MONISM AND THE ONTOLOGY OF LOGIC[1]

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Introduction

What exists in the world? This is a deceptively simple question—and one with an equally simple (if entirely trivial) answer: everything.[3] But if we dig ever-so-slightly deeper, we might recognize that this question can be interpreted in two different ways. We might, first, be asking, 'What *kinds* or *types* of things exist in the world?' The answer could naturally involve ordinary objects—things like tables, people, and cars—or perhaps stranger objects postulated by physicists—things like wave functions, bosons, and quasars.[4] Alternatively, we might be asking 'How *many* objects exist in the world?' Initially, it seems that we are even less well-equipped to answer this second question than we were the first. After all, we are scarcely able to determine the number of planets in the universe—much less the number of mountains, life forms, and molecules on each. Even if there is hope of arriving at an answer, it seems to fall within the purview of science—not philosophy. The armchair is a poor place from where to count everything that exists. Still, we can divide potential answers in two: pluralism—which holds that there are many objects—and monism—which maintains that there is only one.[5]

Monism is radically contrary to common sense. Even a cursory glance at our surroundings reveals a plurality. As I sit here in my chair, I observe my coffee cup, my phone, a lamp, and today's mail. The monist denies these objects—and a great many more.[6] But there is an intriguing path toward monism, one that proceeds on entirely logical grounds.[7]

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[3]This triviality was first noticed in the opening of Quine (1948).

[4]Some posit stranger objects still. Perhaps there is an object consisting of an electron at the end of my nose and the galaxy of Alpha Centauri—and perhaps there are 'outcars' (objects consisting of the portions of a car not presently located in a garage). This debate is orthogonal to my discussion here, and so I will say no more about these views. I direct those interested to Hirsch (1980) and Shoemaker (1988).

[5]There is yet a third answer to this question—*nihilism*—according to which there are no objects. I have nothing to say about nihilism, and so disregard it here.

[6]Alternatively, monism might not deny that these objects exist, but might instead claim that they are all identical, or do not count as objects.

[7]Historically, philosophers have endorsed a great many arguments for monism (though it remains a minority view). Parmenides is often interpreted as endorsing monism on the ground that it is incoherent to

Independently plausible logical assumptions entail that only one object exists. In some ways, I find this to be the most persuasive argument for monism yet. As Schaffer (2018: 18) said, 'The existence monist...need[s] strong arguments for their view. If it could be proven that positing a plurality of concrete objects...led irrevocably to contradiction, this should turn the tide.' What follows is the first sustained attempt to demonstrate that this is the case. If principles of logic entail that at most one object exists, then positing pluralities leads irrevocably to contradiction.

Despite this argument's initial appeal, I ultimately reject its conclusion. Some of the logical assumptions that generate monism are inconsistent. They must be walked back, if not abandoned entirely. But although I do not ultimately endorse the monism that results, I hope that readers will find it valuable to investigate how logic could lead to such a radical place.

Logical Framework

Given the central role that logic plays in this argument, it is important to clarify the assumptions at work. My first assumption is that classical logic holds. Sentences certified to be true by classical truth tables are indeed true—and sentences so certified to be false are indeed false. I assume that proofs carried out in classical first-order logic yield true conclusions if their premises are true: that is, I assume that the conclusions of sound arguments are true. I will provide no defense of this assumption. Those who deny classical logic need read no further.

This assumption is often passed over in silence—but there are several reasons to mention it here. First, it preempts a potential objection. Conversationally, some have rejected this argument on the ground that logic ought to be ontologically neutral—that is, that logic ought to have no implications regarding what exists.[8] Precisely because this logic is not ontologically neutral (in that it entails that only one object exists), we ought to reject this system.[9]

