The Semantic Foundations of Philosophical Analysis

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"The business of philosophy, as I conceive it, is essentially that of logical analysis"—Bertrand Russell

Abstract

I provide an analysis of sentences of the form 'To be F is to be G' in terms of exact truth-maker semantics—an approach which identifies the meanings of sentences with the states of the world directly responsible for their truth-values. Roughly, I argue that these sentences hold just in case that which makes something F also makes it G. This approach is hyperintensional, and possesses desirable logical and modal features. In particular, these sentences are reflexive, transitive and symmetric, and, if they are true, then they are necessarily true, and it is necessary that all and only Fs are Gs. I motivate my account over that provided by Correia and Skiles (Forthcoming), and close by defining an irreflexive and asymmetric notion of analysis in terms of the symmetric and reflexive notion.

1 Introduction

The subject of this paper is a targeted reading of sentences of the form 'To be F is to be G,' which philosophers often use to express analyses, and which have occupied a central role in the discipline since its inception. Examples that naturally lead themselves to this reading include:

- 1. To be morally right is to maximize utility.
- 2. To be human is to be a rational animal.
- 3. To be water is to be the chemical compound H_2O .
- 4. To be even is to be a natural number divisible by two without remainder.
- 5. To be a béchamel is to be a roux with milk.

As these examples indicate, although philosophers frequently utter these sorts of sentences, they do not fall within the exclusive purview of philosophical inquiry. Mathematicians, chemists and even chefs use them as well. And while some are knowable a priori,

others are sensitive to empirical investigation—so there is variation within the phenomenon I seek to describe.

Sentences of this form have been employed since antiquity (as witnessed by 2). Throughout the ensuing history, proposed instances have been advanced and rejected for multitudinous reasons. On one understanding, this investigation thus has a long and rich history—perhaps as long and rich as any in philosophy. Nevertheless, explicit discussion of these sentences in their full generality is relatively recent (for example, see Dorr 2016; Correia 2017; Correia and Skiles Forthcoming; Rayo 2013; Linnebo 2014). Current advances in hyperintensional logic provide the requisite resources to analyze these sentences perspicuously—to provide an analysis of analysis. The significance of this project ought to be apparent: the standards that putative analyses must meet hang in the balance.

A bit loosely, I claim that these sentences are true just in case that which makes it the case that something is F also makes it the case that it is G and vice versa. There is a great deal to say about what I mean by 'makes it the case that.' In some ways, this paper can be read as an explication of that phrase. For the moment, suffice it to say that rather than understanding it modally (along the lines of 'To be F is to be G' is true just in case the fact that something is F necessitates that it is G and vice versa) I employ truth-maker semantics: an approach that identifies the meanings of sentences with the states of the world exactly responsible for their truth-values. It will take time before my account can be stated any more precisely; the details of truth-maker semantics must first be appreciated.

I believe this to be true for several reasons. Two relatively banal ones are that it is intuitive and that it satisfies our theoretical demands with minimal theoretical costs. But perhaps the most intriguing reason I believe that this is so, and a guiding motivation behind the current approach, concerns the analogy between real and nominal definition. On one conception, nominal definitions are a special kind of real definition—they are the real definitions of linguistic objects. Even those who do not conceive of nominal definition in this way often countenance a close connection between it and real definition. It is common practice for the notion of real definition to be introduced as the worldly analogue (or perhaps the metaphysical counterpart) of nominal definition. If the two are so closely related, then the philosophically salient aspects of one ought to resemble the philosophically salient aspects of the other. Of course, this analogy determines little about real definition without an account of nominal definition—a theory of how words, phrases, sentences and the like are defined. A popular conception of nominal definition is semantic essentialism; terms are defined by their semantic content. The definition of a word is given by what the word means, and the definition of a sentence is given by what it is the sentence means.

If we are to take both semantic essentialism and the analogy between real and nominal definition seriously (and I strongly believe that we should), then the logical attributes of philosophical analysis ought to mirror the logical attributes of meaning (or, some might

¹On another conception, nominal definitions can be thought of as putative real definitions which are defective, or in some way of inferior status. I set this use of 'nominal definition' aside.

