THE GROUNDS OF NONGROUND

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Abstract

This paper concerns the grounds of nonground: what it is in virtue of that facts of the form [F_1 does not ground F_2] hold. While the literature on iterated ground is expansive, there has been nothing written on the grounds of nonground; this paper constitutes the first account. I argue that nonground is grounded in distinctness from ground. If F_1 does not ground F_2, then [F_1 does not ground F_2] is grounded in the fact that F_1 is distinct from that which does ground F_2. While this proposal strikes me as natural, it faces puzzles involving conjunction, contingency, and cardinality. The bulk of this paper explores how the view might be precisified, and how these puzzles might be resolved.

Introduction

The facts of the world do not lie in disarray; they exhibit patterns of explanation and dependence. The fact that the Forbidden City's walls are some shade of red is due to the fact that they are vermillion—and the fact that the number of United Nations member states is prime is due to the fact that the number is 193. Not all facts hold in virtue of all others. The fact that Socrates is wise is not due to the fact that {Socrates} contains someone wise—and the fact that the Eiffel Tower is actually tall is not due to the fact that it is four kilometers from Montmartre. Philosophers have dubbed this phenomenon 'grounding.'[2] If one fact holds in virtue of another, we say that the latter grounds the former. If one fact does not hold in virtue of another, we say that the latter does not ground the former.

Given that facts are organized in this way, the grounding relation is of paramount philosophical importance. Puzzles in numerous subfields can be cast as debates over ground. The question of physicalism might be understood as a debate over whether the mental is grounded in the physical—and the question of normative naturalism might be understood as a debate over whether the normative is grounded in the non-normative. It is unsurprising, given this context, that metaphysicians have dedicated substantial effort into studying the nature of ground.

[1]Many thanks to the attendees of the 2024 Metaphysics Workshop at UC San Diego and the Inaugural Meeting of the Association of Analytical Metaphysics in the Italian-Speaking World for their feedback. I am especially indebted to Pauliina Rumm, who first raised the puzzle of the grounds of nonground to me. Though the theory I develop here is my own, credit for identifying the puzzle goes entirely to her.

[2]The literature on ground is too extensive to adequately canvass here. Early converts include Schaffer (2009); Fine (2012). For critiques of theories of ground, see Wilson (2014); Fritz (2022)—for replies, see Berker (2017); Goodman (2023); deRosset (2023). While it is far from uncontroversial that the notion of ground is in good theoretical standing, I will not defend its intelligibility here. This paper is directed towards those who already theorize in terms of ground; it is not intended to convince those who do not.

There is a puzzle of iterated ground. What (if anything) grounds facts about grounding? If one fact grounds another, what is it in virtue of that the grounding relation obtains? And what grounds facts about nongrounding? If one fact does not ground another, what is it in virtue of that the grounding relation does not obtain?

Answers to these questions are not merely academic; they impact traditional philosophical debates. Philosophers armed with a theory of iterated ground have the resources to explain—in a deep sense—what makes their theories true. A physicalist who understood the grounds of grounding could explain not only the physical foundations of the mental, but also why the mental is grounded as it is. A dualist who understood the grounds of nongrounding could not only reject physicalism, but also explain why the mental does not depend on the physical in the manner that the physicalist claims.

The puzzle of iterated ground has generated a large (and rapidly growing) literature. Accounts of what grounds facts that take the form [F_1 grounds F_2] include the following:[3]

Superinternal Grounding	Not only does F_1 ground F_2, but F_1 also grounds the fact that F_1 grounds F_2.
Grounding Essentialism	The essences of the constituents of F_2 together with F_1 ground the fact that F_1 grounds F_2.
Bridge Principles	There are principles akin to metaphysical laws that ground the fact that F_1 grounds F_2.
Grounding Disunity	There is no unified theory to provide; different grounding facts are grounded in different ways.

Superinternalists hold that F_1 performs double-duty.[4] Not only does it explain why F_2 obtains, but it also explains why the explanation itself holds. Essentialists view this explanation as incomplete—noting that specific examples of superinternal grounding sound implausible. When confronted with a question like 'Why does the fact that John's C-fibers are firing explain the fact that John is in pain?' the response, 'Because John's C-fibers are firing' seems inadequate. They believe that the essence of pain—together with the fact that John's C-fibers are firing—provides a complete explanation. Bridge Principlists, for their part, maintain that grounding is (in some respects) analogous to causation. Just as

[3]For defenses of *Superinternal Grounding*, see Bennett (2011) and deRosset (2013, 2023). For a defense of *Grounding Essentialism*, see Dasgupta (2014). For a defense of *Bridge Principles*, see Schaffer (2017). For a defense of *Grounding Disunity*, see Sider (2020).

[4]The term 'superinternal' was coined by Bennett (2011). There is a classic distinction between internal and external relations. A relation R is said to be 'internal' just in case, for any a and b, the characteristics—or intrinsic nature—of a and b determine whether a stands in relation R to b. A relation is said to be 'external' just in case it is not internal. For example, relative mass might be considered to be an internal relation (since the masses of a and b determine whether a is more massive than b), while relative weight might be considered to be an external relation (since locations of other massive objects impact how much a and b weigh). Following Bennett, a relation R is said to be 'superinternal' just in case the characteristics—or intrinsic nature—of *only one of the relata* determine that the relation obtains.

causal explanations are backed by nomological laws, so too grounding explanations are backed by metaphysical laws. And Disunionists hold that the most plausible grounds for grounding facts vary case by case; perhaps we were wrong in supposing that there is a unified story to be told.

The complementary question—what grounds facts that take the form [F_1 does not ground F_2]?—has received far less attention. In fact, there has been *nothing whatsoever* written on the grounds of nonground.[5] This omission is, to my mind, very surprising—as the same arguments that motivate the puzzle of iterated ground apply to the grounds of nonground.

Many philosophers appeal to the principle Purity—according to which fundamental facts only have fundamental constituents.[6] Purity entails that, if the fact that electron *e* is spin-up is fundamental, then both electron *e* and *being spin-up* are fundamental. The problem is that, if facts of the form [F_1 grounds F_2] were fundamental, then the fundamental would proliferate.[7] Suppose that philosophers are having a conference—and that the fact that they are having a conference is grounded in the fact that they are engaging in activities C (perhaps paying registration fees, giving talks, and asking merely tangentially related questions). If the grounding fact [The fact that philosophers engage in C grounds the fact that they are having a conference] were fundamental, then philosophers, engaging in C, and *having a conference* would all be fundamental. To avoid the proliferation of the fundamental, perhaps we ought to hold that grounding facts are grounded in some way or other.

This problem may be worse than it first appears. Arguably, F_2 is a constituent of the fact [F_1 grounds F_2]. If this grounding fact were fundamental, Purity would entail that all of its constituents—including F_2—are fundamental. But, by stipulation, F_2 is *not* fundamental; it is grounded in F_1. The claim that grounding facts are fundamental thus seems to not only be implausible, but bordering on inconsistent (at least, given the assumption that Purity is true).

