THE METAPHYSICS IN COUNTERFACTUAL LOGIC

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Abstract

This paper investigates the metaphysics in higher-order counterfactual logic. I establish the necessity of identity and distinctness and show that the logic is committed to vacuism, which entails that all counteridenticals are true. I prove the Barcan, Converse Barcan, Being Constraint and Necessitism. I then show how to derive the Identity of Indiscernibles in counterfactual logic. I study a form of maximalist ontology which has been claimed to be so expansive as to be inconsistent. I show that it is equivalent to the collapse of the counterfactual into the material conditional—which is itself equivalent to the modal logic TRIV. TRIV is consistent, from which it follows that maximalism is, surprisingly, consistent. I close by arguing that stating the limit assumption requires a higher-order logic.

Introduction

Over the past few decades, philosophers have systematically investigated both counterfactual and higher-order logic. Given the uses for these systems, this focus is unsurprising. Counterfactuals figure in debates ranging from decision theory to the necessity of mathematics—and figure prominently in analyses of causation.[2] Higher-order logic, for its part, has shed light on debates ranging from Leibniz's Law to propositional granularity to metaphysical grounding.[3] The applications for each system are undoubtably broad.

What is surprising is that almost nothing has been written on their interaction; there is no literature discussing systems that both describe what would have been the case and quantify over terms in any syntactic category.[4] At present, higher-order counterfactual logic does not exist.

I aim to remedy this oversight. The resulting system—which I dub HOCL—governs the logic of higher-order counterfactuals: i.e., counterfactuals that embed a higher-order

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[2]For their use in decision theory, see Bradley and Steffánnson (2017). For their use in the necessity of mathematics, see Yli-Vakkuri and Hawthorne (2020). For their use in theories of causation, see Lewis (1973*a*).

[3]For discussions of its implications for Leibniz's Law, see Bacon and Russell (2019); Bacon (2019); Caie, Goodman and Lederman (2020). For discussions of its implications for grounding, see Fritz (2021, 2022); Elgin (Forthcoming). For discussions of its implications on propositional identity, see Dorr (2016); Bacon and Dorr (2024).

[4]However, two papers that use some form of higher-order inferences in counterfactual logic are Goodman and Fritz (2017) and Kocurek (2022*b*). To the best of my knowledge, this list is exhaustive.

claim in either their antecedent or their consequent. Many sentences in natural language are reasonably interpreted as higher-order counterfactuals. For example, 'If Sarah and Jane had nothing in common, then they would not both be Norwegian' appears to assert that if there were not to exist a property borne by both Sarah and Jane, then they would not both bear *is Norwegian*. One reason to investigate this system is to adequately understand the logic of these sentences. It is worth noting, however, that HOCL does not *merely* govern the logic of higher-order counterfactuals; it not only characterizes counterfactuals that embed higher-order claims, but also counterfactuals that are *themselves* embedded in higher-order inferences. For example, from 'If it were raining, the street would be wet,' we may conclude that there exists a relation between 'It is raining' and 'The street is wet.'

But my primary focus is metaphysics, rather than the logic of natural language. When I began this project, I exclusively focused on the higher-order aspect of this system. However, it quickly became clear that there are underdeveloped—yet important—first-order metaphysical implications as well. So, after discussing the relation between modality and counterfactuality—and after axiomatizing HOCL—I discuss its ramifications for the necessity of identity, counteridenticals, the Barcan Formula, and Necessitism. In these debates, the higher-order aspect of the system is largely auxiliary. While there are higher-order instances of these theorems (and so the result is strictly more powerful than any expressible in a first-order language), restricted versions could also be stated in languages with quantifiers that only range over objects. I then turn to debates where higher-order quantification is indispensable: the Identity of Indiscernibles, Maximalism and the Limit Assumption.

A note on this project's aims. Nearly every assumption I make about counterfactual logic is controversial. While these assumptions are widespread, I will not provide a full-throated defense of them here (aside from some brief remarks about their plausibility). As will become clear, what these assumptions entail is at least equally controversial. There is thus ample room to reject HOCL. But there is a sense in which dissidents need not disagree with anything that I say. I do not claim that HOCL settles debates correctly—nor that its axioms are true. Rather, I take it to be a natural starting point (perhaps even *the* natural starting point) for reasoning about higher-order counterfactuals. Those who would rule differently ought to employ a different logic—and even those who ultimately reject this system may find it illuminating to determine what implications it has.

Modality and Counterfactuality

I make a number of assumptions throughout this paper. I assume that classical logic is true. I assume that sentences certified by truth-tables to be true (in the standard way) are indeed true—and that sentences so certified to be false are indeed false. Moreover, I assume that the results of proofs within classical quantified logic are true if their premises are true. That is, I assume that the conclusions of sound arguments are true.

I do not mean to suggest that this assumption is uncontroversial. Every logical assumption that I make is open to debate—and my commitment to classicality is no exception. Nevertheless, I will provide no defense of this assumption here. Those who deny classical logic need read no further.

I also assume the following connection between necessity and counterfactuality:

$$\Box p := \neg p \mathrel{\Box\!\!\rightarrow} \bot$$

For p to be necessary is for it to be the case that, if p were false, then the absurd would be true.[5] I dub this connection between necessity and counterfactuality 'Definition$_1$.' Definition$_1$ is an immediate consequence of the Lewis (1973*b*)/Stalnaker (1968) semantics for counterfactual conditionals—which holds that a sentence of the form 'If p were true then q would be true' is true just in case the closest possible world(s) in which p are true are world(s) in which q is true. After all, if p is true in every possible world, then the closest possible world in which p is false is an absurdity.

I do not assume that the Lewis/Stalnaker semantics is correct—nor do I assume that it is incorrect. While the widespread appeal of their accounts offers some support for Definition$_1$, an arguably stronger motivation occurs in Williamson (2007*b*)—who notes that it follows from the weakest standard modal logic—K—and the following two principles:

Necessity: $\Box(p \rightarrow q) \rightarrow (p \mathrel{\Box\!\!\rightarrow} q)$
Possibility: $(p \mathrel{\Box\!\!\rightarrow} q) \rightarrow (\Diamond p \rightarrow \Diamond q)$

Necessity asserts that strict implication entails counterfactual implication; if it is necessary that if p is true then q is true, then if p were true then q would be true. **Possibility**, for its part, asserts that anything counterfactually implied by a possible proposition is itself possible; if it is possible for p to be true, and if p were true then q would be true, then it is possible for q to be true.

When I first encountered **Necessity**, it struck me as overwhelmingly plausible.[6] If we canvass the entirety of modal space, and find that absolutely every world in which p is true is a world in which q is true, then there is no possible situation—no matter how distant—in which p is true and q is false. Surely, in that case, if p were true then q would be true. I now recognize that matters are not so simple; **Necessity** takes a stand on a contentious debate. In particular, it presupposes *vacuism*: the claim that all counterpossibles (counterfactuals

[5]Two other potential counterfactual definitions of necessity are $\Box p := \neg p \mathrel{\Box\!\!\rightarrow} p$ and $\Box p := \forall q(\neg p \mathrel{\Box\!\!\rightarrow} q)$. As Williamson (2007*b*) established, minimal assumptions entail that the three are equivalent. There is thus no need to choose between competitors. Opting for the formulation I have is merely a stylistic preference.

[6]**Possibility** also strikes me as overwhelmingly plausible. Even Lange (2009), who is loath to commit to any generalizable principles of counterfactual logic, repeatedly endorses **Possibility**. If we determine that a proposition p could be the case, then the counterfactual 'If p were true then q would be true' takes us from one possibility to another. I note, however, that Williamson (2020) denies **Possibility** on the grounds that there could be contextual shifts between 'If p were true then q would be true' and 'It is possible that p.'

with impossible antecedents) are true. Take an arbitrary proposition p that is necessarily false. Because p is false in every possible world, the conditional $p \rightarrow q$ is true in every world (for every q). So, $p \rightarrow q$ holds necessarily. **Necessity**, then, entails $p \Box\!\!\rightarrow q$. Since this holds for every impossible p and for every q, **Necessity** validates vacuism.

Vacuism is controversial—and rightly so. Nonvacuists (who hold that at least some counterpossibles are substantive) maintain that their view better accords with ordinary judgments.[7] Intuitively, the sentence 'If paraconsistent logic were true, then Graham Priest would be incorrect' is false (since Priest has offered an impassioned defense of paraconsistent logic), but vacuists must maintain that it is true.[8] After all, its antecedent could not possibly obtain.[9]

Despite challenging cases, many paths lead to vacuism. Dominant semantics for counterfactuals—such as the Lewis/Stalnaker—entail that all counterpossibles are true. If there are no worlds in which p is true then, trivially, all of the closest p worlds are q worlds. As Lewis said, "Confronted by an antecedent that is not really an entertainable supposition, one may react by saying, with a shrug: If that were so, anything you like would be true!" (Lewis, 1973*b*, pg. 24). Beyond the appeal of particular semantic accounts, vacuists typically highlight the theoretical virtues of their position.[10] In general, vacuist systems are simpler and more elegant than their nonvacuist counterparts.

HOCL is deeply committed to vacuism, but this will only become clear after the system is formalized. For the moment, suffice it to say that **Necessity** offers some support for Definition$_1$, but there remains room for disagreement—as nonvacuists ought to reject it.

