-under-study at individual level.  $\omega$  represents *possibility* (or potential) for a quantum system, despite extreme classical cases ( $\omega = 1$  or  $0$ ). We should better call  $\omega$  quantum probability henceforth.

(4.5) makes sense for many photons. Yet we do not know which photon passes, and which does not. We can not know quantum systems at individual level. Quantum theory is many system theory.

For three different cases:  $\phi_{\text{FIN}} \parallel \psi_{\text{IN}}$ ,  $\phi_{\text{FIN}} \angle \psi_{\text{IN}}$  and  $\phi_{\text{FIN}} \perp \psi_{\text{IN}}$ , (4.8) yields

$$\begin{cases} \text{Tr}(\pi) = 1 & \text{for } \textit{parallel} \text{ polarizers } \phi_{\text{FIN}} \parallel \psi_{\text{IN}}; \\ \text{Tr}(\pi) \in (0, 1) & \text{for } \textit{oblique} \text{ polarizers } \phi_{\text{FIN}} \angle \psi_{\text{IN}}; \\ \text{Tr}(\pi) = 0 & \text{for } \textit{perpendicular} \text{ polarizers } \phi_{\text{FIN}} \perp \psi_{\text{IN}}. \end{cases} \quad (4.9)$$

In view to **Def. 4.3**, (4.8) and (4.9),  $\text{Tr}(\pi)$  corresponds to quantum probability  $\omega$ ,

$$\text{Tr} : \pi \longrightarrow \omega, \quad \omega = \text{Tr}(\pi) = \langle \phi_{\text{FIN}} | \pi | \psi_{\text{IN}} \rangle = |\langle \phi_{\text{FIN}} | \psi_{\text{IN}} \rangle|^2. \quad (4.10)$$

Bimodal probability density  $\pi$  represents “potential” for mode probability  $\omega$ . Transition probability  $P$  is tacitly equivalent to mode probability  $\omega$ .

Recall (4.7) and **Def. 4.2** for unimodal quantum theories, where  $\psi$  and  $\phi$  form orthonormal bases, and  $\text{ME}\{\hat{\mathcal{A}}\} = \text{Tr}(\hat{\mathcal{A}})$  for an arbitrary Schrödinger operator  $\hat{\mathcal{A}}$ . A trivial case in Malus experiment with *polarizer*  $|\psi_{\text{IN}}\rangle$  and *analyzer*  $\langle\psi_{\text{FIN}}|$  with same polarization angle  $\theta$  ( $\Delta\theta = 0$ ) resembles unimodal quantum theories. Here,  $\text{Tr}(\hat{\mathcal{I}}) = \text{ME}(\hat{\mathcal{I}}) = \langle\psi_{\text{IN}}|\hat{\mathcal{I}}|\psi_{\text{IN}}\rangle = \rho_{\text{IN}}$ . We preempt  $\hat{\rho} \equiv \hat{\mathcal{I}}$ , and dispense with “toy” identity operator. We endow  $\hat{\rho}$  with physical semantics that  $\hat{\mathcal{I}}$  lacked. We have Schrödinger probability density eigenvalue equation

$$\hat{\rho}|\psi\rangle = \rho|\psi\rangle, \quad (4.11)$$

with Schrödinger “*unimodal* probability density operator”

$$\hat{\rho} = -i\hbar \frac{\partial}{\partial S}. \quad (4.12)$$

Its *unimodal trace* ( $\text{Tr}$ ) returns “*unimodal* probability density”

$$\rho = \text{Tr}\{\hat{\rho}\}, \quad \rho = |\psi\rangle \otimes \langle\psi|. \quad (4.13)$$

$\hat{\rho} = -i\hbar \partial/\partial S$  and  $\rho = |\psi\rangle\langle\psi|$  have little utility in bimodal quantum theories, than  $\pi = |\psi_{\text{IN}}\rangle\langle\phi_{\text{FIN}}|$ .

