Ancient Philosophy 35 (2015)

What is a Perfect Syllogism in Aristotelian Syllogistic?

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Aristotle called certain syllogisms in his logical system 'perfect'. In his assertoric logic, treated in *Prior Analytics* i 4-7, all four first figure syllogisms get this title, i.e., *Barbara*, *Celarent*, *Darii*, and *Ferio*, and no syllogism in any of the other figures is termed perfect.[1] Hence in the assertoric syllogistic all and only first figure syllogisms are perfect. In Aristotle's modal syllogistic, however, it is still only syllogisms in the first figure that are perfect, but here some first figure syllogisms are explicitly characterized as imperfect. The question what is the criterion to tell a perfect syllogism from an imperfect one had led to heated debates among Aristotelian scholars already in antiquity.

The proposals made in the course of history are by now only of historical interest. It was one of the merits of Patzig 1968 to elucidate this part of Aristotle's philosophy by pointing out that the distinction between a perfect and an imperfect syllogism has nothing to do with the distinction between valid and invalid syllogisms, but rather that it is based on the evidence of certain valid ones.[2] Here are Aristotle's definitions:

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[1] These designations for the valid first figure syllogisms, invented by medieval logicians, contain information about the logical form of a syllogism: the vowels used in these words are taken from the vowels in the Latin words for affirm (*affirmo*) and deny (*nego*); *a* indicates a universal affirmative proposition, *e* a universal negative one, *i* a particular affirmative proposition and *o* a particular negative one. An Aristotelian syllogism consists of three predicative propositions of the *a, e, i*, or *o* form, one serving as conclusion, the two others as premisses. This set of propositions contains three terms, expressions standing for subject and predicate, and each of these three terms occurs in two different propositions. The term occurring in the two premisses is called the middle term. The term that is used as predicate term in the conclusion is called the major term, the one used as subject in the conclusion is called the minor term. The premiss containing the major term is then called the major, the other one the minor premiss. It is a writing convention to arrange the three propositions as major, minor, conclusion. In a first figure syllogism the subject term in the conclusion is also used as a subject term in the major premiss and the predicate term of the conclusion is also used as a predicate in the minor premiss.

[2] In recent literature on Aristotle's logic, the question of perfection is not a hotly debated topic. The scholarship dedicated to Aristotle's modal logic—and an important part of the prominent literature on Aristotle's logic over the last years has been dealing with his modal syllogistic—can safely ignore the question of what distinguishes a perfect from an imperfect syllogism. The discussion of Aristotle's modal syllogistic is mainly concerned with ways to make this part of Aristotle's logic consistent, and in order to do so, there is no need to decide the question of perfection. Barnes 2007, 378-447 offers a lengthy discussion of perfection, but Barnes does not discuss any modern author, he is concerned only with the ancient commentators and authors such as Galen and the Stoics. And it is not clear what answer Barnes wants to offer to the question of the criteria for a perfect/an imperfect syllogism.

> I call a syllogism perfect if it requires nothing, apart from what is comprised in its assumptions, to make its necessity evident (πρὸς τὸ φανῆναι τὸ ἀναγκαῖον); imperfect if it requires one or more steps which, although they are necessary because of the terms laid down, have not been assumed in the premisses. (*APr*. i 1.24b22-26)

So Patzig rightly stressed the words φανῆναι τὸ ἀναγκαῖον, 'to make the necessity evident'. He also tried to explain wherein the evidence of the perfect syllogisms consists. And he claimed, again quite correctly, that in order to see the evidence of the perfect syllogisms, their wording is of paramount importance. Their evidence does not become apparent if you formulate, e.g., *Barbara* in the traditional manner as:

> All *B* is *C*.
> All *A* is *B*.
> Therefore all *A* is *C*.

Yet if you stick to the formulations Aristotle mostly uses, the 'Aristotelian' formulations as Patzig 1968, 57 calls them, the evidence of *Barbara* is indeed quite conspicuous:

> If *A* belongs to every *B*, and *B* belongs to every *C*, then *A* belongs to every *C*.

As Patzig has pointed out, the position of the middle term here is indeed in the middle, and one can see that the conclusion follows evidently from the premisses. Why? Because of the transitivity of the 'belongs to every'-relation, the *a*-relation. This relation is as clear as is the relation of being-in: If the closet is in the room, and the room is in the house, then the closet is in the house.

This works well in the case of *Barbara*, but unfortunately, so it seems, not in the case of any of the other three perfect syllogisms, i.e., *Celarent, Darii*, and *Ferio*. Because in their case we are dealing with more than one logical relation, and transitivity presupposes one relation only. And none of the other logical relations occurring in these syllogisms, neither *e* nor *i* nor *o,* is transitive. Thus Patzig 1968, 52 claims that for the other three first figure syllogisms, 'their alleged evidence cannot lie in the transparency of a (non-existent) transitivity. Then were does it lie? This question can be readily answered if we call a theorem of the logic of relations to our aid.' Patzig points out that Aristotelian syllogisms can be written as relative products, e.g., for first figure syllogisms in the form $xR|Sz \equiv \exists y\ (xRy\ \&\ ySz)$,[3] and that a first figure syllogism, when written as a relative prod-

[3] Since I sometimes use the symbolic language of first order predicate logic, some explanation of this symbolism might be appropriate. A predicative proposition like 'Socrates is a man' would be written, using upper case letters for predicate expressions and lower case letters for (names of) individuals, as '*Ms*', with '*M*' for 'man' and '*s*' for 'Socrates'. The negation sign '~', standing for 'it is not the case that', so '~*Ms*' would mean 'it is not the case that Socrates is a man'. Other symbols are for 'and', '&'; 'if, then', '⊃'; 'if and only if', '≡'; the letters '*R*' and '*S*' represent two-place-predicates, indicating relations. Thus a complex proposition such as 'if Socrates is a man, then Socrates is an animal' would be '$Ms \supset As$', with '*A*' for 'animal'. Now if we drop the '*s*' in this formula and write '*M*... ⊃ *A*...', we have no longer a proposition that could be either true or false, but only the form

uct, does have its middle terms next to each other, whereas this is not so in the other figures: '*Datisi*, however, written as a relative product, would have the form "xRy & zSy"' (Patzig 1968, 53). And Patzig claims that written this way the order of terms in first figure syllogisms makes them more transparent than their counterparts in the other figures. Their terms are, as Patzig 1968, 53 has it, borrowing a technical term from P. Lorenzen, 'normed':

> It is as a matter of fact easier to see that *x* must be the grandfather of *z* if we are told that *x* is the father of *y* and *y* the father of *z*, than if we are told that *x* is the father of *y* and *z* is the son of *y*. In exactly the same way *Darii* (I) is more evident than *Datisi* (III). (Patzig 1968, 54)

We may agree with Patzig, even if the greater transparency of the grandfather example could at least partly be due to the fact that we are dealing twice with the same relation (i.e., father).[4] Yet even so, we still would have to grant a greater transparency to *Barbara* than to the other three first figure syllogisms. For *Barbara*, so it seems, over and above the 'normed' ordering of its terms, has the advantage of the transitivity of the *a*-relation. It thus seems to outshine all the other three first figure syllogisms. However, nowhere in Aristotle is there any sign of a position of *Barbara* that gives it a status over the other perfect syllogisms.

Thus Patzig's explanation, relying on what he has called the Aristotelian formulation of the premisses, has the drawback that it establishes in the assertoric syllogistic a difference of evidence between *Barbara* and the other three first figure syllogisms.[5] If, as Patzig 1968, 67 states, 'all evident syllogisms must be equally evident', then this condition is not met even by the assertoric perfect syl-

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of a proposition, a proposition 'with gaps', as it were. However, to express the thought that whoever is a man, is also an animal, logicians have found an ingenious device: they fill these gaps with lower case letters taken from the end of the alphabet, *x*, *y*, *z* to serve as variables or placeholders for proper names, and put a symbol in front of this propositional form, the universal quantifier '∀' attached to a variable. The job of the quantifier it is to bind the variable in its range, indicated by brackets. In this way a propositional form is turned into a proper proposition, one that can be true or false. Thus '∀x ($Mx \supset Ax$)' says: anything that is a man, is an animal. A similar device has been developed for particular propositions, the so-called existential quantifier: '∃'. Stating that some Germans are philosophers is equivalent to saying that there is at least one thing that is both German and a philosopher, which yields (using '*G*' for 'German' and '*P*' for 'philosopher') '∃x (*Gx* & *Px*)'. To be able to state an inference as well as a conditional proposition, logicians use '⊢' as a logical symbol for 'therefore'.

[4] Would we still grant that it is as a matter of fact easier to see that *x* must be the uncle of *z* if we were told that *x* is the brother of *y* and *y* the father of *z*, than if we were told that *x* is the brother of *y* and *z* the son of *y*? This example is certainly closer to the Darii/Datisi case than the grandfather example.