With a definition of necessity to hand, possibility can be defined as:

$$\Diamond p := \neg \Box \neg p$$

For it to be possible that p is for it to be false that p is necessarily false. Let this definition of possibility be dubbed 'Definition$_2$.' Definition$_2$ is standard in the literature. If we interpret 'Necessarily p' as the claim that p is true in every possible world, then

[7]Examples of nonvacuists include Zagzebski (1990); Nolan (1997); Brogaard and Salerno (2007); Kment (2014).

[8]At least, on the assumption that paraconsistent logic is not merely actually false, but necessarily false.

[9]A stronger motivation for nonvacuism is given by Jenny (2018)—who argues that mathematics employs substantive counterpossibles. There are pairs of problems p and q, such that neither p nor q are computable (in that no algorithm given a finite time could solve them), but that q is computable relative to p (in that any solution to p generates a solution to q). For example, although neither the validity problem for First-Order Logic nor the halting problem are computable, the validity problem is computable relative to the halting problem. For this reason, 'If the halting problem were computable, then the validity problem for First-Order Logic would be computable' is true. These sentences are nontrivial; it takes mathematical work to establish that one problem is computable relative to another. However, given that uncomputable problems are necessarily uncomputable, they are also counterpossibles. To account for the substance of relative computability theory, perhaps we ought to endorse nonvacuism.

[10]Examples of vacuists (beyond Lewis and Stalnaker) include Kratzer (1979); Bennett (2003); Williamson (2007*b*, 2010, 2015); Emery and Hill (2017).

$\neg\Box\neg p$ asserts that it is false that p is false in every possible world. This naturally seems to require p to be true in at least one possible world—and so it is possible for p to be true.[11]

The significance of this is the following: in addition to accepting classical logic, I endorse both the claim that $\Box p = \neg p \;\Box\!\!\rightarrow \bot$ and that $\Diamond p = \neg\Box\neg p$. While these assumptions are not uncontroversial, they have enough support to make this discussion worthwhile. I also note that those who reject the definition of either necessity or possibility may have a use for the system that follows. These definitions serve one purpose: to translate claims involving counterfactuals to claims involving modals. Without these definitions, such translations are impossible—but the remainder of the theorems still hold, and claims that purely involve counterfactuals may be of interest in their own right.

Higher-Order Counterfactual Logic

The system I employ is not the propositional counterfactual logic I have employed thus far—or even a first-order extension of that system. Rather, I operate with a *higher-order* counterfactual logic: one that allows for quantification over terms in any syntactic category. At the outset of the analytic tradition, higher-order systems played a pivotal role in philosophical inquiry. However, following Quine (1970)'s impassioned insistence on the primacy of first-order logic, these systems largely fell out of favor. A few years ago, it would have been incumbent to provide a general introduction to higher-order logic before this project could commence. Fortunately, matters have improved; there are now excellent overviews of higher-order logic—and the system is widespread enough that little introduction is needed.[12] My discussion of the non-counterfactual fragment of this system will be brief; I dedicate the bulk of my attention to counterfactual logic.

The Syntax of HOCL

I will operate in a simply-typed language with λ-abstraction.[13] This language has two basic types: a type e for entities and a type t for sentences. 'Socrates' and 'The Mona Lisa' are of type e, while 'Roses are red' and 'Violets are blue' are of type t. Additionally, there are complex types that consist of functional relations between the basic ones; for every types τ_1 and τ_2, $(\tau_1 \rightarrow \tau_2)$ is a type. Nothing else is a type.

[11] To the best of my knowledge, the only philosophers who reject Definition$_2$ are intuitionists—such as Bobzien and Rumfitt (2020). I myself find the consequences of intuitionism untenable. While intuionists claim that not all propositions are either true or false ($\neg\forall p(p \vee \neg p)$), they cannot claim that some propositions are neither true nor false ($\exists p(\neg p \wedge \neg\neg p)$) on pain of contradiction. Intuitionists thus lose the inference from $\neg\forall x\phi$ to $\exists x\neg\phi$—an unacceptable loss in my view. Suffice it to say that my first assumption—that classical logic is true—rules out this strategy.

[12] See Bacon (2023); Fritz and Jones (2024) for introductory texts.

[13] The simply-typed λ calculus differs from the pure-type theory of Berardi (1989); Terlouw (1989), which has the power to perform operations on the types themselves.

We can regiment terms of diverse syntactic categories in the standard way. Monadic first-order predicates are identified with terms of type $(e \to t)$; they are functions with entities as their inputs and sentences as their outputs. For example, 'is wise' is treated as function that generates sentences like 'Socrates is wise' and 'Aristotle is wise.' Diadic first-order predicates are terms of type $(e \to (e \to t))$, monadic second-order predicates are of type $((e \to t) \to t)$, etc. The negation operator $\neg$ is of type $(t \to t)$, and the binary connectives $\wedge, \vee, \to$ and $\leftrightarrow$ are all of type $(t \to (t \to t))$.

There are also terms for identity and the standard quantifiers. For every type τ there is a term $=$ of type $(\tau \to (\tau \to t))$. We allow there to be infinitely many variables of every type, as well as λ-abstracts that serve to bind these variables. We also introduce the quantifiers $\forall$ and $\exists$ of type $((\tau \to t) \to t)$ for every type τ. Effectively, first-order quantifiers are second-order properties: the property of *having every object in its extension* and of *having an object in its extension* respectively. There are the modal operators $\Box$ and $\Diamond$ of type $(t \to t)$ and, lastly, the counterfactual conditional $\Box\!\!\to$ of type $(t \to (t \to t))$. It represents sentences like 'If the shampoo were cheaper, I would have bought it' and 'If kangaroos had no tails, they would topple over.'[14]

The Axioms and Rules of HOCL

The nonmodal axioms and inferential rules I employ are the following:

Nonmodal Axiom Schemes:

PC: $\vdash \phi$ if ϕ is a theorem of classical propositional logic
UI: $\vdash \forall F \to Fa$
EG: $\vdash Fa \to \exists F$
UD: $\vdash \forall(P \to Q) \to (P \to \forall Q)$ for P with no free variables
ED: $\vdash \forall(P \to Q) \to (\exists(P) \to Q)$ for Q with no free variables
Ref: $\vdash a = a$
LL: $\vdash a = b \to (\phi \leftrightarrow \phi^{[a/b]})$
Eβ: $\vdash \lambda x.F(a) \leftrightarrow F^{[a/x]}$

Nonmodal Rules:

MP: If $\vdash p \to q$ and $\vdash p$ then $\vdash q$
Gen: If $\vdash p$ then $\vdash \forall p$

[14]In what follows, I occasionally omit various symbols when ambiguity does not result. I also omit the types of terms where the type is either contextually evident, or the term is taken as a schema with applications in every type. I also suppress the λ terms that immediately follow the quantifiers $\forall$ and $\exists$.

Many of these axioms either are included in—or are natural extensions of—First-Order Logic. **PC** and **MP** jointly ensure that classical propositional logic holds within HOCL. **UI**, **EG**, **UD** and **ED** likewise stipulate that quantifiers act classically (though here the axioms should be interpreted as schemata; quantifiers of arbitrary type obey analogues of first-order inferences). Likewise, **Ref** and **LL** govern the logic of identity. Everything is identical to itself—and terms that co-denote can be substituted for one another in any formula.[15] The most novel axiom is **E**β—the principle of β-reduction. This permits the inference from $\lambda x.Fx(a)$ to Fa.

Some small points about this system. First, the axioms and inferences have instances involving free variables, as well as constants. Thus, $x = x$ is a theorem of this system. However, we will only speak of formula as being 'true' when they contain no free variables. To that end, **Gen** can be applied to *formula* with free variables to arrive at *sentences* that lack them. Second, this system is extremely weak in some respects. In particular, it sidesteps many controversial debates over propositional granularity. For example, **E**β merely stipulates that $\lambda x.Fx(a)$ and Fa have the same truth-value; it does not take a stand on whether the two are identical.[16] It is thus available to many metaphysicians.

The counterfactual axioms and rules I employ are the following:

Counterfactual Axiom Schemes:

ID: $\vdash p \mathrel{\Box\!\!\rightarrow} p$
Vac: $\vdash (\neg p \mathrel{\Box\!\!\rightarrow} p) \rightarrow (q \mathrel{\Box\!\!\rightarrow} p)$
B$\Box\!\!\rightarrow$**:** $\vdash p \rightarrow ((p \mathrel{\Box\!\!\rightarrow} \bot) \mathrel{\Box\!\!\rightarrow} \bot)$

Counterfactual Rules:

Closure: If $\vdash p \rightarrow q$ then $\vdash (r \mathrel{\Box\!\!\rightarrow} p) \rightarrow (r \mathrel{\Box\!\!\rightarrow} q)$
REA: If $\vdash p \equiv q$ then $\vdash (p \mathrel{\Box\!\!\rightarrow} r) \equiv (q \mathrel{\Box\!\!\rightarrow} r)$

ID is the principle of reflexivity for counterfactuals.[17] It reflects the thought that when we construct a counterfactual supposition, we start with the supposition itself.[18] **Vac** (or

[15] Some may be more accustomed to a version of Leibniz's Law according to which identicals bear all of the same properties. That formulation is provably equivalent to the version given here.

[16] For an argument that they are identical, see Dorr (2016). For an argument that they are not, see Rosen (2010); Fine (2012*b*).