But classical logic is not ontologically neutral. A standard axiom is $\forall x(x = x)$: all

imagine change or coming-to-be. Spinoza argued that it is impossible for objects to bear the same property—as this would involve the same property existing in distinct places simultaneously. Since God bears the property *exists*, nothing else does. More recently, Horgan and Potrč (2000) argue that if numerous objects were to exist, it would be vague which objects there were. Since they cannot interpret vague existence, they reject numerous objects. Schaffer (2007) provides (but ultimately rejects) another defence on the ground that it is a simpler theory than one that posits a plurality—and simpler theories are preferable to complex ones. Della Rocca (2021) provides yet another argument on the ground that pluralism leads to *regress*. If multiple objects existed, they would stand in some relation R to one another. Each object would then stand in a relation to R, etc. To avoid the threat of regress, perhaps we ought to reject pluralism.

[8]My thanks to Jonathan Schaffer for pressing me on this point.

[9]Many (though not all) nonclassical systems also entail that at least one object exists—and so are not ontologically neutral. See Quine (1954) for a logic that avoids this result—as do many free logics (see Lambert, 1958; Nolt, 2021).

objects are self-identical. From this, one may infer that $a = a$—that object a is self-identical. This, in turn, entails that $\exists x(x = a)$—that there exists an object identical to a. But this requires at least one object to exist—object a.

So, classical logic itself is not ontologically neutral; it has implications regarding what exists. By accepting classicality, we thus lose the standing to object to an argument on the ground that it employs a non-neutral logic.

Another reason to mention my commitment to classical logic is that I appeal to Leibniz's Law. In particular, I assume that identical properties have the same objects in their extensions. If the property *is even* is identical to the property *is divisible by two without remainder*, then the two properties have the same numbers in their extension: a number is even if and only if it is divisible by two without remainder. This version of Leibniz's Law is somewhat stronger than the traditional formulation. The Law is sometimes described as the claim that identical objects bear identical properties. This version is not useful for my purposes, as I am concerned with attributes of identical properties, rather than those of identical objects. Indeed, first-order logic lacks the resources to even *express* the claim that properties are identical. So, it cannot license inferences from the claim that property F is identical to property G.

The trouble is that 'F is identical to G' states that F and G stand in a relation to one another (in particular, they stand in the relation of *being identical* to one another). For this reason, the version of Leibniz's Law at issue must describe properties and relations of properties themselves. But once we talk of 'properties of properties', the threat of self-referential paradox looms large.

Intuitively, some properties are properties of themselves—while others are not. The property *is a property* is a property of itself (since it is a property), as is the property *has an infinite extension* (since there are infinitely many properties with infinite extensions). In contrast, the property *is an onion* is not a property of itself (after all, it is a property, not an onion) nor is the property *has a finite extension* (since there are infinitely many properties with finite extensions). But if some properties are properties of themselves—while others are not—it is natural to think that there is a property is not a property of itself. It would have is an onion and has a finite extension in its extension—but not is a property or has an infinite extension. Is this property a property of itself? From each answer, the other arises: it is a property of itself just in case it is not. We have entered the realm of paradox, and have lost whatever grip on reality we had. We must retreat, and begin the investigation of properties again.

Fortunately, there is an elegant system with the ability to describe properties of properties that is immune to paradox—the λ-calculus.[10] λ is a mathematical system with applications that span numerous fields. It is heavily employed both in theoretical computer science and in formal linguistics. (Indeed, the realization that we can represent

[10] For a comprehensive introduction to the λ-calculus, see Barendregt (1984). The first suggestion of using this sort of theory to avoid paradox occurs with Russell (1903)—but the more sophisticated version of it that we see here was developed later, also by Russell (1908).

natural languages like English in λ—and prove results about them analogous to the theorems about artificial languages like arithmetic—constituted one of the biggest advances in our understanding of natural language in history.)[11] But our present interest in this system is driven, not by its uses for linguistics or computer science, but rather its uses for logic and metaphysics. The remainder of this section consists of a brief overview of λ; those already familiar with this system are free to skip to the next section.