[5] Smith 1989, 110 explains the difference between a perfect and an imperfect syllogism as 'the difference between a valid argument and an *evidently* valid argument', without elaborating on the nature of the evidence of the perfect syllogisms. Striker 2009, 82 refers to Patzig's explanation—'Patzig points out that Aristotle's version highlights the transitivity of the a-relation'—without any reference to the difficulty already noticed by Patzig concerning the other three perfect assertoric first figure syllogisms.

logisms. Patzig's explanation becomes even more problematic when we come to Aristotle's modal logic. For here several of those modal syllogisms that Aristotle recognises as perfect, four out of six (of the form *Barbara*), do not, according to Patzig, deserve this title and, hence, Aristotle was wrong: 'Our investigations have shown that when Aristotle designates certain syllogisms as 'perfect', he is asserting that they are substantially more transparent than all other syllogisms. This assertion is false' (Patzig 1968, 67). How did Patzig arrive at this somewhat disappointing conclusion?

Here is his list of modal syllogisms in *Barbara* recognized as perfect by Aristotle. (I shall use Patzig's symbols: *N* stands for necessity, *P* for two-sided possibility or contingency; however, in what follows I shall replace Patzig's *P1* for one-sided possibility [not impossible] by *M,* and his arrow → by the horseshoe ⊃ as the symbol for implication. In characterizing the modal structure of a syllogism, I shall use *A* for an assertoric proposition. I shall take it for granted that all terms in Aristotelian syllogisms are non-empty.):

(1) *NAaB* & *NBaC* ⊃ *NAaC* (*Barbara NNN*, i 8.29b36-30a5)
(2) *NAaB* & *BaC* ⊃ *NAaC* (*Barbara NAN,* i 9.30a17-23)
(3) *AaB* & *NBaC* ⊃ *AaC* (*Barbara ANA,* i 9.30a23-32)
(4) *PAaB* & *PBaC* ⊃ *PAaC* (*Barbara PPP*, i 14.32b38-33a1)
(5) *PAaB* & *BaC* ⊃ *PAaC* (*Barbara PAP*, i 15.33b33-36)
(8) *PAaB* & *NBaC* ⊃ *PAaC* (*Barbara PNP*, i 16.36a2-5)

There are two that are explicitly called imperfect:

(6) *AaB* & *PBaC* ⊃ *MAaC* (*Barbara APM*, i 15.34a34-b2)
(7) *NAaB* & *PBaC* ⊃ *MAaC* (*Barbara NPM*, i 16.35b38-36a2)

Patzig 1968, 62 takes the letters expressing modality to be 'determining the *predicate* of a proposition'. He then compares syllogisms (1) and (2) and comments on them as follows: 'In fact only the second of these syllogisms has the property which we have discussed before and have seen to generate evidence—that is the *identity* of the last member of the first relation and the first member of the second.' Besides (2) it is only (5) that has this property and can justly be called perfect. In all the other syllogisms, thus Patzig, certain operations are needed to arrive at this identity of the middle term. In (1), (3), and (8) the minor premiss has to be changed to *BaC*, using the law 'that *NBaC evidently entails BaC*' (Patzig 1968, 62). (4) is a special case, since '*PBaC* does not evidently entail *BaC*—it evidently does *not* entail it' (Patzig 1968, 63).

So Aristotle here makes use of a new definition of ἐνδέχεσθαι ὑπάρχειν, the one given at i 13.32b23-37, 'that the proposition "*A* can belong to all *B*" may be taken as equivalent to the proposition "*A* can belong to all things to which *B* belongs" (*PAaB*), but it may also be construed as "*A* can belong to all things to which *B* can belong" (*PAaPB*)' (Patzig 1968, 63). So this stipulation makes it possible to add the modal operator to the subject term of the major premiss and hence establish the identity of the two occurrences of the middle term. 'Thus (1), (3), (4), and (8) each need one modal operation to become perfect syllogisms ((1), (3), and (8) an operation based on law (I) *NBaC* ⊃ *BaC* ; (4) one based on law (II), *PAaB* ⊃ *PAaPB*)' (Patzig 1968, 66). The syllogisms called imperfect by Aristotle, i.e., (6) and (7), do need three such operations (Patzig 1968, 66). Over and above the two laws (I) and (II), they need law (III): *AaB* ⊃ *MAaB*. What is actually the case is not impossible.

Thus Aristotle, according to Patzig, seems to allow syllogisms to be perfect that need only one operation, whereas syllogisms needing more than one operation are imperfect. That would be a rather arbitrary division. Hence Patzig wants to restrict perfection only to cases (2) and (5). The other ones should be deemed imperfect.

I

Patzig's criticism of Aristotle's theory of perfect syllogisms has not gone unchallenged. Richard Patterson agrees with Patzig that the transitivity of the relation 'applies to all' makes *Barbara* transparent. This however, as Patzig had already acknowledged, does not help us with the other three assertoric first figure syllogisms since the other syllogistic relations, *e* or *i* or *o* are not transitive.

> Thus in generalizing from Barbara one arrives not at a single principle involving transitivity, but at the rather less exciting principle that in all four cases 'the end of the … step from A to B and the beginning of the … step from B to C coincide'. (Patterson 1993, 362; Patzig 1968, 52 quoted)

Now Patterson 1993, 362, as to the group of assertoric syllogisms, wants to give the *dictum de omni et nullo* a priviledged position for a general explanation of perfection: 'it can in fact play, with respect to Barbara, Celarent, Darii, and Ferio, the kind of role played by the the definition of transitivity with respect to Barbara'. And he provides us with a formulation of 'the general *dictum*…slightly reworded' that will fit all four first figure syllogisms:

> If A applies (fails to apply) to erverything to which B applies
> and
> B applies to all (or some) of the Cs,
> then A applies (fails to apply) to all (or some) of the Cs.'
> (Patterson 1993, 363)

This may in fact be a general schema that covers all four first figure syllogisms, yet it is also clear that the beauty of transitivity has vanished.

However, it is with Patzig's treatment of the modal syllogisms that Patterson most strongly disagrees. He first takes issue with Patzig's discussion of modal *Barbara* (4) above and points out that his law (II) *PAaB* ⊃ *PAaPB* that he used to turn (4) into a perfect syllogism is simply not a valid implication: 'the fact that possibly-and-possibly-not-A applies to every actual B does not entail that that same predicate applies to every possibly-and-possibly-not-B' (Patterson 1993, 369). It can be refuted by the following counter-example: '(Let A = Walking, B = White Thing on the Mat, and let all actual Bs be cats, but some ppBs be cloaks.)' [pp is Patterson's symbol for Patzig's P]. Hence this implication cannot be used to turn Patzig's syllogism (4) into a perfect syllogism. Moreover, the syllogism

(4) in Patzig's reading, i.e., $PAaB$ & $PBaC \supset PAaC$, is not valid either. Here is Patterson's counter-example: 'Let A = Walking, B = Thing on the Mat, C = Cloak, in a situation in which all actual Bs are cats' (Patterson 1993, 370). Yet the syllogism with the modal operator *P* added to the subject term of the major premiss, according to Aristotle's new definition of 'possibly belonging' at i 13.32b23-37, i.e., $PAaPB$ & $PBaC \supset PAaC$, is perfect right from the start (Patterson 1993, 370).

As to the three cases of *Barbara* syllogisms (1), (3), and (8) that all, according to Patzig, need the law (I), i.e., $NBaC \supset BaC$, Patterson 1993, 370 claims that 'it would be perfectly natural for Aristotle to declare syllogisms (1), (3), and (8) obviously valid *just as they stand*—that is, on the basis of what is explicitly and directly stated in the premises'. The application of Patzig's law (I) 'would be, in Aristotle's view, completely superfluous'. It seems indeed rather natural to go from 'applies necessarily' to 'applies', and the law $NBaC \supset BaC$ is certainly different from the rules of conversions like $AeB \supset BeA$, for which Aristotle offers proofs.

At the same time, Patterson wants to use the *dictum* that he had used to find a general principle for the first figure assertoric syllogisms to formulate a general principle for modal syllogisms. However, the formulations he offers (Patterson 1993, 373) are all read off from *Barbara* and hence do not cover any other of the four first figure modal syllogisms. Moreover, the wording of these formulas is so contorted that it is hard to see how it could make the necessity of the perfect modal syllogisms obvious or transparent.

II

Thus Patzig's as well as Patterson's treatment of Aristotle's perfect syllogisms seem to be somewhat disappointing. Transitivity, which is certainly apt to make the necessity of a syllogism apparent, in their accounts is restricted to *Barbara* alone and of no help with the other first figure syllogisms. However in their discussions, both Patzig and Patterson have failed to notice an important point in Aristotle's presentation of his perfect syllogisms, and I shall argue that taking this point into account will give Patzig's basic idea, i.e., transitivity, a wider application than he and Patterson realized. This explanation will also allow all and only syllogisms claimed to be perfect by Aristotle a status of equal evidence.