An analogous argument applies to nongrounding facts. As before, suppose that philosophers are having a conference—but also suppose that the fact that they are having a conference is *not* grounded in the fact that they are engaging in activities D (perhaps ignoring undergraduate emails). If the nongrounding fact [The fact that philosophers engage in activities D does not ground the fact that they are having a conference] were fundamental, then philosophers, engaging in D, and *having a conference* would all be fundamental. This example generalizes. Arguably, everything figures in *some* nongrounding

[5]There are fully general accounts of the logic of ground that take a stand on the grounds of these facts—see, e.g., Goodman (2023). However, there is neither a defense nor discussion of the grounds of nonground in these texts.

[6]In what follows, 'fundamental fact' should be interpreted as 'ungrounded fact.' Arguably, the most canonical discussion of Purity occurs in Sider (2011). However, see Fine (2010); deRosset (2013); Raven (2016) and Litland (2017) for other defenses of Purity. See Barker (2023) for an argument against Purity.

[7]A sophisticated version of this point occurs in deRosset (2023)—who argues that if grounding facts were fundamental, everything would be fundamental.

fact; for an arbitrary fact F_1, there is some other fact F_2 that F_1 does not ground (after all, no fact grounds absolutely everything). If nongrounding facts were fundamental, Purity would entail that the constituents of F_1 (and F_2) are all fundamental. Because the choice of F_1 was arbitrary, Purity would thus entail that absolutely everything is fundamental.

As with the puzzle of ground, the puzzle of nonground can be extended. Take an arbitrary derivative fact F: one that is grounded in some-fact-or other. If grounding is irreflexive, F does not ground itself. So, there is a nongrounding fact [F does not ground F]—a fact that presumably has F as a constituent. If this nongrounding fact were fundamental, Purity would entail that F is fundamental. But, by stipulation, F is *not* fundamental; it is derivative. Just as the claim that grounding facts are fundamental bordered on inconsistency, so too the claim that nongrounding facts are fundamental borders on inconsistency.

Schaffer (2010) and Bennett (2011) provide another argument that relies on a principle of free modal recombination. Derivative facts cannot be freely recombined; possible worlds that contain the same fundamental facts also contain the same facts that depend upon them. For example, possible worlds cannot contain different conjunctive facts without also containing different atomic facts; conjunctions supervene upon their conjuncts. The reason that recombination fails, Schaffer and Bennett argue, is that conjunctive facts are grounded in their conjuncts. This does not hold for fundamental facts themselves. So, perhaps for any collection of fundamental facts F_1, F_2, ... there are worlds that disagree with respect to F_1, F_2, ... but agree with respect to all other fundamental facts.

Suppose that nothing grounds facts about what grounds what; facts that take the form [F_1 grounds F_2] are fundamental. According to Recombination, there would then be pairs of possible worlds that disagree about the grounding facts, yet agree with respect to all other fundamental facts. In particular, there is a world that is identical to this one, except that none of the actual grounding relations obtain. If, in the actual world, disjunctions are grounded in their true disjuncts, in this alternate world disjunctions are *not* grounded in their true disjuncts. And if, in the actual world, the fact that a substance is made of water is grounded in the fact that it is made of H_2O, in this alternate world the fact that a substance is made of water is *not* grounded in the fact that it is made of H_2O. This leads to an odd sort of skepticism. What makes us think that we occupy a world in which facts are grounded, rather than one in which they are not? After all, a nongrounding world would be phenomenally indistinguishable from ours—and the arguments that lead philosophers to endorse grounding would apply there as well. Finding this possibility implausible, Schaffer and Bennett conclude that grounding facts are grounded in one way or another.

Once again, a parallel argument applies to nongrounding facts. Suppose that nothing grounds facts about what does *not* ground what; facts that take the form [F_1 does *not* ground F_2] are fundamental. According to Recombination, there would be pairs of worlds that disagree about the nongrounding facts, yet agree with respect to all other fundamental facts. There would then be a world identical to this one, except that the nongrounding facts do not hold. If it is actually the case that F_1 does not ground F_2, in this alternate world F_1 *does* ground F_2. This is a world that radically violates the logic of ground—one where seemingly unrelated facts ground one another with abandon. If grounding is actually irreflexive, no fact actually grounds itself. In this alternate world, *every* fact grounds itself. And if the fact that the sky is blue does not actually ground the fact that chocolate contains cocoa, in this world the fact that the sky is blue *does* ground the fact that chocolate contains cocoa. Because all other fundamental facts obtain, this world would be phenomenally indistinguishable from ours—and the same arguments concerning grounding would apply. What makes us think that we occupy the world where grounding is well-behaved—one in which it complies to intuitive logical principles and where facts are relevant to that which they ground—rather than the world where it is not? To avoid this problem, perhaps we ought to hold that nongrounding facts are grounded in one way or another.

The arguments that motivate theories of iterated ground thus motivate theories of the grounds of nonground as well; the reasons to hold that nongrounding facts are derivative are just as strong as the reasons to hold that grounding facts are derivative. Simultaneously, these arguments do not strike me as conclusive. An available response is to reject both Purity and Recombination—after which they lose much of their force. But the arguments are strong enough to justify an exploratory venture; they warrant the development of a theory of the grounds of nonground, if for no other reason than to determine what progress or pitfalls might arise.

The Grounds of Nonground

It is worthwhile to precisify our notation. One distinction that impacts formal theories of grounding is that between full and partial ground. A collection of facts is said to fully ground another if they suffice (in the relevant sense) to make that fact true. A fact is said to partially ground another if it is a member of a collection of facts that fully ground.[8] Thus, the fact that roses are red fully grounds the fact that roses are red or violets are blue—but merely partially grounds the fact that roses are red and violets are blue. Following standard notation, I use the variably polyadic operator ‘$<$’ to denote full ground and the binary sentential operator ‘$\prec$’ to denote partial ground.

A significant choice-point concerns the relata of the grounding relation. While I have operated with the (somewhat orthodox) factive conception thus far, some hold that it is a relation between propositions instead.[9] As is already evident, I denote facts with [ ]

[8]While this formulation of the distinction between full and partial ground is standard, see Trogdon and Whitmer (2021) for an argument that fully ground should be defined in terms of partial ground, rather than the reverse.

[9]Fine (2012) employs both a factive and propositional conception of ground—while Schaffer (2009) opts for an entity conception. See Wilhelm (2020) for an argument for the entity-conception and Lo (2022) for a reply. See, also, Elgin (2024) for a use of the grounding operator for all terms across the type-theoretic hierarchy.

notation—and will refer to propositions without the use of brackets. Thus, '$[p] < [q]$' asserts that the fact that p fully grounds the fact that q, while '$p < q$' asserts that the proposition p fully grounds the proposition q. I primarily operate with a factive notion of ground, but explore the advantages of a propositional notion at the end of this paper.