[17] Williamson (2007*b*) dubs this principle '**Reflexivity**.' I depart from his terminology in order to avoid ambiguity with my axiom of **Ref**—according to which terms are self identcal.

[18] While I take **ID** to be extraordinarily intuitive, I note that some have argued that it fails in at least some

Vacuity) generates vacuous counterfactuals. The basic thought is that a situation in which p is false is the 'worst' situation from the perspective of p. If p would be true *even in a* $\neg p$ *situation*, then p is true in every situation whatsoever. So, in any situation in which q is true, p is true (for an arbitrary q).

There are two rules within this system. **Closure** allows us to generate counterfactual conditionals from (provable) material conditionals, while **REA** (or Replacement of Equivalent Antecedents) allows for the substitution of logically equivalent expressions in the antecedents of counterfactuals.[19] I also assume polyadic extensions of these axioms and rules hold. **Closure**, in particular, licenses the inference from 'If $\vdash (p_1 \wedge p_2 \wedge ... \wedge p_n) \rightarrow q$, to $\vdash ((r \boxright p_1) \wedge (r \boxright p_2) \wedge ... \wedge (r \boxright p_n)) \rightarrow (r \boxright q)$.'

REA is particularly controversial; given other plausible principles about counterfactual logic, it fails at least some of the time.[20] However, I will not appeal to the most controversial instances of **REA**. Its only occurrences take the form $(p \boxright q) \leftrightarrow (\neg\neg p \boxright q)$.[21] Prominent semantic accounts that deny **REA** in its full generality still license this particular instance.[22] Despite the controversy surrounding **REA**, I doubt that rejecting it is particularly promising.

B$\boxright$ is so-named because it is the counterfactual analog of the **B** modal axiom $p \rightarrow \Box\Diamond p$.[23] Given the other axioms and rules, we can establish that **B**$\boxright$ entails **B** as follows:

i.	$p \rightarrow ((p \boxright \bot) \boxright \bot)$	**B**$\boxright$
ii.	$p \equiv \neg\neg p$	**PC**
iii.	$(p \boxright \bot) \equiv (\neg\neg p \boxright \bot)$	*ii*, **REA**
iv.	$(\neg\neg p \boxright \bot) \equiv \neg\neg(\neg\neg p \boxright \bot)$	**PC**
v.	$(p \boxright \bot) \equiv \neg\neg(\neg\neg p \boxright \bot)$	*iii*, *iv*, **PC** and **MP**
vi.	$((p \boxright \bot) \boxright \bot) \equiv ((\neg\neg(\neg\neg p \boxright \bot)) \boxright \bot)$	*v*, **REA**
vii.	$p \rightarrow ((\neg\neg(\neg\neg p \boxright \bot)) \boxright \bot)$	*i*, *vi*, **PC** and **MP**
viii.	$p \rightarrow ((\neg\neg\Box\neg p) \boxright \bot)$	*vii* and Definition$_1$
ix.	$p \rightarrow ((\neg\Diamond p) \boxright \bot)$	*viii* and Definition$_2$
x.	$p \rightarrow \Box\Diamond p$	*ix* and Definition$_1$

cases. See, for example, Lowe (1995); Nolan (1997); Kocurek (2022*a*).

[19]Williamson (2007*b*) refers to this as 'Equivalence.' I note that (Polluck, 1976, pg. 11) states that **Closure** is "So obvious as to need no defense."

[20]The principles I allude to are **Simplification**: $\vdash ((p \vee q) \boxright r) \rightarrow (p \boxright r)$ and the **Failure of Antecedent Strengthening**: $\nvdash (p \boxright r) \rightarrow ((p \wedge q) \boxright r)$. This conflict was noted independently by Fine (1975) and Nute (1975). I do not want to cast too much doubt on **REA**; plausible principles also entail that it holds in full generality. If necessarily equivalent propositions are identical, and Leibniz's Law is true, then **REA** universally succeeds.

[21]There is one exception—I also appeal to it once when addressing vacuous β-conversion in the discussion of Maximalism. I take it that this is also uncontroversial.

[22]See, e.g., Fine (2012*a*).

[23]Williamson (2007*a*) operates with the provably equivalent axiom **BS**: $\vdash (p \boxright (q \boxright \bot)) \rightarrow (p \rightarrow (q \boxright \bot))$. I opt for my axiom due to its comparative simplicity.

This proof can be more-or-less reversed to demonstrate that **B** entails **B**$\Box\!\!\rightarrow$:

i.	$p \rightarrow \Box\Diamond p$	**B**
ii.	$p \rightarrow ((\neg\Diamond p) \Box\!\!\rightarrow \bot)$	*i* and Definition$_1$
iii.	$p \rightarrow ((\neg\neg\Box\neg p) \Box\!\!\rightarrow \bot)$	*ii* and Definition$_2$
iv.	$p \rightarrow ((\neg\neg(\neg\neg p \Box\!\!\rightarrow \bot)) \Box\!\!\rightarrow \bot)$	*iii* and Definition$_1$
v.	$p \equiv \neg\neg p$	**PC**
vi.	$(p \Box\!\!\rightarrow \bot) \equiv (\neg\neg p \Box\!\!\rightarrow \bot)$	*v*, **REA**
vii.	$(\neg\neg p \Box\!\!\rightarrow \bot) \equiv \neg\neg(\neg\neg p \Box\!\!\rightarrow \bot)$	**PC**
viii.	$(p \Box\!\!\rightarrow \bot) \equiv \neg\neg(\neg\neg p \Box\!\!\rightarrow \bot)$	*vi*, *vii*, **PC** and **MP**
ix.	$((p \Box\!\!\rightarrow \bot) \Box\!\!\rightarrow \bot) \equiv ((\neg\neg(\neg\neg p \Box\!\!\rightarrow \bot)) \Box\!\!\rightarrow \bot)$	*viii*, **REA**
x.	$p \rightarrow ((p \Box\!\!\rightarrow \bot) \Box\!\!\rightarrow \bot)$	*iv*, *ix*, **PC**, **MP**

These axioms and rules are too weak to constitute 'counterfactual logic' in any comprehensive sense of the term. I do not include the weak-centering axiom $(p \Box\!\!\rightarrow q) \rightarrow (p \rightarrow q)$—nor axioms that correspond to other standard axioms of modal logic.[24] I do not omit these because I have any principled objection to them—but rather because they play no role in the theorems that follow. For our purposes, weak axioms are enough.

The Necessity of Identity and Distinctness

A natural starting point is the necessity of identity—due to both the simplicity of the proof and the significance of the result. We can establish the necessity of identity using **Closure**, **ID** and Definition$_1$ as follows:

i.	$x = x$	**Ref**
ii.	$\neg(x = x) \rightarrow \bot$	*i*,**PC** and **MP**
iii.	$(\neg(x = x) \Box\!\!\rightarrow \neg(x = x)) \rightarrow (\neg(x = x) \Box\!\!\rightarrow \bot)$	*ii*, **Closure**
iv.	$\neg(x = x) \Box\!\!\rightarrow \neg(x = x)$	**ID**
v.	$\neg(x = x) \Box\!\!\rightarrow \bot$	*iii*, *iv*, **MP**
vi.	$x = y \rightarrow ((\neg(x = x) \Box\!\!\rightarrow \bot) \leftrightarrow (\neg(x = y) \Box\!\!\rightarrow \bot))$	**LL**
vii.	$x = y \rightarrow (\neg(x = y) \Box\!\!\rightarrow \bot)$	*v*, *vi*, **PC** and **MP**
viii.	$\forall x, y(x = y \rightarrow (\neg(x = y) \Box\!\!\rightarrow \bot))$	*vii*, **Gen** (x2)
ix.	$\forall x, y(x = y \rightarrow \Box(x = y))$	*viii*, Definition$_1$

It is thus possible to establish the necessity of identity purely in the language of counterfactual logic. While higher-order resources are unneeded in this proof, the result holds for terms of arbitrary type; identical properties are necessarily identical, identical

[24]For many of these axioms, see Williamson (2007*a*).

sentential operators are necessarily identical, and identical connectives are necessarily identical.