λ allows us to formalize the talk of 'properties of properties' while avoiding the paradox. At its core, it is a hierarchical system: properties occur on different 'levels.' Just as there are properties of objects, so too there are properties of properties of objects, properties of properties of properties of objects, etc. Importantly, every property is only a property of items lower on the hierarchy than itself. Because no property applies to items on its own level, it is not a property of itself. Indeed, this language cannot even *state* that a property is (or is not) a property of itself. Thus, the sentence 'The property *is not a property of itself* is not a property of itself' is neither true nor false; it simply is not a sentence of our language. Because there are no sentences that predicate terms of themselves, the threat of paradox is removed.

This hierarchy is regimented with the use of *types*. Each constant (and variable) is assigned a type, which corresponds to wherever on the hierarchy it falls. In any typed language, we start out with a limited number of basic types and then identify the rest with functional relations between the basic ones. Here, we will work with a language that has two basic types—a type e for entities and a type t for sentences. Terms of type e denote ordinary objects; 'Socrates', 'The Eiffel Tower', and 'The Iliad' are all constants in this category. Terms of type t, for their part, denote sentences (both simple and complex). 'Socrates was a philosopher', 'Roses are red and violets are blue', and 'All humans are mortal' are all of type t.

We also allow for composite types—ones that are functions constructed out of the two basic elements.[12] For example, there is a type $(e \to t)$ that takes entities as its inputs and has sentences as its outputs. Typically, we identify first-order monadic predicates with terms of this type. Thus, 'is tall' is treated as a function that takes names like 'John' and 'Sarah' as inputs, and that has 'John is tall' and 'Sarah is tall' as respective outputs. Effectively, monadic predicates are functions that generate sentences that predicate the relevant property of the input.

Relatedly, dyadic first-order predicates are terms of type $(e \to (e \to t))$. They are functions with entities as their inputs, and first-order predicates as their outputs. So,

[11] See Montague (1970).

[12] When formally describing the syntax of this language, we say that, for any types τ_1 and τ_2, $(\tau_1 \to \tau_2)$ is a type. This type is a function that takes terms of type τ_1 as its inputs and has terms of type τ_2 as its output. We can use this syntax to regiment the talk of 'hierarchy of types'. Let us define a function *Rank* from the types to the natural numbers such that $Rank(e) = 1$ and $Rank(t) = 1$ and, for any types τ_1 and τ_2, $Rank(\tau_1 \to \tau_2) = Rank(\tau_1) + Rank(\tau_2)$. The claim '$\tau_1$ lies higher on the hierarchy of types than τ_2' amounts to the claim that $Rank(\tau_1) > Rank(\tau_2)$. We can prove (by induction) that if τ_2 is the functional input of τ_1, then $Rank(\tau_1) > Rank(\tau_2)$.

for example 'is next to' takes terms like 'Moe' and 'Larry' as its inputs, and has—as its outputs—terms like 'is next to Moe' and 'is next to Larry' (depending upon the initial inputs). These first-order predicates can themselves be supplemented with entities to generate sentences like 'Curly is next to Moe' and 'Curly is next to Larry.' And so we are off to the races: we can use these functions to describe second-order properties, third-order properties, and all the rest.[13]

In first-order languages, quantifiers perform two functions simultaneously. They serve both to bind variables and to make claims about generality. But in higher-order languages, these tasks come apart. In order to express 'The property *is F and G* is identical to the property *is G and F*', there must be a way of binding variables without thereby making universal or existential claims. That is to say, in order to describe '*is F and G*', the variable that falls under the scope of F must take the same values as the variable that falls under the scope of G—and so there must be a device to bind variables here—but there are no quantifiers in this sentence. In this language, λ performs this crucial function: it serves to bind variables. Quantifiers, then, do not bind variables. They are used only to make claims about generality. Thus, we express the claim that something is red with '$\exists\lambda x.(Red(x))$' rather than '$\exists x.(Red(x))$.' The quantifiers are of type $((\tau \rightarrow t) \rightarrow t)$' for every type τ. Effectively, first-order quantifiers are second-order properties—the property of *having at least one object in its extension* and the property of *having all objects in its extension*.