Let a nongrounding fact be any that takes the form:

$$[\neg([p] < [q])]$$

This is intended to be interpreted as: the fact that it is not the case that the fact that p grounds the fact that q. While I begin by discussing non-*full*-ground, I address non-partial-ground as well. And I will begin by addressing examples involving a single fact that serves as the nonground—but will explore challenges arising from polyadic extensions as well.

To develop a theory of the grounds of nonground, it helps to start with a concrete example. Let $[p]$ and $[q]$ be fundamental facts, and consider the disjunction $[p \vee q]$. Most metaphysicians hold that disjunctions are grounded in their true disjuncts. In this case, both p and q are true, so $[p \vee q]$ is fully grounded in $[p]$ and fully grounded in $[q]$. In principle, of course, the disjunction could have further grounds. Many maintain that ground is transitive—so if either $[p]$ or $[q]$ were derivative, the disjunction would also be grounded in whatever it is that grounds them.[10] Even granting that $[p]$ and $[q]$ are fundamental, the disjunction could be grounded in additional ways—at least in principle. However, for the sake of simplicity, let us assume that these are the only grounds. $[p \vee q]$ is fully grounded in $[p]$, fully grounded in $[q]$, and nothing else.

What grounds the fact that $[\neg([r] < [p \vee q])]$? That is, if $[p \vee q]$ does not hold in virtue of $[r]$, what explains the fact that it does not hold in virtue of $[r]$? An appealing suggestion is that the reason $[r]$ does not ground the disjunction is because it isn't that which *does* ground the disjunction. $[r]$ *isn't* $[p]$—and $[r]$ *isn't* $[q]$. $[p]$ and $[q]$ are the only things that ground $[p \vee q]$; because $[r]$ isn't either of those facts, it does not ground $[p \vee q]$. The fact that $[[r] \neq [p]]$ and $[[r] \neq [q]]$ grounds the fact that $[r]$ does not ground $[p \vee q]$.

This particular example can be generalized. Take an arbitrary fact $[q]$ fully grounded in each of the facts $[p_1]$, $[p_2]$, ... (that is, a fact $[q]$ such that $[p_1] < [q], [p_2] < [q], ...$). What grounds the fact that $[r]$ does not ground $[q]$? On my view, the fact that $[r]$ is distinct from all of $[q]$'s grounds: the fact that $[r] \neq [p_1]$, the fact that $[r] \neq [p_2]$, etc. In a slogan, nonground is grounded in distinctness from ground.

While I personally find this proposal intuitive, it might strike others as implausible.[11] After all, if $[p]$ doesn't ground $[q]$, surely that's a relationship that holds in virtue facts about $[p]$ and $[q]$; why is $[p]$'s distinctness from *some other fact* relevant to the grounds of nonground? What matters, after all, is that $[p]$ fails to stand in the correct relationship to $[q]$, not that it stands in a relation to something else.

[10]For potential counterexamples to the transitivity of ground, see Schaffer (2012)—and for a reply, see Litland (2013).

[11]My thanks to Ezra Rubenstein for pressing me on this point.

In some sense, this is a difficult challenge to rebut. Intuitions are hard things to cultivate—and if a philosopher denies that my theory has intuitive appeal, there is little I can say that will bring it about. But it might help to emphasize the oddity of denying my account in explanatory contexts. Suppose that the fact that grass is green is grounded in the fact that its microphysical structure has feature F. Consider a context where two philosophers are discussing why grass's color is not grounded in other ways. Suppose one were to ask, 'Why isn't grass's greenness grounded in the fact that it has microphysical structure with feature G?' and were met with the response 'As it turns out, grass's greenness is only grounded in the fact that its microphysical structure has feature F. The fact that this structure has feature G isn't the fact that it is has feature F—so the fact that grass has microphysical structure G doesn't ground its greenness.' It is very difficult to see why this explanation would be deficient—or to imagine what a more satisfying one would look like. Distinctness seems to be a good stopping point for metaphysical explanation; once someone is convinced that [Grass's microphysical structure has feature G] differs from the grounds of [grass is green], there is little else to be confused about.

Nevertheless, this theory faces a number of puzzles involving conjunction, contingency, fundamentality and cardinality. In what remains of this paper, I will explore various ways the theory might be developed, and how problems might be avoided. I do not take the resulting account to be the only viable candidate—and I will not argue that it is preferable to all others. My aim is to show that this is a *good* theory, not necessarily the best. One advantage to being the first mover in this area is that there is no need to dedicate space to rebutting available alternatives—as none are well-developed at this time.

Pluralities and Conjunctions

Let us revisit the initial example: the fact that $[r]$ does not ground $[p \vee q]$ is grounded by $[[r] \neq [p]]$ and $[[r] \neq [q]]$. While the underlying theory is (hopefully) intuitive, there is a subtle ambiguity in my use of the word 'and'; in this context, it could be interpreted in one of two ways. It might, firstly, be taken to denote a conjunction. On this interpretation, the nongrounding fact is grounded in a single conjunctive fact—where each conjunct asserts that $[r]$ is distinct from a ground of $[p \vee q]$. Alternatively, 'and' could signify separate elements of a list. On this second interpretation, the grounds of nonground consist of a plurality of distinctness facts, rather than a conjunction of them. More formally, we can represent the first alternative as:

$$[([r] \neq [p]) \wedge ([r] \neq [q])] < [\neg([r] < [p \vee q])]$$

and the second as:

$$[[r] \neq [p]], [[r] \neq [q]] < [\neg([r] < [p \vee q])]$$

This example can be generalized. The grounds of nonground can take one of two forms: either a single conjunctive fact or a collection of distinctness facts.[12] Is there any reason to favor one formulation over the other?

I believe that there is: we ought to accept the list formulation rather than the conjunctive formulation because there is a sense in which it is more likely to be true.

On the standard logic of ground, the conjunctive formulation entails the list formulation. It is typically held that conjunctions are grounded in their conjuncts; $[p \wedge q]$ is grounded in the plurality of $[p], [q]$. In this case, the conjunction of distinctness facts—$[([r] \neq [p]) \wedge ([r] \neq [q])]$—is grounded in both distinctness facts—$[[r] \neq [p]], [[r] \neq [q]]$. If the nongrounding fact is grounded in a conjunction of distinctness facts, and if the conjunction of distinctness facts is grounded in their plurality, then the plurality of distinctness facts grounds the nongrounding fact. We can represent this more formally as:

i.	$[([r] \neq [p]) \wedge ([r] \neq [q])] < [\neg([r] < [p \vee q])]$	Conjunctive Account
ii.	$[[r] \neq [p]], [[r] \neq [q]] < [([r] \neq [p]) \wedge ([r] \neq [q])]$	Grounds of Conjunction
iii.	$[[r] \neq [p]], [[r] \neq [q]] < [\neg([r] < [p \vee q])]$	*i*, *ii*, Transitivity of Ground

By contrast, the list formulation does not entail the conjunctive formulation. Nothing in the logic of ground dictates that if a plurality of facts collectively grounds $[p]$, then their conjunction also grounds $[p]$. Indeed, not only does the standard logic *not license* this inference, but the inference *cannot be consistently added* to the logic of ground.[13] So, it could be the case that a plurality of distinctness facts grounds both their conjunction and a nongrounding fact—while the conjunction of distinctness facts does not ground the nongrounding fact. Thus, while the conjunctive formulation entails the list formulation, the list formulation does not entail the conjunctive formulation.