## 4.1 Unimodal Mathematical Expectation

For a predicate  $\mathcal{P}$  of the system-under-study, “*unimodal* mathematical expectation” (ME) is given by

$$\mathcal{P} \stackrel{\text{def}}{=} \text{ME} \{ \widehat{\mathcal{P}} \} = \frac{\text{Tr}(\widehat{\mathcal{P}}\rho)}{\text{Tr}(\rho)} = \frac{\text{Tr}(\widehat{\mathcal{P}}\widehat{\rho})}{\text{Tr}(\widehat{\rho})} = \frac{\langle \psi | \widehat{\mathcal{P}} | \psi \rangle}{\langle \psi | \psi \rangle}, \quad (4.14)$$

where  $\widehat{\mathcal{P}}$  is Schrödinger operator of predicate  $\mathcal{P}$ ;  $\text{ME} : \widehat{\mathcal{P}} \rightarrow \mathcal{P}$ . Here  $\widehat{\rho} = -i\hbar \partial/\partial S$  and  $\rho = |\psi\rangle\langle\psi|$ ;  $\psi$  (4.1) is *initial* mode vector, often deceptively envisaged unimodal dogmatic *statevector* of the system-under-study. For *pure* mode vectors  $\rho^2 = \rho$ ; for *mixture* of mode vectors  $\rho^2 \leq \rho$ ; and  $\text{Tr}(\rho) = 1$  for both.

## 5 Complementarity between Quantum Probability and $\Psi$ -Wave

For an arbitrary *initial* modevector  $|\psi\rangle$ , such as (4.1), *unimodal* mathematical expectation (ME) of commutator  $[\widehat{S}, \widehat{\rho}]_-$  yields

$$\text{ME} \{ [\widehat{S}, \widehat{\rho}]_- \} = \langle [\widehat{S}, \widehat{\rho}]_- \rangle = \langle \psi | [\widehat{S}, \widehat{\rho}]_- | \psi \rangle = i\hbar \langle \psi | \psi \rangle, \quad (5.1)$$

or

$$\text{ME} : [\widehat{S}, \widehat{\rho}]_- \rightarrow i\hbar, \quad \langle [\widehat{S}, \widehat{\rho}]_- \rangle = i\hbar. \quad (5.2)$$

**[Caution!]** Our deduction follows from unimodal quantum theories, where *ket*  $|\psi\rangle$  and *bra*  $\langle\psi|$  are both *initial* mode vectors, and orthonormal. We follow mathematical expectation criterion of Von Neumann’s [6] or Dirac’s [7] unimodal quantum theories, not from Section 6.2] In view to generalized uncertainty relation

$$\sigma_S^2 \sigma_\rho^2 \geq \left( \frac{\langle [\widehat{S}, \widehat{\rho}]_- \rangle}{2i} \right)^2,$$

we obtain

$$\sigma_S \sigma_\rho \geq \frac{\hbar}{2} \quad \text{or} \quad \Delta S \Delta \rho \geq \frac{\hbar}{2}, \quad (5.3)$$

where  $\Delta$  measures *spread* in  $S$  and  $\rho$ .

Quantum unimodal probability density ( $\rho$ ) and Action ( $S$ ) are complementary; knowledge of one precludes that of another. For an *apparent* system-under-study or *object* (with  $\rho = 1$ )<sup>3</sup>, action  $S$  is ambiguous, and does not represent property of the system. For a *partial* or quantum system-under-study (with  $\rho \in (0, 1)$ ),  $S$  is known with definite precision (5.3). In either case,  $S$  is an epistemic variable, and represents experimenter’s action on the system-under-study. In classical theory,  $S$  was *functional* of the path, not function of the system.  $S$  represents action of epistemic system on the system, not systemic variables.