The upshot is this: the following derivation of monism depends on a version of Leibniz's Law that applies to properties—one that entails that identical properties have identical extensions. First-order logic lacks the ability to express the claim that properties are identical, and so we need a language with more expressive power. Care must be taken, as there is a risk of a self-referential paradox when describing properties of properties and relations themselves. However, once we adopt a typed higher-order language, we can indeed describe properties of properties with no threat of paradox. This language departs slightly from first-order logic, as λ terms—rather than quantifiers—bind variables. But in other respects, it functions exactly like classical logic. The rules of conjunction elimination,

[13]If our goal were to regiment the syntax of this language, it would be necessary to specify the types of the other logical terms in our language. However, these details have a minimal role in the overall argument here. Still, I have included some important ones in this note in case it is instructive for the reader. The sentential negation operator $\neg$ is a function with sentences as inputs and sentences as outputs—that is, it is of type $(t \rightarrow t)$. In particular, the output of this function is the negation of the input. If its input is 'Grass is green,' then its output is 'Grass is not green.' The binary operators $\wedge, \vee, \rightarrow$ and $\leftrightarrow$ (which represent conjunction, disjunction, the conditional, and the biconditional) are all of type $(t \rightarrow (t \rightarrow t))$. They take sequences of two sentences as their inputs and have a single sentence as their outputs—the conjunction, disjunction, conditional, or biconditional of the inputs respectively. That is to say, they generate sentences like 'Roses are red and violets are blue', 'Roses are red or violets are blue', 'If roses are red then violets are blue', and 'Roses are red if and only if violets are blue.' We also introduce variables for terms of every type. Not only are there variables for objects, but there are variables for properties, relations, sentential operators and the like. The only other constant worth mentioning in our language is used to express identity. For every type τ, there is a term $=$ of type $(\tau \rightarrow (\tau \rightarrow t))$. So, we represent the claim that Hesperus is identical to Hesperus with 'Hesperus = Hesperus', and the claim that conjunction is identical to conjunction with '$\wedge = \wedge$'.

disjunction introduction, and all the rest hold within this system. For the remainder of this chapter, this is the language that we will employ.

We are (finally) in a position to describe the higher-order formulation of Leibniz's Law as follows:

$$\alpha = \beta \rightarrow (\phi \leftrightarrow \phi^{[\alpha/\beta]})$$

If α is identical to β then a sentence ϕ is true if and only if ϕ is true when occurrences of α are substituted for occurrences of β. For example, because Superman is identical to Clark Kent, this principle entails that 'Superman can fly' holds if and only if 'Clark Kent can fly' holds. This principle thus entails the object-restricted interpretation of Leibniz's Law. If object a is identical to object b, we may use this to infer $Fa \leftrightarrow Fb$ (for any property F). So, it entails that identical objects bear all of the same properties. This principle also applies to identical properties: it entails that identical properties have identical extensions—as $\lambda x.Fx = \lambda x.Gx$ entails $Fa \leftrightarrow Ga$. This is the instance of Leibniz's Law that the following derivation of monism employs.

Two Logical Assumptions

In addition to my commitment to classical logic, I make two further assumptions. The first is that β-equivalent propositions are identical (a principle that I dub 'β-identification'). Specifying what this principle means requires care, as closely related principles can easily become confused.

One of the basic inferential resources of the λ-calculus is β-conversion. This is the inference from $\lambda x.Fx(a)$ to Fa. The proposition that *being an F* applies to an object a entails the proposition that Fa.[14] So, for example, the proposition that *is short* applies to Napoleon entails the proposition *Napoleon is short*. A bit more formally, if a term with a variable applies to an object, β-conversion allows us to replace occurrences of the variable with the name of that object. This inference (from a term to its β-conversion) is relatively uncontroversial.[15] A somewhat stronger (but nevertheless orthodox) assumption is that the two terms denote the same proposition. That is, not only may we infer Fa from $\lambda x.Fx(a)$, but they express *the very same thing*. The proposition that Mark bears *is bald* is identical to the proposition *Mark is bald*—and the proposition that Jack stands to Jill in the relation *is next to* is identical to the proposition *Jack is next to Jill*.