²See, e.g., Fine (1994).

say, the logical attributes of analyticity). After all, on this view, real definitions resemble the way in which terms are defined, and terms are defined by their meanings. To put it only slightly differently, if the logic of real and nominal definition functioned radically differently, I would conclude that they are disanalogous in the relevant sense. As it turns out, we already possess a logic of meaning (see Angell (1977, 1989, 2002)). It is a virtue of the present approach that the logic of analysis aligns with this system.

The structure of this paper is as follows. In section 2, I isolate the targeted reading of 'To be F is to be G' at issue. In section 3, I discuss current developments in truthmaker semantics in considerable detail. The various ways in which the semantics might be constructed bear on philosophical analysis in that the logical features of analysis turn on which alternative we select. For example, the truth-value of sentences of the form 'To be F is to be both $F \wedge F$ ' depends upon a choice-point in truth-maker semantics—or so I will argue. In section 4, I provide the details of my account and demonstrate that it has appropriate logical features; it is transitive, reflexive and symmetric and has the resources to distinguish between the meanings of predicates with (classically) logically equivalent extensions (sentences of the form 'To be F is to be both F and G or not G' are typically false). I present the resulting logic of analysis, and highlight advantages my proposal has over that defended by Correia and Skiles (Forthcoming), the closest account available in the literature. I conclude in section 5 by sketching an account of an irreflexive and asymmetric notion of analysis in terms of the reflexive and symmetric notion.

2 Generalized Identity

'To be F is to be G' may admit of multiple readings. There may be a reading of 'To be a politician is to be indebted to one's constituents,' 'To be a scientist is to be curious about the natural world,' or 'To be early is to be on time' on which these sentences are true. Perhaps closer to my intended target, there may also be a reading of 'To be a bachelor is to be male' or 'To be crimson is to be red' on which these sentences are true. If so, these are not the readings I address. The reading of 'To be F is to be F' that I am concerned with is synonymous with (or, at the very least, close to synonymous with) 'To be F just is to be F' or, perhaps, 'being F is the same as being F.'

This reading has borne multiple labels in the literature. Some refer to these sentences as 'generalized identities' (e.g., Linnebo (2014); Correia (2017); Correia and Skiles (Forthcoming)), others prefer 'identifications' (e.g., Dorr (2016); Caie, Goodman and Lederman (2019)), while still others refer to them as "just-is'-statements' (e.g., Rayo (2013)). Nothing philosophically significant turns on which label we select—so long as the targeted reading itself is clear. For the purposes of this paper, I will refer to these sentences as 'generalized identities.' My use of the term 'analysis' differs from 'generalized identity' only in that it is slightly more expansive: it captures both generalized identities and sentences with different syntactic structures that resemble generalized identities in philosophically significant

respects.

It is often possible to express analyses using verbs (and verb phrases) in various forms. Examples of these sorts of sentences include:

- 6. To know that p is to have a justified true belief that p.
- 7. To die is to cease to live.
- 8. Shrinking is decreasing in size.
- 9. To resemble is to be similar to.

Perhaps an exhaustive discussion of analysis ought to address these types of sentences as well. I presently have very little to say about them, and largely disregard them here.

However, I maintain that sentences of the form 'For a to be F is for a to be G' belong to the same family as generalized identities—primarily because of their use in expressing analyses of 0-ary predicates. 'To be F is to be G' may, and often does, express analyses of predicates of various adicities. Both 'To be a vixen is to be a female fox' and 'To be adjacent is to be next to' are grammatically correct, although 'vixen' is a monadic predicate while 'adjacent' is dyadic. But sentences of this form are typically ungrammatical when applied to 0-ary predicates: i.e. predicates that apply to no objects. For example, 'To be John is a bachelor is to be John is an unmarried male' is ungrammatical. However, 'For John to be a bachelor is for John to be an unmarried male' is perfectly grammatical, and conveys the information expected of an analysis.