Quite generally, if p entails q but q does not entail p, our beliefs are more likely to be true if we believe q but remain agnostic about p—as compared to believing p outright.[14] After all, if we *do* believe that q (but remain agnostic about p) our beliefs are true in situations where q is true but p is false. (Of course, in these situations our beliefs would have been false if we had committed to p). By contrast, there are no situations where p is true and q is false—so our beliefs are also true in all p situations. We are thus more likely to be correct

[12]Note that these alternatives coincide whenever the grounds of fact are not overdetermined. That is, if $[q]$ has only a single full ground—$[p]$—both alternatives maintain that $[r \neq p] < [\neg([r] < [q])]$. This puzzle arises only for facts with overdetermined grounds.

[13]Suppose that if $[p_1], [p_2], ... < [q]$ then $[p_1 \wedge p_2 \wedge ...] < [q]$. Given that one of the things that $[p_1]$, $[p_2]$, ... ground is their conjunction, this principle would license the inference to $[p_1 \wedge p_2 \wedge ...] < [p_1 \wedge p_2 \wedge ...]$—in violation of the irreflexivity of ground.

[14]This point is cashed out in terms of parsimony in Sober (2015). The 'razor of agnosticism' might be understood as a dictum to remain agnostic about propositions we need not take a stand on in order to increase the odds of a theory's truth. One complication arises from the fact that we presumably deal with necessary truths; my theory is presumably either necessarily true or necessarily false. Nevertheless, I take it that our credences for my theory ought to be neither 0 nor 1, so the point stands.

by committing to q and remaining agnostic with respect to p, rather than taking a stand on whether p is true.

The upshot is this. The nongrounding account could be formulated in two ways: either as a conjunction or as a list. On the standard logic of ground, the conjunctive formulation entails the list formulation, while the list formulation does not entail the conjunctive formulation. In cases of unilateral entailment, our beliefs are more likely to be true if we commit ourselves only to the logically weaker claim. For this reason, I prefer to accept the list formulation, and remain strictly agnostic about the conjunctive formulation.

A Puzzle for Full Nonground

The distinctness view gives rise to a pair of puzzles for full and partial ground. These dilemmas become most apparent with the help of two auxiliary principles:

> NECESSITY OF DISTINCTNESS:
> If $[p]$ is distinct from $[q]$, then $[p]$ is necessarily distinct from $[q]$.
>
> GROUND AND STRICT IMPLICATION:
> If $[p]$ grounds $[q]$, then, necessarily, if p is true then q is true.

Each of these principles is controversial, yet eminently defensible. The necessity of distinctness is a theorem in modal logics containing the B axiom $p \rightarrow \Box\Diamond p$.[15] If we model modality with possible-worlds semantics, B corresponds to the assumption that accessibility is symmetric; if world w can access w', then world w' can also access world w.

Suppose, for reductio, that distinctness were contingent; $[p]$ and $[q]$ are actually distinct, but could have been identical. Therefore, there exists a possible world w where $[p]$ is identical to $[q]$. The necessity of identity is a theorem in even the weakest of the standard modal logics—so, within w, it is provable that $[p]$ is necessarily identical to $[q]$.[16] Every world that w can access is one in which $[p]$ is identical to $[q]$. Given the B axiom, this includes the actual world—so $[p]$ is (actually) identical to $[q]$. This contradicts the assumption that $[p]$ and $[q]$ are (actually) distinct. Therefore, facts that are actually distinct are necessarily distinct.

[15] This was first established in Prior (1956). I have included a brief informal discussion of the reasoning behind the proof; I direct those interested in the technical details to the original text. An additional argument for the necessity of distinctness (that employs an actuality operator, rather than the B axiom) occurs in Williamson (1996). However, for an argument that distinctness is contingent (on the grounds that principles of free modal recombination entail contingent distinctness) see Roberts (2021). Much of this debate centers on the distinctness of objects—rather than facts (so, if we were to formulate these debates in type-theory, it would be represented by terms of type e, rather than t). I take it that the arguments translate; it would be odd for a philosopher to hold that the distinctness of objects was necessary, but the distinctness of facts contingent (or vice-versa).

[16] The necessity of identity was first proven in Barcan (1947)—and the assumptions were weakened and the proof simplified in Kripke (1980).

Ground and Strict Implication is the orthodox connection between grounding and necessity.[17] It asserts that if $[q]$ holds in virtue of $[p]$, there is no possible situation where p is true and q is false. It reflects the thought that full ground provides a complete explanation; if the truth of p does not settle the truth of q, then a complete explanation for why it is the case that $[q]$ requires more than merely citing $[p]$. After all, $[p]$ fails to rule out the possibility that q is false. So, if $[p]$ fully grounds $[q]$ it is necessary that if p then q.

While these principles both strike me as plausible, they remain controversial—so it is worthwhile to clarify the dialectical role that they play. Here, they serve to generate challenges to my account. In a sense, those who reject them need not disagree with anything that I say; rather than objecting to my view, such philosophers have another path to accepting the claim that distinctness grounds nonground.

Consider a variant of the initial example: suppose that p is contingently true and q is contingently false; suppose that both $[p]$ and $[q]$ are fundamental—and consider $[p \vee q]$. There is then a fact $[p]$ but there is no fact $[q]$. In this case, the grounds of the disjunction are not overdetermined. It is grounded by $[p]$, and nothing else.

Because $[p \vee q]$ is only grounded in $[p]$, the nongrounding view holds that the reason $[q]$ does not ground $[p \vee q]$ is that $[q]$ is distinct from $[p]$.[18] That is:

$$[[q] \neq [p]] < [\neg([q] < [p \vee q])]$$

Given Ground and Strict Implication, this entails the following:

$$\Box([[q] \neq [p]] \rightarrow [\neg([q] < [p \vee q])])$$

Necessarily, if $[q]$ is distinct from $[p]$, then $[q]$ does not ground $[p \vee q]$. The weakest standard modal logic—K—includes a distribution axiom; necessity distributes over the material conditional: $\Box(p \rightarrow q) \rightarrow (\Box p \rightarrow \Box q)$. This axiom thus entails:

$$\Box[[q] \neq [p]] \rightarrow \Box[\neg([q] < [p \vee q])]$$

If it is necessary that $[q]$ is distinct from $[p]$, then it is necessary that $[q]$ does not ground $[p \vee q]$. Because q and p differ in their truth-values (since p is true and q is false), $[q]$ is actually distinct from $[p]$. Given the Necessity of Distinctness, we have:

$$\Box[[q] \neq [p]]$$

This, then, entails:

[17] To the best of my knowledge, the only philosopher who denies Ground and Strict Implication is Skiles (2015)—who argues that ground could accomplish much of its theoretical work even if it failed to necessitate.