At first, it can be difficult to state β-identification in natural language—as natural language often does not distinguish between a term and its β-conversion. In ordinary use,

[14]Since we operate here with a typed higher-order language, this also applies to terms other than properties and objects. Generally, if a function is applied to an argument, β-conversion allows us to infer whatever it is that results from applying that function to that argument.

[15]As Dorr (2016: 52) notes, this inference may fail for opaque predicates (those involving terms like 'believes'). Suffice it to say that, by endorsing classical logic, I set aside these concerns.

it would be odd to distinguish 'that which results from applying *is wise* to Socrates' from 'Socrates is wise.' It is a distinction that arises in our formalism—not in ordinary thought. But the very fact that the terms appear interchangeable in natural language might be taken to indicate that they refer to the same thing.

β-identification is defended most explicitly by Dorr (2016). The main reason to believe that it is true is that it does not seem to be false. Intuitively, the proposition *Sarah is as tall as herself* is the same as the proposition *Sarah is as tall as Sarah*—and this seems to have nothing to do with facts about Sarah or the relation *is as tall as*. Quite generally, the claim that an object a stands in a relation R as itself seems to be the same as the claim that a stands in relation R to a. This inference holds if β-identification is true: the proposition $\lambda x.Rxa(a)$ is the same as the proposition $\lambda x.Rxx(a)$, according to β-identification.

My second assumption (beyond my commitment to classical logic) is that identical propositions contain identical properties. That is, if the proposition that Fa is identical to the proposition that Gb, then property F is identical to property G. (I dub this the 'Principle of Singular Extraction'—the PSE, for short). For example, if the proposition *John is a brother* is identical to the proposition *John is a male sibling*, then the property *is a brother* is identical to the property *is a male sibling*. And if the proposition *the number four is even* is identical to the proposition *the number four is divisible by two without remainder*, then the property *is even* is identical to the property *is divisible by two without remainder*.

I take it that the PSE is extraordinarily intuitive. One reason to accept the PSE is simply to recognize its initial intuitive appeal. But beyond this principle's initial appeal, some accounts of propositions entail that it is true. If these accounts are correct, then we have independent reasons to accept the PSE.

Some philosophers believe that propositions are *structured*, being built out of worldly items in much the way that sentences are built out of words.[16] Perhaps the proposition *grass is green* is built out of material concerning greenness and grass—and perhaps the proposition *the sky is blue* is built out of material concerning blueness and the sky. Propositions that are composed of different materials are distinct, even if they necessarily have the same truth values. The proposition *roses are red or not red* is presumably built out of material concerning roses, while *violets are blue or not blue* is built out of material concerning violets. Although these propositions are true in the same possible worlds (that is, each proposition is true in *every* world), they are distinct due to their differing compositions. Structured accounts appeal to those who would use propositions to make fine-grained distinctions—that is, to distinguish between propositions that necessarily have the same

[16]Some readers may be more accustomed to discussing *facts*—rather than propositions—as constructed out of worldly items. (This may be so, especially given Russell's Philosophy of Logical Atomism (1919)—which held that facts are constructed out of worldly items in much this way.) I do not wish to delve too deeply here into the distinction between facts and propositions. In many respects, facts seem to resemble true propositions (and it may be valuable for us to describe those propositions that are false—as well as those that are true). For our purposes, what matters is only that we are talking about the 0-ary analogue of monadic properties, binary relations, etc. Whether these items bear the label of 'fact' or 'proposition' is not a topic with which I will engage in much depth.

truth-values. For our purposes, the important point is this: *if propositions are structured in this way*, then there is a reason to endorse the PSE.[17] Because propositions that are composed of different material are distinct—and because some of this material concerns the properties that occur within propositions—propositions that are identical must contain the same material. That is, they must contain the same properties.