<sup>3</sup>In unimodal quantum theories, quantum probability  $\omega = \text{Tr}(\rho)$ . Roughly  $\rho = \sqrt{\omega}$ ;  $\rho = 1, 0$  for  $\omega = 1, 0$  and  $\rho \in (0, 1)$  for  $\omega \in (0, 1)$ .

We adduce, in view to (4.1), (4.10) and (5.3),

Quantum probability  $\omega$  and  $\psi$  function are complementary.

Knowledge of one precludes that of another.  $\psi$ , being a function of  $S$ , represents action of episytem on the system, and  $\omega$ , *trace* of  $\rho$ , represents quantum probability.  $\psi$  constitutes experimenter's frame  $\mathfrak{F}_e\{\psi_i, \phi_f; \tau\}$ , and represents no property of the system-under-study at all.

Complementary entities are not apparent simultaneously. For an apparent system-under-study (one with  $\omega = 1$ ),  $\psi$  is *partial*. ‘Apparent’, ‘proper’ and ‘characteristic’ are synonymous, at least in our context. Apparent systems do not have characteristic or *eigen*  $\psi$ . Apparent systems (or absolute objects) have *states of being*, but not the  $\psi$ . Statements like “eigenvector  $|\psi\rangle$  of the system-under-study” or “state function  $\psi$  of the system-under-study” are oxymoronic. A statement like “ $\psi$  is maximal informative function for the system-under-study” is more relevant for what quantum experiments ascertain.

Nevertheless, we could know  $\psi$  (4.1) during *initial* and *final* modes of quantum experiments (with  $\omega \in (0, 1)$ ), with definite precision controlled by (5.3). It may not be assimilated in that  $\psi$  represents the system-under-study *partially*, incompletely or with finite precision; instead, we see the system-under-study as a *counterpart* of the experimenter, not as an object in its own essence. For isolated systems or objects,  $\psi$  (4.1) is ambiguous; uncertainty diverges  $\Delta S, \Delta\psi \rightarrow \infty$ .

*Knowing* and *doing* are complementary.

Schrödinger conceived  $\psi$  (4.1) as “particle like wavefunction” to reconcile wave-particle *duality* and path *discreteness* of particles in Wilson cloud chamber [10]. Bohr called *particle* and *wave* complementary modes of the system-under-study.  $\psi$  (4.1) can not be known from *particulate* systemic variables alone in the sense of complementarity.

Von Neumann (1932) adduced that we can not describe quantum systems *causally* even though we know their wavefunction. For him, state variables  $(q, p)$  and wavefunction  $\psi$  are essentially different structures [6, Ch. III]. To describe system's states-of-being, one needs to supplement  $\psi$  with additional parameters that were called *hidden*. Hidden parameters were preempted by classical mechanicians to reduce *statistical* relations to *causal* ones, for example, in kinetic theory of gases. It became philosopher's stone of physics, and many endured in vain to search it, including Einstein [12], Bohm [13, 14] and many others. Von Neumann adduces that to reduce quantum theory from statistical to causal interpretation, by means of supplementing hidden parameters, is *impossible* [6, Ch. IV.2]. He renders statistical interpretation of Born as only consistent interpretation of quantum theory.

## 6 Experimental Quantification

Quantification transcends one *system* theories to *many*. Here, I quantify *experiment*, not the system-under-study. Unimodal *initial* density operator  $\rho_{\text{IN}} = |\psi_{\text{IN}}\rangle\langle\psi_{\text{IN}}|$  represents “partial” or incomplete experiments. It also lacks modal structure, and owes

