Transitivity of preferences and well-known counterexamples

If you prefer option A over option B and B over C, must you prefer A over C in order to be rational? Would it be irrational to nevertheless prefer C over A – to have cyclic preferences? Below are two arguments for a yes answer, followed by four well-known counterexamples or kinds of counterexample. Note that it may be useful to model rational actors as having non-cyclic preferences despite these examples.

The inconsistency argument. (P1) If it would be inconsistent to have cyclic preferences, then it would be irrational to have cyclic preferences. (P2) It would be inconsistent to have cyclic preferences. Therefore: it would be irrational to have cyclic preferences. Mathematician Leonard Savage says that when he is found to have cyclic preferences, he feels much the same discomfort as when he is found to have contradictory beliefs (1954: 21).

The money pump argument. According to this argument, a person with cyclic preferences can be subject to a money pump, hence such preferences are irrational. For example, a stamp collector prefers stamp A to B, B to C, and yet C to A. The collector owns A and B. Someone selling C can sell it to the collector in exchange for A and some money, since the collector prefers C to A. The seller can then sell A to the collector in exchange for B and some money, since the collector prefers A to B. They can then sell B to the collector in exchange for C and some money, since the collector prefers B to C. But now the collector has A and B but not C. We are back to where we started and the same pattern can be repeated until all the collector's money has been spent (Davidson et al. 1955).

Condorcet's paradox. This paradox was discovered by the Marquis de Condorcet, who lived at the time of the French revolution. To illustrate it, imagine that a certain tribe consists of three individuals on a remote island and that they are voting on an issue that matters to the group as a whole. First they vote on whether they would prefer death by snake bite or death by drowning. Then they vote on whether they would prefer death by drowning or death by being shot with an arrow. Finally, they vote on whether they would prefer death by snake bite or death by being shot with an arrow. Below are the results.

	Snake or drowning	Drowning or arrow	Snake or arrow
Individual 1	Snake	Drowning	Snake
Individual 2	Drowning	Drowning	Arrow
Individual 3	Snake	Arrow	Arrow

Each individual has preferences of the form A over B, B over C and A over C. But if we count the votes in each column to determine what the group as a whole prefers, given a pair of options, we find that it prefers drowning over death by arrow, death by arrow over death by snake bite and yet death by snake bite over drowning. The group has cyclic preferences, but because each individual has non-cyclic preferences and because the rule for determining what the group prefers is fair, this does not seem irrational.

Multiple values. Sometimes an individual has cyclic preferences that seem rational because they are evaluating options according to multiple values. There are many examples in the literature. Recently, Johanna Thoma has presented an example in which she is looking for an apartment to rent and there are three she can afford. She cares about size, views, and length of commute. Here are descriptions of the three apartments.

Apartment A: 40m² large; view onto a garden; 5 minute commute.
Apartment B: 70m² large; view onto a sky line, lake, and woods; 60 minute commute.
Apartment C: 100m² large; view onto the brick wall of the building next door; 30 minute commute.

Thoma justifies her cyclic preferences as follows: "I prefer Apartment B over Apartment A because Apartment B is larger and has such a lovely view, and this outweighs the fact that it has a longer commute. I prefer Apartment C over Apartment B because Apartment C is even larger, and has a shorter commute, and this outweighs the fact that it does not have a good view. And I prefer Apartment A over Apartment C, because it has an even shorter commute, and a better view, and this outweighs the fact that it is smaller." (2018: 6)

External cyclicity. This is a term for a kind of counterexample (Bar-Hillel and Margalit 1988: 129). To illustrate it, imagine that a princess is required by conventions in her land to marry a prince from one of the neighbouring territories. There are three unmarried princes in the neighbouring territories and she will be offered a choice of two. Let us refer to the three princes as A, B, and C.

The princess decides that she will pick in the following way: she will consider the possibility of a boxing match between the two princes involved in the choice and if she has more reason to think that a certain prince would win, she will pick that prince. The three princes have boxed before so she consults the boxing records. She finds that prince A and prince B have fought ten times and prince A won on all of these occasions, so she reasons that given a choice of A or B, she will choose A. She finds that prince B and prince C have fought ten times and prince B won on all these occasions, so she reasons that given a choice of B or C, she will choose B. She also finds that prince A and prince C have fought ten times, but prince C won on all of these occasions, so she reasons that given a choice of A and C, she will choose C. Granting her criterion for choosing between princes, her preferences are rational.

Gradual increments. This kind of case requires us to consider more than three options. There is a medical device that allows doctors to apply electric current to a human body. The device has settings from 0 to 1000, 0 being when no current is being applied and 1000 when the maximum amount of current is being applied. The patient cannot experience any difference between two consecutive levels, such as between 664 and 665. The patient starts at 0 and is offered £10000 each day if he will move to the next level; otherwise the device will be used at the level he is at. Since he cannot experience the difference between two levels, he always agrees to move up a level. He prefers level 1 over 0, 2 over 1, and so on, up to 1000 over 999. But if he were given a choice of 0 or 1000 and all the money, he would prefer 0. This example comes from Warren Quinn (1990: 79-80) and is a variation on a well-known example regarding indifference between two options.

References

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