Structured propositions provide a path to the PSE—but a controversial one. In recent years, theories of structured propositions have come under sustained assault. One problem concerns their cardinality—that is, how many propositions there are. This problem was first discovered by Russell (1903) and noted (apparently independently) by Myhill (1958). One of the core commitments of a theory of structured propositions is that any differences in the syntactic structure of language correspond to differences in propositions. The fact that 'Mark is tall' differs syntactically from 'Mark is not not tall' ensures that the sentences express different propositions. The problem is that, for every collection of propositions, we can construct a sentence asserting that precisely those propositions are true. For this reason, there exists a mapping from every element of the powerset of propositions to a unique sentence (that is, we can map every element of the powerset of propositions to the sentence asserting that precisely the elements within that set are true).[18] If each of these sentences were to correspond to a different proposition, then there would exist a mapping from every element of the powerset of propositions to a unique proposition. But Cantor's Theorem entails that there is no such mapping. For every set s, there exists no mapping from every element of the powerset of s to a unique element of s. And so, for this reason, it cannot be that every difference in syntactic structure corresponds to a unique proposition.[19] Accounts of structured propositions which entail that differences in syntactic structure correspond to differences in the proposition are inconsistent. And if we are to reject structured accounts, then these accounts do not provide a reason for us to endorse the PSE. While I myself take no stand on propositional granularity in this chapter, I do reject all accounts of propositions that are inconsistent.

Higher-Order Monism

The stage is set. No further assumptions are needed to demonstrate that monism is true. Our logic has strength enough for that result. We can derive monism in the following way:

[17] For an account of structured proposition that does *not* license the PSE, see Bacon (2023). On his view, propositions are structured pictorially—rather than syntactically—and so a difference in our syntax need not correspond to a difference in proposition.

[18] The reference to set theory in the Russell-Myhill problem is purely expository. The problem can be generated while making no reference to sets.

[19] For a conception of structured propositions that avoids the Russell-Myhill problem, see Fritz, Lederman and Uzquiano (2021).

i.	$\lambda y.(y = y)(a) = \lambda y.(y = a)(a)$	β-Identification
ii.	$\lambda y.(y = y) = \lambda y.(y = a)$	*i,* PSE
iii.	$\forall \lambda y.(y = y)$	Classical Logic
iv.	$\forall \lambda y.(y = a)$	*ii, iii,* Leibniz's Law
v.	$\exists \lambda x.\forall \lambda y.(y = x)$	*iv,* Classical Logic

Less formally, β-identification entails that the proposition that *a is self-identical* is identical to the proposition *a is identical to a*. After all, the second proposition is merely the β-conversion of the first, and β-identification asserts that β-equivalent propositions are identical. Given the PSE, the properties contained within these propositions must be identical as well. That is, the property *is self-identical* is, itself, identical to the property *is identical to a*. Classical logic entails that all objects are self-identical: every object falls within the extension of *is self-identical*. Because all objects are self-identical—and because *is self identical* is, itself, identical to *is identical to a*—it follows that all objects are identical to *a*. (After all, identical properties must have identical extensions). And so there exists an object *x* (namely *a*) that all objects are identical. But the claim that all objects are identical to one object *just is* a statement of monism. So, monism is true.

This particular derivation concerns objects—but we can apply an argument of this structure to properties as well. That is, just as we can establish that there is only one object, we can also establish that there is only one property, as follows:

i.	$\lambda Y(Y = Y)(F) = \lambda Y.(Y = F)(F)$	β-Identification
ii.	$\lambda Y.(Y = Y) = \lambda Y.(Y = F)$	*i,* PSE
iii.	$\forall \lambda U.(Y = Y)$	Classical Logic
iv.	$\forall \lambda Y.(Y = F)$	*ii, iii,* Leibniz's Law
v.	$\exists \lambda X.\forall \lambda Y.(Y = X)$	*iv,* Classical Logic

β-identification entails that the proposition *F is self-identical* is identical to the proposition *F is identical to F*. Given the PSE, the second-order property *is self-identical* must be identical to the second-order property *is identical to F*. Classical logic entails that all properties are self-identical. Because all properties are self-identical—and because *is self-identical* is identical to *is identical to F*—all properties are identical to *F*. Therefore, there is but a single property.