It is also possible to prove the necessity of distinctness using **ID**, **Closure**, **B**$\Box\!\!\rightarrow$, **REA**, **Vac**, and Definition$_1$ as follows:

i.	$x = y \rightarrow (x \neq y \mathrel{\Box\!\!\rightarrow} \bot)$	Previous Theorem
ii.	$\neg(x \neq y \mathrel{\Box\!\!\rightarrow} \bot) \rightarrow x \neq y$	*i*, **PC** and **MP**
iii.	$\neg(\neg(x \neq y \mathrel{\Box\!\!\rightarrow} \bot) \rightarrow x \neq y) \rightarrow \bot$	*ii*, **PC** and **MP**
iv.	$(\neg(\neg(x \neq y \mathrel{\Box\!\!\rightarrow} \bot) \rightarrow x \neq y) \mathrel{\Box\!\!\rightarrow} \neg(\neg(x \neq y \mathrel{\Box\!\!\rightarrow} \bot) \rightarrow x \neq y)) \rightarrow (\neg(\neg(x \neq y \mathrel{\Box\!\!\rightarrow} \bot) \rightarrow x \neq y) \mathrel{\Box\!\!\rightarrow} \bot)$	*iii*, **Closure**
v.	$\neg(\neg(x \neq y \mathrel{\Box\!\!\rightarrow} \bot) \rightarrow x \neq y) \mathrel{\Box\!\!\rightarrow} \neg(\neg(x \neq y \mathrel{\Box\!\!\rightarrow} \bot) \rightarrow x \neq y)$	**ID**
vi.	$\neg(\neg(x \neq y \mathrel{\Box\!\!\rightarrow} \bot) \rightarrow x \neq y) \mathrel{\Box\!\!\rightarrow} \bot$	*iv*, *v*, **PC** and **MP**
vii.	$((\neg(x \neq y \mathrel{\Box\!\!\rightarrow} \bot) \rightarrow x \neq y) \wedge (\neg(x \neq y \mathrel{\Box\!\!\rightarrow} \bot))) \rightarrow x \neq y$	**PC**
viii.	$((\neg x \neq y \mathrel{\Box\!\!\rightarrow} (\neg(x \neq y \mathrel{\Box\!\!\rightarrow} \bot) \rightarrow x \neq y)) \wedge (\neg x \neq y \mathrel{\Box\!\!\rightarrow} \neg(x \neq y \mathrel{\Box\!\!\rightarrow} \bot))) \rightarrow (\neg x \neq y \mathrel{\Box\!\!\rightarrow} x \neq y)$	*vii*, **Closure**
ix.	$(x \neq y \wedge \neg x \neq y) \rightarrow \bot$	**PC**
x.	$((\neg x \neq y \mathrel{\Box\!\!\rightarrow} x \neq y) \wedge (\neg x \neq y \mathrel{\Box\!\!\rightarrow} \neg x \neq y)) \rightarrow (\neg x \neq y \mathrel{\Box\!\!\rightarrow} \bot)$	*ix*, **Closure**
xi.	$\neg x \neq y \mathrel{\Box\!\!\rightarrow} \neg x \neq y$	**ID**
xii.	$(\neg x \neq y \mathrel{\Box\!\!\rightarrow} x \neq y) \rightarrow (\neg x \neq y \mathrel{\Box\!\!\rightarrow} \bot)$	*x*, *xi*, **PC** and **MP**
xiii.	$\bot \rightarrow p$	**PC**
xiv.	$(\neg p \mathrel{\Box\!\!\rightarrow} \bot) \rightarrow (\neg p \mathrel{\Box\!\!\rightarrow} p)$	*xiii*, **Closure**
xv.	$(\neg p \mathrel{\Box\!\!\rightarrow} p) \rightarrow (q \mathrel{\Box\!\!\rightarrow} p)$	**Vac**
xvi.	$(\neg p \mathrel{\Box\!\!\rightarrow} \bot) \rightarrow (q \mathrel{\Box\!\!\rightarrow} p)$	*xiv*, *xv*, **PC** and **MP**
xvii.	$(\neg\neg(x \neq y \mathrel{\Box\!\!\rightarrow} \bot) \mathrel{\Box\!\!\rightarrow} \bot) \rightarrow (\neg x \neq y \mathrel{\Box\!\!\rightarrow} \neg(x \neq y \mathrel{\Box\!\!\rightarrow} \bot))$	Instance of *xvi*
xviii.	$(\neg(\neg(x \neq y \mathrel{\Box\!\!\rightarrow} \bot) \rightarrow x \neq y) \mathrel{\Box\!\!\rightarrow} \bot) \rightarrow (\neg x \neq y \mathrel{\Box\!\!\rightarrow} (\neg(x \neq y \mathrel{\Box\!\!\rightarrow} \bot) \rightarrow x \neq y))$	Instance of *xvi*
xix.	$(\neg\neg(x \neq y \mathrel{\Box\!\!\rightarrow} \bot) \mathrel{\Box\!\!\rightarrow} \bot) \rightarrow (\neg x \neq y \mathrel{\Box\!\!\rightarrow} \bot)$	*vi*, *viii*, *xii*, *xvii*, *xviii*, **PC** and **MP**
xx.	$(x \neq y \mathrel{\Box\!\!\rightarrow} \bot) \equiv \neg\neg(x \neq y \mathrel{\Box\!\!\rightarrow} \bot)$	**PC**
xxi.	$(\neg\neg(x \neq y \mathrel{\Box\!\!\rightarrow} \bot) \mathrel{\Box\!\!\rightarrow} \bot) \equiv (((x \neq y \mathrel{\Box\!\!\rightarrow} \bot) \mathrel{\Box\!\!\rightarrow} \bot))$	*xx*, **REA**
xxii.	$((x \neq y \mathrel{\Box\!\!\rightarrow} \bot) \mathrel{\Box\!\!\rightarrow} \bot) \rightarrow (\neg x \neq y \mathrel{\Box\!\!\rightarrow} \bot)$	*xix*, *xxi*, **PC** and **MP**
xxiii.	$x \neq y \rightarrow ((x \neq y \mathrel{\Box\!\!\rightarrow} \bot) \mathrel{\Box\!\!\rightarrow} \bot)$	**B**$\Box\!\!\rightarrow$
xxiv.	$x \neq y \rightarrow (\neg x \neq y \mathrel{\Box\!\!\rightarrow} \bot)$	*xxxxii*, *xxiii*, **PC** and **MP**
xxv.	$\forall x, y(x \neq y \rightarrow \neg x \neq y \mathrel{\Box\!\!\rightarrow} \bot)$	*xxiv*, **Gen**(x2)
xxvi.	$\forall x, y(x \neq y \rightarrow \Box x \neq y)$	*xxv*, Definition$_1$

I was unable to prove the necessity of distinctness without appeal to each of these axioms. The increased number of axioms (in comparison to those needed to prove the necessity of identity) corresponds to increased controversy; a philosopher who rejected **B**$\Box\!\!\rightarrow$, for example, may deny the necessity of distinctness, but not the necessity of identity (assuming, of course, they accept the remainder of the axioms).

The necessity of distinctness interacts with counterfactual logic in significant ways. In particular, it takes a stand on the debate over counteridenticals: counterfactuals with false identifications in their antecedents.[25] Potential examples of counteridenticals include 'If I were you, I would leave by 4:00 pm' and 'If John were Einstein, he would pass his physics exam.'

By itself, the necessity of distinctness does not force a position on whether counteridenticals are substantive. But—in combination with vacuism—it does.[26] As mentioned above, HOCL entails vacuism; **Closure** and **REA** collectively entail that it holds. This can be shown as follows:

i.	$\bot \rightarrow q$	**PC**
ii.	$(p \Box\!\!\rightarrow \bot) \rightarrow (p \Box\!\!\rightarrow q)$	*i*, **Closure**
iii.	$\forall q((p \Box\!\!\rightarrow \bot) \rightarrow (p \Box\!\!\rightarrow q))$	*ii*, **Gen**
iv.	$\forall q((p \Box\!\!\rightarrow \bot) \rightarrow (p \Box\!\!\rightarrow q)) \rightarrow ((p \Box\!\!\rightarrow \bot) \rightarrow \forall q(p \Box\!\!\rightarrow q))$	**UD**
v.	$(p \Box\!\!\rightarrow \bot) \rightarrow \forall q(p \Box\!\!\rightarrow q)$	*iii*, *iv*, and **MP**
vi.	$p \equiv \neg\neg p$	**PC**
vii.	$p \Box\!\!\rightarrow \bot \equiv \neg\neg p \Box\!\!\rightarrow \bot$	*vi* and **REA**
viii	$(\neg\neg p \Box\!\!\rightarrow \bot) \rightarrow \forall q(p \Box\!\!\rightarrow q)$	*v*, *vii*, **PC** and **MP**
ix.	$\Box\neg p \rightarrow \forall q(p \Box\!\!\rightarrow q)$	*viii* and Definition$_1$

While this particular derivation relies both upon **Closure** and **REA**, I suspect that the real culprit is **Closure**; anyone who accepts this rule will be pressured to endorse vacuism. In effect, **Closure** allows for the substitution of entailments in counterfactuals' consequents (i.e., the inference from $p \Box\!\!\rightarrow q$ and $q \vdash r$ to $p \Box\!\!\rightarrow r$)—and this comes very close to vacuism itself.[27] Given an impossible proposition p, **ID** entails that $p \Box\!\!\rightarrow p$. Because p is impossible, it entails absolutely everything—including an arbitrary q, so **Closure** allows us to infer $p \Box\!\!\rightarrow q$. Those who would reject vacuism will thus presumably also reject **Closure**.

HOCL is thus committed both to the necessity of distinctness and to the vacuity of counterpossibles. Therefore, it is committed to the claim that all counteridenticals are vacuous; philosophers who appeal to substantive counteridenticals are wrong. Adherents of HOCL thus owe a response to plausible examples of substantive counteridenticals.

[25] For defenses of substantive counteridenticals, see Kocurek (2018); Wilhelm (2021). The discussion of counteridenticals far precedes these works—see, e.g., Polluck (1976).

[26] Recall that vacuism is the claim that all counterpossibles hold vacuously—i.e., that $\Box\neg p \rightarrow \forall q(p \Box\!\!\rightarrow q)$

[27] More precisely, if we endorse the **Deduction Theorem**—according to which $p \vdash q$ entails $\vdash p \rightarrow q$, then we can understand Vacuism as following from **Closure**. For an arbitrary $p \vdash q$, the **Deduction Theorem** then entails $\vdash p \rightarrow q$—and, given closure, this in turn entails $\vdash (r \Box\!\!\rightarrow p) \rightarrow (r \Box\!\!\rightarrow q)$.

