Symbols:

Propositional variables: φ, ψ, χ.

Φ: Formula expressing proposition $\varphi, \delta(\Phi)$ for the formula expressing proposition $\delta(\varphi)$.

$\top$: Some tautology.

$\bot$: Some contradiction.

$\delta(\varphi)$: It is good that φ.

$\epsilon(\varphi)$: It is evil that φ.

$\sim(\varphi)$: It is neutral that φ.

$\neg$: Negation.

$\wedge$: Conjunction .

$\vee$: Disjunction.

$\forall, \exists$: Shorthands for *every*, *there exists.* Their occurrence does not mean I am using predicate calculus as a metalanguage etc.

$\vDash$: Logical (semantic) consequence.

$\rightarrow$: Material implication.

$\Rightarrow$: A material implication that is a logical consequence of the empty set.

$\vdash$: Syntactic consequence, $\Gamma \vdash \Phi$ means set of formulas Γ proves formula Φ.

$A(\varphi)$: The value of φ

a, $a_1, \ldots$: Values

min: The lowest value of…

$\Sigma(\ldots)$: Sum of values.

0. Introduction

Does indeterminism enable freedom and, consequently, responsibility? On one hand, these inquiries are pertinent from the perspective of the existence and scope of moral responsibility. On the other hand, it seems nonsensical to assert that evil of *The Holocaust* is dependent on quantum mechanics. This suggests the existence of a distinct form of goodness, logically prior to moral goodness, that entities can exemplify even without being responsible. However, the question arises as to why some beings exhibit goodness while others do not. It appears that goodness and values possess a certain autonomy with respect to facts, yet whenever we attempt to elucidate the reasons behind a specific entity's goodness, we must consider facts about that entity. This most fundamental goodness, its autonomy, and its relation to descriptive properties is the primary focus of the book. The term "axiological logic" is borrowed from Jerzy Kalinowski, who in "The Logic of Norms" contrasts its subject with the subjects of the logic of commands and deontic logic:

There are three types of practical judgments: commands (commands in the proper sense), norms, and value judgments, or evaluations, and thus three categories of practical sentences: imperative sentences in the proper sense, normative or deontic sentences, and evaluative or axiological sentences. With the logic of the latter, called axiological logic, only briefly outlined. (Kalinowski, 1993, pp. 32, my translation)

This contrast is crucial, as I intend to explore a type of values that encompasses everything. It merely defines what is good and what is bad, independent of and primary to what, for instance, is obligatory. The work comprises five chapters. The first chapter delves into the philosophy of logic and the ontological foundations of axiological logic. Its objective is to justify the significance of logical research for axiology and the methodological approach of loosely transitioning between logic and ontology. The second deals with axiological logic proper. The third is about the concept of value, which is part of the semantics of axiological logic. In the fourth part we delve in detail into relations between the many conflicting intuitions behind the naturalistic fallacy. In the last part, we pick up on and reformulate the open question argument in order to argue for a platonic non-naturalism of sorts.

1. The Good within General Ontology

In this chapter, I aim to provide the basic conceptual analysis of the concept of goodness. I denote goodness by the symbol δ, evil by ϵ, axiological neutrality by ~, value by A, necessity by $\Box$, $\top$ tautology, $\bot$ contradiction. I will also use $\Rightarrow, \Leftrightarrow$ for logical implication and equivalence (material implication, equivalence true in all models), while reserving $\rightarrow, \leftrightarrow$ for material implication and equivalence (the ones given by truth tables).

1.1 Logic as an ontology

The subject matter of logic manifests itself in three distinct ways, suggesting three distinct modes of existence. These modes can be briefly characterized as:

- Intentional, characterized by dependence on mental phenomena (beliefs, art, theories, institutions).
- Real, characterized by their independence from the mind, changeability, and their dependence on an external cause for their existence, lacking any inherent necessity. (Stróżewski 2004, pp. 125-128) (Contingent concrete objects)
- Ideal mode of existence, characterized by independence from mental phenomena, immutability, and necessity. (Abstract objects understood platonically) (Zwoliński 1974, pp 22-25)

To begin, I will analyze these intuitions and develop a way to synthesize insights present within them. I intend to broaden the conventional conception of logic as a method of thought (laws of thought, valid reasoning) with ontological thesis, implicit in the assumption that there is any correct method for thinking about things. There are three ways logic appears.

1. Logic, as a tool of human practice, serves as a means of acquiring new information from existing knowledge. It functions as a means of *processing* carriers of this information, which you might consider senses or other psycho-linguistic objects. From this vantage point, logic seems to be about intentional entities originating from humans and secondary to their goals. A tool that serves as a means of organizing the contents of thought.
2. We experience mistakes in the use of logic. At this level, it is no longer sufficient to describe logic as a tool for individuals to perform specific actions. Mistakes can occur, and they are not the same as simply using another tool neutrally or setting different goals. When an individual makes a mistake, it implies either incorrect application of logic or complete absence of its use, while it should have been employed. Logic shifts from purely pragmatic to a domain

of errors, corrections, and external criticism of the individual. As Chmielecki (2004, pp.42-43) suggests, thinking does not operate solely on information but on representations, which refer beyond themselves. These representations refer to objects and the way these objects are set criteria that one must conform to in order to think about them the right way. This phenomenon suggests a non-intentional mode of existence for the subject matter of logic. An example of this is Quine's philosophy of logic. In his theory, logical objects are considered similar to natural laws, differing only in their generality and abstractness. (For a detailed discussion check Levin, 1978)

3. The final phenomenon is the one that is seen relatively late, not through everyday experience, but through the rigorous science of logic. In this realm, the necessity of the truth of logical theorems is unveiled. Their independence from the real and the intentional suggests itself, because the proofs do not in any straight forward way mention anything about the world. This phenomenon suggests an ideal mode of existence for the subject matter of logic. Necessity of logic excludes any grounding in individual intentional acts or anything real and changeable.

Depending on which of these phenomena one deems adequate, a different understanding of logic's subject matter emerges. Let us label the theory that accepts only the intentional aspect of logic as, int-n.real-n.id. This view denies any objective foundation for logic beyond human thought, resulting in a fictionalist interpretation where logical theorems are viewed as tools without any reference outside the way thinking is organized. So, laws might describe thinking by way of deduction but, there is nothing that makes this or that way of thinking more or less adequate.

The combination int-real-n.id implies that logic is grounded in the real world but does not assert the existence of an ideal realm. This perspective aligns with a modified Parmenidean view, where logic reflects the inherent structure of reality itself (being–thinking identity). The inclusion or exclusion of an ideal realm (of abstract objects) in this view can lead to either logic being a general ontology encompassing all of being or a more localized ontology of only concrete objects.

The combination int-real-id is similar to the previous one, but it does not limit logic to the real world. It enables both a real and an ideal grounding, making logic applicable to both the structure of human thought, concrete objects as well as abstract objects. Analogically, before the discovery of Non-Euclidian geometries, Euclidean Geometry could be considered to be a part of mathematics, dealing with abstract objects, but also as a theory of space a part of physics and, as far as spatial form of representation is inherent to human cognitive structures, a part of psychology.

Finally, int-n.real-id expresses pure platonism. On this understanding, logic primarily deals with abstract objects, and the veracity of logical theorems is rooted in these entities, irrespective of any connection to the real world. Analogically, Euclidean Geometry is about mathematical structures, and so remains true irrespective of whether it describes physical space.

Other combinations are considered less significant for the subsequent argument. This work explores axiological logic, so it is appropriate to start by outlining some philosophy of logic. This is in order to argue that a logic can include substantive propositions about the property of goodness. Our argument will conclude that logic examines the necessary regularities within a particular domain. We begin with the classical, ancient conception of logic as the study of correct reasoning, proof, and deduction, exemplified by Aristotle's syllogisms. (Grzegorczyk, 1984, pp. 485-486) Aristotle's entire logical system revolves around the concept of deduction. (*sullogismos*). *A thorough explanation of what a deduction is, and what they are composed of, will necessarily lead us through the whole of his theory.* (Smith, 2018). To support this ontological understanding of logic, we will demonstrate the inconsistency of accepting the first phenomenon (intentional aspect) while rejecting the third (ideal aspect).

The position defended here is precisely the following: The subject matter of logic is the principles of valid reasoning ⇔ The subject matter of logic is the necessary regularities (e.g. laws, orders, similarities) of some domain.

To unravel this thesis, we must consider the three phenomena mentioned earlier. The left side of the equivalence refers to the first phenomenon, while the right side comprises two crucial words: "necessary" and "some." "Necessity" signifies an ideal mode of existence, while "some" carries a negative connotation. Notably, the word "any" is absent. Consequently, a logic can be a local ontology, not necessarily a universal one. Therefore, the combination that encapsulates this position is int-n.real-id. Weirdly, the only part of this work that maintains relative neutrality regarding the philosophy of logic is axiological logic itself. The justification of its claims remains independent of philosophical perspectives on logic itself. Consequently, the disagreement does not lie in the acceptance or rejection of specific logical theorems but rather in a disagreement about their subject.

This study goes beyond axiological logic; it aims to showcase the applications of axiological logic within ontology and axiology. Furthermore, these areas are interconnected. Sometimes, after proving a theorem, I offer a philosophical commentary. This approach is justified by platonism in the philosophy of logic. In this chapter we won't make a detailed argument for platonism but, we must clarify this realist interpretation of logic, where axiological logic is essentially a subset of the ontology of values, and define what a *proposition* is within this framework. I take it as a given, the fundamental characteristic of logic is the necessary truth of its theorems. I will categorize philosophies of logic based on how they conceive of this necessity. A necessarily true proposition can be defined in several equivalent ways. Let $\square(PROP)$ denote the class of necessarily true propositions (true in all models), and $\Diamond\ (PROP)$ the class of consistent propositions (true in some models).

1. A necessarily true proposition is one that is logically implied by any other proposition. That is; $\square(PROP) = \{\varphi : \forall\varphi, \psi \Rightarrow \varphi\}$
2. A necessarily true proposition is one such that it is a logical consequence of the empty set. That is; $\square(PROP) = \{\varphi : \emptyset \vDash \varphi\}$
3. A necessarily true proposition is one whose conjunction with any consistent proposition is a consistent proposition; $\square(PROP) = \{\varphi : \psi \in\Diamond\ (PROP) \Rightarrow (\varphi \wedge \psi) \in\Diamond\ (PROP)\}$.

Although these three definitions are equivalent, it is worth mentioning them all because they can have different implications for the question of whether necessity is a primitive or can be constructed from something more basic. In the first two definitions, the primitive objects are logical implication in the first and the empty set and logical consequence in the second. However, in both logical implication and logical consequence, necessity seems to be already inherent. In the third option, the basic property or relation is consistency – and this appears to be the most promising approach for the analysis of necessity. Philosophical views on logic can be divided based on how they interpret the relationship between logic and necessity. One view, let us call it realistic, recognizes necessity as something metaphysically elevated, requiring a unique ontological category for the subject matter of logic. The other view, let us call it anti-realistic, sees necessity as a secondary side effect that arises peripherally.

The primary anti-realist position posits that the necessity of logical truths stems from language. This position can be further categorized into semantic and formalist approaches. The semantic approach asserts that the necessity of logical truths arises from their analyticity. For instance, if one were to follow Kant's (2010, paragraph 131) example, the analyticity of a proposition *S is P* would be understood as a relationship of subsumption between the sense of *S* (the subject) and the sense of *P* (the predicate). According to this theory, necessity would simply be a byproduct of semantics. Words have senses, and some of these senses overlap, leading to the necessity of logical truths. Kant himself of course, thought there are non-analytic necessary truths, but a view that reduces necessity to analyticity is also well known. A similar idea is evident in the Tractatus Logico-Philosophicus. (Wittgenstein, 2010 pp.53) According to this view, the meaning of propositions is determined by their truth conditions. Necessity, in this context, is merely a consequence of language.

Logical truths, or tautologies, are simply those propositions that are true unconditionally. This perspective suggests that necessary truths are formal and devoid of content. If the sense of a proposition, its content, lies in its truth conditions, and logical truths do not possess such conditions, then they are not truly propositions but merely a contentless form. However, it is important to recognize that this form of antirealism can easily transition into realism. For instance, Frege's philosophy (Zalta, 2018, section 2.7) posits that while logic is analytic, meanings and senses are independent entities, existing even in the absence of real linguistic communities and speech acts. This kind of realism is merely a step away from full-fledged platonism, which disposes with reference to linguistic objects and locates the subject matter of logic in abstract objects, such as properties, propositions, relations, and so on.

An even more radical extension of this idea can be found in formalist nominalism. If we accept that logical truths lack content, a natural development of this position would be the thesis that logic does not even deal with concepts or semantics; it only deals with syntax. This idea was present among positivists, such as Carnap, who understood logical consequence through provability. "Carnap defines the notion of logical consequence in the following way: a statement *A* is a logical consequence of a set *S* of statements if and only if there is a proof of *A* based on the set *S*"(Murzi, section 3) and in Hilbert's formalism (Weyl calls it a meaningless game of formulas (Zach 2016, section 3)), which reduces the existence of mathematical objects to their consistency, but importantly, in the sense of unprovability of both a formula Ψ and its negation $\neg\Psi$ from a formula Φ. According to this perspective, logic focuses on studying the formal (syntactic) characteristics of formal languages. In other words, its subject matter encompasses signs, sequences of signs, and their relationships. Necessity is considered a secondary consequence of the rules for combining (grammar) and transforming signs (inference rules). This type of formalism faced a significant challenge when the incompleteness theorems were discovered, leading to the conclusion that in consistent formal systems powerful enough to express arithmetic, there are always true propositions that cannot be proven within that system. Consequently, logical consequence is distinct from the relationship of provability. All of these variants of antirealism are rejected here, primarily due to their inconsistency with necessity of logical theorems. A necessary truth φ cannot be grounded in something contingent, for then there would be a possibility of the non-existence of those grounds, and thus also the possibility of φ being false. If linguistic objects, such as formal languages, signs, relations between signs, concepts, etc., are created by human linguistic communities, then truths about them are just as contingent as those communities themselves. (Faced with this fact, anti-realism can easily slide into realism. For example, one could say that logic does not speak about languages as they actually exist, but about some ideal languages that are independent of the communities of users.)

Another peculiarity of the anti-realist position is the moment when this lack of content manifests itself. For instance, let us consider an ontological thesis expressed in natural language and translated into the symbolism of a formal language (as I do in this work with the naturalistic fallacy for example). Let us say, *something exists* in first order logic $\exists x(x = x)$. What could potentially cause this ontological thesis to suddenly lose its content? I suggest nothing could happen. In this sense, using formalism to express certain theses is not much different from using abbreviations. Formalisms are crucial in defining the discipline of logic, its rigor, and so on. However, it does not follow that they exhaust the definition of the subject matter of the science of logic. Even if we assumed that logic was a purely formal (syntactic) discipline, devoid of a basis for objective interpretation and, consequently, ontological conclusions, what prevents the creation of a discipline of logic+, which would simply be logic itself, but allowing interpretations for syntax and so one whose truth could be questioned? Indeed, how would logic differ from logic+? To attain a realistic understanding of logic, we must broaden the methodological definition of logic. A methodological definition of logic is one that defines its subject matter as something connected to method. Examples of methodological definitions include statements that logic deals with the laws of thought, valid arguments, or is the theory of deduction. This understanding already exists in Aristotle, where logic is not viewed as part of philosophy but only as a method for philosophizing. It does not provide insights into what there is but rather guides how to investigate it. Expanding this methodological definition involves demonstrating that methodology is intertwined with ontology, meaning that propositions of one always imply propositions of the other. We can draw an analogy between a hammer and a nail. It is impossible to define what a hammer is without knowing what nails are, and vice versa. What nails are implies what a hammer is. To transition from a methodological to a realist and ontological understanding of logic, and thus expand the subject matter of logic, we must then ask what are the ontological implications of the very existence of criteria for correct thinking. What must a nail be like for this logical hammer to drive it in effectively? Again, by a regularity I mean things like laws, ordering relations, similarities.

1. **For there to be laws of thought about a domain, there must be some regularities within the domain.** If there were no regularities in a domain, it would consist of unrelated entities, and moreover, the individual entities themselves would consist of unrelated entities. For if any two entities were similar in some respect, that would already be a kind of regularity. Any object, trivially, is similar to itself, so if objects consisted only of themselves, there would be a regularity within them. The situation of there being no regularities in a domain is therefore contradictory, as it assumes that entities are both unrelated to each other and that each entity is composed of other entities. This is a world of monads composed of monads. In such a world,

there would be no general laws of correct reasoning, because each entity would be so different from everything else that the study of any one of them would require an absolutely unique method.

2. **Necessity.** This is essentially a reiteration of the key premise from which our considerations began. Necessity does not have to be tied to permanence – it can appear at the level of the principle of change. Let us assume we are discussing a domain with variable objects, a domain where no universal law applies to every object at every moment. In such a domain, invariant and necessary conditional dependencies must exist, specifying which law applies to which object in specific situations. Otherwise, there would be no way to determine whether a given object adheres to a given law or not, leading to a lack of criteria for evaluating the validity of a given principle of thought regarding a specific entity.

3. **Domain.** The third characterization of logic is that it pertains to a domain. This term is intentionally vague because it negates a specific thesis and cautions against asserting another. It negates the thesis that logic must necessarily be a fundamental ontology. Logic does not have to be, for instance, some basic ontology of facts or the broadest laws of being. It can speak of a narrower ontology, the ontology of a smaller domain. In the presented work, this domain will be values and value-evaluating properties, such as good, evil, and axiological neutrality. On the other hand, there is the thesis that any domain has its own logic—just as there is a logic of values, obligation, necessity, and so on—so there are also logics of flowers, streets, or bottles.

Hence, we extend the subject matter of logic with necessary regularities (of domains).

1.11 Propositions

A proposition, is an abstract object that is either true or false.

Sider asks (2009, pp. 4): *"The formal sentence P→P is a tautology, but since it is uninterpreted, we probably shouldn't call it a logical truth. Rather, it represents logical truths like "If snow is white then snow is white". A logical truth ought at least to be true, after all, and P →P is not true, since it does not even have a meaning—what's the meaning of P?*"

Thus, two issues arise: first, clarifying what propositions are considered to be in this work, and second, understanding the meaning of a propositional variable. These are essential for interpreting the theses of axiological logic.