Under the target reading, the 'is' of generalized identity shares many logical and modal features with the 'is' of identity. It is reflexive, symmetric and transitive and, if a sentence of this form is true, then it is necessarily true, and necessary that all and only Fs are Gs. An adequate account of generalized identity ought, minimally, to explain the presence of these features.

Some dispute that 'To be F is to be G' is reflexive and symmetric, maintaining that while 'To be a father is to be a male parent' is true, 'To be a male parent is to be a father' is false, and that 'To be a father is to be a father' is trivially false, rather than trivially true. Such philosophers hold that generalized identities form a strict partial ordering over predicates, perhaps maintaining that if 'To be F is to be G' is true, then G is somehow more basic or fundamental than F is.

I confess that I once had such predilections myself. However, I have come to endorse a reading of 'To be F is to be G' that strongly resembles an identity. There is a perfectly intelligible reading of 'To be F is to be F' on which the sentence is manifestly true. After

³I make the standard assumption that sentences are 0-adic predicates. So, just as 'smaller than' is a dyadic predicate and 'smaller than Jones' is a monadic predicate, 'Smith is smaller than Jones' is a 0-adic predicate.

⁴For one such argument, see Cameron (2014).

all, what else could F possibly be?⁵ Of course, uttering this sort of sentence is typically pragmatically infelicitous. When someone inquires, 'What is it to be a bachelor?' the response 'To be a bachelor is to be a bachelor' is likely to be unhelpful. This can straightforwardly be explained in terms of conversational norms. Because 'To be a bachelor is to be a bachelor' is trivially true, it is not the kind of information such an inquirer is after. Trivial responses to substantive questions flout conversational norms.

In addition, interest in one reading need not preclude the import of another. Although I am primarily concerned with the symmetric and reflexive reading of 'To be F is to be G,' on the present approach it is possible to define an asymmetric and irreflexive notion in terms of the symmetric and reflexive one. I outline an account of such a reading in some concluding remarks, and ask readers primarily concerned with this alternate reading to bear with me until then.

Once the reflexive and symmetric reading is embraced, it is natural to treat the 'is' of 'To be F is to be G' literally as the 'is' of identity: i.e., to maintain that a sentence of this form is true just in case the property of being F is identical to the property of being G. However, philosophers have presented several arguments against this suggestion.⁶ First, there is linguistic evidence that 'To be F' is not synonymous with 'To be the property of being F' in some contexts. 'I hope to be a skilled philosopher' is perfectly true, but 'I hope to be the property of being a skilled philosopher' is presumably false; I do not hope to be a property. If so, 'To be F is to be G' may not be synonymous with 'To be the property of being F is to be the property of being G.' Second, some desire an account of generalized identity to be compatible with nominalism—the denial that abstract objects like properties and relations exist. A nominalist may grant that to be a triangle is to be a polygon with three angles, but would presumably deny that the property of being a triangle is identical to the property of being a polygon with three angles on the grounds that she denies the existence of properties. Minimally, some suggest, such a nominalist's position ought not be obviously incoherent. By denying that 'To be F' is synonymous with 'To be the property of being F,' such a nominalist's position can be accommodated. Third, the nominalized version of generalized identity arguably possess the resources to avoid self-referential paradox. It may be that to be a non-self-instantiator is to not instantiate oneself, but it would be paradoxical to claim that to be the property of being a non-self-instantiator is to be the property of not instantiating oneself. And so, some argue, the 'is' of generalized identity strongly resembles, but is not strictly, the 'is' of identity, because the terms flanking it do not denote.

I find these arguments deeply unpersuasive. I am unprepared to settle central metaphysical debates based on the infelicity of 'I hope to be the property of being an accomplished

⁵Likewise, while there are contexts in which 'To be a father is to be a male parent' seems appropriate, there are also contexts in which 'To be a male parent is to be a father' seems appropriate. See Dorr (2016) for a discussion of this point. And even those who primarily address an irreflexive reading of 'To be F is to be G' often countenance a reflexive reading as well (e.g., Rosen (2015)).