semantic error [2]. So does the *final* density operator  $\rho_{\text{FIN}} = |\phi_{\text{FIN}}\rangle\langle\phi_{\text{FIN}}|$  [15]. A complete quantum experiment has (at least) two modes; *initial* mode *injects*, *prepares* or *creates* the system-under-study, and *final* mode *extracts*, *registers* or *annihilates* it. The *intervention* lies beyond the theory. Bimodal density operator  $\pi = |\psi_{\text{IN}}\rangle\langle\phi_{\text{FIN}}|$  represents an individual “*complete*” experiment on the system-under-study. Several quantum experiments are represented by an *assembly* of bimodal density operators  $\pi_e$ ,  $e$  representing *index* of the experiment. The experimenter  $\mathfrak{E}$  *creates* and *annihilates* the system-under-study in several *trials* proper to his frame  $\mathfrak{F}_{\mathfrak{E}}\{\psi_i, \phi_f; \tau\}$ ;  $i$  and  $f$  index *initial* and *final* mode vectors of his choices. Each experimental “*trial*” entails a unique bimodal density operator

$$\pi_e = |\psi_e^{\text{IN}}\rangle \otimes \langle\phi_e^{\text{FIN}}|, \quad (6.1)$$

with unique *initial* and *final* mode vectors indexed  $e$ , randomly drawn from  $\mathfrak{F}_{\mathfrak{E}}\{\psi_i, \phi_f; \tau\}$ .

## 6.1 Experimental Assembly

Complete quantum experiments  $\mathfrak{E}_e$  constitute an experimental *sequence*.

**Definition 6.1.** An experimental sequence SEQ  $\mathfrak{E}_e$  represents experiments  $\mathfrak{E}_e$  on the system-under-study. All experiments in SEQ  $\mathfrak{E}_e$  are *distinguished* and *order relevant*.

Classical experiments constitute a set SET  $\mathfrak{E}_e$ , representing *indistinguished* and *order irrelevant* experiments. We see the same Moon, regardless of whether we *repeat* experiments or *reverse* their order. Quantum experiments are *distinguished* and *order relevant*. Nevertheless, repeating experiments on same photons in Malus experiment gives different results, but changing their *order* does not. Malus experiments constitute a *series* SER  $\mathfrak{E}_e$  with experiments *distinguished*, but *order irrelevant*. Still more fundamental quantum experiments might constitute a sequence SEQ  $\mathfrak{E}_e$ .

## 6.2 Bimodal Mathematical Expectation

For a complete individual experimental “*trial*” on the system-under-study, mathematical expectation (ME) of a predicate  $\mathcal{P}$  of the system-under-study is given by

$$\mathcal{P} \stackrel{\text{def}}{=} \text{ME} \{\widehat{\mathcal{P}}\} = \frac{\text{Tr}(\widehat{\mathcal{P}}\pi)}{\text{Tr}(\pi)} = \frac{\langle\phi_{\text{FIN}}|\widehat{\mathcal{P}}|\psi_{\text{IN}}\rangle}{\langle\phi_{\text{FIN}}|\psi_{\text{IN}}\rangle}, \quad (6.2)$$

where  $\widehat{\mathcal{P}}$  is Schrödinger operator of predicate  $\mathcal{P}$ ;  $\text{ME} : \widehat{\mathcal{P}} \rightarrow \mathcal{P}$ . Here  $\text{Tr}(\pi) < 1$ , except for parallel *polarizer* and *analyzer*.

**Definition 6.2.** For an assembly of experimental “*trials*” on the system-under-study represented by SEQ  $\mathfrak{E}_e$ , experimental average of a predicate  $\mathcal{P}$  of the system-under-study *measured* over  $N$  trials is given by

$$\text{Avg. } \mathcal{P} \stackrel{\text{def}}{=} \frac{1}{N} \sum_e^N \text{ME}_e \{\widehat{\mathcal{P}}\} = \frac{1}{N} \sum_e^N \frac{\text{Tr}(\widehat{\mathcal{P}}\pi_e)}{\text{Tr}(\pi_e)} = \frac{1}{N} \sum_e^N \frac{\langle\phi_e^{\text{FIN}}|\widehat{\mathcal{P}}|\psi_e^{\text{IN}}\rangle}{\langle\phi_e^{\text{FIN}}|\psi_e^{\text{IN}}\rangle}, \quad (6.3)$$

where  $e$  is the *index* of trials. For  $N \rightarrow \infty$ , (6.3) gives maximal informative average.