For our purposes, it is useful to think of propositions as analogical to universals or structures that can be exemplified. One aspect of them is important here, and it can be expressed in three ways:

a. **Aboutness:** Propositions are about something. They come with a relation called *truth value* which relates them to objects, they are about.

b. **Truth value:** This relation is not only a potential proposition–object relation. It is always actual because a proposition is always either true or false. Unfortunately, it is often forgotten that falsity, as well as truth, is also a proposition–object relation. Spinoza reminds us of this in the proof of proposition XI, where he writes: ”*Of everything whatsoever a cause or reason must be assigned, either for its existence, or for its non—existence—e.g. if a triangle exist, a reason or cause must be granted for its existence; if, on the contrary, it does not exist, a cause must also be granted, which prevents it from existing, or annuls its existence.*" He shows that just as truth relates a proposition to what makes it true, so falsity relates a proposition it to what excludes its truth.

c. **Identity through reference:** If we were to discuss the criteria of propositional identity and the properties individual propositions possess, it would be straightforward to identify those they have due to being abstract objects. However, we would soon exhaust the properties that distinguish them. Consequently, we would need to explain the differences between propositions by examining their truth-makers or false-makers. This is what I refer to as identity through reference. However, this reference is not a form of external dependence because, regardless of everything else, it always refers in some way, either through truth or falsity.

To Sider’s question about the content of logical truths and the meaning of propositional variables, I answer that the meaning of a variable is that which is common to all objects within its scope, in this case, the class of propositions. The content of a variable is what’s universal within its scope. Thus, propositional calculus and its theorems might be understood as a theory of propositions, truth and falsity, while axiological logic that extends it with evaluative properties is in addition a theory of goodness etc.

1.2 Basic properties of The Good and The Evil

The Good is normative, referring to something that is rationally desirable. It can be identified as the ultimate goal of rational action. That is only a partial identification, because not all goods are even known and *a fortiori* desired. Therefore, goodness refers to the best of some category, representing the best of the possible. If good meant less than perfect, then for cases where some good, but not perfection, is obtained, there would be no rational reason to desire a better outcome. But this seems paradoxical because higher value is preferable to the lower. Therefore, just as reaching a destination is not the same as being halfway there, so realizing a good is not the same as achieving it partially (to some degree).

The Good is a property. Due to the law of excluded middle, expressions like "less" or "more good" (this does not translate well into English) must be considered as mental shortcuts from everyday language. This kind of gradability will be introduced later using the term "value," but just as when we say that something is gray, we do not mean that something is black and white, but that it is to some extent similar to black and to some extent to white. Just as gray is not white, so less or more good is not good; it is only more or less similar to good.

Good and Evil are opposites. I do not assume that they are contradictory, only that they exclude each other. As part of the initial identification, we can therefore accept the law $\neg\big(\delta(\varphi) \wedge \epsilon(\varphi)\big)$, which is apart from a further theorem, to be proven later, that is $\big(\delta(\varphi) \vee \epsilon(\varphi)\big)$.

There is no necessary evil. If something is evil, then it must be avoidable (false in some models). What is necessary (true in all models) is therefore good. Equivalently, we can say: If something is good, then it must be realizable. What is impossible is therefore evil.

Propositions are the value (good, evil) bearers. I do not take values to be exemplified directly by people, actions etc. This approach is essential to prevent the paradox of non-existence (*Plato's beard*). In normative discourse, sentences often appear to be about something that does not exist. For instance, we might claim that the absence of suffering is good or that the ideal of the Stoic sage is good. Both of these statements refer to unrealized states of affairs. However, when we consider propositions as value bearers, we avoid paradoxes. In saying "The Stoic Sage is good," we do not attribute value to a non-existent Stoic Sage but to the proposition "Something is a Stoic Sage." More specifically, we say that the falsity of this proposition is bad, and the truth would be good, while the negation of "Something is a Stoic Sage" is evil in the sense that its truth is bad, and its falsity would be good. This propositional approach may appear unusual to many, but it does not diminish the expressive power of the classical approach according to which stoic sages are good, instead of propositions about stoic sages. If it makes sense for someone to assign goodness to a stoic sage because, for instance, he exhibits great self-control, then the propositional approach asserts that the proposition "The Stoic sage exists" is good because if "The Stoic sage exists," then "Self-control is exemplified." Ultimately, classically understood values can be easily constructed using the propositional approach. An object *x* is good in the classical sense if and only if, the proposition "*x* exists" is considered good in the propositional sense. However, the propositional approach provides a richer semantic foundation. By separating values from first-order objects and relocating them to the realm of propositions, we can establish the groundwork for conditionals. For instance, one can express the conditional statement, "If governments were just, then politicians would be truthful." In practice, it is challenging to identify either justice or truthfulness among governments, which is why the classical view lacks a suitable foundation (truth-maker) for this conditional statement. In contrast, propositions concerning justice and truthfulness exist regardless of their truth value, enabling the propositional approach to handle conditionals effectively.

Another problem this approach addresses is the avoidance of a certain inconsistent picture of the world that arises when values are not shifted to the realm of necessary beings. This argument is not specifically about The Good; it is a general argument for the existence of unexemplified properties. Let us say we try to ground the domain of values in something contingent, like the fact that rational beings desire something. Then, the natural conclusion would be that if there were no rational beings, nothing would be good (or evil, etc.). However, this is internally contradictory. When we say that in some state of affairs, nothing would be good, it means that in that state of affairs, there are criteria for exemplifying goodness that nothing fulfills. This implies that the property of goodness exists (which establishes objective criteria for its exemplification), but it is simply not exemplified. If one were to adopt a subjectivist or relativist perspective, we could not assert that there is no good in a state of affairs without rational beings, nor could we claim that there is good. This is because such a state lacks criteria for goodness or its absence. Non-realists only understand what they are talking about when they make such assertions, because they borrow criteria for realizing values from the state of affairs in which they claim values originated. Based on these criteria, they argue (for example) that without rational beings, there would be no values.

In essence, when we discuss the objective existence of properties like The Good, we imply that there are objective criteria for its exemplification, which are precisely determined by The Good itself. The statement "nothing is good" implies that there are criteria for exemplifying goodness, and yet, no entity fulfills these criteria. If the existence of the property of goodness were contingent on some external factor, such as the actions of rational beings, then in a state of affairs devoid of rational beings, we could neither assert that something would be good nor that nothing would be good.

Moreover, the conditional statement "If rational beings existed in such a state of affairs, then something would be good" would also be meaningless.

1.21 The Good as a property

The four points presented above are not supposed to constitute a philosophical definition. Instead, my intention is to simply clarify and differentiate the concept. Since a definition is not the primary out goal, we can proceed to clarify certain phrases sufficiently to initiate a mathematical logic. The Good is a property, this thesis serves as the foundation for further exploration. In this limited scope, I will restrict my considerations to the ontology of properties. I understand properties as abstract objects—timeless, spaceless, non-mental, necessary, and immutable—that are exemplified by their instances. (This is a standard understanding, check Rosen, 2017). I also use the term "ideal being" to refer to these abstract entities.

The term "ideal being" is often associated with the acceptance of the concept of *ideal mode of existence*. While this concept is not essential for this work, I believe that the so-called "modes of existence" are actually properties rather than modes of *existence* itself. However, I acknowledge that the term *ideal being* might be more intuitive for some readers. For each property *P*, a proposition of the form "*P* is exemplified" or "*P* has an instance" is associated. This proposition is equivalent to an infinitary disjunction of all consistent propositions of the form "*x* exemplifies *P*." I assume that some properties may exist without being exemplified. This assumption is necessary for The Good to be the truth-maker of counterfactual conditionals (logical implications).

1.3 Normativity of The Good

Let us clarify that we are not referring to the normativity inherent in moral obligations. The good we consider here transcends moral agents and is the highest value a being can attain. Moral agents possess their own unique values that they can realize, and these values give rise to the specific characteristics of moral good that do not apply to all good things in general. Consequently, The Good does not *impose* obligations on anything or presuppose freedom and responsibility on the object of the norm. The Good relates to moral agents insofar as the highest value for them is connected to freedom and responsibility, but other beings may attain their highest value through different means. For instance, the highest value a work of art can realize is beauty, so a good work of art is a beautiful work of art. In ordinary language, we might say that a work of art ought to be beautiful. However, this does not imply that a work of art has an obligation to be beautiful, that it is responsible for it, or so on. Like most beings, a work of art lacks the capacity to have obligations. Nevertheless, we can still describe it as good or evil.

I will now present a few examples, or at least candidates for this sort of non-moral good.

1. Aesthetic values. The consideration of aesthetic values is very useful in the context because in their case, we can easily see the cracks in the descriptive – evaluative dichotomy. Consider, for example, aesthetically valuable qualities such as symmetry or proportionality. These properties can be described mathematically, yet they are aesthetically valuable.
2. Heroic acts. I do not want to get into a detailed argument for the existence of such acts; I will limit myself to defining what they are. A heroic act is an action by which a someone would realize a great value, or even a good, but at the risk of the loss of their own moral agency, which is why at the same time, it is not a morally obligatory act.
3. The value of Reality. If, as Moore (1922, paragraph 122) suggests, the contemplation of the beauty of a being is more valuable when that beauty is actually present rather than when the agent is deceived, then we have evidence of the existence of values independent of the subject. Additionally, we can consider the normative role of truth. If truth is worth seeking and discovering for its own sake, then there must be some fundamental value inherent in being itself.

One of the crucial functions of the normativity of The Good is taken over from its very structure as a property, as an abstract object. This refers to the necessity and immutability of properties. Without these qualities, The Good could not be normative. If The Good were not necessary and immutable, recognizing that some being is evil would not necessarily imply that it should be changed for the better. Instead, it could only suggest that either it should be changed for the better or The Good should be changed. However, this would lead us to conclude that instead of abolishing slavery, we could have just as well changed The Good, in such a fashion as to make slavery acceptable. Without this element of necessity and immutability, the normativity of The Good would lack a foundation and would not bind anything. This does not mean that there are no hypothetical imperatives. It only means that, for such conditionals, at least the truth of the conditional must be necessary. Therefore, if participating in a defensive war against a dictatorship is good, then war participation is not necessarily good, but the conditional – if x is attacked by a dictatorship, then x participates in a war – is necessarily good.

1.4 Descriptive and Evaluative Domains

Having described the concept of goodness in such a manner, we can now specify one of the primary objectives of this work, which is to elucidate the relationship between the descriptive and the evaluative. At this juncture, I will primarily focus on posing the appropriate questions. After all, I do not presume that even the fundamental distinction between the descriptive and the evaluative domains is unambiguous. Moreover, as it will be shown, common folklore expressions, such as "values are not derived from facts," are nonsensical.

Moore proposed a distinction between descriptive and evaluative properties, along a line between natural and non-natural properties. However, the concept of naturalness lacks clarity. Considering Moore's logical positivist background, it seems that the motivation behind this distinction stems from a reductionist intuition about the nature of things. Reductionism posits that natural things are fully dependent on the fundamental level of nature, such as the objects of physics. In this sense, the non-naturalness of goodness implies that the property of goodness is ontologically independent and self-sufficient, not fully dependent on "physics" or natural properties. While this is obviously true in one sense, it leads to absurdity in another. From a platonic perspective, the property of goodness is ontologically independent and self-sufficient. Just like any other property. Only the fact that a being exemplifies goodness can be dependent and non-self-sufficient. But it would be absurd to attribute independence and self-sufficiency to this fact of exemplification. Specifically, independence should be understood with respect to facts about what other properties a given being exemplifies. This work will try to prove that in each specific case, whether a given being exemplifies the property of goodness will fully follow from what non-evaluative properties it possesses. In other words, the thesis states that two entities cannot differ from each other only in values only. It would be absurd for two entities to be identical in all descriptive properties (exclude trivializing properties like "being identical to *x*"), except that one would be good and the other evil. Having distinguished between the descriptive and the evaluative at the property level, and instance level, we can formulate two theses that I will examine. Let Γ be the set of descriptive propositions about some *x*, and $\bigwedge(Val)$ the conjunction of all propositions about evaluative properties exemplified by *x*.

Naive thesis of independence: $\bigwedge(Val)$ does not follow from Γ.

The meaning of "follow from" is ambiguous and breaks down into three separate theses, which we will return to shortly.

Let $ex(P)$ mean the proposition stating that the property P is exemplified by something.

Indefinability of $ex(goodness)$ by means of other properties:

There is no formula Φ, such that:

1. The predicate of goodness does not occur in Φ.
2. Φ expresses the proposition φ
3. $\varphi = \bigwedge(ex(P_1), \dots ex(P_\gamma)$ such that γ is an ordinal and for $1 \leq i \leq \gamma$, $ex(P_i) \nRightarrow \varphi$
4. $\varphi \Leftrightarrow ex(goodness)$

The key thing here is to distinguish the thesis of the indefinability of $ex(goodness)$ from the claim that The Good itself is indefinable. Using $x\varepsilon P$ as an abbreviation for "*x* exemplifies *P*", that thesis is expressed as:

$\exists P_1, \dots, P_\gamma$: $(x = \text{The Good}) = \bigwedge\left(x\varepsilon P_1, \dots, x\varepsilon P_\gamma\right)$ (with non-trivialising restrictions on P_i)

The Good itself exemplifies some properties and of course, if we were to list all its properties in a conjunction, it would constitute a definition of sorts. Since there are an infinite number of properties, it is impossible to complete the list. However, we have a concept of goodness, which allows us to distinguish them sufficiently (at least for some purposes). Therefore, the process is feasible to some extent. The two theses are logically related in such a way that defining all possible instances of a property is sufficient to define that property, and therefore the definability $ex(goodness)$ implies definability of The Good. However, a property can be so multiply realizable that it is a further question whether definability of The Good implies the definability of $ex(goodness)$.

I will refer to laws that show logical relations between the evaluative and the descriptive as intelligibility laws. The name is straightforward. Suppose there is no link between an entity's descriptive properties and its goodness. This leads to absurdities: one in utilitarian (utility) values, another in epistemic normativity, and a third in morality.

1. Good (correct) performance of any simple task would then be inconceivable. For instance, let us say we want to assemble a purchased office chair well and have been provided with instructions. Should we follow them? If what is good does not logically follow from any class of descriptions, then there is no answer to this question. It is possible that one chair assembled precisely according to the instructions will be well-assembled, while another chair assembled with the same level of accuracy will be poorly assembled.
2. Let us consider two proofs of a theorem. Both proofs are rigorous, use the same inference rules, and have premises with the same truth values. If there were no transition from

descriptive properties to values, it would possible that one of these proofs would be correct, while the other would not.

3. Imagine a person whose behavior and intentions are described by properties *P, Q, R…* and whose actions are moral. If there were no transition from descriptive properties to values, the same person in the same situation could do the same thing, but this time their action would be immoral. This kind of independence of the evaluative form the descriptive would turn The Good into a chaotic, capricious, and whimsical deity. No plans would make sense because any action and any result could end up being good or evil.

It is a common belief, for instance, that "facts do not imply values" or "something being good" is not a fact. Let us begin by asking some basic questions about what constitutes a fact. For instance, we could claim, it is a fact that Mercury is closer to the Sun than Venus, and almost everyone would agree. But what exactly is this fact?

There are celestial bodies, the Sun, Venus, and Mercury. There is a specific three-member spatial relation that can be expressed as "is closer to than" and Mercury, Venus, and the Sun are in this relation. But is there, in addition to these celestial bodies and relations, another entity that could be called "The fact that the Sun, Mercury, and Venus are in a given relation"? This is not immediately apparent. Consider Neptune and Pluto instead of Mercury and Venus. Neptune can sometimes be closer to the Sun than Pluto, and yet the objects (Neptune, Pluto) remain the same, and the relation (closeness to the Sun) also. In this case, the fact seems to change. I cannot decide whether facts exist, so I do not want to ask whether "values follow from facts." Instead, let us ask one question from the ontology of facts. Is this fact some physical object in the universe, in addition to planets, the Sun, and space? If so, then phrases like "values do not follow from facts" and similar ones are only true in the most nonsensical way. Implication is a relation between propositions, not between physical objects and values. Of course, values do not follow from facts in the same way that tables follow from speakers. However, this is not a very substantive thesis.

In most cases, this is not what is meant. Phrases like *values do not follow from facts* are shorthand for *evaluative propositions do not follow from facts*, where fact is understood as something that can be in a relation of implication, for example, a true proposition, or a true contingent proposition. This thesis, unlike the previous one, is not nonsensical. However, a few elementary logical considerations are enough to show its falsity. Any evaluative proposition, like all other propositions, belongs to one of three categories: necessarily false, contingent, or necessarily true. If they are necessarily false, then they also follow from other necessarily false propositions, including descriptive ones (2+2=5), which falsifies the thesis. If it is necessarily true, then it follows from all propositions, including facts. If it is a contingent value-laden proposition, $V(\varphi)$, then consider a descriptive proposition, $D(\psi)$ and $\chi = (V(\varphi) \wedge D(\psi))$. If χ is descriptive, then given that $\chi \Rightarrow V(\varphi)$ we have an example of a *value following from a fact*. If χ evaluative, then on assumptions, ψ is descriptive $\Rightarrow \neg\psi$ is descriptive, χ is evaluative $\Rightarrow \neg\chi$ is evaluative, we have $D(\neg\psi) \Rightarrow V(\neg\chi)$ which is another example of a *value following from a fact.*

Another place where the distinction between evaluative and descriptive propositions crumbles is in relation to descriptions of values. Are propositions describing values themselves evaluative? After all, descriptions of values are propositions about values, so it would be absurd to claim that nothing about values follows from descriptions of values. The only alternative that does not seem obviously nonsensical is to assert that there are no propositions, or descriptive propositions about values. On such a view, propositions are about what is true and false, and the evaluative domain is said to be fundamentally different. However, upon closer examination, I believe that this hypothesis cannot be coherently stated. It is internally contradictory in the sense that it denies truth and propositions certain features without which I simply cannot comprehend what "truth" or "proposition" mean. Nevertheless, it is certainly not about what is usually addressed in the theory of truth or logic.