⁶So far as I can tell, Dorr (2016) and Correia (2017) provide precisely the same arguments.

philosopher'—it may be that 'To be F' functions differently in that type of construction than it does in 'To be F is to be G.' And while some accounts of generalized identity rule out forms of nominalism, I do not find this particularly undesirable. It would be naïve to expect a theory of analysis to stay neutral on all other philosophical debates. An account may regularly, perhaps even systematically, force our hand; so long as our hand is forced in the correct way, this seems no cost to me. Furthermore, while the reified approach entails that nominalism is incorrect, it does not entail that nominalism is obviously incorrect, because even if the reified approach is true, it need not be obviously true. And so, nominalists may be making a substantive, rather than a silly, mistake. And while the threat of paradox is real, there are a bevy of alternate responses and maneuvers available in the literature.

These responses are cursory, to be sure. But there is a more direct reason to adopt the reified approach. Theories of generalized identity can be incorrect. While I maintain that my proposal is true, I could be wrong; perhaps the account of generalized identity that I advance is faulty. If, instead, I am correct, then alternate proposals are incorrect. The salient question is this: when an account is false, what about the world does it get wrong? I.e., what is its subject matter, which it inaccurately describes?

Of course, there are various ways for accounts to be incorrect. Accounts can be incorrect by their own lights, for example—one might even employ inconsistent axioms. But internal consistency is not enough. Even if a proposal is internally consistent, it might represent the world inaccurately: after all, there are a vast number of (consistent) logical systems we could construct. But if a proposal does get something about the world wrong, what does it get wrong? Surely not natural language—a theory of generalized identity is not an account of how natural-language predicates function. Just as it is possible for an account to describe the world inaccurately, so too it seems possible for natural language to not track the world in the appropriate way, so we cannot identify correctness with correspondence to natural language.

The obvious answer, it seems to me, is that an account is false if it produces incorrect identity conditions for properties: either if properties F and G are identical and the account entails they are distinct, or if properties F and G are distinct and the account entails they are identical. But in order for this to be how accounts go wrong, there must be properties F and G which an account attributes identity conditions to. A theory of generalized identity thus provides putative identity conditions for properties, propositions and relations. It is correct if the identity conditions it specifies are correct, and it is incorrect if the identity conditions it specifies are incorrect.

⁷Additionally, it is worth noting that the denial that 'To be F' is synonymous with 'To be the property of being F' does not, by itself, avoid paradox. In certain free logics in which terms need not denote, it remains possible to derive a contradiction from the nominalized construction. After all, in such logics it is possible to employ non-denoting proper names for 'To be F.' My thanks to Jon Litland for this point. For the sake of space, I will not investigate which method for avoiding self-referential paradox is best.

⁸It is reasonably to attribute this type of internal inconsistency to the structured proposition view—see, again, Dorr (2016).

The nominalist approach cannot identify mistakes in this way; generalized identities do not attribute identity conditions to properties at all, so they cannot attribute identity conditions incorrectly. Minimally, those who defend nominalized versions of generalized identity owe an explanation of what about the world incorrect accounts get wrong. Until such an account is provided, I am inclined to adopt the reified interpretation of generalized identity instead.

It is my hope that the target phenomenon is, by this point, sufficiently clear. I now turn to developments in truth-maker semantics, which I employ within this account.

3 Truth-Maker Semantics

Truth-maker semantics rests upon the conviction that the meanings of sentences directly depend upon the aspects of the world that are responsible for their truth-values. When stated so generally, this hardly seems controversial. Who would doubt that the meaning of 'grass is green' depends on that which is responsible for the truth of 'grass is green' (and that which would be responsible for the falsity of 'grass is green,' were it false)? And, to be fair, many accounts claim conformity to this mantra—at least when it is understood broadly enough. The conviction that sentences' meanings are associated with their truth-conditions has been around at least since Tarski (1933) and arguably since Frege (1892).