In my view, the best response is that natural language examples are not genuine counteridenticals. Rather, they are paraphrases of sentences that do not involve identity—and whose antecedents are contingent. While they are substantive, they do not conflict with vacuism. We need not hold that all counteridenticals paraphrase in the same way; they may be paraphrases for different sorts of counterfactuals. The example 'If I were you, I would leave by 4:00 pm' seems to gloss 'If I were in your situation, I would leave by 4:00 pm'—and 'If John were Einstein, he would pass his physics exam' seems to gloss 'If John were as smart as Einstein, he would pass his physics exam.'[28]

Some natural-language expressions indisputably involve identity—and do not seem synonymous with putative examples of counteridenticals. Take, for example, 'If I were identical to you, I would leave by 4:00 pm.' This explicitly invokes identity—yet does not appear to mean the same thing as 'If I were you, I would leave by 4:00 pm.' Since the former involves identity and is not synonymous with the latter, there is room to deny that the latter involves identity. Moreover, expressions like 'If I were identical to you' are sufficiently removed from ordinary use that we ought not revise counterfactual logic in light of them.

The upshot is this: HOCL allows us to prove both the necessity of identity and the necessity of distinctness. Because it is also committed to vacuism, HOCL entails that all counteridenticals are true. While this is controversial, there is room to resist putative examples of substantive counteridenticals in the literature.

Necessitism and the Barcan Formula

One of the most pivotal choice-points in quantified modal logic is the Barcan Formula:

Barcan Formula (BF): $\forall x \Box Fx \rightarrow \Box \forall x Fx$

If all objects necessarily bear property F, then—necessarily—all objects bear property F. Because all objects are necessarily self-identical, necessarily, all objects are self-identical. Quantified modal logic regiments inferences concerning both necessity and generality; more importantly, it formalizes the interaction of the two. The **Barcan**—and its converse—describe that interaction.

The **Barcan** is extremely controversial. Consider a world consisting of nothing except for two electrons: ones that repel one another and accelerate in opposite directions for eternity. Quite plausibly, each of these electrons is necessarily negatively charged. That is,

[28] Adherents of substantive counterexamples reject the gloss from 'If I were you, then p' as 'If I were in your situation, then p' due to sentences like 'If I were you, I would not be in your situation.' But there is contextual variation in 'your situation' that allows the gloss to succeed even in this case. Suppose, for example, that you have not begun a consequential assignment until the night before it was due. I might then claim 'If I were in your situation (i.e., the situation of having a consequential assignment due) I would not be in your situation (i.e., the situation of having left it so late).'

it may be *essential* to electrons that they are negatively charged; any particle that was not negatively charged would not be those electrons. In this world (which contains nothing except electrons), all objects are necessarily negatively charged. The **Barcan** would then allow us to infer that, necessarily, all objects are negatively charged. But, intuitively, this need not be so. Although each of the two electrons is (necessarily) negatively charged, there could have existed other particles—protons, neutrons and the like—which are not negatively charged. So, although all objects are necessarily negatively charged, it does not seem necessary that all objects are negatively charged.[29]

Despite the unintuitive implications that the **Barcan** has, many are pressured to endorse it—as minimal assumptions entail that it is true.[30] It is possible to prove the **Barcan** in HOCL from **REA, Closure, Vac,** and **B**$\Box\!\!\rightarrow$. To the best of my knowledge, this has gone overlooked in the literature; debates over **REA** and **Vac** omit their connection to the **Barcan** (for example).

It is helpful to first prove some derived rules that, in turn, allow the proof of the **Barcan**.

Derived Rule 1 (DR1): If $\vdash p \rightarrow q$ then $\vdash (\neg p \Box\!\!\rightarrow \bot) \rightarrow (\neg q \Box\!\!\rightarrow \bot)$

i.	$p \rightarrow q$	Supposition
ii.	$((p \rightarrow q) \wedge \neg(p \rightarrow q)) \rightarrow \bot$	**PC**
iii.	$\neg(p \rightarrow q) \rightarrow \bot$	*i, ii,* **PC** and **MP**
iv.	$(\neg(p \rightarrow q) \Box\!\!\rightarrow \neg(p \rightarrow q)) \rightarrow (\neg(p \rightarrow q) \Box\!\!\rightarrow \bot)$	*iii,* **Closure**
v.	$\neg(p \rightarrow q) \Box\!\!\rightarrow \neg(p \rightarrow q)$	**ID**
vi.	$\neg(p \rightarrow q) \Box\!\!\rightarrow \bot$	*iv, v,* **MP**
v.	$\bot \rightarrow p$	**PC**
vi.	$(\neg p \Box\!\!\rightarrow \bot) \rightarrow (\neg p \Box\!\!\rightarrow p)$	*v*, **Closure**
vii.	$(\neg p \Box\!\!\rightarrow p) \rightarrow (\neg q \Box\!\!\rightarrow p)$	**Vac**
viii.	$(\neg p \Box\!\!\rightarrow \bot) \rightarrow (\neg q \Box\!\!\rightarrow p)$	*vi, vii,* **PC** and **MP**
ix.	$((p \rightarrow q) \wedge p) \rightarrow q$	**PC**
x.	$((\neg q \Box\!\!\rightarrow (p \rightarrow q)) \wedge (\neg q \Box\!\!\rightarrow p)) \rightarrow (\neg q \Box\!\!\rightarrow q)$	*ix,* **Closure**
xi.	$((\neg(p \rightarrow q) \Box\!\!\rightarrow \bot) \wedge (\neg p \Box\!\!\rightarrow \bot)) \rightarrow (\neg q \Box\!\!\rightarrow q)$	*viii* (x2), **PC** and **MP**
xii.	$(q \wedge \neg q) \rightarrow \bot$	**PC**
xiii.	$((\neg q \Box\!\!\rightarrow q) \wedge (\neg q \Box\!\!\rightarrow \neg q) \rightarrow (\neg q \Box\!\!\rightarrow \bot)$	*xii,* **Closure**
xiv.	$\neg q \Box\!\!\rightarrow \neg q$	**ID**
xv.	$(\neg q \Box\!\!\rightarrow q) \rightarrow (\neg q \Box\!\!\rightarrow \bot)$	*xiii, xiv,* **PC** and **MP**
xvi.	$(\neg p \Box\!\!\rightarrow \bot) \rightarrow (\neg q \Box\!\!\rightarrow \bot)$	*vi, xi, xv,* **PC** and **MP**

[29] As I discuss below, the controversial implications of the **Barcan** surpass this example. Given minimal assumptions, it entails *necessitism*, the view that all objects necessarily exist.

[30] Marcus (1947) establishes that the **Barcan** holds in S2—a system I do not discuss in depth here—and the strict conditional. That is, the relevant modal operator was $\Box(A \rightarrow B)$, rather than $\Box$. Prior (1956) proves that the **Barcan** holds in a quantified version of S5. The proof of the **Barcan** in the weaker B system is attributed to John Lemmon in Prior (1967). See, also, Cresswell and Hughes (1996).

Derived Rule 2 (DR2): If $\vdash p \rightarrow q$ then $\vdash \neg(p \Box\!\!\rightarrow \bot) \rightarrow \neg(q \Box\!\!\rightarrow \bot)$

i.	$p \rightarrow q$	Supposition
ii.	$\neg q \rightarrow \neg p$	*i*, **MP** and **PC**
iii.	$(\neg\neg q \Box\!\!\rightarrow \bot) \rightarrow (\neg\neg p \Box\!\!\rightarrow \bot)$	*ii* **DR1**
iv.	$p \equiv \neg\neg p$	**PC**
v.	$(p \Box\!\!\rightarrow \bot) \equiv (\neg\neg p \Box\!\!\rightarrow \bot)$	*iv* and **REA**
vi.	$(q \Box\!\!\rightarrow \bot) \equiv (\neg\neg q \Box\!\!\rightarrow \bot)$	**PC** and **REA**
vii.	$(q \Box\!\!\rightarrow \bot) \rightarrow (p \Box\!\!\rightarrow \bot)$	*iii*, *v*, *vi*, **PC** and **MP**
viii.	$\neg(p \Box\!\!\rightarrow \bot) \rightarrow \neg(q \Box\!\!\rightarrow \bot)$	*vii*, **PC** and **MP**

Derived Rule 3 (DR3): If $\vdash \neg((\neg p \Box\!\!\rightarrow \bot) \Box\!\!\rightarrow \bot)$ then $\vdash p$

i.	$\neg((\neg p \Box\!\!\rightarrow \bot) \Box\!\!\rightarrow \bot)$	Supposition
ii.	$\neg p \rightarrow ((\neg p \Box\!\!\rightarrow \bot) \Box\!\!\rightarrow \bot)$	**B**$\Box\!\!\rightarrow$
iii.	$\neg((\neg p \Box\!\!\rightarrow \bot) \Box\!\!\rightarrow \bot) \rightarrow \neg\neg p$	*ii*, **MP** and **PC**
iv.	$\neg\neg p$	*i*, *iii*, **MP**
v.	p	*iv*, **MP**, **PC**