According to this hypothesis, there exists a domain of values, but there are no genuine propositions about it, such as "there is a domain of values." This contradicts the transparency of truth, which implies that "Snow is white" is equivalent to "It is true that ‚Snow is white'." In other words, the world in which *x exists* is the world in which the proposition "*x exists*" is true. Is is challenging to comprehend how these two concepts could diverge. Even if there are no true propositions about values, what about false ones? If there are no propositions about values, then is the proposition "*There are true propositions about values*" false? It seems to be a basic property of propositions that every proposition has an inverse (usually called a negation) such that $\varphi \text{ is true} \Leftrightarrow \neg\varphi \text{ is false}, \varphi \text{ is false} \Leftrightarrow \neg\varphi \text{ is true}$. But then, if there are falsehoods about values, then there are also truths. In conclusion, one cannot simply assume that the dichotomy between the descriptive and the evaluative exists. Therefore, I formulate the following three theses to be investigated later. Speaking of evaluative properties, I will generally mean three of them: goodness, evil, and axiological neutrality. This is a sort of ostensive definition. This list clearly does not exhaust evaluative propositions because, it does not include propositions about justice, beauty etc.

World independence thesis: Evaluative propositions are not contingent.

Necessarily true/false propositions are world (contingent being) independent in that they remain true/false regardless of which propositions about the world are true/false.

General dichotomy: $\varphi \neq \delta(\varphi)$

Dichotomy of contingents: $\delta(\varphi) \Rightarrow (\delta(\varphi) \nRightarrow \varphi))$ for contingent φ

Mulligan (2017) reports:

What might a fact be? Three popular views about the nature of facts can be distinguished:

- *A fact is just a true truth-bearer,*
- *A fact is just an obtaining state of affairs,*
- *A fact is just a sui generis type of entity in which objects exemplify properties or stand in relations.*

The third understanding appears to have no implications. The first and second understandings of fact merely state that goodness is not synonymous with truth, which is already conveyed in the general thesis of dichotomy and the thesis for contingent propositions. This aligns perfectly with the presented work and is one of the ways in which I intend to validate the intuition behind the fact – value dichotomy. On the first and second theory of facts, The Good is indeed not a fact, but the fact that something is its instance – most certainly.

Two types of dichotomies

It is also necessary to distinguish two types of the fact – value, descriptive – evaluative dichotomy. They can be epistemic or ontological in nature. The presented work aims to deny only ontological dichotomies. Therefore, I claim that what is descriptive of being (properties, relations, quantities) fully determines the evaluative part. This contrasts with the epistemic dichotomy thesis, which states that from knowledge of what is descriptive, one cannot infer knowledge of what is evaluative. In logical terminology, I want to demonstrate the existence of logical (semantic) consequences between non-evaluative and evaluative propositions, not syntactic consequences (provability). The fact that logical consequence holds between these propositions does not necessarily imply the existence of an inference rule, such as a practical syllogism, that enables one to deduce one from the other. The absence of such a syllogism suggests more about our cognitive processes rather than the ontology of The Good. It is worth contemplating what the lack of such a practical syllogism, which in logic manifests as the incompleteness of the formal system, would imply for metaethics. Kurt Gödel believed that proving the incompleteness theorems implied mathematical platonism. Incompleteness demonstrates that there will always be more mathematical truths than mathematical theorems, which is often cited as an argument for the thesis that mathematical truths are not constructed but discovered. Similarly, if we were to demonstrate the absence of this epistemic transition while simultaneously proving that logical consequence holds between evaluative and descriptive propositions, it could potentially serve as an argument for axiological platonism. Among ontological dichotomies, two types can be further distinguished. The first type is based on the thesis that the fact that a being exists does not necessarily imply that it is good. This dichotomy can be further divided into two different meanings of "is." The first sense is the existential sense, where "being is" means the same as "being exists." The second sense is the predicative sense, where the phrase "being is" requires the addition of some property or relation, such as "a being is solid", "a being is colorful", or "a being is complex". I want to clarify that I only deny the theory of the naturalistic fallacy in the second sense. To achieve this, we need to reunify the elements that the theory of the naturalistic fallacy separates with an ontological chasm. Two main strategies can be employed to accomplish this: (1) Just straightforwardly give examples of evaluative propositions implied by descriptive ones. (2) The aforementioned principles of intelligibility, which show that an evaluative proposition always follows from a non-evaluative one. In all these attempts (except for fragments where, for example, I specifically aim to improve Moore's argumentation), I will avoid phrases like "naturalistic fallacy" as much as possible. There is too much ambiguity hidden in it, so it is not worth using this concept. One could conventionally define the position defended in this work as naturalistic in the sense that, I consider the concept of the naturalistic fallacy to be a mistake. However, in detail, I reject only the naive thesis of independence, but I argue for the theses of the indefinability $ex(goodness)$ by means of other properties, the world independence, the general dichotomy, and the dichotomy of contingents.

2. Axiological Logic

2.1 Two Attempts at a Non-Axiological Logic of The Good

The first two attempts at axiological logic and their failures justify the need to distinguish between good, evil (evaluative properties) and values. In the two theories presented below, the semantics of good and evil are created without substantial reference to the concept of value. The first attempt is inspired by the notion of evil as a lack of good, a concept found in many classic texts like Plotinus or Aquinas (Check O'Rourke, 2015). The second attempt draws from the idea of goodness as a lack of evil, as proposed by Schopenhauer (1913, *On the Sufferings of the World*). Alternatively, we can say that the first attempt defines evil as total depravity (counter-perfection), while the second identifies goodness with perfection. In these two attempts, the concept of value plays a minor role.

If evil is defined as a lack of good, then only two classes of values are truly significant. One class contains the value 0, which is shared by all evil things, and the other class contains all other values exemplified by all good things. Conversely, if we consider good to be perfection, then there is a value 1 shared by all good things, while evil things exemplify values different than 1.

Evil as a lack of goodness: $((\varphi \rightarrow \psi) \wedge \delta(\psi)) \rightarrow \delta(\varphi)$

Goodness as a lack of evil: $((\varphi \rightarrow \psi) \wedge \epsilon(\psi)) \rightarrow \epsilon(\varphi)$

The first understanding is unacceptable for two reasons. Abbreviate an inconsistent proposition as $\perp$, and a tautology as $\top$. Then, $\perp \rightarrow \varphi$ for any φ, including some φ such that $\delta(\varphi)$, hence $\delta(\perp)$.

For the second proposition, consider the case where (1) $\varphi = \top$, (2) ψ is true, (3) $\epsilon(\psi)$, (4) ψ is contingent. In this case the truth value of φ = the truth value of ψ, hence $\varphi \rightarrow \psi$ is true, but $\delta(\varphi)$, hence we have provided a counterexample to the principle.

This justifies, why we need an axiological logic, which uses a value operator A for its semantics.

2.2 Axiological Logic with Values

In the following, we will use a relational approach, in the sense that good (δ), evil (ϵ)and axiological neutrality ($\sim$) are constituted by the relationship between values rather than by a rigid tie to a single value. These extend the classical propositional calculus. We can say $\delta(\varphi), \epsilon(\varphi), \sim(\varphi)$ for any proposition φ. Symbols $\top, \perp$ usually mean certain 0-ary connectives, but we use $\top, \perp$as variables ranging over tautologies and contradictions of the propositional calculus. We will index them, if there is a need to talk of more than one tautology/contradiction. We might for example fix $\top_i = (\varphi_i \vee \neg\varphi_i)$ etc. For reasons of readability, I forgo the usual convention, instead of writing;

Formula Φ is true ⇔ Some semantic condition. I just write proposition $\varphi \Leftrightarrow \ldots$

Also, the indented reading for $A(\varphi)$ is the value of the proposition φ, where A is a function from the propositional class to the class of values. I note, strictly speaking in semantics, one is supposed to predicate over syntactic objects, formulas and then match them to some objects. So, first you need a function from the class of well formed formulas to the set of values, and then another function that maps each formula to a proposition it expresses, and then a function that maps propositions to values. Again, for readability, just consider A's domain to be the propositional class, but note that, is it derivative from a basic semantic construction that maps syntactic objects to some objects.

I leave the question of the cardinality of the class of values open.

By ψ is logically related to φ, I mean at least one of the following holds, $\varphi \Rightarrow \psi$, $\psi \Rightarrow \varphi$, $\varphi \Rightarrow \neg\psi$, $\neg\varphi \Rightarrow \psi$. We say, propositions and logically independent otherwise.

These relational structures are as follows:

δ Semantics: $\delta(\varphi) \Leftrightarrow A(\varphi) > A(\neg\varphi)$

Local Peak Property (LPP): $\delta(\varphi) \Leftrightarrow A(\psi) \leq A(\varphi)$ for all logically related ψ

This property postulates that something is good, if it is the best of possibilities.

ϵ Semantics: $\epsilon(\varphi) \Leftrightarrow A(\varphi) < A(\neg\varphi)$

~ Semantics: $\sim(\varphi) \Leftrightarrow A(\varphi) = A(\neg\varphi)$

Relative ~ semantics: $(\varphi)/\psi \Leftrightarrow A(\varphi) = A(\psi)$

The question of what value are will be explored in chapter 3. At this stage, it is crucial to understand that evaluative properties specified in this manner are secondary to value relations.

The primitive notions, are therefore these value relations: $<, >$, better and worse. It should be noted that values are not numbers, so these symbols are used analogically and are not the same relations as in arithmetic. Statements about "counting values" should be understood somewhat metaphorically as meaning that, there is some algebraic structure to values.

2.21 Necessitation Rule and Complex Expressions

Complex expressions include conjunctions, disjunctions, implications, equivalences, and negations. The object of deduction will be the value ratios associated with expressions like "good conjunction," "good implication," etc. The principle used in deducing the content of complex expressions states that all necessary truths are good, and thus all tautologies are good.

Necessity implies goodness: $\top \to \delta(\top)$

It is comparable to the necessitation rule of modal logic, which states: $\vdash \varphi \Rightarrow \vdash \Box\varphi$. And in its deontic version (OB means obligatory): $\vdash \varphi \Rightarrow \vdash OB\varphi$. The deontic version, in particular, offers a valuable perspective on the is–ought problem. Its uniqueness arises from the fact that although these inference rules do not have corresponding laws of logic $\varphi \to \Box\varphi$ or $\varphi \to OB\varphi$. The necessitation rules in modal logics differ from the necessity implies goodness principle in that, whereas necessitation rules of modal logics derive their justification syntactically, from being elements of an adequate sets inference rules, i.e., a set on the basis of which all true statements can be proved, while no false statements can be proved. This is adequate because, if φ is a theorem of logic, then naturally, it is so in virtue of the laws of logic, and since the laws of logic are necessary, every theorem true by virtue of the laws of logic is also necessary. Obligation in deontic logic as sharing structural similarities to necessity, thus inherits this justification. But we prefer a semantic approach, and so, necessity implies goodness principle is not purely syntactical in this way, hence it requires independent justification.

T0.1 $\top \to \delta(\top)$

T0.2 $\bot \to \epsilon(\bot)$

The two theorems follow from the fact that tautologies are logically implied by any proposition and the local peak property. However, I present a different argument that better encapsulates the idea behind these theorems better. The fundamental principle is that any necessary truth is a necessary condition for the existence of anything generally, and The Good specifically. Therefore, if one were to assert that a necessary truth is neutral or evil, consequently it would be evil or neutral whether The Good exists. Hence, we would conclude that the existence of good things is neutral or evil.

(1) Consider three propositions,

(i) A proposition $\top : \forall\varphi, (\varphi \Rightarrow \top)$

(ii) Proposition $\mathfrak{g}$, such that $\mathfrak{g} = \exists x \delta(x)$ (Informally $\mathfrak{g}$ says that something (some proposition) is good)

(iii) Proposition $\mathcal{G}$ such that $\mathcal{G} = (\exists y: y = \delta)$, in this higher order context δ double functions as the name for the property of goodness. (Informally $\mathcal{G}$ says that there exists the property of goodness)

Axiom 1: $\neg\mathcal{G} \Rightarrow \neg\mathfrak{g}$. If there was no property of goodness, nothing would exemplify it.
Axiom 2: $\delta(\mathfrak{g})$. It is good, that something exemplifies the property of goodness.
Axiom 3: $\neg\epsilon(\top)$ [No necessary evil principle]
Axiom 4: $((\varphi \Rightarrow \psi)$ and $\neg(\sim \psi)) \Rightarrow \neg(\sim \varphi)$

(2) $\epsilon(\neg\mathfrak{g})$ [Axiom 2, δ, ϵ semantics]
(3) $\bot \Rightarrow \neg\mathfrak{g}$ [From the principle of explosion]
(4) $\neg(\sim\bot)$ [3, 2, axiom 4]

Conclusion 1: $\epsilon(\bot)$ [4, ϵ semantics, axiom 3]

Conclusion 2: $\delta(\top)$ [Conclusion 1, δ, ϵ semantics]

2.211 Equivalence and Material Implication

Global Peak Proprty (GPP): $\delta(\varphi) \Leftrightarrow A(\psi) \leq A(\varphi)$

By LPP, every good is a local perfection, that is the best among the logically related propositions. The question is, given $\delta(\varphi)$ is there a ψ logically independent of φ, such that $A(\psi) > A(\varphi)$? Intuitively no, because it would make The Perfect in some sense better than The Good, which seems at odds with The Good's normativity. Let us prove it purely logically.

Lemma: LPP $\Rightarrow$ GPP

Consider any $\top$ and any $\varphi: \delta(\varphi)$. By $\delta(\varphi)$, the fact that $\top$ is logically related to all propositions and the LPP, we deduce $A(\top) \leq A(\varphi)$. By T0.1 $\delta(\top)$, hence by the same considerations $A(\varphi) \leq A(\top)$, so $A(\varphi) = A(\top)$.

Implication Semantics:

$$\delta(\varphi \rightarrow \psi) \Leftrightarrow A(\varphi) \leq A(\psi)$$
$$\epsilon(\varphi \rightarrow \psi) \Leftrightarrow A(\varphi) > A(\psi)$$

Equivalence Semantics:

$$\delta(\varphi \leftrightarrow \psi) \Leftrightarrow A(\varphi) = A(\psi)$$
$$\epsilon(\varphi \leftrightarrow \psi) \Leftrightarrow A(\varphi) \neq A(\psi)$$

Note: Implication Semantics ⇒ Equivalence Semantics

Theorem 1: $\neg(\sim \varphi)$

By T0.2 $\epsilon(\varphi \leftrightarrow \neg\varphi)$, hence from the above semantics, we get $A(\varphi) \neq A(\neg\varphi)$. T2 also follows under $\neg(\sim (\varphi \leftrightarrow \psi) \rightarrow \neg(\sim (\varphi)) \vee \neg(\sim (\psi))$

2.212 Conjunction

Conjunction Semantics: $\delta(\varphi \wedge \psi) \Leftrightarrow A(\varphi) > A(\neg\varphi)$ and $A(\psi) > A(\neg\psi)$

We will consider conjunctions by considering the relation of the conjunction's arguments to their negations. We know $\epsilon(\varphi \wedge \neg\varphi)$ hence, it is not sufficient for the conjunction to be good, that only one of its arguments is good.

Conjunction Semantics: $\epsilon(\varphi \wedge \psi) \Leftrightarrow A(\varphi) < A(\neg\varphi)$ or $A(\psi) < A(\neg\psi)$

Consider a contradictory proposition of the form $\top \wedge \bot$. One of its arguments is good. Thus, the evil of a conjunction does not require that all of its arguments be evil; it is sufficient for at least one to be.

2.213 Disjunction

Disjunction Semantics: $\delta(\varphi \vee \psi) \Leftrightarrow A(\varphi) > A(\neg\varphi)$ or $A(\psi) > A(\neg\psi)$

Disjunction Semantics: $\epsilon(\varphi \vee \psi) \Leftrightarrow A(\varphi) < A(\neg\varphi)$ and $A(\psi) < A(\neg\psi)$

The proof will be very similar to the case of conjunction. However, it should be noted that unlike in the case of a good conjunction, several options seem intuitively plausible. If we understand disjunction as something that leaves undetermined, which of its arguments is true, then we could assume that a good disjunction is one that does not allow "things" to go wrong, and thus one in which all arguments are good. On the other hand, it may be intuitive that a good disjunction is one that provides the possibility of good, and thus one in which at least one argument is good. Here, we start by considering that $\delta(\varphi \vee \neg\varphi)$. Only one of its arguments can be good. For a good disjunction to hold, it is therefore sufficient that only one of the arguments be in a certain relation to its negation. Can this be a < (worse) relation? No. Every contradiction is worse than its negation, and the disjunction of two contradictions is itself a contradiction, and thus is evil. If, therefore, the occurrence of *worse* relation between any of the arguments of the disjunction and its negation yielded a good disjunction, then it would be true that $\delta(\bot_k \vee \bot_l)$, and since $\bot_k \vee \bot_l$ itself is a contradiction, we would have a good contradiction, which is absurd. Hence from theorem 1, we infer one of the arguments has to be good.

2.22 Global peak property and the implication semantics

Lemma 1: $A(\psi) \leq A(\top)$

Follows from the implication semantics, T0.1 and the fact that tautologies are implied by everything.

Theorem 2: Global Peak Property $\Rightarrow$ Implication Semantics

Case $\delta(\varphi \rightarrow \psi)$: Assume Implication Semantics fail for the case, then show GPP is false by assuming $\delta(\varphi)$ and deducing $A(\varphi) > A(\psi)$ for some ψ, such that $\delta(\psi)$, thus making ψ a counterexample to GPP. Pick logically related $\varphi, \psi.$ By failure of implication semantics, the are $\varphi, \psi: \delta(\varphi \rightarrow \psi), A(\varphi) > A(\psi)$. By laws of logic, we obtain $\delta(\neg\varphi \vee \psi)$, by disjunction semantics $\delta(\neg\varphi) \vee \delta(\psi)$, by assumption $\delta(\varphi)$, hence $\delta(\psi)$. But by failure of the implication semantics $A(\varphi) > A(\psi)$, so ψ is a GPP counterexample

Case $\epsilon(\varphi \rightarrow \psi)$: Pick logically related φ, ψ. Assume implication semantics failure, hence $\epsilon(\varphi \rightarrow \psi), A(\varphi) \leq A(\psi)$. By laws of logic and disjunction semantics, we get $\epsilon(\neg\varphi) \wedge \epsilon(\psi)$, hence $\delta(\varphi)$. From $A(\varphi) \leq A(\psi)$, $\epsilon(\psi)$ and the local peak property deduce, $A(\varphi) \neq A(\psi), A(\varphi) < A(\psi)$, but $\delta(\varphi)$ and φ, ψ are logically related, so $A(\varphi) < A(\psi)$ is inconsistent with the LPP and *a fortiori* GPP.