## 7 Experimental Phase

*Initial* and *final* statistical operators  $\rho_{\text{IN}} = |\psi_{\text{IN}}\rangle\langle\psi_{\text{IN}}|$  and  $\rho_{\text{FIN}} = |\phi_{\text{FIN}}\rangle\langle\phi_{\text{FIN}}|$  collapse *initial* and *final* “phases”  $\exp(i\theta_{\text{IN}})$  and  $\exp(i\theta_{\text{FIN}})$  of *initial* and *final* mode vectors  $\psi_{\text{IN}}$  and  $\phi_{\text{FIN}}$ . Bimodal statistical operator  $\pi = |\psi_{\text{IN}}\rangle\langle\phi_{\text{FIN}}|$  reserves “bimodal phase”  $\exp\{i(\theta_{\text{IN}} - \theta_{\text{FIN}})\}$ , here termed “experimental phase”.  $\rho_{\text{IN}}$  and  $\rho_{\text{FIN}}$  represent merely transition amplitudes, with no information of intact *initial* and *final* mode vectors.  $\pi$  represents transition amplitude as well as *initial* and *final* phases, collectively  $\exp\{i(\theta_{\text{IN}} - \theta_{\text{FIN}})\}$ . Unimodal statistical operators  $\rho_{\text{IN}}$  and  $\rho_{\text{FIN}}$  represent *partial* quantum experiments, with no information about experimental phase. Experimental phase  $\exp\{i(\theta_{\text{IN}} - \theta_{\text{FIN}})\}$  represents a *complete* quantum experiment; so does the bimodal statistical operator  $\pi = |\psi_{\text{IN}}\rangle\langle\phi_{\text{FIN}}|$ , that preserves it. For Malus experiment, experimental phase is  $\exp\{-i\Delta\theta\}$ .

## 8 Wave Mechanics and Objectivity

### 8.1 Objects and Frame Relativity

Classical objects are absolutistic; all experimenters agree upon their being. Quantum objects (if ever conceived by means of measuring complementary variables simultaneously) are relative; each experimenter creates his own object, that others do not agree upon. Moon is a classical object; all experimenters rely on its being. Quantum objects are yet hypothetical, but differ from classical ones in this semantics. If a hidden-variable theorist theorizes an object proper to each experimenter, he does not recede to classical causality as he plainly believes. Hidden-variable theories do not seem to be counter proposal to Copenhagen theories, even though they ever succeed.

Quantum objects are possible, but relative (proper to an experimenter  $\mathfrak{E}$ ). Classical objects are inevitable, and absolutistic (regardless of experimenter). Quantum experimenter creates or invents quantum objects proper to his frame of experiment  $\mathfrak{E}\{\psi_i, \phi_f; \tau\}$  alone. He lacks transformation of such objects to other ones. Quantum objects are not invariant under quantum transformations; experimenters lack consensus for their being. Quantum “objects” appear for each experimenter’s frame alone, like “time” in special relativity. Quantum theory is *frame relativity* or *quantum relativity*. Dirac called his frame relativity “Transformation Theory” [16]. Special relativity has no absolute “time”; experimenters or their frames have “proper time”, but they lack consensus for it. Each experimenter has his own proper time. Quantum relativity has no absolute “object”; experimenters have “proper or apparent object”, yet each has his own. They lack consensus for its being. Quantum theory relativizes “states-of-being”, as Galilean relativity relativized “space” and special relativity relativized “time”. Observers in special relativity have “unimodal frames”; experimenters in quantum theory have “bimodal frames”. Special relativity relativizes “time”, but retains composite “space-time” as reminiscent absolute [17]. Quantum theory relativizes “objects” and “states-of-being”, but retains the “experimenter” himself as sole absolute. Bohr called relativization of experimenter “painful renunciation” [18], and retained classical concepts for the experimenter alone [4]. Some people misassimilate it as though he retained ontology for the system-under-study, and endow *realistic* in-

interpretations of  $\psi$  to Copenhagen theories. Quantum relativity has deeper semantics than special or general relativity [2]

