# Compliance and Conjunction

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**Abstract**

I provide counterexamples to Kit Fine's semantics for imperative and deontic modals. In particular, I argue that the semantics fails to provide necessary conditions for conjunctive imperatives.

In a recent series of papers, Fine (2018*a*,*b*) has formalized a truthmaker semantics for imperative and deontic logic. This system has quickly garnered substantial interest for a number of reasons. It offers elegant resolutions to puzzles about deontic logic (in particular, to Ross's puzzle)—and, as one of the first applications of truthmaker semantics, functions as a 'proof of concept' for a system with applications ranging from a theory of partial truth (Fine (2013)), the is-ought gap (Fine (2018*c*)), natural language semantics (Moltmann (2020)), epistemology (Elgin (2021*a*)), metaphysics (Elgin (2021*b*)) and the philosophy of science (Elgin (2020)).

There are two central motivations behind a truthmaker semantics for imperatives. The first is that the aspects of the world relevant to imperatives are actions, rather than outcomes. The command 'Raise your hand' thus concerns the act of raising one's hand—rather than the global situation that would result from raising one's hand. The second is that, in the spirit of the truthmaker approach, compliance and contravention are taken to be *exact*. An act which complies with the command 'Shut the door' is entirely relevant to the command; no part of that act is irrelevant. Therefore, the act of both shutting the door and opening the window does not comply with that command (though a part of that act—i.e., the part consisting of shutting the door—does comply with that command). As a result, this approach is hyperintensional. While 'Raise one hand' may be logically equivalent to 'Raise one or both hands,' actions in compliance with the latter need not be in compliance with the former and, for this reason, the imperatives mean different things. That is, while the act of raising both hands complies with the latter imperative, it does not comply with the former.

As is probably already clear, this approach assumes that acts are capable of mereological composition; some acts are composed of others. To reuse a former example, the act of shutting the door and opening the window may be the composite of two acts: the act of shutting the door and the act of opening the window. To capture this structure within our formalism, we may define an act-space as the ordered pair $< A, \sqsubseteq >$ where $A$ is a set of acts, and $\sqsubseteq$ is a binary relation on $A$. '$\sqsubseteq$' is intended to be interpreted as the relation of (improper) parthood, so that '$a \sqsubseteq a'$' means that act $a$ is a part of act $a'$. It is assumed that parthood forms a partial ordering over acts, i.e., that $\sqsubseteq$ satisfies the following criteria:

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| Reflexivity: | $a \sqsubseteq a$ |
| Antisymmetry: | $a \sqsubseteq b \wedge b \sqsubseteq a \rightarrow a = b$ |
| Transitivity: | $a \sqsubseteq b \wedge b \sqsubseteq c \rightarrow a \sqsubseteq c$ |

As defined, many act-spaces are uninteresting. In some, no mereological composition occurs at all; every act is a part of itself, and no act is a part of any other. Here, I will restrict my attention to act-spaces that allow for arbitrary fusion; every collection of acts within $A$ has a fusion within $A$.[1]

To formalize a semantics, we require a language in which to express imperatives. Let us adopt a simple propositional language $L$. Within $L$, there are infinitely many sentence symbols, $s_1, s_2, ...$, the sentential operator $\neg$ and the binary connectives $\wedge, \vee$—all defined in the standard way.[2]

On some approaches, the aim of a semantics is to determine which worlds comply with a given command. But on the truthmaker approach, the goal is to identify the acts within a world that are in exact compliance with (or in exact contravention to) an imperative. It is not assumed that each imperative has a unique act that complies with it. The command 'Bring a raincoat or bring an umbrella' presumably has (at least) two: the act of bringing a raincoat and the act of bringing an umbrella.

Let a model $\boldsymbol{M}$ be an ordered triple $< A, \sqsubseteq, |\cdot| >$ such that $< A, \sqsubseteq>$ is a (complete) act-space and $|\cdot|$ is a valuation functions that takes—as its input—a sentence symbol and has—as its output—an ordered pair $< V, F >$ where both $V$ and $F$ are subsets of $A$—intuitively the acts in compliance with and the acts in contravention to the input respectively. With the definition of a model in place, the semantics is given inductively.