Theorem 3: Implication Semantics $\Rightarrow$ Global Peak Property

Assume Implication Semantics, and assume $\delta(\varphi)$, then show $A(\psi) \leq A(\varphi)$. Then since, Implication Semantics $\Rightarrow \neg(\sim(\varphi))$, we my use $\neg(\sim\varphi)$ to deduce $\delta(\top \rightarrow \varphi) \vee \epsilon(\top \rightarrow \varphi)$. If $\delta(\top \rightarrow \varphi)$, then by implication semantics $A(\top) \leq A(\varphi)$, so together with lemma 1, deduce $A(\varphi) = A(\top)$ hence, we've proven our result. If $\epsilon(\top \rightarrow \varphi)$, then by laws of propositional logic, $\epsilon(\neg\top \vee \varphi)$ and by disjunction semantics $\epsilon(\bot)$ and $\epsilon(\varphi)$, which contradicts the assumption $\delta(\varphi)$.

The Global Peak Property can be deduced from a simpler principle. Call it, the implication inconsistency property.

ImIP: $\delta(\varphi \rightarrow \psi)$ is not consistent with every value relation between φ, ψ

Theorem 4: ImIP $\Rightarrow$ Implication Semantics

(1) $\delta(\bot \rightarrow \top)$, so $\delta(\varphi \rightarrow \psi)$ is consistent with $A(\varphi) < A(\psi)$.

(2) $\delta(\varphi \rightarrow \varphi)$, so $\delta(\varphi \rightarrow \psi)$ is consistent with $A(\varphi) = A(\psi)$.

(3) The only other value relation is $A(\varphi) > A(\psi)$, it must be inconsistent with $\delta(\varphi \rightarrow \psi)$.

Kripke's theorem: $(\delta(\varphi \rightarrow \psi)) \rightarrow (\delta(\varphi) \rightarrow \delta(\psi))$

Assume the antecedent, then by $\varphi \rightarrow \psi \leftrightarrow \neg\varphi \vee \psi$, and disjunction semantics, we obtain $\delta(\neg\varphi) \vee \delta(\psi)$. If $\delta(\psi)$, then since truth is materially implied by any proposition, then the theorem's consequent follows. If $\delta(\neg\varphi)$ then, with the use of Theorem 1 (Nothing is axiologically neutral) we deduce: $\delta(\neg\varphi) \Rightarrow \epsilon(\varphi) \Rightarrow \neg(\delta(\varphi)) \Rightarrow \delta(\varphi) \rightarrow \chi \Rightarrow \delta(\varphi) \rightarrow \delta(\psi)$.

2.23 Further Theorems

In the following parts, I shall use the show examples how to use the provided semantics to analyze value relations *within* propositional tautologies. Each law is obtained from the previous, and the thing indicated in [ ]. I skip, the move from $\top$ to $\delta(\top)$

Laws obtained from $\varphi \rightarrow (\psi \rightarrow \varphi)$

- $\delta(\neg\varphi \vee (\psi \rightarrow \varphi))$[By the law of logic: $\varphi \rightarrow \psi \leftrightarrow \neg\varphi \vee \psi$]
- $\delta(\neg\varphi) \vee \delta(\psi \rightarrow \varphi)$[Disjunction Semantics]
- (4) $\delta(\neg\varphi) \vee \delta(\neg\psi \vee \varphi)$[3, $\varphi \rightarrow \psi \leftrightarrow \neg\varphi \vee \psi$]
- $\delta(\neg\varphi) \vee \epsilon\neg(\neg\psi \vee \varphi)$[*4*, δ, ϵ semantics]
- $\delta(\neg\varphi) \vee \epsilon(\psi \wedge \neg\varphi)$[$(\neg\varphi \vee \psi) \leftrightarrow \neg(\varphi \wedge \neg\psi)$]

Laws obtained from $\varphi \leftrightarrow (\neg\varphi \rightarrow \varphi)$

- $A(\varphi) = A(\neg\varphi \rightarrow \varphi)$ [Equivalence Semantics]
- $\delta(\varphi) \leftrightarrow \delta(\neg\varphi \rightarrow \varphi)$ [1, LPP]

Laws obtained from $((\varphi \rightarrow \psi) \rightarrow \varphi) \leftrightarrow \varphi$

- $A((\varphi \rightarrow \psi) \rightarrow \varphi) = A(\varphi)$ [Equivalence Semantics]

- $(\delta(\varphi \to \psi) \to \varphi) \leftrightarrow \delta(\varphi)) \wedge (\epsilon((\varphi \to \psi) \to \varphi) \leftrightarrow \epsilon(\varphi))$ [Equivalence Semantics, δ Semantics]

- $((A(\varphi \to \psi) \leq A(\varphi)) \leftrightarrow \delta(\varphi)) \wedge (A(\varphi \to \psi) > A(\varphi)) \leftrightarrow \epsilon(\varphi)$[Implication Semantics]

Law obtained from $(\varphi \wedge \neg\varphi) \to \psi$

- $A(\varphi \wedge \neg\varphi) \leq A(\psi)$ [Implication Semantics]

Contradictions exemplify the lowest value.

Laws obtained from Modus Ponens

- $A((\varphi \to \psi) \wedge \varphi) \leq A(\psi)$

- $A((\neg\varphi \vee \psi) \wedge \varphi) \leq A(\psi)$ $[(\varphi \to \psi) \leftrightarrow (\neg\varphi \vee \psi)]$

Law obtained from $\neg(\varphi \wedge \neg\varphi)$

- $\epsilon(\varphi \wedge \neg\varphi)$ [T1]

- $\epsilon(\varphi) \vee \epsilon(\neg\varphi)$ [Conjunction Semantics]

Law obtained from $(\varphi \vee \neg\varphi)$

- $\delta(\varphi) \vee \delta(\neg\varphi)$ [Disjunction Semantics]

Laws obtained from $(\varphi \to \psi) \leftrightarrow (\neg\varphi \vee \psi)$

- $\delta(\varphi \to \psi) \leftrightarrow \delta(\neg\varphi) \vee \delta(\psi)$ [Disjunction Semantics]

- $A(\varphi) \leq A(\psi) \leftrightarrow \delta(\neg\varphi) \vee \delta(\psi)$ [Implication Semantics]

3. Theory of Values

The divine calculus, through which the world is created, elsewhere referred to by Leibniz as mechanica divina, involves an infinite combinatorics of goods. If anyone were surprised that good is understood as subject to calculations, let them recall the basic definition in mathematical decision theory, where it is said that a utility function is defined on a set of actions (i.e., takes arguments from this set) and takes values from the set of real numbers. There is nothing paradoxical about this. Since we speak of greater and lesser goods, of a scale of goods, we are dealing with some kind of ordered set; its infinity could be represented, for example, by the postulate that the scale of goods constitutes an interval of all real numbers between zero and one. (Marciszewski, 2010, pp.140)

In this chapter, the connection between value and ordering relations will be demonstrated in detail. In the previous part, goodness was analyzed as derived from the value ratio between φ and $\neg\varphi$. $\delta(\varphi) \Leftrightarrow A(\varphi) > A(\neg\varphi)$.

In this part of the work, some theses concerning values themselves (valuation function A), will be proven. The main problems I address are:

1. What is a propositions value in relation to its logical consequences?
2. How does one sum values?
3. What are the principles on which relationships between values can be established?

I will use a, if needed with indexes to denote values. 1 is a value such that $a \leq 1$, 0 is a value such that $0 \leq a$. First, it is necessary to extend the propositional treatment we have given to evaluative properties; goodness, evil to values. Suppose that values belong to any objects. But for A to be exemplified by x, there must exist an x such that $A(x)$. However, many important evaluative propositions are about non-obtaining states of affairs. For example, one can say that an ideal person is moral. However, there is no ideal person, so does this statement refer to some strange non-existent entities? No, it speaks of propositions about a perfect way to be a person, regardless of whether they are true (and therefore whether such an ideal person exists) or false. To say that some x exemplifies a is, in fact, to say that the truth of the proposition whose truth-maker is x is valuable.

Here are a few simple theses worth keeping in mind.

A1: $A(\varphi) \leq A(\varphi \vee \psi)$ $\quad$ $A(\psi) \leq A(\varphi \vee \psi)$

The value of any proposition is never greater than the value of a disjunction of which it is a constituent. Both $\varphi \rightarrow (\varphi \vee \psi)$ and $\psi \rightarrow (\varphi \vee \psi)$ are logical laws, from which follows $\delta(\varphi \rightarrow (\varphi \vee \psi))$ and $\delta(\psi \rightarrow (\varphi \vee \psi))$, which, by implication semantics, implies $A(\varphi) \leq A(\varphi \vee \psi)$ and $A(\psi) \leq A(\varphi \vee \psi)$. A slightly different version of A1,

A1.1: $$A(\varphi) > A(\psi \vee \chi) \Rightarrow (\varphi \neq \psi) \wedge (\varphi \neq \chi)$$

Assume that $\varphi = \psi or \varphi = \chi$, then either $\psi \rightarrow (\psi \vee \chi) or \chi \rightarrow (\psi \vee \chi)$, and in both cases $A(\varphi) \leq A(\psi \vee \chi)$.

T1.5: $A(\neg\varphi) \leq A(\varphi \rightarrow \psi)$ $\quad$ $A(\psi) \leq A(\varphi \rightarrow \psi)$

T1.6: $A(\varphi) > A(\psi \rightarrow \chi) \Rightarrow (\varphi \neq \neg\psi) \wedge (\varphi \neq \chi)$

$\varphi \rightarrow \psi$ is equivalent to $\neg\varphi \vee \psi$, so the proof is analogous to the proofs of A1 and A1.1

A2: $A(\varphi \wedge \psi) \leq A(\varphi)$ $\quad$ $A(\varphi \wedge \psi) \leq A(\psi)$

The value of a conjunction cannot be higher than the value of any of its arguments. The proof is analogous to A1.

A2.1: $$A(\varphi \wedge \psi) > A(\chi) \Rightarrow (\varphi \neq \psi) \wedge (\varphi \neq \chi)$$

Assume that $\varphi = \psi or \varphi = \chi$, then in both cases $(\varphi \wedge \psi) \rightarrow \chi$, and in both cases $A(\varphi \wedge \psi) \leq A(\chi)$.

To prove the next theorem, we need the concept of class set of logical consequences of a given formula, as well as the concepts of minimum and maximum. The class of logical consequences of a formula φ will be abbreviated as $LC(\varphi)$.

Let *R* be some ordering relation.

A maximal element of a set *X* is an *x* such that there is no $y \in X$ such that *xRy*.
A minimal element of a set *X* is an *x* such that there is no $y \in X$ such that *yR*x.

Let *Γ* be a set of propositions. We introduce the concept of an axiological minimum.

$$min(\Gamma) = \varphi : (\varphi \in \Gamma), (\psi \in \Gamma \Rightarrow A(\varphi) \leq A(\psi))$$

Analogically for max.

Benatar–Plantinga Principle (BPP): $A(\varphi) = min(LC(\varphi))$

Lemma: $A(\varphi) \leq max(LC(\varphi))$ and $min(LC(\varphi)) \leq A(\varphi)$

Firstly, since $\varphi \in LC(\varphi)$, it is obvious that $A(\varphi)$ is identical to something between the minimum and the maximum.

Let us consider whether for $A(\varphi) = a, min(LC(\varphi)) < a < max(LC(\varphi))$. The common view is that one needs to weigh advantages against disadvantages, which presupposes you can find out what value something has by something like taking the average of the values of its logical consequences, and so *a* would fall somewhere between the minimum and the maximum. From the implication semantics, and $\delta(\perp\rightarrow \varphi)$ for any φ, we deduce $A(\perp) = 0$. Suppose, however, that $A(\varphi)$ is calculated precisely on the basis of the average value from $LC(\perp)$. This contradicts $A(\perp) = 0$, since everything follows from a contradiction, hence the average would be higher than 0.

Hence $\varphi =\perp$ is a counterexample to $A(\varphi) = a, min(LK(\varphi) < a < max(LK(\varphi)$.

Suppose $A(\varphi) = max(LC(\varphi))$. $\top \in LC(\varphi) \Rightarrow max(LC(\varphi) = 1 \Rightarrow A(\varphi) = A(\psi)$. However, then for the case $\psi = \neg\varphi$, $A(\varphi) = A(\neg\varphi)$, which contradicts the previously proven theorem $\neg(\sim(\varphi))$.

Let us now prove BPP with no partial considerations.

Assume for a contradiction $A(\varphi) \neq min(LC(\varphi))$. Then, since $\varphi \in LC(\varphi)$, $A(\varphi) > min(LC(\varphi))$. Consider the semantic counterpart of The Deduction Theorem: $\Gamma \cup \{\varphi\} \vDash \psi \Leftrightarrow \Gamma \vDash \varphi \rightarrow \psi$. Substituting the empty set for Γ in this formula, we get $\emptyset \cup \{\varphi\} \vDash \psi \Leftrightarrow \emptyset \vDash \varphi \rightarrow \psi$.

So $\psi \in LC(\varphi) \Rightarrow (\varphi \rightarrow \psi) \in LC(\emptyset)$, and therefore $\varphi \rightarrow \psi$ is necessarily true, from which it follows that $\delta(\varphi \rightarrow \psi), A(\varphi) \leq A(\psi)$. Which proves $\psi \in LC(\varphi) \Rightarrow A(\varphi) \leq A(\psi)$. Now suppose $A(\varphi) > min(LC(\varphi))$, then there exists a $\psi \in LC(\varphi)$ such that $A(\varphi) > A(\psi)$, which contradicts $\psi \in LK(\varphi) \Rightarrow A(\varphi) \leq A(\psi)$.

Commentary: BPP is thus names after the author of *Better Never to Have Been* (Benatar, 2006) because BPP seems highly pessimistic. We would like to have a sort of *magnum opus* theory of value.

On such a view our worth is determined by our greatest achievements. BPP suggests the exact opposite. The only respite seems to be, that one's *modus ponens* is somebody else's *modus tollens,* so an optimist might take the view that since our worth is high therefore, our lows are not so low. Plantinga (2004) seems to be one such optimist, when he proposes a solution to the problem of evil lies in the unparalleled greatness of the Christian redemption story. In this way *the fall* was justified by the redemption.

The connection between value and the concept of minimum, present in BPP, is at least suggestive of another thesis within the field of axiology.

The most important of these is probably the fact that in the presented theory, the primitive concepts are neither good nor even value, but what is denoted by <, >, and which in everyday language is described as "better", "worse". This will be justified through the connection of value with the idea of order, specifically linear order. To justify this assertion, we must first present arguments for several auxiliary theses. The first of them is the thesis that every proposition exemplifies some value.

A3: $$\forall\varphi\exists a: A(\varphi) = a$$

Let us assume that φ has no value. $\neg\varphi$ also has no value or it does. If $\neg\varphi$ has no value, then, $A(\varphi) = A(\neg\varphi)$ is true (vacuously), which has been previously proven to be impossible. If, however, $\neg\varphi$ exemplifies some value, then similarly, on the basis of vacuous truth, it will be true that both $A(\neg\varphi) > A(\varphi)$ and $A(\neg\varphi) < A(\varphi)$.

A4?: $$\forall a\exists\varphi: A(\varphi) = a$$

I put it as an open question, whether unrealizable values (not goods) are possible.

The concept of axiological minimum and maximum connects the concept of value with ordering. Within the class of values, the value relation ≤ satisfies $(a_i \leq a_i)$, $(a_i \leq a_j$ and $a_j \leq a_k \Rightarrow a_i \leq a_k)$, and $(a_i \leq a_j$ and $a_j \leq a_i \Rightarrow a_i = a_j)$, and thus is an ordering relation.

By A3, the relation ≤ is a linear order on the class of values, that is one that satisfies the above three conditions, as well as $(a_i \leq a_j$ or $a_j \leq a_i)$. The existence of a linear order is thus something that makes all objects of a given domain comparable.

Aquinas (1952, reply in article 1) distinguishes two ways a being can be considered.

(…) *mode can be taken in two ways: first, in so far as it follows upon every being considered absolutely; second, in so far as it follows upon every being considered in relation to another. In the first, the term is used in two ways, because it expresses something in the being either affirmatively or negatively. We can, however, find nothing that can be predicated of every being affirmatively and, at the same time, absolutely, will the exception of its essence by which the being is said to be. To express this, the term thing is used; for, according to Avicenna," thing differs from being because being gets its name from to-be, but thing expresses the quiddity or essence of the being. There is, however, a negation consequent upon every being considered absolutely: its undividedness, and this is expressed by one. For the one is simply undivided being.*

If the mode of being is taken in the second way—according to the relation of one being to another—we find a twofold use. The first is based on the distinction of one being from another, and this distinct-, ness is expressed by the word something, which implies, as it were, some other thing. For, just as a being is said to be one in so far as it is without division in itself, so it is said to be something in so far as it is divided from others.

Analogously, there are two ways of specifying values. One of them is to define them per Aquinas term as *things*. This way is used when, we speak of justice, beauty, freedom, etc. This approach focuses on discovering the properties of these values and creating a description that could be, as it were, suspended in a vacuum, apart from external relations. The second, which is used in axiological logic, is to consider them in relation to others, through their place in the structure, we no longer speak of justice, etc., but of 0, 0.1, ..., 0.9, 1.

- An ordering that establishes the place of φ in relation to values of the elements of $LC(\varphi)$ is called an **internal ordering**.
- An ordering that determines the place of φ in relation to propositions logically independent of φ is called **external**.

Both types of orderings can be generated by any property. Just as for values, the ordering relation is, for example, $\leq$, that is, the relation of being worse or identical in value, so in the case of any property of this type, the order will be a relation measuring the degree of similarity of instances of a property. The boundaries of that ordering are, identity on the one hand, and opposition on the other.

3.1 The Flaw in Axiological Foundationalism

This part of the work addresses why there is no foundational value such that from it and descriptive propositions, you could infer all other values. and why the project of searching for such a premise is epistemically futile. The flaw of foundationalism lies in the attempt to construct a class of values based on a single object. This could be one value, but it could also be any one object, such as the divine will, happiness, desire, pleasure, rationality, beauty, the idea of good, etc. For example axiological reasoning based on the first axiological premise *Happiness is good* looks like this:

1. x increases happiness.

2. Happiness is good.

Conclusion: x is good.