What is unique to truth-maker semantics is its commitment to exact truth-makers. The aspects of the world responsible for the meaning of a sentence s are those aspects which exactly make s true or exactly make s false; i.e., the aspects of the world that determine s's meaning are entirely relevant to s. By design, truth-maker semantics eliminates aspects of sentences' meanings unrelated to their truth-values, and is capable of attributing distinct meanings to necessarily equivalent sentences.

Integral to the truth-maker approach is the idea of a *state*—a fact-like entity that typically has a more restricted scope than a possible world. For example, there is a state of whales being mammals, a state of it raining outside, and a state of 2 being a number. Additionally, there is a state of their being twelve planets in the solar system and a state of 2 being an odd number. So, states are not restricted to those that actually obtain, nor even to those that possibly could obtain. 'State' is intended as a term of art and is as metaphysically neutral as possible. I make no assumptions about what ontological kind of thing a state is, or about whether states figure in the fundamental building blocks of reality. One of the only metaphysical assumptions I make is that states are capable of mereological composition. So, the state of a ball being both red and round may be the composite of the state of the ball being red with the state of the ball being round.

Let a state space be an ordered pair $\langle S, \sqsubseteq \rangle$, where S is a set of states and \sqsubseteq is a

⁹To a very large extent, this paper relies on truth-maker semantics as it was developed by Fine (2013, 2014, 2016, forthcoming). For precursors to this semantics, see Van Fraassen (1969); Hwang and Schubert (1993).

binary relation on S. \sqsubseteq is to be interpreted as the relation of parthood, such that ' $s \sqsubseteq s'$ ' asserts that state s is a part of state s'. I make the standard assumption that parthood is a partial ordering: i.e., that \sqsubseteq satisfies the following criteria:

- a) Reflexivity: For any state $s \in S$, $s \sqsubseteq s$
- b) Antisymmetry: For any states $s, s' \in S$, if $s \sqsubseteq s'$ and $s' \sqsubseteq s$, then s = s'
- c) Transitivity: For any states $s, s', s'' \in S$, if $s \sqsubseteq s'$ and $s' \sqsubseteq s''$, then $s \sqsubseteq s''$

Many state spaces, as defined above, are uninteresting. For example, there are state spaces in which no mereological composition occurs. In these, the extension of \sqsubseteq is restricted to reflexivity; every state is a part of itself, and no state is a part of any other. For the purposes of this paper, let us confine our attention to state spaces that allow for arbitrary fusion, which I call 'complete state spaces.'¹⁰

On some approaches, the end of a semantics is to determine whether statements are true or false in a given world. However, on the current approach, the aim is to determine what precisely it is within a world that is responsible for the truth-values of sentences. Thus, verifiers must be relevant to the sentences that they render true, and they must be entirely relevant. States are the candidate verifiers and falsifiers. So, the state of the republicans controlling the Senate verifies 'The republicans control the Senate' and falsifies 'The republicans do not control the Senate.'

I do not assume that each sentence has a unique verifier and a unique falsifier. Potentially, sentences could have many. The sentence, 'Either Dante wrote *The Divine Comedy* or Aristotle tutored Alexander the Great' presumably has (at least) two verifiers: the state of Dante having written *The Divine Comedy* and the state of Aristotle having tutored Alexander the Great. However, it is important for a sentence's verifiers to guarantee its truth and for its falsifiers to guarantee its falsehood. So, the state of roses being red does not verify 'roses are red and violets are blue,' despite its relevance to the conjunction. Furthermore, verifiers and falsifiers do not contain extraneous information. The state of 2 being both even and prime does not verify 'the number 2 is even,' despite the fact that it guarantees the sentence's truth.¹¹

 $^{^{10}}$ For the most part, this can be achieved by stipulating that any two states within S have a fusion within S. However, this approach fails for infinitely large state-spaces. A restriction appropriate for infinitely large state spaces requires a few more definitions.