[1]For finitely large act-spaces, this may be accomplished simply by assuming that every two acts within $A$ have a fusion within $A$. Defining a condition for infinitely large act-spaces requires a few more definitions. Let an *upper bound* of $B \subseteq A$ be an act $a$ such that, for all acts $b \in B, b \sqsubseteq a$. That is to say, an upper bound of a subset of $A$ is an act which contains—as a part—every act within that subset.

Let a *least upper bound* of $B \subseteq A$ be an act $a$ such that $a$ is an upper bound of $B$ and, for all upper bounds $a'$ of $B$, $a \sqsubseteq a'$. Intuitively, we can think of the least upper bound of $B$ as being the smallest upper bound of $B$—one which is a part of all upper bounds of $B$. Provably, if there is a least upper bound of $B$, then there is a unique least upper bound of $B$. Proof: For an act-space $< A, \sqsubseteq>$, select an arbitrary $B \subseteq A$. Suppose, for reductio, that $B$ has two least upper bounds—$\delta^{T1}$ and $\delta^{T2}$. From the definition of 'least upper bound' we have that $\delta^{T1} \sqsubseteq \delta^{T2}$ and $\delta^{T2} \sqsubseteq \delta^{T1}$. Given antisymmetry, this entails that $\delta^{T1} = \delta^{T2}$.

I denote the least upper bound of $B$ as $\delta^{B}$—and identify the fusion of the elements of $B$ with its least upper bound. Let a 'complete act-space' be any act-act $< A, \sqsubseteq>$ such that every $B \subseteq A$ has a least upper bound in $A$. Here, I am concerned only with complete act-spaces.

[2]Fine also includes a symbol $\top$ for top—which the null act complies with. I omit this additional complication, as it is not relevant to the examples I am concerned with.

i)$^+$ $a \Vdash s$ iff $a \in |s|^{\mathrm{V}}$
i)$^-$ $a \dashVv s$ iff $a \in |s|^{\mathrm{F}}$
ii)$^+$ $a \Vdash \neg A$ iff $a \dashVv A$
ii)$^-$ $a \dashVv \neg A$ iff $a \Vdash A$
iii)$^+$ $a \Vdash A \wedge B$ iff there exist acts $a', a''$ such that $a' \Vdash A$ and $a'' \Vdash B$ and $a = a' \sqcup a''$
iii)$^-$ $a \dashVv A \wedge B$ iff either $a \dashVv A$ or $a \dashVv B$
iv)$^+$ $a \Vdash A \vee B$ iff either $a \Vdash A$ or $A \Vdash B$
iv)$^-$ $a \dashVv A \vee B$ iff there exist acts $a', a''$ such that $a' \dashVv A$ and $a'' \dashVv B$ and $a = a' \sqcup a''$

With this semantics in place, we may define imperative entailment—the conditions in which one imperative entails another. In particular, imperative $A$ entails imperative $B$ just in case:

1) Every act that complies with $A$ contains—as an improper part—an act in compliance with $B$.
2) Every act that complies with $B$ is an improper part of some act in compliance with $A$.

For example, command $A \wedge B$ entails command $A$ because every act in compliance with the conjunctive command contains an act in compliance with each conjunct, and every act in compliance with a conjunct is a part of an act that complies with the conjunction. Relatedly, the command $\neg\neg A$ entails the command $A$ because any act which complies with one also complies with the other.

This semantics has a number of virtues—the first of which is that it is extremely intuitive. An act complies with the command to $\neg A$ just in case it contravenes the command to $A$. So, if the act of not bringing dessert contravenes the command 'Bring dessert,' then that act complies with the command 'Do not bring dessert.'[3] An act complies with the command $A \wedge B$ just in case it is composed of two acts—one of which complies with $A$ and the other of which complies with $B$. So, an act complies with the command 'Go to the bank and go to the supermarket' just in case it is composed of two acts—one of which complies with 'Go to the bank' and the other of which complies with 'Go to the supermarket.' Acts comply with the disjunctive command $A \vee B$ just in case they either comply with $A$ or comply with $B$. So, an act complies with 'Go to the bank or go to the supermarket' just in case it either complies with 'Go to the bank' or it complies with 'Go to the supermarket.'