Quantum objects do not have semantic resemblance with classical objects. I doubt that quantum theory fully corresponds to classical in all logical, syntactic and semantic manner as  $\hbar \rightarrow 0$ . Correspondence principle does not seem to respect *logical*, *syntactic* and *semantic* resemblances all together. Current form of quantum theory seems to have syntactic resemblance alone with classical theory. Our goal is to develop a radical syntax for the theory, that does not respect correspondence.

## 8.2 States of Being

States (of being) pervaded physics since René Descartes, who, emulating Pythagoras and Plato, entailed equivalence between mathematical and physical structures. He eliminated role of observer ‘I’ and ‘God’ from the discussion of ‘Matter’ [18]; his split  $\text{God|World|I}$ . Cartesian objects were “vortices of plenum” that pervaded entire Cosmos. Cartesian world was envisaged an *Object* of objects. Newton brought empiricism to cartesian project, but retained dogmatic states-of-being. States are unimodal mathematical structures; mathematical *idealistic* objects that do not change in perception. We know them as they are.

This dogmatic epistemology ruined physics until advent of quantum theory of Heisenberg (and Bohr), who sooner renounced it together with states-of-being [18]. Copenhagen division is  $\text{God|World-I}$ , where ‘God’ is split from *composite* ‘World-I’ (or *entangled* ‘system–experimenter’). Quantum experimenter has no mathematical structure to represent state-of-being of the system-under-study. His own choices of (*initial* and *final*) mode vectors  $\psi_{\text{IN}}$  and  $\phi_{\text{FIN}}$  constitute his frame of experiment  $\mathfrak{F}_{\mathbf{e}}\{\psi_i, \phi_f; \tau\}$ , being unique to each experimenter alone [2].

Cartesian experimenters had a frame too; their assertions about the system-under-study were communicable to others. Cartesian experiments were idealistic; their transformation (or translation of assertions) entailed absolute objects, things in their own essence. Cartesian experimenters had a consensus for system’s being (in a state); quantum experimenters have none. Quantum experimenters lack this communication or ‘frame transformation’. They lack a dictionary to translate their assertions [2].

A quantum experiment changes system-under-study abruptly and irreversibly, leaving no trace for it to be correlated with subsequent experiments. Quantum systems “jump” from one experiment to another (or one mode to another), lacking causality. This jump is often called *collapse* or *reduction* of the state in hodge-podge classical-quantum language. Quantum theory renounces states-of-being together with possibility for these redundant notions. On the contrary, classical systems evolve from one experiment to another, carrying the germ of state-of-being.

Blochinzhev and Alexandrov objectivized  $\psi$  and ascribed it “state of being” of the system-under-study [18]. Wigner took it too far, and completely deviated from the Copenhagen doctrine [2]. His theory, often called “Orthodox theory”, is completely dogmatic, where  $\psi$ , being statefunction of the system, is an objective reality. Wigner ascribes *consciousness* to the abrupt change in  $\psi$  during measurement, and calls it a breakdown of quantum theory. Finkelstein (1991) calls the oxymora “statefunction  $\psi$  of the system” *syntactic error* in the theory [19]. Ludwig (2006) calls it “Fairy Tale”

or “Myth” [20], that plagues quantum theories of today. States are incompatible with bimodal quantum language. Finkelstein (1996) develops an algebraic “operational” language to study some prototypical “bimodal” quantum experiments [2], and develops its semantics.