[18] While I predominantly focus on the distinctness account, another answer to this question is that $[q]$ does not ground the disjunction because q is false—i.e., that $[\neg q]$. After all, if q *had* been true, $[q]$ would have partially grounded the disjunction. I do not see any obvious way of generalizing this case to a comprehensive theory of the grounds of nonground; I suspect that those tempted by it will have a disunified account—where nongrounding facts are each grounded in their own way.

$$\Box[\neg([q] < [p \vee q])]$$

Necessarily, $[q]$ does not ground $[p \vee q]$. But that's false. $[q]$ could have grounded the disjunction—indeed, it *would* have—if q were true.[19]

While this example strikes me as compelling, there is a way in which it is unsatisfactory. It refers to fact $[q]$, despite the nonexistence of this fact. Classically, one could infer from the nongrounding fact $[[q] \neq [p]] < [\neg([q] < [p \vee q])]$ that:

$$\exists x([x \neq [p]] < [\neg(x < [p \vee q])])$$

But if $[q]$ does not exist, this existential claim is surely false. There *is no fact* $[q]$ to be distinct from $[p]$, so $[q]$ cannot figure in relations—even distinctness relations. This example might appear illicit due to its reference to nonexistent facts.

I myself am unmoved by this objection—but I do not expect all others to share in my reply. My own conception of facts is deflationary; facts are nothing more than true propositions.[20] The proposition q exists even in situations where it is false—and so can figure in distinctness and nongrounding facts. On this way of thinking, a 'factive conception of grounding' is one where the terms that flank the grounding operator '$<$' are all true propositions. False propositions can occur as constituents of these, so long as the propositions that they figure within are true. In this case, the propositions $p \neq q$ and $\neg(q < (p \vee q))$ are both true, so a factive conception permits the former to ground the latter.

Nevertheless, some might endorse a more robust conception of facts than I do—holding that facts are distinct from true propositions, and that if there is no fact $[q]$, then $[q]$ is neither identical to nor distinct from anything.[21] I do not want my argument to hang

[19]I note that there is a sense in which the Necessity of Distinctness is gratuitous within this argument. While it is strictly needed to derive $\Box[\neg([q] < [p \vee q])]$, it is possible to end the proof earlier, and conclude that $\Box([[q] \neq [p]] \rightarrow [\neg([q] < [p \vee q])])$. This strict conditional is extremely implausible—so even those who reject the Necessity of Distinctness will face some pressure to abandon my account. My thanks to Verónica Gómez Sánchez for pressing me on this point.

[20]The reason I am a deflationist is that I am a necessitist; I hold that everything necessarily exists (see Williamson (2013) for the canonical defense of necessitism). Moreover, I am a higher-order necessitist; I hold that terms of arbitrary type (including those of type t) exist necessarily as well. It is difficult for necessitists to speak the language of facts—they venture precipitously close to necessitarianism: the claim that everything actually true is necessarily true. After all, in situations where a fact F exists, the proposition corresponding to F is true. If all facts exist in every possible world, then the propositions corresponding to these facts are necessarily true. Necessitists who countenance contingent truth would do well to be deflationists in my sense; propositions may exist necessarily without having the same truth-value in every possible world. On this conception, the 'language of facts' is nothing more than a language with terms only for true propositions. In this language, '$\forall p.p$' holds, but the term '$\forall$' is implicitly restricted so as to range only over true propositions. Necessitism only holds for the broadest form of necessity—one which lacks this restriction—so deflationists can speak the language of facts without committing themselves to necessitarianism.

[21]Such philosophers may not be without resources to respond to this objection. In particular, those who adopt a free logic may deny the inference from $[[q] \neq [p]] < [\neg([q] < [p \vee q])]$ to $\exists x([x \neq [p]] < [\neg(x <$

on the outcome of this dispute, so it is worthwhile to find another example: one where distinctness only relates extant facts. To that end, I turn to a variant of this puzzle for partial nonground.

A Puzzle for Partial Nonground

On the distinctness view, if $[p]$ does not partially ground $[q]$, the reason it does not do so is that it is distinct from that which does partially ground $[q]$. If $[r]$ does not partially ground $[p \wedge q]$, then the fact that $[r]$ does not partially ground the conjunction is grounded in $[[r] \neq [p]], [[r] \neq [q]]$ (given the standard assumption that conjunctions are grounded in their conjuncts).[22] That is:

$$[[r] \neq [p]], [[r] \neq [q]] < [\neg([r] \prec [p \wedge q])]$$

Suppose that p and q are contingently true and r is contingently false (and that there are no interesting modal connections between p, q and r). Consider the fact $[p \vee (q \wedge r)]$. Because r is false, the only partial ground of the disjunction is $[p]$. What grounds the fact that $[q]$ does not partially ground the disjunction? On my view, the reason $[q]$ does not partially ground $[p \vee (q \wedge r)]$ is that $[q]$ is distinct from that which does partially ground the disjunction—in this case, $[p]$. So, we have that:

$$[[q] \neq [p]] < [\neg([q] \prec [p \vee (q \wedge r)])]$$

Ground and Strict Implication entails:

$$\Box([[q] \neq [p])] \rightarrow [\neg([q] \prec [(p \vee (q \wedge r)])])$$

Necessarily, if $[q]$ is distinct from $[p]$, then $[q]$ does not ground $[p \vee (q \wedge r)]$. The K axiom then entails:

$$\Box[[q] \neq [p]] \rightarrow \Box[\neg([q] \prec [(p \vee (q \wedge r)])]$$

If it is necessary that $[q]$ is distinct from $[p]$, then it is necessary that $[q]$ does not partially ground $[p \vee (q \wedge r)]$. By stipulation $[q]$ is actually distinct from $[p]$; the Necessity of Distinctness entails that $[q]$ is necessarily distinct from $[p]$. So, we have:

$$\Box[\neg([q] \prec [(p \vee (q \wedge r)])]$$

$[p \vee q]])$.

[22] Note that, while the nongrounding fact in question concerns partial ground (in that it holds that $[r]$ does not partially ground $[p \wedge q]$), I provide the full grounds of this fact. Collectively, the fact that $[r]$ is distinct from $[p]$ and the fact that $[r]$ is distinct from $[q]$ fully ground the fact that $[r]$ does not partially ground $[p \wedge q]$.