It should be clear that it is possible to formulate arguments of this structure for terms of every type. We can establish that there is only a single binary first-order relation, that there is only a single monadic second-order property, etc. Interestingly, although it is possible to establish that there is a unique referent for terms of arbitrary type, it is *not* possible to generalize and thereby make a claim about terms of all types. That is to say, there exists no sentence that asserts 'All terms of type τ are identical, for every τ'. Quantifiers only range over terms lower on the hierarchy of types than themselves—and so there is no quantifier that makes a claim about terms of all types generally. Relatedly, although it is possible to establish that there is one object—and it is possible to establish that there is one property—it is not possible to establish that the object is identical to the property. Indeed, the claim '$a = F$' is not even a sentence in our language. Identity only relates to terms of the same type—and, while 'a' is of type e, 'F' is of type $(e \rightarrow t)$.

The fact that this monism generalizes affects the variety of monism at issue. While all existence monists maintain that there is only one object, they disagree about the nature or attributes of that object. Those like Horgan and Potrč (2000), for example, maintain that the object admits of internal variation. That is, it has an internal structure that gives rise to the manifest world that we observe. In some respects, we can think of the object as having a jello-like structure. It congeals in some places—and stretches in others—so that it precisely appears as the world around us does. Spinoza, too, allowed there to be different attributes of the sole object that exists (attributes that we might identify with distinct properties of that object). In contrast, the monism of Parmenides (at least on his monistic interpretations) and Della Rocca (2021) is austere. Not only is there only a single object; the object itself has only a single property. Higher-order monism is much closer in spirit to Parmenides and Della Rocca than to Spinoza, Horgan and Potrč. Because there is only one property *for* the object to bear, we cannot make sense of internal variation within it. It bears but a single property—the property of being identical to itself.

The ability to establish monism on purely logical grounds also suggests that we can comment on its modal status. Nearly every philosopher who countenances modality holds that the results of our theorems are necessarily true. (Indeed, the weakest standard modal logic—**K**—includes the necessitation rule, which explicitly allows us to conclude that the results of our theorems are necessary.) Because we can prove that monism is true, we may also conclude that it is necessarily true. There is no possible situation in which there is more than one object.[20]

It is not merely possible to establish that the PSE entails higher-order monism (in a logic that includes both classicality and β-identification). It is also possible to establish that higher-order monism entails the PSE. This can be shown as follows:

i.	$Fa = Gb$	Supposition
ii.	$\exists \lambda X.\forall \lambda Y.(X + Y)$	Monism
iii.	$H : \forall \lambda Y.(H = Y)$	Supposition for $\exists$-Elimination
iv.	$H : H = F$	*iii*, $\forall$-Elimination
v.	$H : H = G$	*iii*, $\forall$-Elimination
vi.	$H : F = G$	*iv, v,* Classical Logic
vii.	$F = G$	*ii, iii-vi*, $\exists$-Elimination
viii.	$Fa = Gb \rightarrow F = G$	*i-vii*, $\rightarrow$-Introduction

[20]More formally, if we were to expand our language to include the modal operators $\Box$ and $\Diamond$, we could use the necessitation rule to infer $\vdash \Box\exists\lambda x.\forall y(y = x)$ from $\vdash \exists\lambda x.\forall y(y = x)$

In one sense, this result is unsurprising. If higher-order monism is true, then there is only one property *to* figure in any proposition. If proposition *Fa* is identical to proposition *Gb*, then the properties contained within those properties *must* be the same. So, not only does the PSE entail higher-order monism, but higher-order monism also entails the PSE. The claim that identical propositions contain identical properties means *the very same thing* as the claim that higher-order monism is true.