Another virtue is that this resolves a longstanding puzzle about the logic of imperatives—Ross's puzzle. Assertoric sentences typically admit of disjunction introduction. The sentence 'John opened the window' entails 'Either John opened the window or John burned down the building.' But, intuitively, imperatives do not admit of disjunction introduction. If John were told 'Open the window,' and inferred 'Open the window or burn down the

[3]This example assumes that omissions count as acts.

building,' that would be in error. Fine's semantic accommodates this result. Disjunction introduction fails on this semantics; command $A$ does not entail command $A \vee B$ because there are acts in compliance with the disjunction (in particular, acts in compliance with $B$) that are not part of acts in compliance with $A$. And, for this reason, the command 'Open the window' does not entail 'Open the window or burn down the building.'

Despite these virtues, I believe that Fine's semantics is incorrect. In particular, there are counterexamples to his semantics for conjunctive imperatives.[4] Recall that—on this semantics—an act $a$ complies with the command $A \wedge B$ just in case it is the fusion of an act that complies with the command $A$ an act that complies with the command $B$. I do not question this condition's sufficiency, but rather its necessity. That is, I maintain that there are acts that comply with the command $A \wedge B$ that are *not* composites of an act that complies with $A$ and one that complies with $B$. I will provide the general form of this counterexample before describing a case I take to witness this form.

Suppose a person is issued a command $A \wedge B$, and they perform two acts. The first of these acts fully complies with the command to $A$ and partially complies with the command to $B$, while the second act finishes the compliance with the command to $B$. The fusion of these acts, I maintain, complies with the command to $A \wedge B$. Nevertheless, this fusion is not the composite of an act that complies with $A$ and an act which complies with $B$. After all, on the truthmaker approach, compliance is taken to be exact; an act complies with a command just in case it is entirely relevant to the command. The first act is thus not in compliance with the command to $A$, because a part of that act is irrelevant to the command to $A$—it is relevant to the command to $B$. And the second act does not comply with $B$, as it only constitutes a partial compliance with $B$. And so, there are acts that comply with conjunctive commands that are not fusions of acts in compliance with each conjunct.

Here is such a case. Suppose there is a room with two empty beakers and two liquids—Red and Blue. Jack sits in this room when Jill issues a command: "Fill one beaker with Red and fill one beaker with Blue." Jack sees that there is a jug filled with Blue, which he can use to fill either beaker. He looks to his right and looks to his left. No Red jug. Fortunately, he sees a button on the table. Pressing this button does two things (and nothing else). It fills one of the beakers with Red and also fills half of the other beaker with Blue. Jack then performs two acts: he presses the button—thereby filling one beaker with Red and half of the other beaker with Blue—and pours Blue from the jug into the half-empty beaker. As a result of these acts, one beaker is filled with Red and the other beaker is filled with Blue. For the purposes of this example, let us denote Jill's command "Fill one beaker with Red and fill one beaker with Blue" as $c$, and let us denote the fusion of Jack's acts of pressing the button and pouring Blue into the half-empty beaker as $a$.

This example is used in the following argument against Fine's semantics for conjunction:

[4] It is straightforward to use these types of cases to generate counterexamples to his semantics for acts in contravention to disjunctive imperatives—as well as the semantics for conjunction and disjunction in deontic logic. However, for the purposes of this paper I restrict myself to conjunctive imperatives.

*Assumption 1:* Jack has complied with command $c$.

*Assumption 2:* If Jack has complied with command $c$, then Jack has complied with command $c$ by performing act $a$.

Therefore, Jack has complied with command $c$ by performing act $a$.

*Assumption 3:* If Fine's semantics is correct, then Jack has *not* complied with command $c$ by performing act $a$.

Therefore, Fine's semantics is not correct.

This argument is incontrovertibly valid. Its tenability thus rests upon assumptions 1, 2 and 3. Let us take them in turn.

### Assumption 1: Jack has complied with command $c$

The argument for this rests on the intuition that Jack has complied with $c$. When Jill issued her command, the two beakers were empty. Jack then acted, after which one liquid was filled with Red and the other was filled with Blue—precisely as Jill directed. That the beakers were filled was no accident—the filling was directly caused by Jack's acts.