Derived Rule 4 (DR4): If $\vdash \neg(p \Box\!\!\rightarrow \bot) \rightarrow q$ then $\vdash p \rightarrow (\neg q \Box\!\!\rightarrow \bot)$

i.	$\neg(p \Box\!\!\rightarrow \bot) \rightarrow q$	Supposition
ii.	$(\neg\neg(p \Box\!\!\rightarrow \bot) \Box\!\!\rightarrow \bot) \rightarrow (\neg q \Box\!\!\rightarrow \bot)$	*i* and **DR1**
iii.	$((p \Box\!\!\rightarrow \bot) \Box\!\!\rightarrow \bot) \rightarrow (\neg q \Box\!\!\rightarrow \bot)$	*ii*, **REA**, **MP** and **PC**
iv.	$p \rightarrow ((p \Box\!\!\rightarrow \bot) \Box\!\!\rightarrow \bot)$	**B**$\Box\!\!\rightarrow$
v.	$p \rightarrow (\neg q \Box\!\!\rightarrow \bot)$	*iii*, *iv*, **MP**, **PC**

With these rules in place, the **Barcan Formula** can be derived as follows:

i.	$\forall x(\neg Fx \Box\!\!\rightarrow \bot) \rightarrow \neg Fx \Box\!\!\rightarrow \bot$	**UI**
ii.	$\neg(\forall x(\neg Fx \Box\!\!\rightarrow \bot) \Box\!\!\rightarrow \bot) \rightarrow \neg((\neg Fx \Box\!\!\rightarrow \bot) \Box\!\!\rightarrow \bot)$	*i* and **DR2**
iii.	$\neg(\forall x(\neg Fx \Box\!\!\rightarrow \bot) \Box\!\!\rightarrow \bot) \rightarrow Fx$	*ii*, **DR3**, **MP** and **PC**
iv.	$\neg(\forall x(\neg Fx \Box\!\!\rightarrow \bot) \Box\!\!\rightarrow \bot) \rightarrow \forall x Fx$	*iii*, **Gen**, **MP**, **UD** and **PC**
v.	$\forall x(\neg Fx \Box\!\!\rightarrow \bot) \rightarrow (\neg\forall x Fx \Box\!\!\rightarrow \bot)$	*iv*, **DR4**
vi.	$\forall x \Box Fx \rightarrow \Box\forall x Fx$	*v* and Definition$_1$

The **Converse Barcan Formula** is nearly as significant to quantified modal reasoning as the **Barcan** itself:

Converse Barcan Formula (CBF): $\Box\forall x Fx \rightarrow \forall x\Box Fx$

While the **Converse Barcan** is nearly as controversial as the **Barcan** itself, weak principles entail that the it is true. It follows from **REA, Closure** and **Vac**. The proof is as follows:

i.	$\forall x Fx \rightarrow Fx$	**UI**
ii.	$(\neg\forall x Fx \;\Box\!\!\rightarrow \bot) \rightarrow \neg Fx \;\Box\!\!\rightarrow \bot$	*i*, **DR1**
iii.	$(\neg\forall x Fx \;\Box\!\!\rightarrow \bot) \rightarrow \forall x(\neg Fx \;\Box\!\!\rightarrow \bot)$	*ii*, **Gen, UD, PC** and **MP**
iv.	$\Box\forall x Fx \rightarrow \forall x\Box Fx$	*iii* and Definition$_1$

The controversy surrounding the **Barcan** and its converse arises primarily due its implications for *necessitism*: the claim that, necessarily, all objects necessarily exist. More formally, we can represent necessitism as:

Necessitism: $\Box\forall x\Box\exists y(x = y)$

Necessitists hold that this is true—contingentists that this is false.

It is worth acknowledging the limit to the disagreement between necessitists and contingentists. For any particular object, they might agree that that object necessarily exists. Both might claim that God necessarily exists—or that there necessarily exists a prime number between 3 and 7. In principle, contingentists could even accept that I necessarily exist (though they typically do not). They are committed only to the claim that *something-or-other* exists contingently—not to what that something is. So, even if we could (somehow) establish that I necessarily exist, we would not thereby have falsified contingentism. In practice, however, it is useful to work with concrete examples. Since I am as plausible a case of contingent existence as anything, it is natural to suggest that necessitists hold that I necessarily exist, while contingentists deny that I do—so long as we keep in mind that I serve merely as a placeholder for whatever-object-it-is that contingentists maintain might not have existed.

Consider the principle that Williamson dubs 'the being constraint:'

Being Constraint (BC): $\forall x\Box(\exists F(Fx) \rightarrow \exists z(x = z))$

This asserts that all objects are such that, necessarily, if there exists some property that they bear, then they are identical to something-or-other. That is, if there is a way that an object is, then the object must exist. Given the **Barcan** and **Converse Barcan**, this is equivalent to:

$$\Box\forall x(\exists F(Fx) \to \exists z(x = z))$$

This, in turn, follows from **Nec** and

$$\forall x(\exists F(Fx) \to \exists z(x = z))$$

Note that the **Being Constraint**—and the principles that generate it—crucially rely upon higher-order logic; the ability to express higher-order quantifiers is essential to this sort of principle. We can prove the **Being Constraint** in HOCL as follows:

i.	$x = x$	**Ref**
ii.	$\exists y.x = y$	*i*, **EG**, **PC** and **MP**
iii.	$\exists F.(Fx) \to \exists y.(x = y)$	*ii*, **MP** and **PC**
iv.	$\forall x(\exists F.(Fx) \to \exists y.(x = y))$	*iii* and **Gen**
v.	$\neg\forall x(\exists F.(Fx) \to \exists y.(x = y)) \to \bot$	*iv*, **PC** and **MP**
vi.	$(\neg\forall x(\exists F.(Fx) \to \exists y.(x = y)) \Box\!\!\to \neg\forall x(\exists F.(Fx) \to \exists y.(x = y))) \to (\neg\forall x(\exists F.(Fx) \to \exists y.(x = y)) \Box\!\!\to \bot)$	*v* and **Closure**
vii.	$\neg\forall x(\exists F.(Fx) \to \exists y.(x = y)) \Box\!\!\to \neg\forall x(\exists F.(Fx) \to \exists y.(x = y))$	**ID**
viii.	$\neg\forall x(\exists F.(Fx) \to \exists y.(x = y)) \Box\!\!\to \bot$	*vi*, *vii* and **MP**
ix.	$\Box\forall x(\exists F(Fx) \to \exists z(x = z))$	*viii* and Definition$_1$
x.	$\forall x\Box(\exists F(Fx) \to \exists z(x = z))$	*ix* and **Converse Barcan**

The **Being Constraint** thus holds if the **Converse Barcan** holds. Even those who deny (or remain neutral) on the **Barcan** and **Converse Barcan** may feel some pressure to accept the **Being Constraint**.[31] After all, if we were to count the number of objects which are F, we would presumably assume that if something is an F, then it must exist—and so is worthy of being counted. Without the **Being Constraint**, it is difficult to see why the fact that object a is an F would impact the number of objects that are F (since, without the **Being Constraint** the fact that a is F is compatible with a not existing).

The step from the **Being Constraint** to **Neces** is straightforward. All objects bear the property *is self-identical*—from which it follows that there exists some property that every object bears. The **Being Constraint** then allows us to conclude that there exists an object that each object is identical to—and that this holds necessarily. So, those who endorse the **Being Constraint** accept **Necessitism**.

It is also possible to prove **Necessitism** directly in HOCL—without appealing to the **Being Constraint**. This can be shown as follows:

[31] However, for objections to the use of the **Being Constraint** (largely on the grounds that it seems unmotivated for contingentism) see Dorr (2016); Goodman (2016); Litland (Forthcoming).

i.	$x = x$	**Ref**
ii.	$\exists y.(x = y)$	*i*, **EG** and **MP**
iii.	$\neg\exists y.(x = y) \rightarrow \bot$	*ii*, **PC** and **MP**
iv.	$(\neg\exists y.(x = y) \mathrel{\Box\!\!\rightarrow} \neg\exists y.(x = y)) \rightarrow (\neg\exists y.(x = y) \mathrel{\Box\!\!\rightarrow} \bot)$	*iii*, **Closure**
v.	$\neg\exists y.(x = y) \mathrel{\Box\!\!\rightarrow} \neg\exists y.(x = y)$	**ID**
vi.	$\neg\exists y.(x = y) \mathrel{\Box\!\!\rightarrow} \bot$	*iv*, *v*, **MP**
vii.	$\forall x.(\neg\exists y.(x = y) \mathrel{\Box\!\!\rightarrow} \bot)$	*vi*, **Gen**
viii.	$\neg(\forall x.(\neg\exists y.(x = y) \mathrel{\Box\!\!\rightarrow} \bot)) \rightarrow \bot$	*vii*, **PC** and **MP**
ix.	$(\neg(\forall x.(\neg\exists y.(x = y) \mathrel{\Box\!\!\rightarrow} \bot)) \mathrel{\Box\!\!\rightarrow} \neg(\forall x.(\neg\exists y.(x = y) \mathrel{\Box\!\!\rightarrow} \bot))) \rightarrow (\neg(\forall x.(\neg\exists y.(x = y) \mathrel{\Box\!\!\rightarrow} \bot)) \mathrel{\Box\!\!\rightarrow} \bot)$	*viii*, **Closure**
x.	$\neg(\forall x.(\neg\exists y.(x = y) \mathrel{\Box\!\!\rightarrow} \bot)) \mathrel{\Box\!\!\rightarrow} \neg(\forall x.(\neg\exists y.(x = y) \mathrel{\Box\!\!\rightarrow} \bot))$	**ID**
xi.	$\neg(\forall x.(\neg\exists y.(x = y) \mathrel{\Box\!\!\rightarrow} \bot)) \mathrel{\Box\!\!\rightarrow} \bot$	*ix*, *x*, **MP**
xii.	$\Box\forall x.\Box\exists y.(x = y)$	*xi*, Definition$_1$

Those who endorse both **ID** and **Closure** must thus endorse **Necessitism**; contingentists ought to reject at least one of those principles.