This explanation is based on a specific formulation Aristotle uses when he presents for the first time two of his perfect syllogisms in i 4. To quote Aristotle:

> When three terms are so related to one another that the last is contained in the middle as in a whole and the middle is contained or is not contained in the first as in a whole, then it is necessary that there is a perfect syllogism of the extremes. (*APr*. i 4.25b32-35)

Neither Patzig nor Patterson nor any one of the more recent commentators has tried to explain why Aristotle did use this somewhat artificial wording of 'being-contained-as-in-a-whole'.[6] Patzig 1968, 91, of course, notices this phrasing as somewhat deviant from the standard formulation,[7] but he is eager to rewrite Aristotle's wording in this passage in the standard, i.e., 'being-said-of' formulation, without asking why Aristotle made use of this unusual formula in the first place.

Aristotle uses the wording 'being-contained-as-in-a-whole' to formulate a syllogism in *APr*. i only in the passage quoted above. The only other instance to formulate a syllogism occurs at *APr*. ii 1.53a21-24. So why should this rarely used expression have an important part to play in Aristotle's syllogistic and what role could it play with respect to the question of perfection? Although it is rarely used, the importance of this formulation in Aristotle's syllogistic emerges from more than one passage. First of all, in the very first paragraph of the *Prior Analytics* it is listed among the expressions that are earmarked for an explanation later in the chapter, together with concepts that will be central to the subsequent investigation, i.e., 'premiss', 'term', 'syllogism', 'perfect', and 'imperfect syllogism' and 'to be said of every' and '(to be said) of no one'. The explanation given for this expression comes just before the one for 'being said of every' (κατὰ παντὸς κατηγορεῖσθαι): 'For one term to be contained in another as in a whole and (for one term) to be predicated of every one of another is the same' (*APr*. i 1.24b26-28). Thus Aristotle stresses the (logical) identity of the two expressions, and identity seems to be more than mere logical equivalence. For in the case of logical equivalence one might need a proof, e.g., for the equivalence of *e*- or *i*-conversions. That is not the case here. We can always replace the one expression by the other, since they are the same (ταὐτόν).

Second, at the beginning of the modal chapters, we find the two expressions again closely connected: 'We shall give an analogous explanation (ὁμοίως ἀποδώσομεν) of the expression "to be contained as in a whole" and "(to be predicated) of every"' (*APr*. i 8.30a2-3). Moreover, the passage at the beginning of chapter 4 quoted above puts this formulation in a pivotal position: it is used to state the two universal perfect syllogisms when they first occur in this work, explicitly referring to their being perfect.

It emerges from these passages that this expression is doubtlessly of importance to Aristotle's logical investigation. So why is it of such importance? Now, one of the effects of using this expression is that it compels Aristotle to transpose the premisses in order to—as Patzig 1968, 91 had already noticed—keep the middle term in its middle position. And it is worth noting that he switches the premisses around in the same way in the only other passage where this expression is used to formulate syllogisms, i.e., in *APr*. ii 1.53a21-24: 'If *D* is in *B* as in

[6] English-speaking commentators have not seen a reason for Aristotle's use of this artificial wording. Smith 1989, 113 on 25b32-35: 'This joint statement of *Barbara* and *Celarent* is couched in almost deliberately awkward terminology.' Striker 2009, 95 thinks Aristotle 'may have used this version for didactic reasons'; and in commenting on the occurrence of this formulation in chapter 1, she states that Aristotle 'could easily have dispensed with it altogether' (84). Cf. also Barnes 2007, 388).

[7] Patzig 1968, 92 points out that 25b32-35 is 'the *only* place in *A* 1-7 in which a syllogism is expressed by means of the constant "… be contained in …"'.

a whole, and *B* in *A*, then also *D* will be in *A*. Again if *E* is in *C* as in a whole, and *C* in *A*, then also *E* will be in *A*.' As in i 4.25b32-35, Aristotle starts with the last terms, *D* and *E* respectively and ends up with the first, i.e., *A*.

Thus, whenever Aristotle makes use of the 'being-contained' expression in *APr*., he is eager to turn the premisses around so that the two occurrences of the middle term are adjacent to each other, thus stressing a feature that is needed to bring the transitivity of a syllogism to the surface. Since transitivity can also be achieved with the Aristotelian formulations for *Barbara*, we may wonder why he wants a further wording to express transitivity. The answer to this is simple and quite straightforward: Among the formulations to state a universal first figure syllogism available to (or used by) Aristotle, this expression is the only one to allow formulating not only *Barbara* but also *Celarent* so that this mood gets a transitive reading as well. In order to see this it suffices to state *Celarent* using the 'being-contained' formulation: 'If *C* is in *B* as in a whole, and *B* is not in *A* as in a whole, then also *C* will not be in *A* as in a whole.' In the English the 'not' comes before the expression 'as in a whole', thus the transition is not as smooth as in the corresponding Greek. If we extract the wording for *Celarent* from the Greek at i 4.25b32-35, we get the following: ὅταν οὖν ὅροι τρεῖς οὕτως ἔχωσι πρὸς ἀλλήλους ὥστε τὸν ἔσχατον ἐν ὅλῳ εἶναι τῷ μέσῳ καὶ τὸν μέσον ἐν ὅλῳ τῷ πρώτῳ μὴ εἶναι, ἀνάγκη τὸν ἔσχατον ἐν ὅλῳ τῷ πρώτῳ μὴ εἶναι. Here the negative particle together with the infinite form of the verb, i.e., εἶναι, comes after the Greek for 'as in a whole' and is put at the end of the second premiss and at the end of the conclusion. And this syllogism is as transparent as is *Barbara* when formulated using the being-contained expression at ii 1.53a21-24.

It is easy to see that this formulation of *Celarent* is quite close to a presentation of this syllogism as an inference in predicate logic style, starting with the minor premiss:

$$\forall \mathrm{x}\ (Cx \supset Bx)\ \&\ \forall \mathrm{x}\ (Bx \supset {\sim}Ax) \vdash \forall \mathrm{x}\ (Cx \supset {\sim}Ax)$$

Or, replacing the variable by an arbitrary name:

$$(Ca \supset Ba)\ \&\ (Ba \supset {\sim}Aa) \vdash Ca \supset {\sim}\ Aa$$

Here it is the transitivity of the implication, the if-then connection symbolically expressed by the horseshoe, which is responsible for the transitivity of *Celarent*. Comparing the two expressions, the 'being-contained' wording used by Aristotle and the predicate logic style formulation, helps to see what really does the trick to bring transitivity to *Celarent*: first, it is the separation of the negation from the quantifier and thus from the expression responsible for transitivity. Indeed, as long as one keeps the pronoun 'no one' (οὐδείς), mixing negation and quantification together to formulate the *e*-premiss, it is not possible to generate a transitive reading of *Celarent*. Second, it is the placement of the negation at the very end of the premiss pair and hence at the end of the second, i.e., major premiss. If the negation were to pop up in the first, i.e., the minor premiss, transitivity would be lost. Hence it is this ordering of the premisses together with separating the negation from the quantifier and attaching it to the predicate of the major premiss that makes it possible to extend the transitivity belonging to implication or to the phrase 'being-contained-as-in-a-whole' to (a reading of) *Celarent*.

Thus it seems certain that Aristotle chose the somewhat unusual 'being-contained' formulation precisely to bestow transitivity on both the universal syllogisms of his first figure. With this wording he set up a model to test candidates for perfect syllogisms, in particular when it comes to modal syllogisms.

Now, before tackling the problem of how to explain the perfection of *Darii* and *Ferio* (and of certain modal syllogisms), it seems called for to discuss the question of the role of the *explanations* of 'being said of every/of none', of the *dictum de omni et nullo* in this matter. After all, Aristotle refers three times to these explanations in the exposition of his perfect syllogisms and these references are meant to adduce a *reason* for the necessity of these perfect syllogisms.

There is an explicit mention of the *dictum de omni* as well as of the *dictum de nullo* in Aristotle's treatment of the particular first figure syllogisms. In a wording meant to cover both *Darii* and *Ferio*, he presents them both as perfect: 'If the universal is put with the major extreme, be it affirmative or negative, the particular with the minor affirmative, there is necessarily a perfect syllogism' (*APr*. i 4.26a18-20). When the two syllogisms are treated separately some lines further down, each time one of the explanations is referred to, here is what Aristotle has to say about *Darii*: 'Let us assume that *A* is said of every *B*, *B* of some *C*. Now if "being said of every" [reading κατὰ παντός with the Laurentianus] is what we stated at the beginning, then it is necessary that *A* belongs to some *C*' (*APr*. i 4.26a23-25). *Ferio* comes immediately after this: 'If *A* belongs to no *B*, *B* to some *C*, it is necessary that *A* does not belong to some *C*. For it has been defined also how we speak of '[being said] of none'. Thus (ὥστε) there will be a perfect syllogism' (26a25-28). In the second passage we find perfection presented as a consequence of the definition of being said of none, of the *dictum de nullo*! And the first passage presents the necessity with which the conclusion follows from the premisses as being dependent on the *dictum de omni*.