## 9 Correspondence

Quantum logic is bimodal. We see quantum systems in *initial* and *final* modes of experiment; *intervention* lies outside the theory. If one ignores bimodality and adheres to unimodal logic, he entails correspondence  $\mathfrak{Q}_{\text{IN}} \rightarrow \mathfrak{Q}_{\text{FIN}}$ , and *discrete* triangle of Section 2.1.1 collapses to *continuous* line (or channel) connecting  $\mathfrak{E}$  with  $\mathfrak{Q}$ . Such a unimodal (no longer vertex) point  $\mathfrak{Q}$  represents unimodal state-of-being of the system-under-study. In unimodal quantum theories,  $\psi_{\text{IN}}$  represents action vector connecting experimenter  $\mathfrak{E}$  with unimodal system-under-study  $\mathfrak{Q}$ . Specification of  $\mathfrak{Q}$  (with  $\omega = 1$ ) renders the system-under-study an “object”. For an *apparent* system-under-study (with  $\omega = 1$ ),  $\mathfrak{Q}$  represents phase space of a classical system with coordinates  $(q, p)$ . Heisenberg [9] set out a limitation for their *simultaneous* measurement  $\Delta q \Delta p \geq \hbar/2$ , and  $q$  and  $p$  were called *complementary* variables in Copenhagen theory. Some adduce that  $\mathfrak{Q}$  indeed represents *partially* the state-of-being of system-under-study, and as  $\hbar \rightarrow 0$ ,  $\mathfrak{Q}$  represents it *fully*. Quantum theory corresponds to classical in the limit  $\hbar \rightarrow 0$ . Quantum theory was conceived an evolution of classical theory, and recedes to it as  $\hbar \rightarrow 0$ . It created an “interpretational problem” for quantum “measurement”; quantum theory became a *problem* itself, rather than a *solution* [2].

Such absurdity arises from the hodge-podge classical-quantum language. Quantum theory is based on bimodal logic, and has no logical counterpart in unimodal classical theories. Quantum experiments are *pragmatic* and *bimodal*. Classical experiments are *dogmatic* and *unimodal*. Quantum experiments are incompatible with classical language. Quantum language is *subjective*, *bimodal* and *verb mode* (Bohm’s *rheomode* [14], Heisenberg’s *pragmatic* [18] or Finkelstein’s *praxic* [2]), that assimilates bimodal actions. Some people misinterpret *subjectivity* with experimenter’s *consciousness*, and attribute it to experimenter’s *will*. In Copenhagen theory, subjectivity is restricted merely to non-objectivity. Classical language is *objective*, *unimodal* and *noun mode* (*dogmatic* or *ontic*), that assimilates ultimate reality. They contradict each other in their usage, and one scarcely finds a mutual correspondence. Quantum experimenter puts “name” to his actions on the system-under-study. Classical experimenter puts “name” to the system-under-study itself, like “Moon”.

Quantum theory does not *correspond* to classical theory.

Some variants of quantum theory correspond to classical theory in the limit  $\hbar \rightarrow 0$ . This correspondence is mere syntactic. They do not seem to correspond in semantics, while they lack one. These theories are based on hodge-podge classical-quantum language that Weizsäcker [1] calls semantic inconsistency. These theories are based on quantum syntax but classical semantics. Some people call it interpretational problem. Their correspondence to classical theories is obvious. Copenhagen theory does not correspond to classical theory.