Necessarily, $[q]$ does not partially ground $[p \vee (q \wedge r)]$. Once again, this is false. $[q]$ *could* have—indeed, *would* have—partially grounded $[p \vee (q \wedge r)]$ if r were true. In this example, both p and q are true, and so facts $[p]$ and $[q]$ both exist—and can thus stand in relations to one another. This problem cannot be avoided by denying that nonexistent facts are distinct.

Potential Solutions

Contingency generates a problem for the grounds of nonground. Of course, one option is to simply deny the assumptions that lead to conflict: to abandon either the Necessity of Distinctness or Ground and Strict Implication—or else to reject the distinctness account entirely. Perhaps unsurprisingly, more modest approaches strike me as preferable.

One option is to add a totality fact to the distinctness account. Return to the problematic example for full ground: p is contingently true and q is contingently false. On this modification, what makes it the case that $[q]$ does not ground $[p \vee q]$ is not merely the fact that $[q]$ is distinct from $[p]$—but also the fact that $[p]$ is the only thing that grounds $[p \vee q]$. We could represent this option as:

$$[[q] \neq [p]], [\forall x(([x] < [p \vee q]) \rightarrow ([x] = [p]))] < [\neg([q] < [p \vee q])]$$

This suggestion does not seem entirely implausible. If someone were to ask 'Why it is the case that the disjunction does not hold in virtue of $[q]$?', the reply, 'Because $[p]$ is the only thing that the disjunction holds in virtue of, and $[q]$ is not $[p]$' sounds relevant, explanatory and non-redundant. Including a totality fact might be seen to 'fill out' the nongrounding explanation.

Which totality fact is included depends on how the disjunction is grounded. In situations where the disjunction is only grounded in $[p]$, the totality fact only references $[p]$; in situations where the disjunction is only grounded in $[q]$, the totality fact only references $[q]$; and in situations where the grounds of the disjunction are overdetermined, the totality fact references both $[p]$ and $[q]$. Totality facts thus hold contingently—they vary world by world.

The totality inclusion avoids the contingency problem. Given Ground and Strict Implication, facts necessitate that which they ground. In this case, it is necessary that if $[q]$ is distinct from $[p]$—*and if* $[p]$ *is the only ground of* $[p \vee q]$—then $[q]$ does not ground $[p \vee q]$. Crucially, the totality fact holds contingently, so it does not follow that $[q]$ necessarily does not ground $[p \vee q]$. Moreover, this necessity seems entirely unproblematic; there ought to be no possible worlds where $[p]$ and $[q]$ are distinct, $[p]$ is the only ground of $[p \vee q]$ and yet $[q]$ grounds $[p \vee q]$ as well.[23]

[23] In an analogous way, this inclusion avoids the problem for partial ground. Consider $[p \vee (q \wedge r)]$ for contingently true p and q and contingently false r. On this version of the account, $[\neg[q] \prec [p \vee (q \wedge r)]]$ is

Another potential resolution is to opt for a propositional—rather than a factive—conception of ground. On this modification, the truth or falsity of propositions is largely irrelevant to the grounding relations that they stand in; falsehoods ground other propositions. Regardless of whether p or q is true, both propositions ground the disjunction $p \vee q$. That is:

$$p < (p \vee q)$$

$$q < (p \vee q)$$

Not only is it *possible* for q to ground $p \vee q$ on the propositional model, but q *actually does* ground $p \vee q$—even in situations where q is false.

Of course, just as false propositions are capable of grounding, so too false propositions are capable of being grounded. There is thus a (false) nongrounding proposition $\neg(q < (p \vee q))$. What prevents us from carrying out similar reasoning on the propositional model as on the factive model, and reaching a similarly troubling result?

On the distinctness view, the proposition that q does not ground $p \vee q$ is grounded in the propositions that q is distinct from the grounds of $p \vee q$. In this case, we have:

$$(q \neq p), (q \neq q) < \neg(q < (p \vee q))$$

The proposition asserting that q is not p and the proposition asserting that q is not q ground the proposition that q does not ground $(p \vee q)$. Ground and Strict Implication entails that:

$$\Box((q \neq p \wedge q \neq q) \rightarrow \neg(q < (p \vee q)))$$

Necessarily, if q is distinct from p and distinct from itself, then q does not ground $p \vee q$. The K axiom then entails:

$$\Box(q \neq p \wedge q \neq q) \rightarrow \Box\neg(q < (p \vee q))$$

But here there is a crucial difference from the derivation on the factive conception. The antecedent of this conditional asserts that, necessarily, q is distinct from p and distinct from itself. But this is false. q is necessarily *identical to*—not *distinct from*—itself. So, we cannot use this derivation to establish that, necessarily, q does not ground $p \vee q$.[24]

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grounded both in the fact that $[p]$ is not $[q]$ and the fact that $[p]$ is the only partial ground of $[p \vee (q \wedge r)]$. Ground and Strict Implication entails that it is necessary that if $[p]$ is distinct from $[q]$ and if $[p]$ is the only partial ground of $[p \vee (q \wedge r)]$, then $[q]$ does not partially grounds $[p \vee (q \wedge r)]$. As before, this necessity seems entirely unproblematic—and cannot be used to infer that $[q]$ necessarily does not partially ground the disjunction.

[24] A similar resolution applies to the puzzle for partial ground. On the propositional conception $p \vee (q \wedge r)$ is partially grounded in p, q, r and $q \wedge r$ —regardless of these propositions truth values. And, as with the

Once again, we face a choice-point: ought we include a totality fact, or opt for the propositional conception of ground?[25] I personally prefer the propositional conception. However, it only will become clear why this is preferable after considering yet another puzzle.

Nongrounding the Fundamental

Consider a fundamental fact. For the sake of concreteness, suppose that [electron *e* is spin-up] is fundamental—and so not grounded in anything whatsoever.[26] What grounds the fact that [grass is green] does not ground [electron *e* is spin up]? On my view, this nongrounding fact is grounded in the fact that [grass is green] is distinct from the positive grounds of [electron *e* is spin-up]. But because [electron *e* is spin-up] is fundamental, there are no positive grounds for [grass is green] to be distinct from. This an instance of a

puzzle for full ground, the derivation of necessity fails; the proposition $\neg(q \prec (p \vee (q \wedge r)))$ is grounded in $q \neq p, q \neq q, q \neq r, q \neq (q \wedge r)$. Ground and Strict Implication entails that, necessarily, if all of these distinctness propositions are true then q does not ground the disjunction. But it is not necessary that q is distinct from itself (indeed, it is necessary that it isn't), so this strict conditional does not preclude the possibility that q grounds the disjunction.