The first mention of the *dictum de omni* occurs with the universal syllogisms, more precisely, with the first statement of *Barbara*:

> When three terms are so related to one another that the last is contained in the middle as in a whole and the middle is contained or is not contained in the first as in a whole, then it is necessary that there is a perfect syllogism of the extremes. [Leaving out the definitions of 'middle' and 'extremes' in 25b35-37.] For if *A* is predicated of every *B* and *B* of every *C*, then it is necessary that *A* is predicated of every *C*. For it has been said above how we speak of '[being said] of every'. (25b32-40)

Aristotle here uses twice the word 'for' (γάρ). The first occurrence of this 'for' has the task to assign to the subsequent sentence the job of spelling out the *Barbara*-part in the statement at 25b32-35. The word ἀνάγκη ('it is necessary') at 25b38 takes up the ἀνάγκη ('it is necessary') at 25b34. This 'for' could hardly be meant to connect this sentence to the preceding definitions of 'middle' and 'extremes' in 25b35-37. The second 'for', however, is meant to explain the immediately preceding claim that the conclusion of *Barbara* is a necessary consequence of its premisses, or to put it differently, that *Barbara* has the character of necessity. Thus the job of this reference to the *dictum de omni* is strictly parallel to its mention at 26a24-25, where it is presented as a sufficient condition for the necessity (of the conclusion) of *Darii*.

Two observations emerge from these three passages:

> (1) The two explanations of the phrases 'being-said-of-every/of-none' here are always used to explain the necessity of syllogisms that are explicitly characterized as perfect in the immediately preceding or following text.
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> (2) The references to these explanations appear invariably in all three passages immediately after a syllogism has been stated that uses the Aristotelian formulations, i.e., those expressions that, in contrast to the 'being-contained' formulation, do not allow presenting these syllogisms in a wording *generally* apt (i.e., including *Celarent*) to make their transitivity and hence their perfection apparent.

As we have seen, among the wordings used by Aristotle to state syllogistic propositions, it is only the 'being-contained-as-in-a-whole' expression that allows formulating *Celarent* as well as *Barbara* in a transitive reading. This suggests, and the two observations just made would support this suggestion, that Aristotle is eager to find a way to translate, as it were, his usual wordings for (perfect) syllogisms into formulations that are in accordance with the 'being-contained' expression for first figure syllogisms.

A close look at these explanations bears out this suggestion. The *dictum de omni*, the definition of the formula 'being-said-of-every' (κατὰ παντὸς κατηγορεῖσθαι), is stated as follows: 'We say that one term is predicated of every one of another when no thing of those under the subject term (μηδὲν…τῶν τοῦ ὑποκειμένου, the τῶν is not in Ross, but in the Urbinas) can be found of which the other cannot be said' (24b28-30).[8] Aristotle does not give a definition of the universal negative proposition, he simply states, immediately after the passage just quoted: 'and for [being said] of no one, in the same way', καὶ τὸ κατὰ μηδενὸς ὡσαύτως (24b30). However, it is clear how this is meant to be spelled out: 'We say that one term is predicated of no one of another when no thing of those under the subject term can be found of which the other can be said.' The first thing to notice about the explanation of the 'being-predicated-of-every' and 'of-none' expression, is that, in contrast to the Aristotelian formulations and like the 'being contained' wording, it puts the logical subject in first position, with the consequence that the premisses would be placed in reverse order: minor first, major second, so that the occurrences of the middle term would be adjacent to each other. Hence, in this respect, the two formulations, the 'being-contained' word-

[8] For further explanation on this point, cf. Ebert/Nortmann 2007, 229-230. Malink 2008 has pleaded convincingly for retaining the plural τῶν instead of Ross's τῷ in several of the passages where the mss. offer both readings.

ing and the explanation of the 'being-predicated-of-every' with respect to 'of-none', are closely parallel.

The second point to notice is the following: whereas in the Aristotelian formulations or in the use of the copula, the pronouns used to express quantification are attached to the subject term, here the terms of the proposition themselves remain free of these pronouns, the only quantifying expression being attached to the items *under the subject term* ('no thing of those under the subject term'). In this respect, this wording again is similar to the 'being-contained' formulation that also keeps the subject and predicate term of the proposition free of any quantifying expression.

These two observations would suggest that the job of the explanations given for 'being-said-of-every' and 'being-said-of-none' is to offer a way to translate the Aristotelian formulations for his perfect syllogisms into a wording close to the 'being-contained' one.

The third and final observation on these explanations concerns something that has no direct parallel in the 'being-contained' wording, namely, the 'ecthetic' reading of the formulation 'being-said-of-every' and 'of-none'.[9] These formulations treat both the subject and the predicate term of the *a*-proposition (or the *e*-proposition) as if they were predicates of an underlying subject, i.e., 'of the things under the subject term'. You cannot find any item under the subject term of which the predicate term cannot be said.[10]

This formulation of the *a*-proposition seems to correspond to the negated existential proposition:

$$\sim\exists x(Cx \,\&\, \sim Bx),$$

which again is equivalent to

$$\forall \mathrm{x}\ (Cx \supset Bx).$$

That Aristotle here has in mind *universally quantified* propositions is made clear by his talk of 'no thing (μηδέν) of those under the subject term'. Now the 'being-contained' formulation is transitive and since in this respect it functions like an implication, we may read an *a*-proposition (e.g., *BaC*) as,

$$\forall \mathrm{x}\ (Cx \supset Bx),$$

even if talk of an implication with a quantifier binding the variable as a logical technique was not available to Aristotle. In any case, he may have taken the wording provided by the two *dicta* as supplying a transitive reading of the two first figure universal syllogisms in the following way: In case of *Barbara* we should take the subject term of the minor premiss as a predicate of a randomly chosen individual (or subset) under the subject term *C*, a 'thing of those under the subject term', let us call it *n*. Now *n* has the predicate *C*, since it is one of the

[9] I use the term 'ecthetic' with respect to the method of ecthesis employed in *APr*. i 6.28a23-29 in the proofs for *Darapti* and *Felapton*. There Aristotle picks one of the things under the middle term, i.e., *S*, which is the subject term in both premisses, to arrive at the *i*- or *o*-conclusion.

[10] I have argued for the ecthetic reading of this definition in Ebert 1995, 230-232. This reading (together with its use for the explanation of perfection) was endorsed in Drechsler 2005, 290-296. Barnes 2007, 409 calls this reading the 'orthodox interpretation' of the *dictum,* and so does Malink 2009, 109.

things under the subject term, and, because of the account given of 'being-predicated-of-every', n must have the predicate B as well; and having the predicate B also as one of the individuals (or subsets) under the subject term of the major premiss, i.e., B, n must have for the same reason as before also the predicate A. Therefore any n, any item under C, must also have the predicate A.

One can easily see that an analogous account works for *Celarent*. In this case one arrives with n as easily as before at the subject term of the major premiss, and here, because of the account of being predicated of no one, n cannot have the predicate A. Hence, in the case of *Celarent*, any item 'under those of the subject term' of the minor premiss that has the predicate C cannot have the predicate A.

Thus by treating all three terms involved in the two universal first figure syllogisms as predicates of an underlying subject, it is easy to go from the first, i.e., the subject term of the minor premiss, to the third, i.e., the predicate term of the major one, if the being-contained operator is used to connect the terms and the premisses are put in the appropriate order. To use a metaphoric and at the same time simple expression for the idea of this passage from the subject term of the minor to the predicate term of the major premiss via the identical minor predicate and major subject term, I have called it a 'predicate relay race' ('Prädikatenstafette' in German). It is a metaphor meant to express the transitivity involved in these two syllogisms.

Since the being-contained operator for all practical reasons works like the implication of predicate logic, we can represent *Barbara* as

$$(Cn \supset Bn) \,\&\, (Bn \supset An) \vdash Cn \supset An$$

where n is to be taken as an arbitrary name. To get a similar formula for *Celarent*, $\sim An$ has to be substituted for An.

It should be clear that this passage from the subject term of the minor premiss to the predicate term of the major one is not possible in any of the two other Aristotelian figures. They lack the concatenation between the predicate term of the minor and the subject term of the major premiss, since these two terms now contain two different dummy letters or concrete words. There is no transitivity in syllogisms of the other figures.