I denote the fusion of states $T \subseteq S$ as $\bigsqcup T$. Let an upper bound of $T \subseteq S$ be a state s such that, for all states $t \in T, t \subseteq s$. That is to say, an upper bound of a subset of S is a state which contains—as a part—every state within that subset. Let a least upper bound of $T \subseteq S$ be a state s such that s is an upper bound of T and, for all upper bounds s' of T, $s \subseteq s'$. Intuitively, we can think of the least upper bound of T as being the smallest upper bound of T—one which is a part of all upper bounds of T. Provably, if there is a least upper bound of T, then there is a unique least upper bound of T. For a state space S, S, select an arbitrary S and S suppose, for reductio, that S has two least upper bounds—S fixen antisymmetry, this entails that S has a state space S be complete just in case every subset S has a fusion within S. Here, I consider only complete state spaces.

¹¹This differentiates truth-maker semantics from the approach outlined by Barwise and Perry (1983) and

For obvious reasons, propositional languages are of limited relevance to claims of the form 'To be F is to be G.' Let us restrict our attention to a language capable of expressing predication. Let I be the (potentially infinite) set of all objects. Let a language L contain a unique name for each object such that $\underline{i_1}$ denotes $\underline{i_1}$, $\underline{i_2}$ denotes $\underline{i_2}$, etc.. Additionally, let L contain infinitely many predicates $P_1, \overline{P_2}, \ldots$ of fixed adicity, as well as the operators \neg , \land and \lor , defined in the standard way.

Let a model M be an ordered quadruple $\langle S, \sqsubseteq, I, |\cdot| \rangle$ such that $\langle S, \sqsubseteq \rangle$ is a complete state space, I is the set of individuals and $|\cdot|$ is a valuation function that takes each n-adic predicate P and each ordered combination of n objects $\langle i_1, i_2, ..., i_n \rangle$ to an ordered pair $\langle V, F \rangle$ where both V and F are subsets of S, with the intended interpretation that V is the set of $P(i_1, i_2, ..., i_n)$'s verifiers, and F is the set of its falsifiers. The semantics is given inductively:

i)⁺
$$s \Vdash P(i_1, i_2, ..., i_n) \text{ iff } s \in |(P(i_1, i_2, ..., i_n))|^{V}$$

i)
$$s \dashv P(\underline{i_1}, \underline{i_2}, ..., \underline{i_n}) \text{ iff } s \in |(P(i_1, i_2, ..., i_n))|^F$$

$$ii)^+$$
 $s \Vdash \neg A \text{ iff } s \dashv A$

ii)
$$s \dashv \neg A \text{ iff } s \Vdash A$$

iii)⁺
$$s \Vdash A \land B$$
 iff for some states t and $u, t \Vdash A$ and $u \Vdash B$ and $s = t \sqcup u^{12}$

iii)
$$s \dashv A \land B$$
 iff either $s \dashv A$ or $s \dashv B$

$$iv)^+$$
 $s \Vdash A \lor B$ iff either $s \Vdash A$ or $s \Vdash B$

iv)
$$s \dashv A \lor B$$
 iff for some states t and u, $t \dashv A$ and $u \dashv B$ and $s = t \sqcup u$

It is my hope that readers find this semantics to be extraordinarily intuitive. Nevertheless, it has some surprising results. One of the most unexpected is that a verifier of $A \wedge A$ need not verify A. Every fusion of two distinct verifiers of A is a verifier of the conjunction $A \wedge A$, but these fusions need not themselves verify A. I do not find this result particularly problematic. Indeed, as I discuss in section 4, this is directly responsible for desirable logical attributes of analysis. However, those who object can readily modify the semantics by requiring that verifiers are closed under fusion; the fusion of any two verifiers of A is itself a verifier of A. Clauses iii) and iv) thus become:

iii) *
$$s \dashv A \land B$$
 iff $s \dashv A$ or $s \dashv B$ or $s \dashv A \lor B$

$$iv)^{+*} s \Vdash A \lor B \text{ iff } s \Vdash A \text{ or } s \Vdash B \text{ or } s \Vdash A \land B$$

There are at least two ways to extend the semantics to clauses with quantifiers. One utilizes generic objects, such that verifiers of universal statements are generic states (rather than states about particular objects). A verifier for 'All numbers are either even or odd' is

Kratzer (2007) who employ a notion of truth-making such that verifiers need not be wholly relevant to the sentences that they render true.