From the BPP, we understand that values establish an internal order. This perspective on values, which draws an analogy to a number line, hinges on characterizing values based on their position within the structure of values generated by $\leq$. Just like we can structurally characterize number 2 as n such that, n is a natural number and $1 < n < 3$, we can characterize values through their degrees. The value of a proposition can be inferred from what its logical consequences are, thus values establish an internal ordering. However, this structural concept of value also enables us to compare (values of) logically independent propositions, thereby grounding an external ordering. Consequently, the value exemplified by an object cannot be determined solely in relation to that object. Value, and even more so goodness, necessarily coexist with relationships of better and worse. This is crucial; without the ability to compare an object to something, any object appears indifferent. Therefore, attempting to construct the entire class of values based on a single foundational value raises the question of why that value is good. If, however, one understands it as an independent foundation, primary to all other values, there is nothing to compare it to, rendering it impossible to provide any reason for or against its goodness, and it will consequently appear an indifferent thing.

If one treats values as atomic entities without considering their relationships, such as better-worse, values will appear as mere brute facts, devoid of normative significance and indifferent to the observer like a pile of rubble. By separating values from the overall structure of the class of values, one inevitably falls into increasingly radical errors. Examining only individual objects cannot yield their value. Entrenched in axiological foundationalism, one seeks the sources of this indifference in the characteristics of the individual object postulated as the foundation. This leads to a series of assumptions: first, that the value of an object does not derive from its descriptive properties; then, a broadening of this category because, by committing this fundamental error, value cannot be derived from anything. Consequently, through facts, we arrive at the broadest category: being. However, it turns out that values cannot be anything, leading the consistent foundationalist to embrace nihilism.

The source of the error lies not in a specific characteristic of the proposed foundation, such as whether it is natural or non-natural, but in foundationalism itself. Imagine someone attempting to construct an infinite set of natural numbers via construction of each finite number one by one and being surprised that they could not succeed. They might conclude that there is no transition from the finite to the infinite, and therefore, an infinite set of natural numbers cannot be made up entirely of finite numbers. But of course, the error lies in the method. Natural numbers come all at once, in that one needs an axiom system for the whole set to understand what it means for any number to be natural. There is no foundational number. Even 0 can only be understood as a natural number that is not a successor of any natural number.

By understanding values structurally, we can identify what ordinary language refers to as a degree of value. This reveals their mutual entanglement. Values are values when considered together; however, when viewed separately, they appear as a pile of rubble, a mere being that exists. Values are only intelligible as an entire class, a network, not as a tree with roots. Within this network of relationships, we can discern that some values are higher than others and some are lower. Superiority (>) and inferiority (<) are the bases of comparison, but comparisons only make sense with at least two objects present. Consequently, foundationalism will always generate a version of the Moorean Open Question. It postulates one thing as an axiological foundation and so without the possibility of comparison, the foundation will seem indifferent.

Before I proceed to justify why all possible proposals for first premises must be fruitless and devoid of any justification, it is necessary to demonstrate that the problem of the first premise for axiology is not merely about being a "first premise," but specifically about being a "first axiological premise." As for any "first premise," first principle, or axiom, one could, by definition, raise the objection that it is unjustified. After all, every chain of explanations must end somewhere. However, this is a misunderstanding. In the axiomatic method, while axioms are not proven from more fundamental assumptions, this does not negate their rational acceptability. Firstly, intuitively obvious test cases, even more so than axioms, must be provable by a sound axiomatic system. For instance, when Russell and Whitehead constructed the *Principia Mathematica* system, it was evident that the axioms had to be chosen to ensure that 1+1=2, not 1+1=3, and so on. This forms the foundation of the research program known as *Reverse Mathematics*. Secondly, various axiomatizations of the same systems exist. A statement that appears as an axiom in one axiomatization may be a theorem in another. This cross-validation through different approaches provides solid grounds for accepting axioms as true. However, this is not the case for the first axiological premise.

3.12 The First Axiological Premise is arbitrarily adopted

A first axiological premise, serving as the foundation of all values, would also establish epistemic norms (and other norms, such as moral norms). If its acceptance were based on some of its epistemic merits, like explanatory power and simplicity, then a given epistemic norm, such as "High explanatory power is good," would serve as a reason for accepting the first axiological premise. However, if this were the case, then this epistemic norm would precede the first premise, which is inherently contradictory.

3.13 Any hypothesis of a First Axiological Premise excludes rational criticism

For similar reasons, the adoption of the first premise cannot be rationally accepted but can only be the outcome of a dogmatic act. It cannot be criticized or rejected. To criticize a given hypothesis, one must suspend the assertion and examine its epistemic standing. However, this is impossible because the first premise forms the foundation of all values. Suspending the first premise suspends all values, leaving no criteria for comparing any hypothesis.

3.14 Any hypothesis of the existence of a First Axiological Premise leads to dogmatism

Not only can there be no reasons for accepting something as a first axiological premise, but moreover, if there were any reasons for accepting a proposition as a first axiological premise, it would prove that that proposition is not a first axiological premise. For then it would be grounded in the strength of those reasons, and thus would not be prior to them. Paradoxically, therefore, the fewer arguments there are for accepting some value as foundational, the better. From the perspective of a dogmatist, the absence of reasons for accepting a value as the foundation of a first premise only underscores the necessity of dogmatism. If there is no justification, then so much the worse for justifications.

3.14 All hypotheses of a First Axiological Premise are equally futile and arbitrary

If any one hypothesis were less futile, for instance, if it embodied more theoretical virtues, there would be reasons to accept it instead of alternative hypotheses. However, the fact that there are reasons for a particular first axiological premise candidate implies that it is not a first axiological premise.

3.2 Valuations

At this point, it is crucial to reiterate that only The Good is normative, not any arbitrary value. This reminder is essential because without it, it might appear that the very concept of values as objects connected by better or worse relationships has already introduced normativity. At this stage of the argument, there is no normativity yet. Currently, thanks to the focus on the degree aspect of values, we only have the missing link between the non-normative and The Good. The missing link is comparability and non-indifference. To demonstrate how The Good *becomes* normative, only a few more steps are required. Let us define valuation as follows:

Let Γ be a non-empty set of propositions, let $Val(\Gamma)$ denote the class of values of propositions in Γ. Ordering V is a valuation on Γ, abbreviate $V(\Gamma)$ under the following condition:

$$V(\Gamma) \Leftrightarrow (V(\varphi, \psi) \Rightarrow A(\varphi) \leq A(\psi))$$

This prompts the question, why consider valuations apart from just the order of values? It is justified by the need to make the following key distinctions. The condition of being a valuation can be fulfilled by many different relations. For example, among wines, *older than* is a valuation, among philosophers, the relation of *having a longer beard*, etc.

$$Local(V) \Leftrightarrow \text{For some } \Gamma, not\ V(\Gamma)$$
$$Global(V) \Leftrightarrow \text{For every } \Gamma, V(\Gamma)$$

Examples of local valuations are easy to find; such valuations are used whenever we speak of a good person, a good tree, a good dog, etc. Here, the domain of valuation is always limited, either to propositions about people, trees, dogs. Other examples of such valuations are propositions expressed by statements with a subjectivist clause like "something is good for me, from my point of view", etc. Related to this type of local valuation is the one that refers to utility values - they denote goods for something.

Local valuations are critically limited. As stated before, value is only intelligible as an element of the structure of all values, which in turn is the basis for the linear ordering on the whole Propositional Class. Therefore, any adequate valuation is global. This does not mean, however that, we can do without local valuations. A human being does not have such a powerful ability as to grasp the order of the entire Propositional Class, so we approximate by creating ever wider local valuations. Local valuations are simplifications, but they can be cognitively useful.

The second limitation inherent to local valuations is that, they come with only a sort of quasi-good. Quasi-good shares with The Good the characteristic expressed in $\delta(\varphi) \Leftrightarrow A(\varphi) > A(\neg\varphi)$. Quasi-good, like The Good, is at the value peak, in the domain of a given local valuation. Yet, it lacks normativity. This is perhaps best seen in the example of utilitarian values, but examples with subjective clauses are also very telling. The good derived from local valuations is not normative because this valuation does not exhaust all possible options, and does not valuate the clause itself. In other words, whenever we have some quasi-good, i.e., good for something, good for me, good in this culture, good if you want this or that, etc., in such cases the axiological status of that goal, that person, that culture etc. remains undetermined. As long as it is not known whether this "for" is worthy, or whether this desire is rational, etc., we are dealing with a quasi-good that is not normative. In summary, local valuation orders propositions isomorphically to the order of their values, but it is insufficient for normativity. Global valuation is a valuation that compares all possibilities. Based on what has been said about global valuations, one can point out its key property – they are all *consistent* with each other. (This is not a strict logical (in)consistency)

An important property of global valuations is that they make consistent claims about value peaks. To see this thesis, consider the way local valuations can be inconsistent. They distinguish different maximums and minimums. Thus, according to one local valuation, one value will be the highest, and according to another, a different one. This can happen because local valuations coincide with only some segment of the ordered sequence of values. For example, if we talk about good dogs, we mean a local valuation whose domain is the class of propositions about dogs. Let us say values within this class range from a_n to a_m. Based on that valuation, good propositions are propositions of the value a_m. This (class ordered by) local valuation coincides only with that interval of values $a_n \leq ,\ldots, \leq a_m$. But for some superset of it, its local valuation might range to a maximum of $a_k > a_m$. Based on this local valuation propositions of the value a_m are not good. Hence a sort of inconsistency. Global valuations, however, take not a segment but the entire ordered sequence of values, so such a situation cannot occur. These sequences cannot also differ at points between the minimum and the maximum, because then there would have to be subsequences than differ at those points. Therefore, all global valuations are mutually consistent.

3.3 How many values are there?

Analogies between values and numbers are useful, but one should not take that analogy too far. First of all, note since all propositions have a value, then the function A, mapping propositions to values is surjective, hence there is no more propositions than values. But how many is that? In classical propositional calculus, we consider propositional sets that are no larger than the set of natural numbers. However, there are various infinitary logics that do not impose such a restriction. The key point is that analogies with numbers should be approached with caution. The *choice* of which logic is true can lead to a radically different structure for the class of values compared to the particular class of numbers we drew our analogy from.

3.4 Theory of values summarized

We can summarize the presented theory in this manner: values form a network of entangled and codependent objects, rather than a pyramid. While there are various hierarchies, such as the degree of values, in the axiological network, there is no single point upon which everything rests. In other words, there is no single value or group of values that makes everything else a value; rather, they are all inextricably intertwined. From an axiological perspective, the entire network is significant. The Duhem–Quine thesis about propositions finds its reflection in axiology. According to this thesis, isolated propositions are not falsifiable, but entire theories. Valuations are not possible for isolated propositions, and values are not intelligible in isolation.

3.5 Normativity of The Good

Having the entire conceptual apparatus that discusses types of valuations, including which ones are adequate, allows us to finally answer the question of where the normativity of The Good originates. However, we can further refine this question. Instead of asking where the normativity of The Good comes from, we can ask why global valuations yield fully normative goods, while local valuations yield non-normative quasi-goods that are binding only under certain conditions. This leads us to the question of what component must be added to these goods with clauses like "good for" or "good for me" to obtain the simply good, unconditionally good.

However, at this juncture, this question itself must be scrutinized, as it is profoundly misleading. To grasp this, we must delve into a profound insight that underpins Kant's theory of aesthetic judgment. Kant encountered a dilemma akin to the logical connection between descriptive and normative elements – how to bridge the gap between the subjective and the general and necessary aspects of aesthetic judgment? Judgments of taste, according to Kant, are inherently subjective, as the criterion of beauty lies in a thing's ability to evoke aesthetic pleasure. Naturally, one might wonder what additional elements are required to establish generality, necessity of aesthetic judgment. The brilliant wisdom Kant unveiled was that he realized that nothing additional was needed to be added to subjectivity; rather, something needed to be removed. Aesthetic pleasure, therefore, must be disinterested (not driven by any specific goals) and culminate in the ultimate disinterestedness, known as derealization. In this state, we cease to be concerned with the tangible benefits of the beautiful object and we do not even consider its existence. We no longer care if a fictional entity deviates from reality, experiencing a "self-forgetfulness." (Mordka, 2008, pp. 212-214) (Kant, 1987, §§1-22) Consequently, despite its subjective nature, judgments of taste are not derived from any particular interests of individual agents but, from their general features as rational agents. Consequently, rational agents should concur in their aesthetic judgments, making them necessary (*a priori*) and universal.

Let us draw some insights from Kant's solution. To transition from a quasi-good to a good, one must eliminate the clause. As mentioned earlier, the clause and its value remain outside the determination of what is better or worse. Local valuations lack reflexivity in this regard. Consequently, local valuation reveals the value relationships within its domain but fails to define the place of that domain within the broader structure of values. It indicates what is better and worse, but it provides no information about the degrees of value. For instance, the highest value within a set could be relatively low, while the lowest could be relatively high. We can identify which dogs are better and which are worse, but we are unaware of the inherent value of being a dog itself. When there is no clause, nothing exists outside the valuation. Therefore, when we speak of unconditional good, good without clauses, it already encompasses the valuation of all possible clauses, goals, perspectives, and so on. It determines which of these elements are inherently good. Global valuations serve as the foundation for unconditional goodness because they encompass every conceivable condition. Consequently, the transition from non-normativity to normativity does not involve adding new elements to local valuations; instead, it entails removing restrictions that constrain their scope. Similar to Kant's theory, judgments of taste are universal and necessary because they are not based on anything unique to an individual agent. Similarly, The Good is unconditionally normative because no specific clause can alter its prescription, and it cannot change it because every conceivable clause point is already accounted for.

Underestimating Kant's idea can lead to a series of wrong questions that systematically exclude the possibility of an answer. Such questions are all those of the type, 'what is the transition from the fact that a being is something to the fact that it is good/bad/has some value/etc.' when one assumes a certain image of this 'transition'. Namely, the meaning of the claim that exemplifying goodness, etc., results from the properties that a being has is that, propositions about x exemplifying these properties are sufficient to logically imply whether a given being is an instance of The Good. That is, x exemplifies descriptive properties $P_1, \dots, P_\gamma$, and they alone are sufficient to determine whether x is good or not, in them we must find something that implies evaluative propositions about x. The mistake begins when, in asking about this transition, it is treated as something besides $P_1, \dots, P_\gamma$ that must be added to the properties, so that only $P_1, \dots, P_\gamma, Q$ determines whether x is good or not.

Yet, it is precisely the thesis of naturalism that nothing needs to be added, that propositions about Q are themselves a consequence of propositions about $P_1, \ldots, P_\gamma$, and not something added from the outside. When an anti-naturalist poses the problem, thus assuming the *transition* as something that needs to be added to the descriptive properties of a being, then from this very concept of *transition* present in posing the problem, it follows that naturalism is false, and thus commits a *petitio principii* against naturalism, and also makes the solution to the posed problem impossible. Even if there were such a transition between the descriptive properties of a being and good, it would after all be nothing other than either something value-laden and so, we would have just pushed the problem one stair up or something descriptive, but then it could not help us as long as we accepted naturalistic fallacy.

The goodness of a being does not stem from something in addition to its descriptive properties, but from the specificity of class of properties it exemplifies. The exemplification conditions for exemplifying all these properties determine whether it describes and exemplifies The Good or not. We're discussing relations between properties and between propositions, not between concepts. While evaluative and non-evaluative propositions are deeply related, we still might need various concepts to grasp them, and their lexicons might not be reducible to each other. This suggests more about our perspective than propositions. Despite a unity of these aspects in things, we might not be able to use descriptive properties to inquire about evaluative ones.

Of course, axiological laws derived from the property of goodness take different logical forms. For instance, they might take the form of an implication: people are different, times change, and therefore different people should do different things. These are all obvious truths, but no conditionality of The Good follows from this. Only that many evaluative propositions have the form of implication, where the antecedent specifies what the prescription in the consequent refers to. For example, the proposition "It is good when a person is free." or "A person should be free." has the logical form $\delta(\varphi \rightarrow \psi)$, where φ contains information about what a *person* is, and ψ about what *freedom* is.

3.6 Perfection and Harmony

Axiological logic proposes three suggestions for contemplating the highest value. The first suggests that necessary truths exemplify the highest value. This leads to the second suggestion, which proposes that duties have a negative character (Niemczuk 2016, pp.125-149). The third suggestion states that global valuations have a metaphysical nature. This means that, from an axiological perspective, only Reality as a whole, encompassing all that exists, holds significance. The value of a particular being is determined relative to everything else, in relation to the whole of existence. The fact that necessary truths possess the highest value ensures that these highest values are consistently realized by Reality. The goal is not to achieve perfection, but rather to ensure the existence of contingent beings that do not diminish the value of Reality. Necessary truths are what must be in Reality, regardless of any other existence. They can be likened to the substance of the world, guaranteeing the existence of Reality, in which accidental beings can only emerge. However, what is this value of necessary truths, so great that it is perfection? I propose that it is harmony. Necessary truths do not eliminate any possibilities. As the substance of the world, they are not inconsistent with any possible state of Reality, thus not excluding the realization of any value except counter-perfection. Benatar–Plantinga Principle, in a certain sense, alludes to the negative character of duties. This implies that the highest values are already realized, and the objective is to maintain such beings that do not cause Reality as a whole to lose its foundational value. Therefore, I suggest that the highest value is axiological harmony, manifested when the existence of one being does not diminish the value of other beings nor does it exclude any more valuable state of affairs.

3.7 Summation of Values

In this section, we'll delve into the concept of adding up values. This fundamental idea forms the basis of numerous ethical dilemmas, particularly the trolley dilemma, and is relevant to any situation where quantity is a crucial factor. For instance, does the collective well-being of many individuals outweigh the needs of a single person?

When we add two values together, what is this operation of addition? I will denote the operation of adding values as Σ, and it is an operation performed on non-empty sets of propositions. $\Sigma(\varphi_1, \ldots, \varphi_\gamma)$, therefore, symbolizes the axiological sum of γ (ordinal) number of propositions. In other words for finite *n* we could write $\Sigma(\varphi_1, \ldots, \varphi_\gamma) = A(\varphi_1) + \ldots + A(\varphi_n)$. These propositions do not have to be different, one can add the value of a proposition to itself, which only causes a slight complication when we understand summation as an operation on sets. The fact that the concept of summation is associated with arithmetic, however, obscures the true depth of the problem. After all, values are not numbers, and at this point in the book, almost nothing is known about what this summation is.