With transitivity as a feature of appropriate wordings of both *Barbara* and *Celarent*, it remains all the more urgent to explain the perfection of the two particular first figure syllogisms, i.e., *Darii* and *Ferio*. That Aristotle wants them to be treated as perfect emerges from his remarks at *APr*. i 4.26a17-20 characterizing both *Darii* and *Ferio* as perfect, and from 26a28 where *Ferio* alone is called perfect. Unfortunately, Aristotle has not indicated how he thinks the transparency of these syllogisms can be made evident in a way similar to the one we are supposed to use for *Barbara* and *Celarent*. The main difficulty lies in the fact that in both *Darii* and *Ferio* their minor premiss is an *i*-proposition, hence we do not have a continuous sequence of implications. The minor premiss of *Darii*, when written in predicate logic style, with the arbitrary name replacing the variable, is of conjunctive form, and *Darii* looks like this:

$$(Cn \,\&\, Bn) \,\&\, (Bn \supset An) \vdash Cn \,\&\, An.$$

However, Aristotle seems to be aware of the fact that for any *i*-proposition, hence also for *BiC,* there is a subset *D* such that

$$\exists x\ (Cx\ \&\ Bx) \equiv \exists D(\forall x\ (Dx \supset Cx)\ \&\ \forall x\ (Dx \supset Bx)).$$

Patzig already has noticed that Aristotle in i 28.43b43-44a1 comes 'very close to' this logical law. Here are Aristotle's words in Patzig's translation:

> If we want to prove that one term belongs to some of another, we must consider the terms to all of which each term belongs: if a term to all of which the one term belongs is identical with a term to all of which the other belongs, then the first term must belong to some of the second. (Patzig 1968, 162)

In this passage Aristotle states only one half of this equivalence, i.e.,

$$\exists D(\forall x\ (Dx \supset Cx)\ \&\ \forall x\ (Dx \supset Bx)) \supset \exists x\ (Cx\ \&\ Bx)$$

However, if from the assumption (of the existence) of the subset *D* we are entitled to infer *BiC*, it seems reasonable to assume that Aristotle knows that we can infer from *BiC* the existence of a subset *D* such that

$$\forall x\ (Dx \supset Cx)\ \&\ \forall x\ (Dx \supset Bx)$$

or, replacing the variable by an arbitrary name,

$$(Dn \supset Cn)\ \&\ (Dn \supset Bn).$$

If we then replace the minor premiss of *Darii* by the second of these propositions, i.e., $Dn \supset Bn$, we get a *Barbara* syllogism leading to the conclusion $Dn \supset An$. Only with the second of these propositions do we stay within the term arrangement of the first figure, the first proposition, $Dn \supset Cn$, does not connect with the major premiss, i.e., $Bn \supset An$. However, since *D* may be only a part of *C* as well as of *B*, so that there may be *C*s that are not embraced by *B*, we can only infer that there are *some* *C*s that are also *A*s. Hence *Cn* & *An* or *AiC*.

A similar consideration is applicable to *Ferio*. Here we will end up with $Dn \supset {\sim}An$ via a syllogism in *Celarent*. Taking into account that *D* may be only a part of *C* as well as of *B*, so that there may be *C*s that are not embraced by *B*, we can only infer that there are *some* *C*s that are not *A*s. Hence *Cn* & ~*An* or *AoC*.

This reading of the minor premiss $\exists x(Cx\ \&\ Bx)$ gives *B* a stronger position over *C*, since only a *part* of *C* is used to reach the desired conclusion. Whereas the usual reading of the *i*-proposition treats its two terms as strictly symmetrical, hence the use of 'and' (&) in the symbolic notation, the representation used above, turning *B* into the predicate of a universally quantified proposition, introduces an asymmetry into this proposition. Now it is worth noticing that Aristotle sometimes uses a wording for the *i*-proposition that has a similar effect. He talks of '*C* being under *B*' (τὸ Γ ὑπὸ τὸ Β ἐστίν, i 9.30a40), thus introducing an asymmetry between the two terms with *C* being the subject term. This formulation is certainly appropriate when we are dealing with an *a*-proposition, with a case of the subsumption of a *C* under *B*. Aristotle uses it indeed to render a universal proposition, e.g., at i 10.30b13 or at i 14.33b34-36. Yet quite often, at least in the chapters discussing his syllogistic theory, he employs it when he renders an *i*-proposition, and almost always when this proposition is the minor premiss of a syllogism.

The first occurrence of this phrase is to be found at i 4 where it is used to define the minor term in a syllogism, or to be more precise, the minor term in first figure perfect particular syllogisms, i.e., in *Darii* and *Ferio*. It is worthwhile quoting the complete context:

> If one term is universal, one particular in relation to the other, then when the universal is put with the major extreme, be it affirmative or negative, while the particular is put with the affirmative minor, it is necessary that there will be a perfect syllogism, but when it [i.e., the universal] is put with the minor or when the terms are put in some other relation, this is impossible. I call major extreme the one in which the middle is (contained), minor the one being under the middle. (*APr*. i 4.26a17-23)

Notice first that Aristotle when defining the major term relies on the being contained wording that allows him to talk about an *a*- as well as an *e*-premiss. However, Aristotle does not continue (as he might have done, if he had a syllogism in *Barbara* in mind), '(I call) minor the one contained in the middle', but he uses the 'being under' phrase instead. Thus he clearly wants to take into account the specific relation expressed by an *i*-proposition. Since we have seen that this phrase covers also the term-relation expressed in an *a*-proposition, we ought to understand him as pointing to the fact that, when dealing with an *i*-prosition, there is always a *part* in the subject term of this proposition that can be treated as a proper part of (the extension of) the predicate term.

Its second use occurs at i 9.30a40 in the discussion of *Darii NAN*:

> Let us first assume that the universal premiss is necessary, so that *A* belongs to every *B* of necessity and *B* only belongs to some *C*. Then it is necessary that *A* of necessity belongs to some *C*. For *C* is under *B* and *A* belonged to every *B* of necessity.[11] (30a37-30b1)

Aristotle thus discusses *Darapti NAN*:

> Since *B* belongs to every *C*, *C* will also belong to some *B* because of the conversion of the universal to the particular premiss, hence if *A* belongs to every *C* of necessity and *C* to some *B*, then *A* belongs to some *C* of necessity. For *B* is under *C*. So the first figure comes about. (i 11.31a26-30)

Notice that we are again dealing with a case of *Darii NAN*, the first figure syllogism to which *Darapti NAN* here is reduced.

In the same chapter *Felapton NAN* is reduced to *Ferio NAN*:

> Now since by conversion we get that *C* belongs to some *B*, and *A* belongs of necessity to no *C*, then *A* will of necessity not

[11] Commenting on 30a37-40, Smith 1989, 122 explains the meaning of 'being under' as follows: '"under" in this context usually means either "within the extension of" or "a subject of predication of". In the present case, it has to mean something like "part of *C* falls under *B*", if Aristotle's argument is to work'. This supports the interpretation given in the text above.

belong to some *B*. For *B* is under *C*. (31a35-37)

Some lines further down *Disamis ANN* as well as *Datisi NAN* are discussed and reduced to *Darii NAN*. (31b17-21).[12] And each time the minor premiss of the first figure syllogism is presented with its subject term 'being under' the predicate term. Moreover, in each of these passages the relation of being under established between two terms is used to support the character of necessity of the syllogism under discussion; the formula is almost invariably: *For* (γάϱ) *B* is under *C*.

So if we are right to read the *i*-premiss as a sort of restricted *a*-proposition, then the transitivity of both *Barbara* and *Celarent* can be used to bring transparency also to *Darii* and *Ferio*. Thus Patzig's fundamental idea that perfection of first figure syllogisms is based on the transitivity of the wording chosen to represent these syllogisms is even more fruitful than Patzig himself had thought. It would cover all four first figure assertoric syllogisms.

To conclude: The criterion for a perfect assertoric syllogism is indeed the transitivity provided by the 'being-contained' operator as used in Aristotle's first statement of *Barbara* and *Celarent*.

III

As said at the beginning, Aristotle in his modal syllogistic treats first figure syllogisms that are valid, but not perfect. We shall see that these imperfect first figure modal syllogisms are all characterized by a lack of transitivity. Yet before we can discuss this part of Aristotle's logic in detail, a word about the interpretation of Aristotelian modal operators seems called for. Since the seminal study Becker 1933, there has been an ongoing debate as to whether Aristotle uses these operators in a *de re* or in a *de dicto* sense. His conversion rules seem to presuppose the *de dicto* interpretation that takes modal operators to modify a complete sentence, whereas the syllogisms seem to be working only with a *de re* reading that takes the modal operators to be part of a syllogistic term, mostly a predicate. This seems to make Aristotle's modal syllogistic incoherent, a criticism levelled against Aristotle already by Albrecht Becker and most recently by Gisela Striker.[13] However, other scholars have argued that even a *de dicto* reading of the modal operators need not make Aristotle's syllogistic incoherent, provided one does rely on logical systems as S4 or S5, hence allowing modal operators to fall into the scope of other modal operators (see, e.g., Nortmann 1996 or Schmidt 1989 and 2000). Moreover, Rini 2011 made, on the basis of an essentialist semantics, a strong argument for a *de re* interpretation of Aristotle's modal operators throughout, using only first order predicate logic and methods of proof that were known to Aristotle or that can be plausibly attributed to him (see Ebert 2014).