¹²Let ' \sqcup ' denote mereological fusion—so ' $t \sqcup u$ ' denotes the fusion of state t with state u.

the state of a generic number being either even or odd. The semantics I prefer is instantial: verifiers of universal and existential statements are states concerning their instances. The introduction of predicates is currently more significant than the introduction of quantifiers is, so I opt for the most easily intelligible approach to quantification. The instantial approach treats the semantics of universal statements like large conjunctions. The meaning of the claim 'Everything is F' is treated similarly to the conjunction of the claims 'F(i),' for all \underline{i} . More formally, we have:

- $(v)^+$ $s \Vdash \forall x A(x)$ iff there is a function f from I into S such that $f(i) \Vdash A(i)$ for each $i \in I$, and $s = |\{f(i) : i \in I\}$
- v) $s \dashv \forall x A(x)$ iff for some $i \in I, s \dashv A(i)$
- $vi)^+$ $s \Vdash \exists x A(x)$ iff for some $i \in I, s \Vdash A(i)$
- vi) $s \dashv \exists x A(x)$ iff there is a function f from I into S such that $f(i) \dashv A(i)$ for each $i \in I$, and $s = \bigsqcup \{f(i) : i \in I\}^{13}$

Exact equivalence can thus be defined as follows: sentences A and B are exactly equivalent just in case their verifiers and falsifiers are identical. Many logically equivalent sentences (from a classical perspective) are not exactly equivalent. For example, $A \land \neg A$ need not be exactly equivalent to $B \land \neg B$. Verifiers of the first sentence are fusions of a verifier of A with a falsifier of A, while verifiers of the second sentence are fusions of a verifier of B with a falsifier of B. And, as mentioned above, A is not even exactly equivalent to $A \land A$ unless appropriate modifications are made. Nevertheless, exact equivalence differs from syntactic identity. $A \land B$ is exactly equivalent to $B \land A$, and A is exactly equivalent to $\neg \neg A$.

¹³One further complication arises if we assume that a verifier ought to necessitate the sentence it verifies, and allow for a variable domain of objects that could exist. Suppose, for example, that all cats have tails. On the instantial approach, a verifier of 'all cats have tails' is the fusion of a verifier of the claim that each cat has a tail—i.e., that Fluffy is a cat with a tail, Scamp is a cat with a tail, etc.. This fusion could obtain, and yet 'all cats have tails' be false in a possible situation in which there were more cats than actually exist—one of which lacks a tail. The obvious solution to this problem is to incorporate a totality state—a verifier of a universal statement is the fusion of verifiers of all instances with a verifier of the claim that those are all of the objects that exist. Interestingly, once the notion of a totality state is employed, it is possible to define the distinction between inclusive and exclusive quantification. For if we insist that there be a nonempty fusion of instances with totality states, the resulting quantifiers will be exclusive, but if we allow for empty fusions of instances with totality states, the resulting quantifiers will be inclusive. In the case of an empty domain with inclusive quantification, a verifier of the claim that everything is F will be the state of there being nothing which exists.

¹⁴This itself is a desirable outcome; it would be something of a bizarre coincidence if the granularity of properties, propositions and relations perfectly aligned the granularity of our syntax.

4 The Semantic Foundations of Philosophical Analysis

Truth-maker semantics provides the resources to account for 'To be F is to be G' perspicuously: to elucidate what I meant when I claimed that these sentences hold just in case that which makes something F is that which makes something G. A sentence of this form is true just in case, for any name \underline{i} , 'F(i)' is exactly equivalent to 'G(i).' 'To be a person is to be bound by the categorical imperative' holds just in case the verifiers and falsifiers of the claim that someone is a person are identical to the verifiers and falsifiers of the claim that she is bound by the categorical imperative, and 'To be morally right is to maximize utility' holds just in case the verifiers and falsifiers of the claim that an act is morally right are identical to the verifiers and falsifiers of the claim that it maximizes utility.