Therefore, it is necessary to seek out very basic intuitions. Firstly, it seems that there must be some limit, at which point the multiplication of a given type of value no longer increases the degree of realized values. Consider what mistake Judas made when he sold out The Messiah for 40 pieces of silver. 40 pieces of silver have some value, perhaps it allows Judas a few meals, perhaps a little sense of security, etc. Whatever the value of Jesus' life was, is it enough to multiply the value of 40 pieces of silver a sufficient number of times to eventually obtain a value greater than or equal to the value of Jesus? If there were no limit on value summation, then Judas' mistake would amount to taking too little money – which seems absurd. It seems rather that the value of the type of value of 40 pieces of silver never adds up to the type of value that Jesus possesses. Thus, the sum of values is not always higher than any single argument of that summation. That is, we have provided a counter example to $\Sigma(\varphi_1, \ldots, \varphi_\gamma) > A(\varphi_i)$ for $1 \leq i \leq \gamma$.

Another example might be the value of actions. Take any mediocre action and any heroic action. If there were no limit to summation, then summing a sufficient number of values of mediocre actions would give a value greater than or equal to a heroic action. What would then be the mistake of mediocrity? Only in too small a quantity of produced mediocrity. This example is all the more interesting because, in contrast to the story of Judas and Jesus, it operates within the framework of one type of value, the value of actions, moral values, while in the story of Jesus we had the utilitarian value of money and the independent (autotelic) value of Jesus. This illustrates that the problem does not concern the relationship between different types of values, but the operation of summation itself.

The second piece of evidence, suggests that for some propositions, it is true that $\Sigma(\varphi_1, \ldots, \varphi_\gamma) > A(\varphi_i)$ for $1 \leq i \leq \gamma$. It is given to us by Moore in his considerations of the organic whole. I quote from Moore at length, begin quite:

The paradox, to which it is necessary to call attention, is that the value of such a whole bears no regular proportion to the sum of the values of its parts. It is certain that a good thing may exist in such a relation to another good thing that the value of the whole thus formed is immensely greater than the sum of the values of the two good things. It is certain that a whole formed of a good thing and an indifferent thing may have immensely greater value than that good thing itself possesses. It is certain that two bad things or a bad thing and an indifferent thing may form a whole much worse than the sum of badness of its parts. And it seems as if indifferent things may also be the sole constituents of a whole which has great value, either positive or negative. Whether the addition of a bad thing to a good whole may increase the positive value of the whole, or the addition of a bad thing to a bad may produce a whole having positive value, may seem more doubtful; but it is, at least, possible, and this possibility must be taken into account in our ethical investigations. However we may decide particular questions, the principle is clear. The value of a whole must not be assumed to be the same as the sum of the values of its parts.

A single instance will suffice to illustrate the kind of relation in question. It seems to be true that to be conscious of a beautiful object is a thing of great intrinsic value; whereas the same object, if no one be conscious of it, has certainly comparatively little value, and is commonly held to have none at all. But the consciousness of a beautiful object is certainly a whole of some sort in which we can distinguish as parts the object on the one hand and the being conscious on the other. Now this latter factor occurs as part of a different whole, whenever we are conscious of anything; and it would seem that some of these wholes have at all events very little value, and may even be indifferent or positively bad. Yet we cannot always attribute the slightness of their value to any positive demerit in the object which differentiates them from the consciousness of beauty; the object itself may approach as near as possible to absolute neutrality. Since, therefore, mere consciousness does not always confer great value upon the whole of which it forms a part, even though its object may have no great demerit, we cannot attribute the great superiority of the consciousness of a beautiful thing over the beautiful thing itself to the mere addition of the value of consciousness to that of the beautiful thing. Whatever the intrinsic value of consciousness may be, it does not give tothe whole of which it forms a part a value proportioned to the sum of its value and that of its object. If this be so, we have here an instance of a whole possessing a different intrinsic value from the sum of that of its parts; and whether it be so or not, what is meant by such a difference is illustrated by this case.

19. There are, then, wholes which possess the property that their value is different from the sum of the values of their parts; and the relations which subsist between such parts and the whole of which they form a part have not hitherto been distinctly recognised or received a separate name. Two points are especially worthy of notice. (1) It is plain that the existence of any such part is a necessary condition for the existence of that good which is constituted by the whole. And exactly the same language will also express the relation between a means and the good thing which is its effect. But yet there is a most important difference between the two cases, constituted by the fact that the part is, whereas the means is not, a part of the good thing for the existence of which its existence is a necessary condition. The necessity by which, if the good in question is to exist, the means to it must exist is merely a natural or causal necessity. If the laws of nature were different, exactly the same good might exist, although what is now a necessary condition of its existence did not exist. The existence of the means has no intrinsic value; and its utter annihilation would leave the value of that which it is now necessary to secure entirely unchanged. But in the case of a part of such a whole as we are now considering, it is otherwise. In this case the good in question cannot conceivably exist, unless the part exist also. The necessity which connects the two is quite independent of natural law. What is asserted to have intrinsic value is the existence of the whole; and the existence of the whole includes the existence of its part. Suppose the part removed, and what remains is not what was asserted to have intrinsic value; but if we suppose a means removed, what remains is just what was asserted to have intrinsic value. And yet (2) the existence of the part may itself have no more intrinsic value than that of the means. It is this fact which constitutes the paradox of the relation which we are discussing.

It has just been said that what has intrinsic value is the existence of the whole, and that this includes the existence of the part; and from this it would seem a natural inference that the existence of the part has intrinsic value. But the inference would be as false as if we were to conclude that, because the number of two stones was two, each of the stones was also two. The part of a valuable whole retains exactly the same value when it is, as when it is not, a part of that whole. If it had value under other circumstances, its value is not any greater, when it is part of a far more valuable whole; and if it had no value by itself, it has none still, however great be that of the whole of which it now forms a part. We are not then justified in asserting that one and the same thing is under some circumstances intrinsically good, and under others not so; as we are justified in asserting of a means that it sometimes does and sometimes does not produce good results. And yet we are justified in asserting that it is far more desirable that a certain thing should exist under some circumstances than under others; namely when other things will exist in such relations to it as to form a more valuable whole. It will not have more intrinsic value under these circumstances than under others; it will not necessarily even be a means to the existence of things having more intrinsic value: but it will, like a means, be a necessary condition for the existence of that which has greater intrinsic value, although, unlike a means, it will itself form a part of this more valuable existent.

20. I have said that the peculiar relation between part and whole which I have just been trying to define is one which has received no separate name. It would, however, be useful that it should have one; and there is a name, which might well be appropriated to it, if only it could be divorced from its present unfortunate usage. Philosophers, especially those who profess to have derived great benefit from the writings of Hegel, have latterly made much use of the terms 'organic whole,' 'organic unity, ' 'organic relation. 'The reason why these terms might well be appropriated to the use suggested is that the peculiar relation of parts to whole, just defined, is one of the properties which distinguishes the wholes to which they are actually applied

with the greatest frequency. And the reason why it is desirable that they should be divorced from their present usage is that, as at present used, they have no distinct sense and, on the contrary, both imply and propagate errors of confusion.

To say that a thing is an 'organic whole 'is generally understood to imply that its parts are related to one another and to itself as means to end; it is also understood to imply that they have a property described in some such phrase as that they have 'no meaning or significance apart from the whole'; and finally such a whole is also treated as if it had the property to which I am proposing that the name should be confined. But those who use the term give us, in general, no hint as to how they suppose these three properties to be related to one another. It seems generally to be assumed that they are identical; and always, at least, that they are necessarily connected with one another. That they are not identical I have already tried to shew; to suppose them so is to neglect the very distinctions pointed out in the last paragraph; and the usage might well be discontinued merely because it encourages such neglect. But a still more cogent reason for its discontinuance is that, so far from being necessarily connected, the second is a property which can attach to nothing, being a self-contradictory conception; whereas the first, if we insist on its most important sense, applies to many cases, to which we have no reason to think that the third applies also, and the third certainly applies to many to which the first does not apply. (Moore, 1922)

Incidentally, these observations seem to confirm speculations about the highest value being harmony. For what is this *organic wholness* that Moore speaks of, if not axiological harmony between all parts and the whole? In any case, these two clues give us reason to believe that sometimes the sum is not greater than the value of the summed parts, and sometimes it is. Thirdly, I will assume that there is something justified in the arithmetic intuition, which says that there is some analogy in both summations. These three clues are sufficient, therefore, to formulate theorems that can be more rigorously proved. I take *Singleton* as obvious and, and then show commutativity and associativity.

Singleton:	$\Sigma\{\varphi\} = A(\varphi)$
Σ commutativity:	$\Sigma\{\varphi, \psi\} = \Sigma\{\psi, \varphi\}$
Σ associativity:	$\Sigma(\Sigma\{\varphi, \psi\}, \Sigma\{\chi\}) = \Sigma(\Sigma\{\varphi\}, \Sigma\{\psi, \chi\})$

Commutativity is trivial, it just follows from the fact that order of elements does not matter for set identity. For associativity let us start with a lemma.

Lemma: $\Sigma\{\varphi_1, \dots, \varphi_\gamma\} = \Sigma\{\Sigma\{\varphi_1, \dots \varphi_{\alpha_1}\}, \Sigma\{\varphi_{\alpha_1+1}, \dots \varphi_{\alpha_2}\}, \dots, \Sigma\{\varphi_{\alpha_\sigma+1}, \dots \varphi_\gamma\}\}$for $1 \leq \alpha_i \leq \gamma$

Alternatively put; $\Sigma(\Gamma) = \Sigma(\Sigma(\Delta_1), \dots, \Sigma(\Delta_\gamma))$ for $\Delta_i \Delta_j, i \neq j, \Delta_i \cap \Delta_j = \emptyset, \Delta_i \subseteq \Gamma, \Delta_j \subseteq \Gamma$. That is, these different sequences of deltas are different partitions of the set Γ. For example for $\Gamma = \{\varphi_1, \varphi_2, \varphi_3, \varphi_4, \varphi_5, \varphi_6\}$; $\Delta_1 = \{\varphi_1, \varphi_2\} \Delta_2 = \{\varphi_3, \varphi_4, \varphi_5, \varphi_6\}$ ... There can of course be many of these partitions, so that in another partition, for example, the first subset in a sequence, might have, for example, 5 elements, and not two. The basic case occurs when all Δ_i of the sequence are singletons. In this case, T20 is obviously true, because then

$$\Sigma(\Sigma(\varphi_1)\Sigma(\varphi_2)\dots\Sigma(\varphi_\gamma)) = \Sigma(A(\varphi_1,),\dots,A(\varphi_\gamma)).$$

There are many ways to partition Γ, so that the sets into which it is divided will have different cardinalities. For example, consider $\Gamma = \{\varphi,\psi,\chi\}$ and a partitions

$$\Delta_1 = \{\varphi,\psi\}, \Delta_2 = \{\chi\}, \Theta_1 = \{\varphi\}, \Theta_2 = \{\psi,\chi\}$$

Then, $\Gamma = \Delta_1 \cup \Delta_2 = \Theta_1 \cup \Theta_2 \Rightarrow \Sigma(\Gamma) = \Sigma(\Delta_1 \cup \Delta_2) = \Sigma(\Theta_1 \cup \Theta_2)$.

Σ is therefore commutative and associative. So in this respect, the arithmetic intuition has been confirmed. In the next part, the subject of research will be the neutral element for Σ , and here it will turn out where the arithmetical intuition diverges from the truth.

3.71 Neutral Element

The next issue to consider is the existence of a neutral element for summation (identity element) and what it is. I will use *I* to denote the neutral element. The question is, therefore, whether and, if so, what value satisfies $\Sigma(a,I) = a$ for arbitrary a. To discover something about the neutral element, we must first consider the existence of negative and positive values, and determine at what level these negativities or positivities occur. Does it make sense to talk about negative or positive values *per se*? Firstly, I will translate the thesis that there are no negative values into the language of axiological logic.

I caution against equivocation between negative in an axiological sense and metaphysical senses such as when we talk of negative facts or negative properties. I will consider the negativity of value here purely in terms of its function during the summation operation. I say a value is negative, abbreviate $-(a)$ under this condition $-(a_n) \Leftrightarrow \Sigma(a_n, a_i) = a_i$.

$\Sigma 1$?: $$\Sigma(\Gamma) \geq max(\Gamma)$$

$\Sigma 1$ is a translation of the thesis that there are no negative values. (As it turns out, this thesis is false.) The translation is correct because, firstly, Σ is commutative, and secondly, the axiological sum of given formulas cannot be less than the most valuable of the summed formulas if and only if there are no such values, whose addition to another value *a* can result in a sum less than *a*. Let us assume some results of operations on values as true. First, if we denote the value of counter-perfection by 0 and perfection by 1; $0 + 0 = 0, 1 + 1 = 1$. These two principles show that the result of summation can be equal to the axiological maximum, e.g., when all $\varphi_1, \ldots, \varphi_\gamma$ are contradictions, or when they are necessary. $\Sigma(\Gamma) = max(\Gamma)$ is true for some Γ.

Let us ask if the axiological sum can be less than the maximum. Is $\Sigma(\Gamma) > max(\Gamma)$ true for some Γ. That is, the question of the existence of negative values. If there are no negative values, then it cannot be the case that $\Sigma(a_n, a_m) < a_m$. The issue of negative and positive values would be simple if we were to operate at the level of instances of values. At this level, such negativity certainly exists. A certain sound may be aesthetically valuable, but nevertheless not fit into a certain composition, because it would disrupt the harmony – the value of that sound would therefore appear negatively. However, speaking of negative values seems to be a misunderstanding here, because we do not mean this or that value is negative, but its instance. The summation of values does not take place at the level of instances of values, but at the level of values themselves. Therefore, to check $\Sigma 1$, we must examine whether there are any negative or positive values *per se*, regardless of whether their instances are good or not. Let us define negativity and positivity as follows;

1. A value is negative in a given state of affairs if it excludes a higher value.
2. A value is positive in a given state of affairs if it does not exclude the realization of a higher value.

The concept of exclusion is necessary to formulate negativity and positivity due to BPP. $A(\varphi) = min(LC(\varphi))$. If we used some phrases such as *increase* or *decrease in value* the *positive* or *occurrence of a value in a given state of affairs* must *increase* or *decrease its value*, it would suggest that the value of some state of affairs/proposition is the result of some averaging or other weighting of the values realized in it, and we have already shown it is not the right way to think. It follows from BPP that any axiological minimum of a set of propositions about some *x*, occurs negatively insofar as it is not perfection. If this minimum is not perfection, then removing it while leaving the rest would open the possibility to there being something of a higher value. However, the existence of this kind of negativity at the level of instances directly implies where this kind of negativity cannot exist at the level of values themselves. Take any value $a > 0$. *a* negative for propositions that do not logically imply propositions with value lower than *a*. However, it s positive otherwise. Specifically, for any value $a > 0$, a is positive for any contradiction. Similarly with positivity. Any value $a > 0$ is positive for a contradiction. Consider the aesthetic value of eloquent speech, oratory. It is positive when realized in a skilled teacher but negative when possessed by a demagogue. Therefore, negativity and positivity primarily appear not in an abstract value but in its relationship to what it is realized in. Only two values can be considered positive or negative in themselves (not at the level of instances): perfection and counter-perfection. Perfection never excludes a higher value, while negative values are those whose realization currently excludes the existence of something of a higher value, i.e., the axiological minimum of a given state of affairs. Consequently, counter-perfection always occurs as negative because it always occurs as a minimum. Apart from counter-–perfection, there are therefore no negative values such that always $\Sigma(a_n, a_i) < a_i$. Nevertheless, to the previous principles 0+0=0 and 1+1=1, we can now add another, which falsifies $\Sigma 1$. Namely, since 0 always occurs negatively and 1 always positively, we have; $\Sigma(a, 0) \leq a and \Sigma(a, 1) \leq 1$

After obtaining these two results, we can return to the neutral element question. They convey important information. Namely, neither 0 nor 1 are neutral elements, although intuitively 0 would seem to be the ideal candidate. The number 0 is the neutral element for arithmetic summation – so the arithmetical analogy ends here. The only sensible thesis about the neutral element that seems to remain is the thesis that for any value, it itself is a neutral element for itself. That is; $\Sigma(a, a) = a$. However, such idea is incorrect because the neutral element is supposed to be unique. Moreover, it would be paradoxical. Returning to the story of Judas, it would turn out that 80 pieces of silver have the same value as 40, that a million human lives – as much as one, that in the trolley dilemma it does not matter what you do with the lever. This is not proof, but an argument for the conclusion that there is no neutral element. This can therefore be used to formulate the last theorems. For $n \leq \gamma,\ i \leq \gamma$;

$\Sigma 2$: $$(A(\varphi_n) \neq 0 and A(\varphi_n) \neq 1) \Rightarrow \Sigma\{\varphi_1, \ldots, \varphi_\gamma\} \neq A(\varphi_n)$$

$\Sigma 2.1$: $$\Sigma\{\varphi_1, \ldots, \varphi_\gamma\} = A(\varphi_n) \Rightarrow A(\varphi_n) = 0 or A(\varphi_i) = 1$$

From 0+0=0 it follows that if all summed values have a value of 0, then their sum has a value of 0. From 1+1=1 it follows that if all summed values have a value of 1, then their sum has a value of 1. Considering the rejection of $a + a = a$, and therefore the fact that there is no neutral element for summation, 0+0=0 and 1+1=1 are the only cases where the sum of values is identical to values summed.

$\Sigma 2.5$: $$\Sigma\{\varphi_1, \ldots, \varphi_\gamma\} \neq 0 and \Sigma\{\varphi_1, \ldots, \varphi_\gamma\} \neq 1 \Rightarrow A(\varphi_i) \neq \Sigma\{\varphi_1, \ldots, \varphi_\gamma\}$$

3.72 A few remarks on negative values

Another understanding of negative values would be the literal translation that negative value is the inverse of *A*, or the opposite of *A*. However, it would be very strange to understand the negative value in this way. Negation works in a clear way in the domain of truth values, it is simply an inverse operation, the negation of falseness gives truth and vice versa. This clarity comes from the fact that the domain of truth values is two-element. However, this is not the case with axiological values. Because if, for example, some $a = 0.6$, what is its inverse? If inverse it to behave like the logical negation, inverse of *a* is the compliment of a with in the class of values, including higher values. But there is also a view that imagines a world of mirror-like values (Ingarden 1966, pp.168), where negative value would be some kind of opposite. This is the case when values are given in pairs, beauty – ugliness, wisdom – stupidity, positive values – negative values. This creates the need to introduce some neutral resting point between the two. First, it has been shown that there is nothing absolutely axiologically neutral- there is no indifference. Secondly, let us imagine such a mirror. If, let us say, beauty is a value 0.8, then in such a mirror structure the ugliness will have a value of 0.2, and this neutral point between them will be 0.5. But why would this point between them be indifferent? Third, in this view, the negative value would correspond to what is called an inverse element in abstract algebra. The inverse element for the operation Σ, however, is the element for any a satisfying the equalities $\Sigma(a, inverse(a) = I$, where I is a neutral element. For the Σ operation, however, there is no neutral element.