Often necessitists claim that there are the same *number* of objects in every world; if there are n objects in the actual world, then there are n objects in all. However, this numerical claim does not follow from necessitism itself; the necessity of identity and distinctness are needed as well. For example, if the necessity of distinctness were false, it could be that every object necessarily exists, but some worlds have fewer objects that the actual world because objects that are distinct in the actual world are identical in another.

However, we have already established both the necessity of identity and distinctness in HOCL. Not only does everything necessarily exist, but it is necessary that there are many things as there actually are.

The Identity of Indiscernibles

The Principle of the Identity of Indiscernibles (hereafter, the **PII**) is the principle that objects cannot differ only numerically. Distinct objects must differ from one another in some non-numerical respect. Despite the presence of apparent counterexamples (most notably, Black (1952)'s pair of indiscernible spheres), many maintain that one interpretation of this principle is trivially true: the claim that objects bearing all of the same properties are identical.[32] This is held to be trivial due to the existence of haecceities: properties like *is identical to a*. Any objects that bear the same properties (in general) bear the same

[32] The first derivation of this triviality occurs in Whitehead and Russell (1952). One philosopher who denies that there is a trivial version of the PII is Rodriguez-Pereyra (2022)—on the grounds that objects could differ 'only numerically' while not bearing all of the same properties.

haecceities (in particular). And, clearly, all objects that bear the property *is identical to a* are identical to one another.

Like the **Being Constraint**, the **PII** is primed for higher-order modal (or counterfactual) logic. The sentence 'Objects *cannot* differ only numerically' has modal force. It is not only a claim about what is actually so, but rather about what must be so.[33] Moreover, 'Objects that bear all of the same properties' overtly quantifies over properties themselves—so we cannot hope to reconstruct this proof in a first-order language. Establishing this principle requires reasoning with both modals and higher-order quantifiers.

We can prove the **PII** in HOCL in the following way:

i.	$\forall X.\forall x,y.(Xx \leftrightarrow Xy) \rightarrow \forall x,y.(\lambda z.(x = z)(x) \leftrightarrow \lambda z.(x = z)(y))$	**UI**
ii.	$\forall x,y.(\lambda z.(x = z)(x) \leftrightarrow \lambda z.(x = z)(y)) \rightarrow (\lambda z.(x = z)(x) \leftrightarrow \lambda z.(x = z)(y))$	**UI**
iii.	$\lambda z.(x = z)(x) \leftrightarrow x = x$	**E**β
iv.	$x = x$	**Ref**
v.	$\forall X.\forall x,y.(Xx \leftrightarrow Xy) \rightarrow \lambda z.(x = z)(y)$	*i, ii, iii, iv,* **PC,** and **MP**
vi.	$\lambda z.(x = z)(y) \leftrightarrow x = y$	**E**β
vii.	$\forall X.\forall x,y.((Xx \leftrightarrow Xy) \rightarrow x = y)$	*v, vi,* **PC** and **MP**
viii.	$\neg(\forall X.\forall x,y.(Xx \leftrightarrow Xy) \rightarrow x = y) \rightarrow \bot$	*vii,* **PC** and **MP**
ix.	$(\neg(\forall X.\forall x,y.(Xx \leftrightarrow Xy) \rightarrow x = y) \Box\!\!\rightarrow \neg(\forall X.\forall x,y.(Xx \leftrightarrow Xy) \rightarrow x = y)) \rightarrow (\neg(\forall X.\forall x,y.(Xx \leftrightarrow Xy) \rightarrow x = y) \Box\!\!\rightarrow \bot)$	*viii,* **Closure**
x.	$\neg(\forall X.\forall x,y.(Xx \leftrightarrow Xy) \rightarrow x = y) \Box\!\!\rightarrow \neg(\forall X.\forall x,y.(Xx \leftrightarrow Xy) \rightarrow x = y)$	**ID**
xi.	$\neg(\forall X.\forall x,y.(Xx \leftrightarrow Xy) \rightarrow x = y) \Box\!\!\rightarrow \bot$	*ix, x,* **MP**
xii.	$\Box\forall X.\forall x,y.((Xx \leftrightarrow Xy) \rightarrow x = y)$	*xi* and Definition$_1$

So, if **Closure** and **ID** hold, then it is necessary that objects that bear all of the same properties are identical.

Maximalism

Metaphysicians subscribe to various forms of **maximalism**: a profligate ontology that holds that the world is as full as it could be. Mereological Universalists, for example, hold that any collection of objects composes another.[34] In addition to ordinary objects

[33] However, for contingent versions of the PII, see Casullo (1984)—and French (1989) for a response.

[34] There are far too many universalists to provide a comprehensive list here. Notable adherents include Lewis (1986); Sider (2001).

like tables, cars and chairs, there are also strange objects—like the object composed of an electron at the end of my nose and the galaxy Alpha Centauri. And defenders of essential plenitude hold that for any object that bears a collection of properties *FF*, there exists an object that bears *FF* essentially and all other properties accidentally.[35] While there is an object co-located with me that is contingently seated, there is another which is essentially seated—one that ceases to exist the moment I stand

There are various arguments for different forms of **maximalism**. Many hold that there is no non-arbitrary way to restrict which objects exist—and, in the absence of a non-arbitrary restriction, we ought to accept no restriction at all. There seems no principled reason why my body exists, but an object consisting of my body an an electron hovering next to my left thumb does not. And many accept that the statue is distinct from the clay (because the statue is essentially shaped thus-and-so, while the clay is only accidentally shaped thus-and-so)—but find no principled reason to deny that properties other than shape give rise to coincident objects as well.

Philosophers tempted by this line of argument may wonder how 'full' the world could be. One view is the following:

> "What maximalism says is that for any type of object such that there can be objects of that type...there are such objects" Eklund (2008)[36]

We might formally represent this type of **maximalism** as the following:

$$\forall X.(\Diamond(\exists x.Xx) \rightarrow \exists x.Xx)$$

As stated, **maximalism** faces serious problems—the most well known of which is the bad-company problem.[37] We can define the property *being an xheart* as *being a heart and such that there are no livers* and the property *being an xliver* as *being a liver and such that there are no hearts*. While it seems possible for there to be xhearts, and seems possible for there to be xlivers, there cannot be both xhearts and xlivers.

HOCL does not entail **maximalism**; it is compatible both with the claim that **maximalism** is true and the claim that **maximalism** is false. However, it can be used to demonstrate that **maximalism** has an important implication for counterfactual logic; it entails the converse of Lewis (1973*b*)'s Weak Centering Axiom: $(p \mathrel{\Box\!\!\rightarrow} q) \rightarrow (p \rightarrow q)$. Converse Weak Centering is the principle that material implication entails counterfactual implication; the conditional 'if p then q' entails that if p were true then q would be true. We can establish that **maximalism** entails Converse Weak Centering as follows:

[35]Some who subscribe to plenitude include Fine (1999); Johnston (2006); Koslicki (2008).

[36]Eklund also includes the modifier 'given that the empirical facts are exactly what they are'—a modification that he acknowledges requires clarification.

[37]In addition to Eklund (2008), see Thomasson (2015); Fairchild (2019)

i.	$\forall X.(\Diamond(\exists x.Xx) \to \exists x.Xx)$	**Maximalism**
ii.	$\forall X.(\neg(\exists x.Xx \mathrel{\Box\!\!\to} \bot) \to \exists x.Xx)$	*i*, Definition$_1$ and Definition$_2$
iii.	$\neg(\exists x.\lambda y.\neg(p \to q)(x) \mathrel{\Box\!\!\to} \bot) \to \exists x.\lambda y.\neg(p \to q)(x)$	*ii*, **UI**
iv.	$\neg(p \to q) \equiv \exists x.\lambda y.\neg(p \to q)(x)$	**E**β, **EG**, **PC** and **MP**
v.	$\neg(\neg(p \to q) \mathrel{\Box\!\!\to} \bot) \to \neg(p \to q)$	*iii*, *iv*, **REA**, **PC** and **MP**
vi.	$(p \to q) \to (\neg(p \to q) \mathrel{\Box\!\!\to} \bot)$	*v*, **PC** and **MP**
vii.	$(p \wedge (p \to q)) \to q$	**PC**
viii.	$((p \mathrel{\Box\!\!\to} p) \wedge (p \mathrel{\Box\!\!\to} (p \to q))) \to (p \mathrel{\Box\!\!\to} q)$	*vii*, **Closure**
ix.	$p \mathrel{\Box\!\!\to} p$	**ID**
x.	$(p \mathrel{\Box\!\!\to} (p \to q)) \to (p \mathrel{\Box\!\!\to} q)$	*viii*, *ix*, **PC** and **MP**
xi.	$\bot \to (p \to q)$	**PC**
xii.	$(\neg(p \to q) \mathrel{\Box\!\!\to} \bot) \to (\neg(p \to q) \mathrel{\Box\!\!\to} (p \to q))$	*xi*, **Closure**
xiii.	$(\neg(p \to q) \mathrel{\Box\!\!\to} (p \to q)) \to (p \mathrel{\Box\!\!\to} (p \to q))$	**Vac**
xiv.	$(p \to q) \to (p \mathrel{\Box\!\!\to} (p \to q))$	*vi*, *xii*, *xiii*, **PC** and **MP**
xv.	$(p \to q) \to (p \mathrel{\Box\!\!\to} q)$	*x*, *xiv*, **PC** and **MP**

While important in its own right, this result also relates to the strongest consistent modal logic: TRIV.[38] It is characterized by the axiom $p \leftrightarrow \Box p$. If we were to describe modality in terms of world accessibility, TRIV corresponds to the assumption that accessibility is reflexive and unique; the actual world can access itself, and nothing else.