[25] Both the totality amendment and propositional conception resolve the puzzles of full and partial ground simultaneously. However, I note that there is another solution to the puzzle of partial nonground in particular. Thus far, I have assumed that full and partial nongrounding facts are grounded in an analogous way: each account is given in terms of distinctness. However, some might hold that the two sorts of facts are grounded in different ways. Partial ground is often defined in terms of full ground. For [p] to partially ground [q] is for there to exist a collection of facts including [p] that fully grounds [q]. Given this definition of partial ground, it is natural to suggest that the reason [p] does not partially ground [q] is that there exists no such collection of facts. That is to say:

$$\neg\exists xx([p] \in xx \wedge (xx < [q])) < [\neg([p] \prec [q])]$$

(Note the use of plural quantification. While formalisms for plural quantification often uses '$p \prec xx$' to express the claim that p is one of the xxs, I have opted for '$p \in xx$' to avoid ambiguity—since '$\prec$' is used for partial ground. However it is important to note that '$\in$' is not intended to refer to set membership, as 'xx' refers to the plurality of xxs, not a set containing them.)

What makes it the case that [p] does not partially ground [q] is that it is not part of a collection of facts that fully grounds [q].

This resolves the puzzle for partial ground. Given Ground and Strict Implication, it follows that—necessarily, if [p] is not a member of a collection of facts that fully ground [q], then [p] does not partially ground [q]. But this necessity seems unobjectionable; it allows for [p] to partially ground [q] in some worlds but not in others. After all, it may be contingent whether there exists such a collection of facts.

One shortcoming of this solution is that it does nothing to resolve the puzzle of full ground; if full nongrounding is grounded in distinctness, while partial nongrounding is grounded in terms of the definition of partial ground, the puzzle of full ground remains. I do not mean to discount this possibility entirely, but it strikes me as sufficiently unsatisfactory as to be set aside; full and partial nongrounding facts would be grounded in entirely different ways—and one of the seemingly related puzzles remains.

[26] As should be clear from the previous section, there is now a debate over wether to operate with a factive or propositional conception of ground. My use of 'the fact that [p]' is due to ease of prose and for consistency with the previous discussion. It could be replaced with 'the proposition that p' if the reader prefers.

general puzzle. If a fact $[q]$ is fundamental, what grounds the fact that $[p]$ does not ground $[q]$? I have argued that nonground is grounded in distinctness from positive grounds. But what if there are no positive grounds?

The answer that strikes as most natural is that these facts are zero-grounded. If $[q]$ is fundamental, the empty plurality of facts grounds $[[p]$ does not ground $[q]]$. This holds for both the list and conjunctive formulations of my account. On the list formulation, the fact $[[p]$ does not ground $[q]]$ is grounded in a plurality of distinctness facts—each holding that $[p]$ is distinct from a ground of $[q]$. The number of facts within this plurality depends upon the number of positive grounds of $[q]$. If $[q]$ has a single ground—$[r_1]$—then the plurality consists of a single fact: $[[p] \neq [r_1]]$. If $[q]$ has two grounds—$[r_1]$ and $[r_2]$—then the plurality consists of two facts: $[[p] \neq [r_1]], [[p] \neq [r_2]]$. In the case where $[q]$ is fundamental, there are no positive grounds—so the plurality ought to contain zero facts. But the plurality consisting of zero facts *just is* zero-grounding, so the fact $[[p]$ does not ground $[q]]$ is zero-grounded.

Similarly, on the conjunctive formulation, the fact $[[p]$ does not ground $[q]]$ is grounded in a conjunction of distinctness facts—where each conjunct asserts that $[p]$ is distinct from a ground of $[q]$. The length of the conjunction is determined by the number of positive grounds of $[q]$. If $[q]$ has a single ground, then there is one conjunct; if $[q]$ has two grounds, then there are two, etc. As before, if $[q]$ is fundamental, there are no positive grounds—so the length of the conjunction ought to be zero. But a conjunction consisting of no conjuncts is the empty fact, so $[[p]$ does not ground $[q]]$ is zero-grounded.

Perhaps some suspect that one version of my account avoids appealing to zero-ground: the formulation that includes a totality fact. Recall that, on this view, $[[p]$ does not ground $[q]]$ is not only grounded in the fact that $[p]$ is distinct from $[q]$'s grounds—but also that those are all of the grounds of $[q]$ that there are. In the case where $[q]$ is fundamental, what grounds the fact that $[p]$ does not ground $[q]$ is the fact that nothing grounds $[q]$; the totality fact asserting that $[q]$ has no grounds grounds the fact that $[p]$ does not ground $[q]$.

At first blush, this might seem plausible. Confronted with the question 'Why is it the case that $[p]$ does not ground $[q]$?,' the answer 'Because nothing grounds $[q]$' sounds appropriate. I believe this to ultimately be mistaken. The natural way to formalize this view is:

$$[\forall x(\neg([x] < [q]))] < [\neg([p] < [q])]$$

Notice that the grounding fact takes the form of a universal: the fact that, for all x, x does not ground $[q]$. The standard logic of ground holds that universal facts are grounded in their instances: what makes it the case that everything is F is the fact that a is F, that b is F, etc. In the present case, the fact that nothing grounds $[q]$ is grounded in the fact that $[r]$ does not ground $[q]$, $[s]$ does not ground $[q]$, etc. One instance concerns the fact that $[p]$; the fact that $[p]$ does not ground $[q]$ partially grounds the fact that nothing grounds $[q]$. That is to say:

$$[\neg([p] < [q])] \prec [\forall x(\neg([x] < [q]))]$$

Given the transitivity of ground, the fact that $[p]$ does not ground $[q]$ would thus partially ground itself. The totality account thus violates the standard logic of ground; for this reason, I prefer the propositional formulation.

Cantor's Hell

I close by addressing a final puzzle for the distinctness account—one that strikes me as far more troubling than any considered thus far. Indeed, there was a time when I feared that any reasonably general version of my theory was inconsistent, and that it therefore ought to be abandoned.

So far, I have predominantly considered monadic nongrounding; I have addressed cases of the form $[[p]$ does not ground $[q]]$.[27] But ground is variably polyadic; any number of facts can serve as grounds—and any number of facts can serve as nongrounds. Just as we can explain why $[p]$ does not ground $[q]$, so too we can explain why the plurality $[p_1], [p_2], ...$ does not ground $[q]$.

On the distinctness account, there seems to be a natural explanation for why a given plurality of facts does not ground $[p]$: because it is distinct from the plurality of facts that *does* ground $[p]$. For example, we might explain why the plurality $[r], [s]$ does not ground the conjunction $[p \wedge q]$ with:

$$[[r], [s] \neq [p], [q]] < [\neg([r], [s] < [p \wedge q])]$$

The reason that the plurality $[r], [s]$ does not ground $[p \wedge q]$ is that it differs from the plurality $[p], [q]$: the plurality that grounds $[p \wedge q]$. For this explanation to be viable, distinctness must relate not only individual facts, but also pluralities of facts. Not only must we introduce facts of the form $[[p] \neq [q]]$, but also facts of the form $[[p_1], [p_2], ... \neq [q_1], [q_2], ...]$.