For our purpose, there is no need to attempt a definite answer to the question of how to read Aristotelian modal operators in general. In what follows, I shall pre-

[12] *Disamis ANN* may not be a valid syllogism, see on this question Rini 2000.

[13] See Becker 1933, 42ff.; cf. Striker 2009, 115: 'Given this situation, one can hardly avoid the conclusion that the system of modal syllogisms as it stands is logically incoherent.'

suppose a strict *de re* reading of these operators, mainly for one reason: There seems to be an almost unanimous agreement among scholars that, as far as syllogisms and not conversion rules are concerned, these operators have to be taken in a *de re* sense, hence as modifying terms and not as modifying complete sentences. Thus, agreeing with Patzig 1968, 62 and Becker 1933, 19f. in this matter, I shall take the modal operator to be *part of a term* in a syllogistic proposition, *not as a factor modifying a whole proposition*.

First figure syllogisms that involve only necessary propositions or that mix necessary and assertoric premisses are discussed in *APr*. i 8 and 9. *APr*. i 8 claims that for all three figures, if the necessity operator is added to every proposition, things are the same as in the assertoric syllogistic (with an exception for *Baroco* in the second and *Bocardo* in the third figure, which can be ignored for the present investigation). However, if we spell out the syllogism in *Barbara* with three necessary propositions, we see that Aristotle needs some additional step to make transitivity work. *Barbara* written in predicate style logic as an inference looks like this, using *N* as a symbol for the necessity operator:

$$\forall \mathrm{x}\ (Cx \supset NBx)\ \&\ \forall \mathrm{x}\ (Bx \supset NAx) \vdash \forall \mathrm{x}\ (Cx \supset NAx)$$

or, replacing the variable by an arbitrary name

$$(Cn \supset NBn)\ \&\ (Bn \supset NAn) \vdash Cn \supset NAn.$$

(For *Celarent* we get the following:

$$\forall \mathrm{x}\ (Cx \supset NBx)\ \&\ \forall \mathrm{x}\ (Bx \supset N{\sim}Ax) \vdash \forall \mathrm{x}\ (Cx \supset N{\sim}Ax)$$

and

$$(Cn \supset NBn)\ \&\ (Bn \supset N{\sim}An) \vdash Cn \supset N{\sim}An.)$$

To make transitivity work, we must allow modal weakening, i.e., the step from

$$Cn \supset NBn \text{ to } Cn \supset Bn.$$

If we grant Aristotle this step as something that goes, as it were, without saying, we can regard this syllogism as perfect. Its necessity can be seen without anything further needed.[14]

Aristotle does not explicitly state that *Barbara NNN* is perfect, yet since he claims that for syllogisms with three necessary propositions things are similar to assertoric ones (cf. i 8.29b36-30a2), we may assume that perfection also applies to all of the first figure syllogisms with all their propositions necessary. However, when it comes to premisses where one is necessary, the other assertoric, cases treated in i 9, things are different. Only if the major premiss is necessary, do we get (a perfect syllogism with) a necessary conclusion.

Aristotle supposes that *A* is taken to belong or not to belong of necessity to *B*, and *B* merely to belong to *C*, hence he is talking about *Barbara* as well as about *Celarent*. With the premisses thus chosen, *A* will belong or not belong to *C* of necessity (i 9.30a17-20). He then goes on to give a reason for this claim:

> For since *A* of necessity belongs or does not belong to *B*, and C is one of *B*s, it is evident that one or the other will also of necessity hold of *C*. (30a21-23)

[14] The main reason to attribute this step to Aristotle comes from i 16.36a2-7. The syllogism *Barbara PNP* is explicitly called perfect and this presupposes a step from $Cn \supset NBn$ to $Cn \supset Bn$.

The use of the word 'evident' (φανερόν) indicates that Aristotle is talking about a perfect syllogism. One further point deserves attention: when Aristotle first states this syllogism (in lines 30a17-20) he puts the common minor premiss of the two syllogisms as '*B* merely belongs to *C*'. Yet he has now chosen the formulation '*C* is one of the *B*s' (τὸ δὲ Γ τι τῶν Β ἐστί), thus availing himself of a wording leaning upon the definition of 'being-said-of every', a wording turning around subject and predicate terms of the premiss and presenting *C* as one of the things under the term *B*, hence translating the first formulation of the minor premiss into one that makes it parallel to the being-contained formulation.

However, if in this premiss combination the minor premiss is necessary, the major one assertoric, the conclusion will not be necessary (30a23-25). This is Patzig's number

(3) *AaB* & *NBaC* ⊃ *AaC* (*Barbara ANA,* i 9.30a23-32).

Written in predicate style notation as an inference, we get

$\forall x\ (Cx \supset NBx)\ \&\ \forall x\ (Bx \supset Ax) \vdash \forall x\ (Cx \supset Ax)$

or, replacing the variable by an arbitrary name:

$(Cn \supset NBn)\ \&\ (Bn \supset An) \vdash Cn \supset An.$

Allowing modal weakening from $Cn \supset NBn$ to $Cn \supset Bn$, we still have a case of transitivity, hence a perfect syllogism. However since the predicate of the major premiss does not carry the necessity operator, the conclusion will not be necessary either. Most of the work done by Aristotle on this case is to refute the claim that *Barbara ANN* and *Celarent ANN* are valid syllogisms.[15] For *Barbara ANN*, Aristotle offers a *reductio*-proof as well as a set of concrete terms to show that the conclusion cannot be necessary (30a23-32) and he states that the for the rejection of *Celarent ANN* the proof would be the same (30a32-33).

The particular syllogisms with one premiss necessary, the other assertoric are treated in quite the same way:

> if the universal premiss is necessary, then also the conclusion will be necessary; if the particular premiss (is necessary), then (the conclusion will) not be necessary, neither with a negative nor with an affirmative universal premiss. (30a34-37)

After this general claim, Aristotle looks at *Darii NAN* more closely, first stating this syllogism and then giving a proof for it:

> Let first the universal (premiss) be necessary, so that *A* should belong to *B* of necessity, *B* merely belong to some *C*. Then it is necessary that *A* belongs to some *C* of necessity. For *C* is under *B*, and *A* did belong to every *B* of necessity. (30a37-b1)

So this syllogism yields a necessary conclusion, and the same will hold for *Ferio NAN* (30b1-2). It should be clear that these syllogisms will be as perfect as their assertoric relatives.

Aristotle does not explicitly state that the syllogisms with a necessary minor and an assertoric major premiss, which do not have a necessary conclusion yet must have an assertoric one, are perfect. However, if we allow in this case the

[15] For the intricate problems of this chapter, cf. Ebert and Nortmann 2007, 382-406.

step from $Cn \supset NBn$ to $Cn \supset Bn$ as in the *NNN* case, we should take them to be perfect. That the syllogisms *Darii ANN* and *Ferio ANN* do not yield a necessary conclusion, in this respect similar to the universal cases, is stated and shown by a set of terms (i 9.30b1-6).

As long as Aristotle is dealing with first figure premisses of which at most one is necessary, the other assertoric, we do not hear of any imperfect syllogisms. This changes as soon as he allows assertoric or apodeictic premisses to mix with premisses characterized by contingency (two-sided possibility, i.e., 'possible' in the sense of neither impossible nor necessary).

The first cases ruled out as imperfect are those where the combination of logical relations in the premisses does not correspond to a premiss combination of valid first figure syllogisms in the assertoric logic. These cases are the combinations in *APr*. i 14: *ae PP* (33a5-12), *ee PP* (33a12-20) and *ao PP* (33a27-34) (using the letter *P*, as stated above, to indicate contingency). Here we can at best get an imperfect syllogism (cf. 33a17-20). What hinders these premiss pairs from being premisses of a perfect syllogism is simply the fact that they do not have a valid 'relative', as it were, in the assertoric part of Aristotle's logic. They can be turned into (premiss pairs of) a perfect syllogism by 'modal conversion', i.e., by exploiting the fact that any proposition characterized by the factor of modal contingency implies also the opposite of what it states as possible. If it is possible (in the sense of two-sided possibility) that it will rain tomorrow, then it is also possible that it will not rain tomorrow. Hence, one can move from *PAeB* to *PAaB* and from *PAoB* to *PAiB* (cf. i 13.32a29-b3). Thus the first two cases, *ae PP* and *ee PP*, can be turned into premisses of *Barbara PPP*, or perhaps, in the case of *ee PP*, into premisses of *Celarent PPP*—Aristotle's wording is not clear at this point (cf. 33a12-17), the plural used at 33a16 may indicate that he thinks of conversion into *Barbara PPP*—and the third one, *ao PP*, into premisses of *Darii PPP*, all of which are considered to be perfect syllogisms. (Why *Barbara PPP* and the other first figure syllogisms are treated as perfect, will be discussed further down when syllogisms with *PP* premiss pairs are investigated.)