This concludes the study of summation. If I have shown one thing about these waters, it is their murkiness. Under terms such as positive or negative value, whole families of more or less valid concepts that have been wrongly glued together can stand. In every day life we pull on concepts of naive concepts summation. It casts strong doubt at our everyday evaluative reasoning.

4. Bridging the fact–value gap and incompleteness of axiological logic

With value theory, we can make claims about intelligibility. These claims overcome the naturalistic fallacy and Hume's guillotine. They show the relationship between descriptive and evaluative propositions. However, they also reveal the gap between goodness and truth. If we interpret Hume's guillotine syntactically, it asks for a reliable inference rule to deduce an evaluative proposition from a descriptive one. This gap is not bridged. We only overcome theories that find semantic gaps between descriptive and evaluative concepts.

4.1 Logical preliminaries

I will distinguish between different types of results, as they differ so much that the thesis that evaluative propositions are implied by descriptions oscillate between obvious and false. Without practicing these subtleties of mathematical logic, phrases like "you cannot get an ought from an is" can fall into nonsense, leading to arbitrary conclusions. In the next part, I will prove that from what is descriptive, it is impossible to deduce all evaluative propositions. Let us start with a bad argument. What is good does not follow from what is, evaluative from descriptive, so the value characteristics are independent of descriptive ones. The error of this type of reasoning is contained in the inaccurate use of the word "follow". If it follows, i.e. it results syntactically (proves), then in the absence of such results, it is not possible to conclude about semantic consequence, nor can you draw ontological conclusions on this basis. There are four types of results that I have to mention here. For material implication, logical implication, logical (semantic) consequence, and syntactic consequence.

Material implication (→)

The material implication is an extensional connective. The truth value of a material implication is determined by the truth value of its arguments. In this type of implication, the content of propositions is entirely disregarded; only their truth values matter, not their subject matter. This approach leads to well-known paradoxes, such as the conclusion that (people die of hunger) implies (Warsaw is the capital of Poland). The sense of paradox arises from the expectation that the material implication establishes a connection between the contents of propositions, but it does not. The philosophical question behind material implication is to what extent it accurately expresses the natural language conditional. Naturally, the level of interest in results about material implication will vary depending on the way one answers this question. For a witty exposition of these paradoxes check (Brandom, 1983) Some of these paradoxes are:

$$\varphi \rightarrow (\psi \rightarrow \varphi), \varphi \rightarrow (\neg\varphi \rightarrow \psi), (\psi \rightarrow \varphi) \vee (\varphi \rightarrow \psi)$$

In the context of bridging the evaluative – descriptive gap, an important property of material implication is that; $\varphi \rightarrow (\psi_1 \vee \psi_2 \vee\ldots\vee \psi_n) \Rightarrow (\varphi \rightarrow \psi_1) \vee (\varphi \rightarrow \psi_2) \vee\ldots\vee (\varphi \rightarrow \psi_n)$

Logical Consequence (semantic) $\vDash$

$$\Gamma \vDash \varphi \Leftrightarrow \mathfrak{M} \Vdash \Gamma \Rightarrow \mathfrak{M} \Vdash \varphi$$

Model $\mathfrak{M}$ when working with propositional calculus means a truth assignment , so read $\mathfrak{M} \Vdash \Gamma$ as Model $\mathfrak{M}$ assigns truth to every element of Γ.

Logical implication ($\Rightarrow$)

Based on the logical consequence, it is possible to determine what the law of logic is, and on the basis of this term and material implications, what logical implication is.

$$\varphi \Rightarrow \psi \Leftrightarrow \emptyset \vDash \varphi \rightarrow \psi$$

Material implication vs logical implication

The logical implication is not extensional, which precludes the occurrence of the previously mentioned paradoxes of material implications. While there are other paradoxes that I have not explicitly mentioned, such as the assertion that contradictions imply everything or that tautology is implied by everything, still the statements about logical implication are stronger and more intriguing because they align more closely with our intuitive understanding of the conditional. What is crucial for the intelligibility theorems is that;

$$\varphi \Rightarrow (\psi_1 \vee \psi_2 \vee\ldots\vee \psi_n) \Rightarrow (\varphi \Rightarrow \psi_1) \vee (\varphi \Rightarrow \psi_2) \vee\ldots\vee (\varphi \Rightarrow \psi_n)$$

Is not true for logical implication. Therefore, both the statement of intelligibility for ⇒ and its proof will need to be formulated and proven in a fundamentally different manner than the corresponding theorem for material implication.

Syntactic consequence (⊢)

Γ proves Φ, abbreviate $\Gamma \vdash \Phi$. We're discussing provability in logic, focusing on signs and transformations of signs. We do not consider their content or truth values. No proof of φ from Γ is valid in itself. Proofs rely on inference rules, which define valid moves in a formal language.
$\Gamma \vdash \Phi$ based on the set of inference rules Inf ⇔ there is a sequence of well formed formulas $\Phi_1, \ldots, \Phi_\gamma$, such that $\Phi_\gamma = \Phi$ and for any $\Phi_i, (1 \leq i \leq \gamma)$ either $\Phi_i \in \Gamma$ or the initial subsequence $\Phi_1, \ldots, \Phi_i$ of $\Phi_1, \ldots, \Phi_\gamma$, is a proof of Φ_i based on set of formulas Γ.

The difference between logical and syntactic consequence

The occurrence of syntactic consequence is always relative to a specific formal language and its inference rules. This type of relativity is distinct from logical consequence; it does not refer to a particular model or set of models, but rather to all models of Γ. Hence, $\Gamma \nvdash \varphi \not\Rightarrow \Gamma \nvDash \varphi$; the absence of proof is a matter of inference rules, not truth and models. So these two categories do not overlap. Logical consequence is about truth and falsehood and is exactly connected with logical implication, so one can try to draw some ontological conclusions about the property of goodness on this basis. However, extracting them from syntactic consequence is nonsense; this one talks about our ways of proof, it is about epistemology. And when commenting on the possibility of inferring *good* from *is*, you should always specify what formal system is being investigated. Here are some examples of the intuition behind the concept of naturalistic fallacy, which, however, mix different issues.

Let us assume that from the fact that someone is sick, it does not necessarily follow that they should be treated. This reasoning lacks an evaluative premise: is it good to be healthy. Even if the example is accurate, it is just an example, at best a demonstration, not a proof. But what does it even show? No one asserts that every descriptive proposition implies every evaluative proposition. Descriptive propositions that, I claim to logically imply evaluative propositions are for example, full description of x, probably together with a description of all logical possibilities. In the example presented, it would be necessary, therefore, not to say that "someone is sick, so they should be treated", but,

(1) Someone is sick,
(2) A full description of this someone along with his talents, etc., and what she could do instead of getting sick,
(3) A description of all logical possibilities describing what could've happened if she'd realized these things instead of being sick, and other possible worlds where she might not even exist.

Based on these premises, I maintain that it is logically implied whether a person should or should not be treated (in the sense of it being good, not necessarily in a moral sense). You could request a proof based on these premises to determine whether someone should or should not be treated a certain way. While I cannot provide such proof, it does not imply that there is no logical implication between the premises and the conclusion. Firstly, clearly it would require infinite knowledge to unpack these premises. Secondly, $\Gamma \nvdash \varphi \nRightarrow \Gamma \nvDash \varphi$(known since Gödel's discoveries). So, the mere impossibility of formulating something like a practical syllogism (absent a completeness theorem for a given system) does not show much about the possibility of descriptions implying evaluations, or about some ontic gap between values and facts. Fortunately, there are non-constructive methods of proof, that do not require us to provide an example, so we can use them to nevertheless say something about the logic of evaluative propositions.

Naive thesis of independence: $\bigwedge(Val)$ does not follow from Γ.

Now, we can fully appreciate the way, this thesis breaks down into different ones. The syntactic and the semantic.

They are very different. So different that in this part of the work, I aim to show: that for some φ, where $\mathcal{L}$ is an adequate axiom system of axiological logic.

$$(\delta(\varphi) \Rightarrow (\varphi \Rightarrow \delta(\varphi)) \text{ and } \mathcal{L}. \nvdash \delta(\Phi).$$

4.2 Material intelligibility

$\rightarrow$ intelligibility.: $\varphi \rightarrow ((\delta(\varphi) \vee \epsilon(\varphi))$

One of $>, <, =$ holds between any values. The consequent of the implication is a law of axiological logic, by δ, ϵ semantics and theorem 1 (nothing is axiologically neutral). A true proposition is materially implied by any proposition, which proves the theorem. By $\varphi \rightarrow (\psi_1 \vee \psi 2 \vee \ldots \vee \psi_n) \Rightarrow (\varphi \rightarrow \psi_1) \vee (\varphi \rightarrow \psi_2) \vee \ldots . \vee (\varphi \rightarrow \psi_n)$, we obtain:

$$(\varphi \rightarrow (\delta(\varphi)) \vee (\varphi \rightarrow \epsilon(\varphi))$$

The truth of material intelligibility is obvious, but how substantial is it? I do think, it is somewhat interesting. Let us contrast these propositions with analogous theorems of deontic logic. One of these is called *exhaustion* (McNamara, 2014) and says that something is either obligatory, optional, or forbidden. $(OB(\varphi) \vee OP(\varphi)) \vee IM(\varphi))$ This principle might be interesting from the point of view of the axiomatics of deontic logic, but it is difficult to notice any deeper philosophical meaning in it. The same move that was used in the proof of material intelligibility can be used on the basis of *exhaustion* to obtain the theorems:

$\varphi \rightarrow ((OB(\varphi) \vee OP(\varphi)) \vee IM(\varphi))$ $(((\varphi \rightarrow (OB(\varphi)) \vee (\varphi \rightarrow OP(\varphi))) \vee (\varphi \rightarrow IM(\varphi))$

which seems to similarly provide a counter example to the naturalistic fallacy. However, there has not been significant attention paid to this principle in relation to the descriptive – evaluative gap. $OP(\varphi) \Leftrightarrow (\neg OB(\varphi) \wedge \neg OB(\neg\varphi))$. So *OP* it might seem like more of a lack of deontic status – something is optional if the moral law is silent on the matter. On this view, *exhaustion* says nothing more than that it follows from φ that φ has a deontic status or that it does not. Which is nothing more than an instance of the law of an excluded middle. So this law is trivial if, we consider *OP* to not quite capture any deontic status. However, there is no corresponding issue for axiological logic, both good, evil and even axiological neutrality express the relationship of values. The key point is that axiological neutrality, expresses identity of values, so neutrality does not express the lack of value and value relations. It is truly about values. If that was not enough, due to theorem 1, we do not even need neutrality to state material intelligibility. There is no corresponding move in deontic logic.

The intelligibility laws guarantee that always one of the evaluative propositions follows from of a descriptive one. Now let us attempt to prove a stronger theorem, Whatever status, good or evil, belongs to any proposition, this status is a logical consequence of φ. I want to note to facts. First, it would be a misunderstanding to ask for a deduction of a specific axiological status. Such a deduction would constitute a proof that everything is evil/good and that is not consistent with axiological logic. Secondly intelligibility laws do not tell us anything about whether we can deduce axiological status φ of from φ. However, this is an epistemic (syntactic) gap, not an ontic (semantic) one.

4.3 Logical intelligibility

In this part, I will show that evaluative propositions are logically implied by descriptive ones. I will use a similar argumentative strategy as for the material intelligibility. The proof of material intelligibility uses the fact that a true proposition is materially implied by any other, and that if $\varphi \rightarrow (\psi_1 \vee \psi 2 \vee \ldots \vee \psi_n) \Rightarrow (\varphi \rightarrow \psi_1) \vee (\varphi \rightarrow \psi_2) \vee \ldots \vee (\varphi \rightarrow \psi_n)$, which implies $(\varphi \rightarrow (\delta(\varphi)) \vee (\varphi \rightarrow \epsilon(\varphi))$. In the case of the logical implication the corresponding proposition $\varphi \Rightarrow (\psi_1 \vee \psi_2 \vee \ldots \vee \psi_n) \Rightarrow (\varphi \Rightarrow \psi_1) \vee (\varphi \Rightarrow \psi_2) \vee \ldots \vee (\varphi \Rightarrow \psi_n)$ fails. However, necessary truths are logically implied by any other proposition. My proof strategy will therefore be to demonstrate that the evaluative propositions δp or εp, if true, are necessarily true.. Let us recall, after all, that the law of logic is the logical consequence of an empty set, and so laws of axiological logic also share this status.

⇒ intelligibility: $\delta(\varphi) \Rightarrow (\varphi \Rightarrow \delta(\varphi))$

At this point we shall really begin to pit different intuitions behind the naturalistic fallacy theory against each other. Remember, the world independence thesis, that evaluative propositions are not contingent. We put it in a different way here.

Independence Property (IP): $\delta(\varphi) \Leftrightarrow \top$ or $\delta(\varphi) \Leftrightarrow \bot$

Necessity lemma: $\delta(\varphi) \Rightarrow (\delta(\varphi) \Leftrightarrow \top)$

Because logical equivalence plays a central role in IP, these principles are only stable in metalogic. Or maybe a sort of higher order axiological logic? We could also formulate axiological logic as an extension of modal logic S5 and so we would simply write $\Box(\delta(\varphi)) \vee \Box(\epsilon(\varphi))$. I must admit, I have not achieved sufficient clarity in these matters. Nevertheless, let me offer some arguments in support of IP.

Argument from the ontology of propositions

For this argument, I use $\Box\varphi$ to abbreviate $\varphi \Leftrightarrow \top$ and $\Diamond\, \varphi$ to abbreviate $\varphi \not\Leftrightarrow \top \; and \; \varphi \not\Leftrightarrow \bot$.

Axiom 1: $(\Diamond\, \psi \Rightarrow \Diamond\, (\varphi \wedge \psi)) \Rightarrow \Box\varphi$,

Axiom 2: $\exists P(\Box(x \text{ exemplifies } P)) \Rightarrow \Box(x\ \textit{exists})$

Note: Axiom 2 does not imply there are unicorns, because no proposition of the form (x exemplifies being a unicorn) is necessarily true. Also note, we assume the meaning of variable x is fixed.

Axiom 3: $\Box$(x *exists*) ⇒ $((x \text{ exemplifies } P) \Leftrightarrow \Box(x \text{ exemplifies } P))$

Axiom 4: Truth and falsity are relations between propositions and truth/false makers.

Definition: Let P be a property such that (x exemplifies P) ⇔ (x is true or x is false)

Theorem 1: By excluded middle, every proposition φ exemplifies P.

Theorem 2: $\Box$(φ *exists*) [Theorem 1, Axiom 2]

Assumption: $\delta(\varphi)$

Theorem 3: $\Box(\delta(\varphi))$ [Assumption, Theorem 2, Axiom 3]

Theorem 4: $\varphi \Rightarrow \delta(\varphi)$ [Theorem 3, necessary truths are logically implied by all propositions]

Argument from universal propositions

Recall:

General Dichotomy (GD): $\varphi \neq \delta(\varphi)$

Dichotomy of contingents: $\delta(\varphi) \Rightarrow (\delta(\varphi) \nRightarrow \varphi))$ for contingent φ

Theorem: GD ⇒ There is a descriptive proposition $\mathfrak{U}_n$: $\mathfrak{U}_n \Rightarrow \delta(\mathfrak{U}_n)$

Let us consider maximally consistent sets of propositions, abbreviate $\mathcal{MC}(\Gamma)$. A set of propositions is maximally consistent if for some φ, $\Gamma \nvDash \varphi$, and $(\varphi \notin \Gamma) \Rightarrow \Gamma \cup \{\varphi\} \vDash \psi$ for any ψ. Consider $\{\Gamma_1, \dots, \Gamma_\gamma\}$ such that $\mathcal{MC}(\Gamma_i)$and $\Gamma_i \nvDash \Gamma_j (1 \leq i, j \leq \gamma)$. Now we construct a class of universal propositions, $\mathcal{U} = \{\mathfrak{U}_1, \dots \mathfrak{U}_\gamma\}$ such that $\mathfrak{U}_i \vDash \Gamma_i, \Gamma_i \vDash \mathfrak{U}_i$. Any such proposition is a complete description of the whole of what is. Clearly, exactly one of them must be true. Note: $\neg\mathfrak{U}_i \Rightarrow \mathfrak{U}_k \mathit{for\ some k} \neq i$. Hence from $\neg(\sim(\varphi))$, it follows that $\delta(\mathfrak{U}_n)$ for some n. By maximality $\mathfrak{U}_n \Rightarrow \delta(\mathfrak{U}_n)$. $\mathfrak{U}_n$ is contingent, hence by GD, it is a descriptive proposition. Hence, we have seen an example of a descriptive proposition that logically implies an evaluative one.

Theorem: IP ⇒ There is a descriptive proposition $\mathfrak{U}_n$: $\mathfrak{U}_n \Rightarrow \delta(\mathfrak{U}_n)$

In the previous proof in order to show $\mathfrak{U}_n$ is descriptive, one may just as well use IP instead GD.

Theorem: $$\mathrm{DOC} \Rightarrow \mathrm{IP}$$

Consider $\psi = \delta(\varphi)$. If ψ was not necessarily true, then either it would be a contradiction, and hence would logically imply any proposition including contingent ones, or ψ would be contingent, but obviously any contingent proposition logically implies many contingent propositions.

4.4 Incompleteness Theorem

Let $\mathcal{L}$ range over any formal system that is an adequate formalization of axiological logic (every theorem of $\mathcal{L}$ is a true proposition of axiological logic), recall Φ is the formula expressing the proposition φ.