But there's a problem. We can't. It is impossible to introduce a distinctness fact for every plurality of facts that are distinct. That is to say, there cannot be a different fact $[[p_1], [p_2], ... \neq [q_1], [q_2], ...]$ for each pair of pluralities such that some fact is a member of $[p_1], [p_2], ...$ but not a member of $[q_1], [q_2], ...$

The problem stems from the cardinality of the set of distinctness facts. How many such facts are there? Let's start with a relatively simple case: how many distinctness facts involve $[p]$? Well, there is the fact that $[q] \neq [p]$, the fact that $[r] \neq [p]$, the fact that $[q], [r] \neq [p]$ etc. As it turns out, nearly *every* plurality of facts (except for the plurality

[27] For ease of prose, I have continued to speak in terms of facts; the reader should be aware from the previous discussion that I ultimately interpret these cases propositionally. Nothing in this section hangs on that difference.

consisting just of $[p]$) is distinct from $[p]$. So, the cardinality of the set of distinctness facts involving $[p]$ is thus the cardinality of the set of pluralities of facts.[28]

The cardinality of the set of pluralities of facts is the cardinality of the powerset of the set of facts; there are as many pluralities as there are combinations of facts. If there were distinctness facts corresponding to each of these pluralities, there would then be an injection from the powerset of facts to the set of facts; for every collection of facts $[q_1], [q_2], ...$, there would exist a fact that $[p] \neq [q_1], [q_2], ..$ (except for the case where the plurality consists of $[p]$). We could then map each element of the powerset of facts to the distinctness fact which held that that plurality is not $[p]$.[29] Since the distinctness facts are, well, *facts*, we could therefore map every element of the powerset of facts to a unique fact.

There cannot be such a mapping. Cantor's Theorem entails that there is no injection from the powerset of facts to the set of facts. For this reason, this no distinctness fact for each plurality of facts that are distinct. Because the distinctness view depends upon generalizing distinctness so that it is a relation between pluralities—and because we *can't* generalize distinctness in this way—the nongrounding view appears to be untenable.

As it turns out, cardinality constraints impact not only the number of distinctness facts, but also the number of nongrounding facts. Just as we cannot generate a distinctness fact for each plurality, so too we cannot generate a nongrounding fact for each plurality. Select an arbitrary fundamental fact $[p]$; how many nongrounding facts involve $[p]$? There is the fact that $[p]$ does not ground $[p]$, the fact that $[q]$ does not ground $[p]$, the fact that $[p], [q]$ does not ground $[p]$ etc. In fact, for every plurality of facts $[p_1], [p_2], ..$, there ought to be a nongrounding fact asserting that $[p_1], [p_2], ...$ does not ground $[p]$. If this were so, then the *nongrounding* facts would generate an injection between the powerset of facts and the set of facts—for the exact same reason that distinctness facts did. For each collection of facts, we could generate a nongrounding fact that maintains that that collection does not ground $[p]$. Since this injection cannot exist, we cannot countenance as many nongrounding facts as there naturally seems to be.

Notably, this restriction has *nothing whatsoever* to do with the distinctness account. Regardless of what—if anything—grounds nongrounding facts, *every* theory of nonground faces this limitation. Cantor's Theorem restricts the number of nongrounding facts that there can be.

The upper bound of the cardinality of the distinctness facts is thus the cardinality of facts. For the same reason, the upper bound of the cardinality of the nongrounding facts is also the cardinality of facts. So, there is no reason (or, at least, no *cardinality* based reason) to deny that we can provide a distinctness fact for each nongrounding fact that needs to be grounded. In this sense, the distinctness account remains a live candidate; given a nongrounding fact, there may be a distinctness fact that serves to ground it.

The position we find ourselves in is far from comfortable. There can neither be as

[28] This cardinality claim holds on the assumption that there are infinitely many facts.

[29] More precisely, we could use distinctness for most of this injection, and identity for the fact that $[p] = [p]$.

many distinctness nor nongrounding facts as there naturally seems to be. It could be that distinctness facts that ought to differ from one another are in fact the same; perhaps $[p], [q] \neq [r], [s]$, but $[[p], [q] \neq [t], [u]] = [[r], [s] \neq [t], [u]]$—or perhaps there are collections of facts such that no distinctness fact involving them exists.[30] Analogously, it could be that nongrounding facts that ought to differ from one another are in fact the same; perhaps $[p], [q] \neq [r], [s]$, but $[\neg([p], [q] < [t])] = [\neg([r], [s] < [t])]$—or perhaps there are collections of facts such that no nongrounding fact involving them exists. These options are far from intuitive, but they are options forced upon anyone who countenances nongrounding facts. What matters, for our purposes, is that there is no *special problem* for the distinctness view—no problem that would prevent us pairing a distinctness fact with each nongrounding fact that needs to be grounded—that other accounts might hope to avoid. Cantor's Theorem reveals odd aspects of our theory, but ultimately provides no reason to reject the distinctness account.

Conclusion

The case is more-or-less complete. An underdeveloped puzzle concerns the grounds of nonground—and a very natural solution appeals to distinctness from positive grounds. As we have seen, a number of puzzles emerge when this view is carefully considered—but a version survives scrutiny. I believe that the best version of this view is propositional, rather than factive, takes the form of a list of distinctness propositions, rather than a conjunction of them, and holds that the nongrounds of the fundamental are zero-grounded.

If this is correct, there are several immediate implications—ones that surpass the puzzle at hand. First, this pushes us toward a propositional, rather than factive, conception of ground more generally. It would be odd if the grounds of nongrounding were to be understood propositionally, but the rest of ground to be understood factively. Second, there is theoretical work for distinctness to accomplish. Far from being a theoretically idle relation, distinctness performs substantive work.

Lastly, while I take it that my view is plausible, it is far from the only potential candidate. It could be that, if $[p]$ grounds $[q]$, then $[p]$ also grounds the fact that $[r]$ does not ground $[q]$; that $[q]$ grounds the fact that $[r]$ does not ground $[q]$; that $[p]$ and $[q]$ collectively ground the fact that $[r]$ does not ground $[q]$;[31] or that all facts of the form $[[r]$ does not ground $[q]]$ are zero-grounded. While this is the first paper on the grounds of nonground, I hope that it will not be the last.

[30] The reason that the first modification resolves the problem is that the trouble with Cantor's Theorem stemmed from generating a *different* fact for each distinct pluralities of facts. If some of these pluralities correspond to the *same* distinctness fact, then distinctness need not generate an injection between the powerset of facts and the set of facts.

[31] Conversationally, both Louis deRosset and Isaac Wilhelm have suggested this to me—which might indicate that it is an option worth taking seriously

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