The second class of imperfect, yet valid first figure syllogisms are those with a contingent minor and an assertoric major premiss. Here all the syllogisms with a contingent minor are imperfect, whereas all those with a contingent major are perfect:

> If one of the premisses is assertoric and the other one contingent, when it is the major premiss that expresses contingency, all the syllogisms will be perfect…, but when it is the minor premiss, they will be imperfect. (*APr*. i 15.33b25-29)

If we write these syllogisms in predicate logic style, this yields for the perfect case in *Barbara PAP*:

$$\forall x\,(Cx \supset Bx)\ \&\ \forall x\,(Bx \supset PAx) \vdash \forall x\,(Cx \supset PAx)$$

or written with an arbitrary name replacing the variable:

$$(Cn \supset Bn)\ \&\ (Bn \supset PAn) \vdash Cn \supset PAn$$

Here perfection can be read off from the transitivity of the relations expressed by the premisses: the predicate term of the minor and the subject term of the major premiss are strictly identical, hence the necessity of the syllogism is clear without requiring any further steps:

> For let it be assumed that *A* possibly belongs to every *B*, and let *B* be posited to belong to every *C*. Now since *C* is under *B* and *A* belongs possibly to every *B*, it is evident that it also possibly belongs to every *C*. So a perfect syllogism comes about. (i 15.33b33-36)

Notice that here again it is the 'being-under' wording that is used to support the evidence of the syllogism *Barbara PAP*. In the next lines, Aristotle states that *Celarent PAP* is perfect as well (33b36-40). Clearly transitivity, and hence perfection, is preserved when negation is added to the predicate of the major premiss and to the predicate of the conclusion.

Yet, when contingency is joined to the minor premiss, transitivity, and hence perfection, is lost. For from $Cn \supset PBn$ we cannot go to $Cn \supset Bn$ to achieve the identity of the minor predicate and the major subject term needed for transitivity. Aristotle states that for *Barbara* and for *Celarent* with premiss combination *AP*, the validity of the syllogisms has to be shown using a *reductio*-proof (διὰ τοῦ ἀδυνάτου δεικτέον, 34a3), and he adds that at the same time, i.e., because this way of proof has to be taken, it will also be clear that they will be imperfect (34a3-4).[16] The situation is virtually the same with *Darii* and *Ferio*: If the major premiss is contingent, the minor one assertoric, this yields perfect syllogisms (35a31-35), if it is the other way around, we get imperfect ones (35a35-40).

The third class of imperfect, though valid first figure syllogisms, is discussed in *APr*. i 16 where premiss pairs with one contingent and one necessary premiss are investigated. The cases discussed are in a way similar to the assertoric/contingent cases, for here, too, perfection is achieved when contingency is joined to the major premiss, and we get imperfect syllogisms when it is joined to the minor one:

> When one of the premisses has an apodeictic and the other a contingency sense, there will be a syllogism if the terms are related in the same way as before; and it will be perfect when necessity goes with the minor term. (35b23-26)

Necessity going with the minor term in this case is as much as saying contingency is going with the major one. Hence perfect *Barbara PNP* in predicate logic will look like this:

$$\forall \mathrm{x}\ (Cx \supset NBx)\ \&\ \forall \mathrm{x}\ (Bx \supset PAx) \vdash \forall \mathrm{x}\ (Cx \supset PAx)$$

or written with an arbitrary name replacing the variable:

$$(Cn \supset NBn)\ \&\ (Bn \supset PAn) \vdash Cn \supset PAn.$$

[16] Aristotle claims that the 'privative ones of the syllogisms' (i.e., the first figure syllogisms with a contingent minor and an assertoric major premiss) do have conclusions with one-sided possibility (33b29-31). Yet the same would be true of *Barbara* and *Darii* with the same arrangement of premisses. Cf. on this problem Ebert and Nortmann 2007, 536-539 and Striker 2009, 141f. For the question of the criterion of perfection, this issue can be ignored.

To achieve transitivity, we have to allow for modal weakening, i.e., for the step from $Cn \supset NBn$ to $Cn \supset Bn$. This yields two identical occurrences of the middle term, hence all we need for transitivity and thus for perfection.

Yet when necessity goes with the major term and therefore contingency with the minor one, there is no way to get two identical occurrences of the middle term, since from $Cn \supset PBn$ there is no logically safe way to reach $Cn \supset Bn$ and thus to two identical occurrences of the middle term. Transitivity breaks down and so first figure syllogisms with a necessary major and a contingent minor premiss can only yield imperfect syllogisms. Thus *Barbara* with *NP* premisses is explicitly called imperfect and yields a conclusion (of one-sided possibility) (35b38-36a1); Aristotle adds that its imperfection is clear from the proof that is similar to the proofs used before (36a1-2). He will have in mind the proofs at 34a36-b1 and 34b19-27 for syllogisms in *Barbara* and *Celarent*, both with an *M*-conclusion. Yet *Barbara* with *PN* premisses leading to a contingent conclusion is called 'perfect, not imperfect. For it is made perfect straightaway through its original premisses' (i 16.36a5-7).

Similarly, *Celarent* with *NP* premisses is imperfect, yet it leads to an assertoric conclusion:[17]

> In case the premisses are not of the same quality, let first the negative one be necessary, such that *A* cannot belong to any *B* and let *B* possibly belong to every *C*. Then it is necessary that *A* belongs to no *C*. (36a7-10)

Its imperfection is not explicitly stated but emerges from the fact that a proof is needed (36a10-17). *Celarent PNP* however is perfect:

> Let the affirmative premiss be necessary, and let *A* possibly belong to no *B* and *B* belong of necessity to every *C*. The syllogism then will be perfect, yet it will conclude not to not belonging, but to possibly not belonging. (36a17-21)

Since his readers may find it astonishing that from the two moods in *Celarent* with a mixture of contingent and necessary premisses, the perfect one leads to a contingent conclusion whereas the imperfect one has an assertoric one, Aristotle adds an argument to explain that the conclusion here can only be contingent (36a21-25). So this argument is not meant to offer a proof for this (perfect) syllogism as such, but only for the modal character of the conclusion.

Things are similar with *Darii* and *Ferio* (36a32). For when in *Ferio* the major premiss is necessary (and hence the minor contingent), the conclusion will be negative and assertoric (36a33-34), just as in the *Celarent NPA* case. *Ferio NPA* is then spelled out using letters (36a34-36) and a *reductio*-proof is given for its validity (36a36-39). Its imperfection is not explicitly mentioned, yet the proof needed for this mood should be sufficient to show that it is imperfect.

Aristotle then turns to the cases where the minor premiss is necessary (and hence the major one contingent). Although this is not explicitly said, these syllo-

[17] For the problems connected with the logical status of the conclusion, assertoric or only one-sided possibility cf. Ebert and Nortmann 2007, 597-601.

gisms should be perfect. However, the text as it stands looks suspicious:

> When the particular affirmative premiss is necessary, the one in the negative syllogism, e.g., *BC*, or the universal in the affirmative (syllogism), e.g., *AB*, there will not be a syllogism yielding an assertoric conclusion. The proof will be the same as before. (36a39-b2)

Aristotle's aim here seems to be clear, namely, to show that *Ferio PNP* (and probably *Darii PNP*) have a contingent conclusion, not an assertoric one as with *Ferio NPA*. *Ferio* with *PN* premisses, should yield a contingent conclusion, and hence be a perfect syllogism (thus already Alexander 1883, 212, 33-35). Bringing a universal necessary major premiss into this context would turn us back to the imperfect *NP* cases of which *Ferio NPA* has been dealt with at 36a34-39. Moreover, if Aristotle wants to talk here of this (imperfect) syllogism, i.e., *Darii NP*(*M*?) in addition to perfect *Ferio PNP*, his referring to a proof that will be the same as before is quite strange. For the proof of *Darii* with *NP* premisses would in all probability be a *reductio*-proof like the one given for *Ferio NPA* at 36a36-39, whereas for *Ferio PNP* as a perfect syllogism a proof for its validity will not be needed, what it may need is a proof that its conclusion can only be contingent, not assertoric, hence a proof similar to the one provided at 36a21-25 to show that (perfect) *Celarent PNP* can only have a contingent, not an assertoric conclusion. So these would be two quite different proofs. And talking of a proof to show that 'there will not be a syllogism yielding an assertoric conclusion' fits this latter case much better. All this speaks against the assumption that Aristotle at this point wants to bring in *Darii* with *NP* premisses.