TRIV is implausible in many cases—but some philosophers advocate its use. Yli-Vakkuri and Hawthorne (2020) argue that it holds in the language of pure mathematics. That is, in a language capable only of expressing mathematical claims, a sentence p is true just in case it is necessarily true. Chen (Forthcoming) argues that there is only one physically possible world—so TRIV holds for nomological possibility. And necessitarians—who hold that the world necessarily is as it actually is—presumably accept TRIV for metaphysical modality.

Converse Weak Centering entails TRIV. To see why this is so, select an arbitrary true proposition p. Because $\neg p$ is false, **PC** entails that $\neg p \to \bot$ is true—and Converse Weak Centering then entails $\neg p \mathrel{\Box\!\!\to} \bot$. Given Definition$_1$, this is equivalent to $\Box p$. So, p entails $\Box p$. Because Maximalism entails Converse Weak Centering—and Converse Weak Centering entails $p \to \Box p$, **maximalism** entails $p \to \Box p$.

As it turns out, we can also show the reverse: that TRIV entails **maximalism**.[39] If the only possible world is the actual world, then if it is possible for an object to bear property

[38] See (Cresswell and Hughes, 1996, pg. 67) for proof that this is the strongest consistent modal logic.

[39] Because TRIV is consistent, and TRIV entails **maximalism** we can be confident that **maximalism** is consistent.

F, then some object actually does bear property *F*. This holds for every property; so, **maximalism** is true. It may seem to be a surprising coincidence that philosophers motivated by plentitude have stumbled upon the same logic employed in the philosophy of mathematics, philosophy of physics and metaphysics. But, in some respects, the connection between TRIV and **maximalism** ought to be unsurprising. The maximalist is guided by the thought that the world is as full as it could be; it takes only a slight shift in emphasis to arrive at the view that the world could only be as full as it (actually) is.

The Limit Assumption

Thus far, my formal approach has been conservative by design.[40] I have focused on what counterfactual logic can prove, not on what can be proven about counterfactual logic. As such, I have largely avoided discussions over the semantics of HOCL. However, I close by addressing a debate that has occurred almost entirely within counterfactual semantics: the **Limit Assumption**. This assumption cannot even be *stated* in a first-order counterfactual logic. The upshot is that philosophers who would state this assumption (either to endorse or to reject it) ought to operate with a higher-order system like HOCL. Within this system, we can express what the debate is about.

At its core, the **Limit Assumption** concerns whether, given an entertainable supposition p, there is a most-similar possible world in which p is true. The **Limit** allows for ties; there can be two (or more) p worlds that are equally—and maximally—similar to the actual world. However, it does not allow an infinite sequence of worlds, each of which approaches the actual world with arbitrary similarity.[41] At some point or other, we must arrive at a 'limit': a maximally similar possible world in which p is true. The **Limit** is thus of particular interest to philosophers who analyze counterfactuals in terms of world similarity.

A classic counterexample was introduced by Lewis (1973*a*). Suppose there were a line that was exactly one inch in length, and consider counterfactuals of the form 'If that line were longer than one inch, it would be of length $1 + x$.' If the **Limit Assumption** were true, every such sentence seems to be false. After all, a world in which the line is of length $1 + x$ is more dissimilar from the actual world than one in which it is of length $1 + \frac{x}{2}$—for every x. There is thus an infinite sequence of worlds that arbitrarily approaches the actual world. Because there is an infinite sequence of increased similarity, Lewis claims, the **Limit Assumption** is false.[42]

[40] My thanks to Jeremy Goodman for suggesting a discussion of the **Limit Assumption** in this paper.

[41] Adherents include Stalnaker (1968); Polluck (1976); Herzberger (1979); Warmbrōd (1982). Dissidents include Lewis (1973*b*); Hàjek (Forthcoming). For a discussion of various ways to precisify the assumption, see Kaufman (2017).

[42] Lewis does not find this example to be definitive, stating, "This and other examples are not quite decisive; but they should suffice at least to deter us from rashly assuming there *must* be a smallest antecedent-permitting sphere." (Lewis, 1973*b*, pg. 20)

Intuitive as this example is, it has unappealing implications. Polluck (1976) and Herzberger (1979) argue that it conflicts with an independently appealing principle of counterfactual logic: the claim that if p were true, then everything counterfactually implied by p would be true simultaneously. More formally:

$$p \boxright \forall q((p \boxright q) \rightarrow q)$$

(Note that if the counterfactual excluded middle—$p \boxright q \vee p \boxright \neg q$—holds, then the consequent is a world-proposition: one that determines the truth-value of every proposition whatsoever). For reasons Lewis discussed, for every length x, if the line were longer than one inch, the line would not be of length $1 + x$. Given the Polluck/Herzberger, it follows that if the line were greater than one inch, it would not be of length $1 + x$ for every x. But, intuitively, it follows from the claim that the line would not be of length $1 + x$ inches (for every x) that the line would not be longer than one inch. So, it follows that if the line were longer than one inch it would not be longer than one inch—an absurdity.

This reasoning appeals to a version of **Closure**. It follows from the claim that, for every x, the line would not be of length $1 + x$, that the line would not be greater than one inch in length. This entailment allowed the inference that if the line were greater than one inch it would not be greater than one inch.

However, **Closure** does not license this inference as it stands. This is because every instance of **Closure** only takes *finitely-many* premises before it can be applied, and this case involves infinitely many premises. For this reason, Polluck and Herzberger claim that adherents of the **Limit Assumption** grant infinite instances of **Closure**, while dissidents only grant finite instances. More precisely, Polluck argues that the **Limit Assumption** is equivalent to the claim that, for an infinite $\Gamma \models r$, if $\forall q \in \Gamma, p \boxright q$, then $p \boxright r$.

Already, we ought to be skeptical of our ability to distinguish finite from infinite cases—at least in a first-order language. Take a satisfiable and infinite Γ such that $\Gamma \models r$. $\Gamma \bigcup \{\neg r\}$ is therefore not satisfiable. The compactness theorem for first-order logic states that an infinite collection of sentences is satisfiable just in case every finite subset of sentences is satisfiable. Therefore, there must exist a finite $\Delta \subset \Gamma : \Delta \bigcup \{\neg r\}$ that is not satisfiable. Δ must be satisfiable, so we have that $\Delta \models r$. Given the completeness of first-order logic, we then have $\Delta \vdash r$. But because Δ is finite, a finite instance of **Closure** will allow us to infer that, for every $q \in \Delta$, if $p \boxright q$ then $p \boxright r$. So, in a first-order language, any infinite instance of **Closure** entails the existence of a finite instance of **Closure**. The upshot is that in order to distinguish finite from infinite cases (which we need to in order to distinguish opponents from adherents to the **Limit Assumption**, we require a language where compactness fails. Higher-order logic fits the bill.

Let us represent single-proposition entailment with $\leqslant$, so that $p \leqslant q$ iff $p \vdash q$. It is straightforward to define finite propositional entailment in terms of single-propositional entailment. We say that a collection Γ entails that p just in case the conjunction of Γ single-proposition entails that p. For infinite collections of propositions, this may not succeed if there are no infinite conjunctions.

Higher-order logic provides the resources to define infinite entailment. Effectively an infinite collection Γ entails p just in case every proposition that entails every element of Γ also entails p. More precisely, we represent the infinite collection of propositions with a propositional operator X of type $t \rightarrow t$ (which asserts that a proposition is a member of the relevant connection. Infinite entailment can then be represented as:

$$\leqslant_{\infty} := \lambda X.\lambda p.\forall r((\forall q(Xq \rightarrow r \leqslant q) \rightarrow r \leqslant p)$$

Armed with this definition of entailment, the infinite extension of Closure is:

$$(\Gamma \leqslant_{\infty} r) \rightarrow ((\forall q \in \Gamma p \mathrel{\Box\!\!\rightarrow} q) \rightarrow (p \mathrel{\Box\!\!\rightarrow} r))$$

Those who endorse the **Limit Assumption** claim that this is true; those who deny it claim that it is false.

Conclusion

If nothing else, I hope to have piqued the reader's interest in HOCL. This paper merely scratches the surface of what can be proven. Metaphysically significant results follow from extremely weak assumptions. I suspect that much more of interest could be proven in a stronger system—and I hope that others will explore what follows in a higher-order counterfactual logic.

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