The case is more or less complete. The logic that we have adopted entails an extreme form of monism, applying not only to objects, but to properties, relations, and the like. Because it is possible to prove that monism is true on the basis of purely logical assumptions, many metaphysicians will be pressured to accept that monism is necessarily true. And because it is also possible to establish that the PSE entails monism, the two can be seen as different formulations of the very same principle.

From Monism to Contradiction

The remainder of this chapter consists of an objection and a reply.[21] The objection is—in my view—extremely strong. The logical assumptions at play must be restricted, if not abandoned entirely. They have a consequence that is even less tenable than monism itself—contradiction.

The problem concerns the higher-order nature of this type of monism. Just as it is possible to establish that there is but a single object, property, and binary relation, so too it is possible to establish that there is but a single proposition.[22] For this reason, the proposition p is identical to the proposition that $\neg p$. An application of Leibniz's Law then entails that the two have the same truth-value: p is true just in case p is false. We can establish the inconsistency more formally as follows:

i.	$\lambda y.(y = y)(p) = \lambda y(y = p)(p)$	β-Identification
ii.	$\lambda y.(y = y) = \lambda y.(y = p)$	*i*, PSE
iii.	$\forall \lambda y.(y = y)$	Classical Logic
iv.	$\forall \lambda y.(y = p)$	*ii, iii,* Leibniz's Law
v.	$\neg p = p$	*iv*, $\forall$-Elimination
vi.	$\neg p \leftrightarrow p$	*v*, Leibniz's Law
vii.	$\bot$	*vi,* Classical Logic

It cannot be that the PSE, β-identification and classical logic are all true, for they are jointly inconsistent.

The significance of this result surpasses particular assumptions made at the outset. Not

[21] My thanks to Alex Roberts for pressing me on this point.

[22] As before, if the reader prefers to think of facts or states of affairs as playing the role of 0-ary relations, we could rephrase this to state but that there is a single fact or state of affairs.

only is this derivation of monism flawed, but higher-order monism *is itself* inconsistent. The claim that there is a unique referent for terms of every type leads inexorably to contradiction, for such a view *must* maintain that p is identical to $\neg p$. While classical logic is committed to the existence of at least one object, higher-order logic is committed to the existence of at least two propositions—one of which is true, the other of which is false. Furthermore, since the PSE is equivalent to higher-order monism, it too is inconsistent. What is to be done in light of this problem? Need monism be rejected entirely?

It is possible to weaken the premises that generate contradiction, without entirely abandoning them. Here, the PSE is treated as a schema that applies to terms of every type—to propositions that apply properties to objects, as well as ones that apply properties to properties. But we need not treat this as a general schema. Instead, we could treat it as a principle that *only* concerns propositions that predicate properties of objects (and says nothing at all about terms of other types). With this restriction in place, the former derivation of contradiction fails. Because the principle does not apply to sentential operators (such as $\neg$), it cannot be used to derive that $p \leftrightarrow \neg p$. Nevertheless, the principle still may be used to derive first-order monism. It entails that there is only a single object—but not that there is only a single proposition.

I close this chapter with a question: ought we to accept this restricted form of the PSE? The principle has much intuitive appeal, but if its general form is to be rejected then why accept this particular case? Much turns on the answer. If the PSE is to be rejected (not only in its general form but also in its particular form), then pluralism remains a live possibility. But if we accept this restricted claim—as well as classical logic and β-Identification—then we must accept that monism is true. I doubt that many will accept such a radical conclusion, but those who do must revise many of their beliefs. Our common conception of the world—a conception of objects like cars, tables, rocks and people—is false. This extends beyond our beliefs: our desires, fears, hopes and wants must all be reconfigured if there is but a single object to occur within them. Principles of logic may radically reshape our perspective on the world.

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