So I suspect that talk of a necessary universal premiss is an intrusion into Aristotle's text.[18] However, for the problem of the criterion of perfect syllogisms there is no need to decide this question. The outcome of the discussion of first figure syllogisms with premisses of different modality of which one at least is a contingent one, has confirmed our findings so far: Whenever the minor premiss is contingent, there can be no transitivity allowing a transparent connection between the minor subject and the major predicate, hence we have an imperfect syllogism. Whenever contingency goes with the major premiss, so that a transitive relation is expressed in the premisses, the syllogism comes out as perfect.

Finally, what about the remaining cases, i.e., those premiss pairs with two contingent premisses? We should expect them to be imperfect, since the contingent minor premiss should block transitivity and hence perfection, as in all the cases discussed in chapters 15 and 16 with contingent minor premisses. However, as

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[18] Here is the Greek text of 36a39-b2 (with line breaks as in Ross):
ὅταν δὲ τὸ ἐν μέρει καταφατικὸν ἀναγ-/καῖον ᾖ, τὸ ἐν τῷ στερητικῷ συλλογισμῳ, οἷον τὸ B Γ, ἢ τὸ κα-/θόλου τὸ ἐν τῷ κατηγορικῷ, οἷον τὸ A B, οὐκ ἔσται τοῦ ὑπάρχειν/ συλλογισμός. ἀπόδειξις δ᾽ ἡ αὐτὴ ἣ καὶ ἐπὶ τῶν πρότερον. One of the oldest ms., the Urbinas, omits the words τὸ B Γ, ἢ τὸ κα-/θόλου τὸ ἐν τῷ κατηγορικῷ. The τό before ἐν τῷ κατηγορικῷ is attested only in two mss., the Laurentianus und the Ambrosianus, and in the Syrian translation. Without the words τὸ καθόλου and without οἷον τὸ A B the text would make perfect sense, claiming that for both *Ferio* and *Darii* with *PN* premisses the conclusion will not be assertoric.

stated at the beginning of chapter 14, Aristotle wants these syllogisms to be perfect: 'When *A* possibly applies to every *B*, and *B* to every *C*, there will be a perfect syllogism that *A* possibly applies to every *C*' (32b38-40). 'Possibly' is not repeated in the minor premiss, but it should be understood, otherwise he would discuss a case of a *PA* combination, which he is going to discuss in i 15. Hence *Barbara PPP*, according to Aristotle, is a perfect syllogism. A similar claim is made for *Celarent PPP* at i 14.33a1-3, for *Darii PPP* at 33a21-24 and for *Ferio PPP* at 33a25-27. So will this destroy the hypothesis that telling a perfect from an imperfect syllogism we need to look for a transitive reading of the syllogisms in question? No. On the contrary, these cases will be a perfect confirmation of this suggestion. For in order to get these syllogisms into the class of perfect syllogisms, Aristotle, as already noticed by Patzig 1968, cf. 63, resorts to a device that commentators have (I think, correctly) dubbed as logically somewhat dubious: He relies on a distinction of 'possibly being applied to', the device called 'ampliation' in scholastic terminology:

> Since for one term possibly to apply to another can be taken in two different senses, namely, either that it applies to a subject to which the other term applies or that it applies to a subject to which the other term possibly applies, (for saying that *A* possibly is predicated of the subject of *B* means one of two things: either that it may be predicated of the subject of which *B* is predicated or that it is possibly predicated of the subject of which *B* is predicated.) To say that *A* is possibly predicated of the subject of *B* is in no way different from saying that *A* can belong to every *B*. (i 13.32b25-32)

Although Aristotle has only explained a distinction, he refers to the second of the two senses of 'possibly applying' in i 13 as to a definition: The sentence quoted above from i 14 continues: 'This is evident from the definition. For we explained "possibly applying to every" in this way' (i 14.32b40-33a1). This seems to be a reference back to the passage just quoted from i 13.

It is clear what the upshot of this manoeuvre is: Aristotle takes the *subject* term in a proposition expressing contingency as modified by a contingency operator. And this again has the consequence, if applied to first figure premiss pairs with two contingent premisses, that the minor predicate and the major subject term now are identical, as they were in assertoric logic. Hence we have transitivity restored to these cases as well. In predicate logic presentation *Celarent PPP* would look like this:

$$\forall x\ (Cx \supset PBx)\ \&\ \forall x\ (PBx \supset P{\sim}Ax) \vdash \forall x\ (Cx \supset P{\sim}Ax)$$

or written with an arbitrary name replacing the variable:

$$(Cn \supset PBn)\ \&\ (PBn \supset P{\sim}An) \vdash Cn \supset P{\sim}An.$$

IV

To come to a conclusion. The criterion that allows us to tell a perfect from an imperfect syllogism is quite simple and straightforward: If the syllogism can be written, starting with the minor premiss and using the 'being-contained-as-in-a-whole' formulation, corresponding to a predicate logic representation, in such a way as to yield a transitive relation, the syllogism is perfect, if this cannot be done, imperfect. However, to see that the criterion of transitivity can be applied to all four first figure syllogisms, it is essential to realize that Aristotle's 'being-contained-as-in-a-whole' wording is able to cover syllogisms with an *e*-proposition as major premiss as well as those with an *a*-proposition, and that the 'being-under' phrase used for the *i*-premiss is meant to turn an *i*-premiss into a sort of restricted *a*-proposition.[19]

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BIBLIOGRAPHY

Alexander of Aphrodisias. 1883. *In Aristotelis Analyticorum Priorum librum I commentarium*. Maximilianus Wallies ed. CAG II, 1. Berlin: Reimer.

Barnes, J. 2007. *Truth, etc*. Oxford: Clarendon Press.

Becker, A. 1933. *Die aristotelische Theorie der Möglichkeitsschlüsse*. Berlin: Junker und Dünnhaupt

Drechsler, M. 2005. *Interpretation der Beweismethoden in der Syllogistik des Aristoteles*. Frankfurt/M.: Lang.

Ebert, Th. 1995. 'Was ist ein vollkommener Syllogismus des Aristoteles?' *Archiv für Geschichte der Philosophie* 77: 221-247.

Ebert, Th. and U. Nortmann. 2007. Aristoteles, *Analytica Priora Buch I*, übersetzt und erläutert. Berlin: Akademie Verlag.

Ebert, Th. 2014. rev. of A. Rini 2011. *Archiv für Geschichte der Philosophie* 96: 140-144.

Malink, M. 2008. 'ΤΩΙ vs. ΤΩΝ in *Prior Analytics* 1.1-22' *Classical Quarterly* 58: 519-536.

Malink, M. 2009. 'A Non-Extensional Notion of Conversion in the *Organon*' *Oxford Studies in Ancient Philosophy* 37: 105-141.

Nortmann, U. 1996. *Modale Syllogismen, mögliche Welten, Essentialismus. Eine Analyse der aristotelischen Modallogik*. Berlin: de Gruyter.

Patterson, R. 1993. 'Aristotle's Perfect Syllogisms, Predication, and the *Dictum de omni*' *Synthese* 96: 359-378.

Patzig, G. 1968. *Aristotle's Theory of the Syllogism*. Dordrecht: Reidel.

Rini, A. 2000. '*Hupo* in the *Prior Analytics:* a note on Disamis XLL' *History and Philosophy of Logic* 21: 259-264.

Rini, A. 2011. *Aristotle's Modal Proofs*. Prior Analytics A8-22 *in Predicate Logic*. Dordrecht/Heidelberg/London/New York: Springer.

Ross, W.D. 1965. *Aristotle's Prior and Posterior Analytics*. A revised text with introduction and commentary. Oxford: Clarendon Press.

Schmidt, K.J. 1989. 'Eine modal prädikatenlogische Interpretation der modalen Syllogistik des Aristoteles' *Phronesis* 34: 80-106.

Schmidt, K.J. 2000. *Die modale Syllogistik des Aristoteles*. Paderborn: Mentis.

Smith, R. 1989. Aristotle, *Prior Analytics* translated, with introduction, notes, and commentary. Indi-

[19] This article is in some parts the translation of a reworked and improved version of Ebert 1995. I am indebted to an anonymous reader for *Ancient Philosophy* for advice and criticism and grateful to Wolfgang Preuß of Erlangen for deleting mistakes and inaccuracies from my English.

anapolis: Hackett.

Striker, G. 2009. Aristotle, *Prior Analytics* Book I. Translated with an Introduction and Commentary. Oxford: Clarendon Press.

Thom, P. 1996. *The Logic of Essentialism. An Interpretation of Aristotle's Modal Syllogistic*. Dordrecht/Boston/London: Kluwer.

Tredennick, H. 1962. Aristotle, *Prior Analytics*. Greek text with English translation. London: Heinemann.