IT: $$\exists\varphi(\varphi \Leftrightarrow \top) \text{ and } \mathcal{L} \nvdash \Phi)$$

The meaning of $\mathcal{L} \nvdash \Phi$ requires attention. As we have noted, it only makes sense to speak of $\vdash$ in context of some formal system. But we have not given an axiomatization of AL. We will therefore consider $\vdash$ to range over any formalization of AL such that it does not prove falsehoods, and then we will consider what inference rules for introducing (the symbol) δ it may contain. Let us also for simplicity's sake say, we do not allow ourselves to introduce δ based on ϵ (so, we do not just push back the problem a step further).

Let us fix some contingent, that is a φ: $(\varphi \not\Leftrightarrow \top), (\varphi \not\Leftrightarrow \bot)$. Due to $\delta(\psi) \vee \epsilon(\neg\psi)$, we know there exists a contingent proposition φ: $\delta(\varphi)$. Consider what inference rule could let us deduce the formula expressing $\delta(\varphi)$, abbreviate $\delta(\Phi)$. Besides evaluative properties in AL, we may consider propositions in light of two things.

(1) Logical form.

(2) Truth value.

(3) Combinations of (1), (2).

Ad (1): We show a counterexample to each logical form. Recall: Φ express a contingent φ.

(i) $\Phi = \neg\Phi'$. Obviously, there are contingent propositions such as $\neg\varphi$, such that $\epsilon(\neg\varphi)$

(ii) $\Phi = \Psi \to X$. Pick contingent ψ, χ: $\delta(\psi), \epsilon(\chi)$.

All other logical forms are expressible by $\neg, \to$.

Ad (2): There exists a contingent φ such that $\epsilon(\varphi)$, therefore $\Phi \vdash \delta(\Phi)$ is inadequate.

Ad (3): It is also not sufficient to have a certain logical form and a certain truth value. For true/false negations construct counterexamples using double negation. For implications assume $\Phi = \Psi \to X, \Phi \vdash \delta(\Phi)$. Let X express a contradiction, then since $\Psi \to X$ is contingent, Ψ is contingent.

Case (i): If $\delta(\neg\Psi')$, then for $\Psi = \neg\Psi' \Rightarrow A(\psi) > A(\chi) \Rightarrow \epsilon(\psi \to \chi) \Rightarrow \epsilon(\Phi)$, therefore $\neg\Psi'$ is a counterexample.

Case (ii) If $\delta(\Psi')$, then for $\Psi = \Psi' \Rightarrow A(\psi) > A(\chi) \Rightarrow \epsilon(\psi \to \chi) \Rightarrow \epsilon(\Phi)$, therefore Ψ' is a counterexample.

By the independence property of AL for some contingent φ, $\delta(\varphi) \Rightarrow ((\mathfrak{M} \Vdash AL) \Rightarrow (\mathfrak{M} \Vdash \delta(\varphi))$, but $\mathcal{L} \nvdash \delta(\Phi)$, which ends the proof.

5. Platonism from a logical point of view

In this section of the work, I will argue that Platonism is the only logically appropriate position in axiology. However, my justification will be unique because I will completely abstract away from the contents of alternative theories to Platonism. I suggest that their flaws lie in their logical form itself. From this perspective, there are three types of axiological theories: platonism, nihilism, and everything else. The differences between these theories are irrelevant to the arguments presented. Therefore, the argument will have the following structure: first, I will justify the that either platonism or nihilism is true; then, I will argue that it is not nihilism.

Platonism

These three types of metaethical theories are indeed answers to the question about the modal status of evaluative propositions. They can be classified as necessary, contingent, or perhaps they do not have any modal status because they (evaluative propositions) do not exist. When I refer to evaluative propositions, I mean those that express judgments about whether something is good or bad. Therefore, Platonism is defined by the following thesis:

Def. Platonism: For any true evaluative proposition φ and any pair of contingent propositions

$$\psi, \neg\psi, \varphi \text{ is consistent with } \psi \text{ and } \varphi \text{ is consistent with } \neg\psi$$

We leave open the question of what these contingent propositions are about. They might be descriptions concerning people's preferences, cultural conventions, psychological facts about what causes pleasure in a species, or goals set by individuals, whatever else one's favorite axiology says. It is crucial to highlight the subtle distinction between contingent propositions and propositions that at least indirectly describe a contingent entity. For instance, "Kowalski derives pleasure from something" is contingent because Kowalski is not a necessary being. However, some propositions in the form "Something is pleasant to people" may be necessary because they are grounded in the relationship between abstract objects such as the property of being human and the property of being pleasureful. Regarding the condition of consistency between both propositions describing the contingent being, it essentially means that the true evaluative proposition φ must be necessarily true, as claimed in the independence property. This subtlety encompasses two distinct aspects. Let us start with pointing out some problems with grounding value in contingent beings, but I will also consider whether it is possible to reduce The Good to any property like pleasure, happiness, compliance with cultural norms, and so on.

All the rest – Contingentism

A theory is a type of contingentism if it asserts any evaluative proposition is true only if some contingent proposition is true. The specific contingent proposition varies depending on the theory, and we can abstract away from it. Some such theories even claim to represent universal and objective moral values. For instance, theories that derive morality from human species characteristics or nature, understood as an internal aspect rather than an abstract object, fall into this category. An intriguing example of this theory is presented in Chomsky's argument against cultural relativism. (Marshall 2023) He views morality as the biological ability inherent in humans analogical to language acquisition. The acquisition of culture (which also encompasses morality) must be rooted in some biological capacity, a rich internal structure that enables the development of the moral system despite limited input. However, biological capacity sets the range of variability and limits. Just as the ability to see in humans allows for the creation of diverse visual systems but also restricts certain possibilities, such as the inability to develop an insectoid visual system, humans can possess different eye colors but will never produce an insectoid visual system. Relativists draw the conclusion that there are no universal norms based on the existence of diverse moral systems. Conversely, Chomsky reaches the exact opposite conclusion. The spectrum of various moral systems demonstrates the range of variability in moral systems, but this fact also implies the existence of a limit to this possible variability, leading to the existence of universal values. I find the most intriguing aspect of this argument to be the logical connection between the scope of variability and its limitation. This observation is particularly relevant in the context of relationist theories of value, which posits that values are neither subjective nor objective properties but rather relations between subjective and objective elements. However, this position faces the same logical challenges as cultural relativism.

Consider a relation between subjective and objective elements. The question arises about the extent of variability in the objective side. For instance, is it possible for anything to be desired rationally? If yes, the specific desires depend solely on the subjective side because there is no restriction on the objective side. Consequently, relationism collapses to subjectivism. On the other hand, if the scope of variability on the objective side is limited, as in Chomsky's argument, there is a range, implying a limit. Therefore, there exists a set of objective factors that determine what one can (for example) rationally want. Apart from these factors, there are entities outside the range variability. In this case, relationism collapses to objectivism.

In other words, let us consider the entire range of possible value systems to choose from. Even the silliest inventions, such as those in which the most valuable thing is standing on one leg while eating kiwi with Chinese chopsticks or valuing the purity of the Aryan race, fall within this spectrum. By limiting the objective side of the spectrum, we can narrow down the range of *valid* value systems to a more manageable set. It does not matter whether this spectrum narrows to a set of valid systems with cardinality of 1, 17, 100000000, or some transfinite number. What is crucial is that this objective restriction divides the set of possible value systems into two categories: one excluded by the objective conditions, and ones consistent with them. The properties that distinguish these two sets are objectively valuable, regardless of our choices. This is because decisions are only possible within the framework of the set of valid systems, and these properties are common to all of them. Subjective decisions, on the other hand, are only possible within the framework of objective criteria. This type of relationism falls into contradiction because it can only occur on the platonic "substance" of non-relational values, which are common to everything that objective criteria do not exclude.

But back to contingentism. These theories must take the form of conditionals. Since, according to them, there are no independent and necessary evaluative propositions, because some specific contingent condition needs to be satisfied for any evaluation to be true, therefore any evaluative proposition can only occur in such a theory as a consequent of the implication.

$$\varphi \Rightarrow \psi\colon \varphi \text{ is contingent and } \psi \; is \text{ evaluative}$$

And also, so that the theory is not platonic.

$$\neg\varphi \nRightarrow \psi\colon \neg\varphi \text{ is contingent and } \psi \; is \text{ evaluative}$$

Instead of a proposition φ, there may also be a schema of a propositions. For example, a schema of propositions that speak of happiness, pleasure, reason, beauty, the command of a deity, or whatever someone's favorite axiology says. Consider, however, is φ evaluative or descriptive?

Case (1): If φ is evaluative, then the theory is false, because then φ will need some preceding contingent condition φ', but then the theory that postulates φ rather than φ' as a foundation of value is false. In other words, φ can either be evaluative, and what the value of it describes is independent of anything else, and then it is not contingent and the theory collapses to platonism, or is logically dependent on some contingent proposition φ', and then the theory postulating the condition φ as fundamental is false.

Case (2): The second option is that φ is descriptive. Then ask, is φ better than $\neg\varphi$? This is the most important question to which no form of contingentism can give a meaningful answer. This is because if φ is better than $\neg\varphi$, then since according to the theory φ is foundational to every evaluative proposition, $\varphi \Rightarrow \delta(\varphi)$ occurs, but at the same time $\neg\varphi \nRightarrow \delta(\varphi)$, because otherwise the theory collapses to platonism. A theory of this type, especially (although not necessarily) when its foundational proposition φ refers to some human characteristics such as will, happiness, pleasure, rationality, life, etc. is completely infertile for considerations of the most important axiological–existential questions, in which one wants to evaluate not this or that goal, but the basic human condition. Does our happiness count, why is it worth valuing human happiness? Is it better to be a rational animal, or is it better to submit to the base biological instincts? All theories referring to some state that man can achieve, as the foundation for value judgments, presuppose the good of this type of life that man can lead, that something highly valuable can be achieved, and if this cannot be justified, then the whole building intricately built on this premise is as barren as its foundation. Another way of analysis leading to the same conclusion is to ask a question about the axiological status of the conditional $\varphi \Rightarrow \psi$ itself. Such a question, "Is it good that evaluative propositions are founded on this foundation?". If it is not good, then we have the absurdity, then it is said that it is evil or indifferent, that there is a foundation for the goodness of all that is good, and therefore that it is evil or indifferent, or anything good can exist. If, on the other hand, it is good, then we fall into the same triviality as before. By assumption if $\delta(\varphi \Rightarrow \psi)$, $\varphi \Rightarrow (\delta(\varphi \Rightarrow \psi))$ and so we only obtain to barren tautology. If φ is true, it is good that φ is the foundation of the axiological sphere, but if φ was false, it would be good that something else was the foundation.

Platonism but of what sort?

Platonism avoids the above problem, because if axiological truths are primary in relation to a contingent entity, they can play the role of a universal standard. This conclusion was muddily expressed by Wittgenstein (2010), writing;

6:41 The sense of the world must lie outside the world. In the world everything is as it is and happens as it does happen. In it there is no value and if there were, it would be of no value. If there is a value which is of value, it must lie outside all happening and being-so. For all happening and being-so is accidental. What makes it non-accidental cannot lie in the world, for otherwise this would again be accidental. It must lie outside the world.

There are several misunderstandings in this thought. For instance, it creates an opportunity to confuse value with a valuable object, and it also confuses being a value with being valuable. (values and exemplars of values) However, the basic idea seems to suggest that if a value were a contingent entity, it could not serve as a universal standard or a measure of the comparative evaluation of various possible states of the world. This is because in another possible state of the world, there would be other values, and the current values from the perspective of that alternative state would be ineffective in that state. Interestingly, the problems associated with contingentism in a modified form also arise in one platonistic perspective. Recall the thesis indefinability of the proposition $ex(goodness)$ (goodness is exemplified by something) by means of other properties:

There is no formula Φ, such that:

1. The predicate of goodness does not occur in Φ.
2. Φ expresses the proposition φ
3. $\varphi = \bigwedge(ex(P_1), \dots ex(P_\gamma)$ such that γ is an ordinal and for $1 \leq i \leq \gamma$, $egz(P_i) \nRightarrow \varphi$
4. $\varphi \Leftrightarrow ex(goodness)$

For example, is goodness a simple property or a complex one? Can it be defined as for example, good is what leads to happiness, good is beauty, etc. I am in favor of a simple property view, that is the indefinability in the sense specified above, because it is the only one which does not generate a version of Moore's open question.

My version of the open question scheme for any theory that proposes a definition of $ex(goodness)$ is "Is it good that there is nothing better than what the definition of $ex(goodness)$ specifies?”. If one opposes the simple view, such a question is open, at least in the basic sense that asking the question does not indicate semantic incompetence, the answer would require something more than an explanation of the meanings of words. The question is also open in a similar sense as in the problems of contingentism. There is no broader perspective, a standard more basic than goodness, to which one could refer to judge the open question. Although when the axiological sphere is consists of necessary beings, the theoretical situation is not as hopeless as in the case of contingentism. In the case of necessity, one can expect some evidence showing that there would be a contradiction in "something better than *goodness*". The simple view and the presented theory of The Good do not generate this open question, because according to it, "Is it good that there is nothing better than what the definition of $ex(goodness)$ specifies?” is a closed question. The Good is The Perfect, the best of possibilities. Therefore, this question is equivalent to "Is it good that there is nothing better than what the criteria of perfection specify”, which is a closed question.

Nihilism

There are three primary axiological theories: Platonism, Contingentism, and Nihilism. I have previously demonstrated why Platonism is superior to Contingentism. Now, I will elucidate my rejection of Nihilism. I employ the term “elucidation” as a measure of caution, as “justification” may be excessive. I will analyze Nihilism here as a specific form of philosophical skepticism, specifically targeting axiology. I will refer to Strong Nihilism and Weak Nihilism. Weak Nihilism posits that nothing possesses value. Weak Nihilism has been refuted in previous sections of my work. For instance, I provided proof that every proposition must have a value or that every proposition must be good or bad, not indifferent. Even if something was indifferent, it is secondary to the relationship between values. Strong Nihilism must be associated with philosophical skepticism. It must deny the possibility of axiological knowledge and assert that logical reasoning, deduction, and so on, are not sources of knowledge. There must be some inherent illusion in our intellects. However, there is no argument against skepticism or Nihilism. Admittedly, arguments from performative contradiction are possible, such as that a nihilist or skeptic, despite their declared views, still acts as if they are not nihilists or skeptics, or that they implicitly accept certain theses as true. Nevertheless, these arguments are not good arguments against skepticism or nihilism. At best, they constitute arguments against skeptics. This occurs for several reasons.

- They only show hypocrisy, and the hypocrite may also be right.
- The fact that hypocrisy is not avoidable only confirms the skeptical thesis about the unavoidability of axiological illusion.
- Skepticism rejects all theses, and therefore the thesis that one should act on the basis of one's beliefs.
- Nothing arises from nothing, every argument must have premises, skepticism rejects all premises, so an argument against skepticism is impossible.

It is important to note that nihilism and skepticism are not relevant concerns. The foundation for rejecting nihilism and skepticism lies in the principle of phenomenal conservatism (Huemer, 2007), which essentially serves as an epistemological version of the principle of “innocent until proven guilty.” If we accept this principle or an equivalent negation of global skepticism, we can revisit the consideration of Weak Nihilism and the previously presented evidence against it. One cannot argue against Strong Nihilism in this way. However, if Strong Nihilism is not rejected, its rejection does not make a difference. We can still adhere to the principle of phenomenal conservatism. Nihilism posits the rejection of all normativity, including epistemic normativity. If nihilism were true, it would negate the rule against accepting claims without justification. Consequently, we could adopt phenomenal conservatism even without justification and thus return to our initial position. In this manner, nihilism does not have any significant impact.

Summary

My objective was to identify a type of goodness that precedes moral goodness logically and possesses autonomy from empirical facts. I contend that this notion of goodness is fundamental, independent of deontic concepts, yet remains cognizant of the broader framework of ontology and value theory. I argue that logic is not just about organizing thoughts but also about the inherent regularities that exist within specific domains, like the domain of values. This approach enables me to place axiological logic within a broader ontological framework.

Central to this investigation lies the reconceptualization of goodness as a property. Instead of directly attributing it to objects, I propose that goodness is attributed to propositions. For instance, instead of asserting that *Stoic sage is good*, I argue that the proposition, *Something is a Stoic Sage* is good. This propositional approach not only resolves certain paradoxes but also facilitates understanding of conditional value statements. A central theme explored in this work is the intricate relationship between descriptive facts and evaluative properties. I challenge the conventional notion that evaluative properties are not implied by descriptive ones. This rigid separation leads to inconsistencies, making it challenging to discern whether certain actions are virtuous or vile solely based on their descriptive characteristics. I contend that goodness must be logically grounded in the descriptive aspect of objects. I argue that evaluative properties logically follow from descriptive ones, even if the transition does not make proofs possible. In developing axiological logic, I introduced value operators like good, evil, and axiological neutrality. One key idea is the "Local Peak Property", which states that something is good if it is the best among logically related propositions. This property, along with other relational structures, extends classical propositional calculus into a framework capable of expressing nuanced axiological judgments.

An essential aspect of this framework is the principle that all necessary truths are good. I draw an analogy to the necessitation rules in modal logic but offer a semantic justification. If necessary truths were neutral or evil, this would imply that the existence of good things is itself neutral or evil, which is clearly an untenable position. I further explore how axiological logic handles complex expressions like conjunctions, disjunctions, and implications. For example, I propose that a conjunction is good only if all of its components are good, while a disjunction is good if at least one of its constituent propositions is good. These rules provide a systematic way to analyze complex value relations, extending the reach of axiological logic.

In the third chapter of the work, I discussed values themselves (as distinct from good, evil) and show how they are the missing link between the non-normative and the normative. In this part, I propose that foundationalism about values leads to critical philosophical errors and propose to replace it with, the theory of values that emphasizes structural network–like dependence of every value. This

part also tries to formulate some laws about the way to sum values together. The results, however, are mostly negative, which casts doubt on a lot of axiological reasoning which seems to rely on some implicit notion of value summation.

In the last chapter, I picked up what I consider to be the most valuable part of anti-naturalism, which is the *open question* argument and the simplicity of goodness thesis. I argue that a form of platonism that affirms the simplicity of goodness is the only axiology that avoids the *open question*. The logic presented could benefit from further refinement by the community. For example, I am not fully clear on where the logic stops and the metalogic begins. The status of principles such as local, and global peak properties also seems to need further work. At any rate, I hope this work at least opens some questions while avoiding, the *open question*.

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