There are a few ways to target this intuition. If Jill were to observe Jack push the button and pour the jug—thereby filling one beaker with Red and the other with Blue—and then ask "When will you comply with my command?" it would seem odd. If she were to berate Jack for failing to comply with her command, it would seem unreasonable. In my mind, this intuition is robust, and I will take it as given; Jack has complied with command $c$.

### Assumption 2: If Jack has complied with command $c$, then Jack has complied with command $c$ by performing act $a$.

Suppose that Jack has complied with command $c$. Then, there must exist some act of Jack's which is in compliance with $c$. The reason to believe assumption 2 is that there is no act other than $a$ which Jack performs which plausibly complies with $c$.

If we knew nothing of Jack's acts except that he performed act $a$, we could infer that he had complied with $c$; we need not search the room for other beakers which Jack has filled in order to determine whether he has complied with Jill's command. Therefore, either act $a$ or one of its parts must be in compliance with $c$.

The only proper parts of act $a$ are the acts of pushing the button and the act of topping-up the half empty beaker. Neither of these acts is in compliance with $c$. The act of pushing the button did not comply with $c$—because $c$ requires that one liquid be filled with Blue and, after the push of a button, a beaker was only half-filled with Blue. The act of topping up the beaker does not itself comply with $c$, as it does nothing to ensure that the other beaker is filled with Red. Only the fusion of these acts—the act of pushing the button and the act of topping up the beaker—ensures that Jack has complied with Jill's command $c$. And so, if Jack has complied with Jill's command $c$, then Jack has complied with Jill's command $c$ by performing act $a$.

### Assumption 3: If Fine's semantics is correct, then Jack has not complied with command $c$ by performing act $a$.

According to Fine's semantics, an act complies with the command $A \wedge B$ just in case it is composed of two acts—one of which complies with $A$ and the other of which complies with $B$. If act $a$ complies with command $c$, then $a$ must be composed of one act which complies with the command "Fill one beaker with Red" and another which complies with the command "Fill one beaker with Blue."

But act $a$ is *not* composed of an acts which comply with these conjuncts. The act of pushing the button is neither in compliance with "Fill one beaker with Red" nor "Fill one beaker with Blue." This act exceeds one which is in exact compliance with "Fill one beaker with Red"—because it partially fills a beaker with Blue. This is irrelevant to the command "Fill one beaker with Red", and acts that comply with commands are entirely relevant to those commands on the truthmaker approach. And because half of a beaker is left unfilled with Blue, it also does not comply with "Fill one beaker with Blue" (because acts that comply with commands guarantee that the command has been complied with). Similarly, the act of pouring the Blue jug neither exactly complies with "Fill one beaker with Red" nor "Fill one beaker with Blue." This act does nothing to ensure that a beaker is filled with Red, and only goes partway toward ensuring that a beaker is filled with Blue.[5]

Act $a$ is thus not composed of two acts—one of which complies with "Fill one beaker with Red" and the other of which complies with "Fill one beaker with Blue." On Fine's semantics, this means that act $a$ does not comply with command $c$. Therefore, if Fine's semantics is correct, then Jack has not complied with Jill's command $c$ by performing act $a$. It follows from assumptions 1, 2 and 3 that Fine's semantics is incorrect. In particular, his semantics for conjunctive imperatives fails.

I close by noting that this type of example threatens not only Fine's truthmaker semantics for conjunction, but the exact approach to imperatives quite generally. For, suppose that Jill had merely issued the command "Fill a beaker with Red," and Jack had responded by pushing the button—thereby filling one beaker with Red and half of the other beaker with Blue. Intuitively, Jack has complied with that command; his act resulted in a beaker being filled with Red. Nevertheless, the act of pushing the button is not in exact compli-

[5] Act $a$ itself neither complies with the command "Fill one beaker with Red" nor "Fill one beaker with blue." It exceeds an act which complies with the former command because it fills a beaker with Blue, and exceeds the latter command because it fills a beaker with Red.

ance with Jill's command. After all, this act partially filled a beaker with Blue—which is irrelevant to the command. And there seems no plausible part of Jack's act that is in exact compliance with her command—as it cannot be separated into parts, one of which concerns filling a beaker with Red and the other with partially filling a beaker with Blue. And so, Jack has complied with Jill's command without performing an act in exact compliance with her command. Any exact semantics for imperatives will struggle to accommodate this result.

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