## 10 Summary

Unimodal statistical operators  $\rho_{\text{IN}} = |\psi_{\text{IN}}\rangle\langle\psi_{\text{IN}}|$  and  $\rho_{\text{FIN}} = |\phi_{\text{FIN}}\rangle\langle\phi_{\text{FIN}}|$  represent *incomplete* or *partial* quantum experiments, and lack modal semantics [2]. Bimodal statistical operator  $\pi = |\psi_{\text{IN}}\rangle\langle\phi_{\text{FIN}}|$  represents *complete* or *full* quantum experiments, and reserves modal semantics. Unimodal variants of quantum theory, modeled on  $\rho_{\text{IN}} = |\psi_{\text{IN}}\rangle\langle\psi_{\text{IN}}|$ , are based on hodge-podge classical-quantum language, that Finkelstein (1972) calls “hybrid” [cq] theories [21]. Classical theories are [c], Copenhagen theories are [q]. The protocol of progress is [c]  $\rightarrow$  [cq]  $\rightarrow$  [q]. *Ontic* or *causal* quantum theories are [cq], that correspond to [c] in the limit  $\hbar \rightarrow 0$ ; [q] theories do not. Hidden variable theories seem to reduce [cq] and [q] theories to [c]; [cq] theories are more likely to reduce to [c], than [q] theories.  $\rho_{\text{IN}} = |\psi_{\text{IN}}\rangle\langle\psi_{\text{IN}}|$  and  $\rho_{\text{FIN}} = |\phi_{\text{FIN}}\rangle\langle\phi_{\text{FIN}}|$  represent [cq] experiments;  $\pi = |\psi_{\text{IN}}\rangle\langle\phi_{\text{FIN}}|$  represents [q] experiments, and resembles core insights of the Copenhagen theory. Copenhagen theory *jumps* from realm of classical physics, and creates the realm of quantum physics.

Some people claim that Copenhagen theory has no proper form, and could not be assimilated due to lingual ambiguities alone. Heisenberg agreed at this point with Stapp [22]. Stapp adduces that Copenhagen theory is pragmatic. Finkelstein studies its modal semantics [2]. Copenhagen theory differs from other variants of quantum theory in its modal structure. Some theories, being operational, seem to be closer to Copenhagen theory. Theories that incorporate dogmatic states-of-being are unimodal. Some variants of quantum theory are pragmatic but unimodal. They are based on hodge-podge classical-quantum language, that increases ambiguity. These theories fit with experimental inferences, but lack a consistent semantics. Some people call it “interpretational problem”, and many endure in vain to find one. Great many alternative interpretations of quantum theory have been published, and probably none discuss the original one. Some discuss ontological “causal” interpretation of quantum theory [23] (this school is in progress today), and others eliminate “observer” from the discussion [24, 25] as Descartes did. Copenhagen theory has been scarcely discussed after 1970’s [2] and dogmatic “states-of-being” seem to dominate physics again, that we renounced in 1920’s. Other variants of quantum theory are modeled on states-of-being in quantum domain, and discuss measurement problem in their *interpretation*. Measurement is the *central* problem of these theories, and has been widely discussed [26]. It gives rise to redundant concepts like “collapse” and “reduction” of *wavefunction* and *state*; quantum systems have no state to collapse [27]. Copenhagen theory is modeled on experiments, and has no measurement problem. Copenhagen theory is both *pragmatic* and *bimodal*, and owes no semantic error.

Quantum theory has no *interpretational* problem.

Bohr’s insistence for classical concepts was restricted merely to the *experimenter* alone. He theorizes an *absolute* experimenter, and calls its relativization “painful renunciation”. A theory of *relative* experimenters is in progress [4]. Some people misinterpret it as though Copenhagen theory insists classical concepts in *dogmatic* and *causal* sense, and endow states-of-being and realistic interpretations of  $\psi$  to Copenhagen theory.

This work aims at assimilating core insights of the Copenhagen theory.

## Acknowledgements

Work is not a new interpretation of quantum theory, yet an insight for the actual one. In the intervening moment, since its advent, many theorists seem to have deviated from what quantum theory is about, including author's earliest endurance. In apology to it, this work resembles core insight of Copenhagen theory. I owe this revelation to seminal writings of Werner Heisenberg, David Bohm and David Finkelstein. This work emerges from a "Talk" given at *Max-Planck-Institut für Mathematik in Die Wissenschaften*, Leipzig. I am indebted to Larry Horwitz, Matej Pavsic and Detlef Dürr for support and encouragements on various occasions.

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