It may seem odd, given that I am ostensibly concerned with what makes it the case that an object is F, that I am sensitive to states that make it the case that something isn't F—i.e., that this account is sensitive to falsifiers as well as to verifiers. On some semantic approaches, the inclusion of falsifiers is gratuitous. If the meaning of 'Fa' is given by the possible worlds in which a is F, this collection also determines the worlds in which a is not F. I suspect this gratuity is responsible for the apparent oddity of mentioning falsifiers explicitly. But on the truthmaker approach this is not the case. Sentences may be alike in what makes them true, but differ in what makes them false. It seems to me that there is nothing to be gained from the exclusion of falsifiers, and a great deal is lost. For, as we shall see, including falsifiers allows one to infer from the fact that the falsifiers of 'Sarah is a bachelor' differ from the falsifiers of 'Sarah is vixen' that 'To be a bachelor is to be a vixen' is false, where an approach sensitive only to verification cannot justify this inference.

Some might suspect that sentences of the form 'To be F is to be G' are true just in case ' $\forall xFx$ ' is exactly equivalent to ' $\forall xGx$.' This is false. ' $\forall xFx$ ' is exactly equivalent to ' $\forall xGx$ ' just in case the sentences' verifiers and falsifiers are identical. On the current approach, universal statements are treated instantially. That is to say, the claim that everything is F is treated like the conjunction of the claims that each individual is F. Suppose that there were two objects i_1 and i_2 such that a verifier for ' $F(i_1)$ ' was a verifier for ' $G(i_2)$,' and a verifier for ' $F(i_2)$ ' was a verifier for ' $F(i_1)$ '. In this case, the conjunction ' $F(i_1) \land F(i_2)$ ' has an identical verifier to ' $F(i_1) \land F(i_2)$ '—the verifier for each is the fusion of two identical states. Nevertheless, it would be absurd to take this to lend support to the truth of 'To be F is to be F.' After all, a state which renders ' $F(i_1)$ ' true is a state which renders ' $F(i_1)$ ', rather than ' $F(i_1)$ ', true. It is essential to preserve the verifiers and falsifiers for each instance—and not merely for the fusion of their instances.

¹⁵It is straightforward to generalize this account to apply to predicates of any fixed adicity. For any pair of n-adic predicates F and G, the sentence, 'To be F is to be G' is true just in case, for all collections of n names, $\underline{i_t}$ — $\underline{i_n}$, ' $F(i_1, i_2, ..., i_n)$ ' is exactly equivalent to ' $G(i_1, i_2, ..., i_n)$.'

¹⁶The first truth-maker approach sensitive to falsification occurs in Fine (2016).

¹⁷This thought predates Montagovian semantics. It is evident in Wittgenstein (1922), who says "For the totality of facts determines both what is the case, and also what is not the case" (1.12).

¹⁸This problem may be peculiar to the instantial approach to quantification. On the generic approach—

This account accommodates the aforementioned logical features of generalized identities, primarily because exact equivalence is an equivalence relation. For any F and any i, 'F(i)' is exactly equivalent to 'F(i),' so sentences of the form, 'To be F is to be F' are universally true. Exact equivalence is symmetric; if 'F(i)' is exactly equivalent to 'G(i)' then 'G(i)' is exactly equivalent to 'F(i).' So, if 'To be F is to be F' is true, then 'To be F(i)' is exactly equivalent to 'F(i)' is exactly equivalent to 'F(i)' and 'F(i)' is exactly equivalent to 'F(i)' is exactly equivale

Of course, with a semantics in place we can establish more than that exact equivalence is an equivalence relation: it determines a complete logic of analysis. The present account entails that sentences of the following forms (which correspond to axioms in a logic of exact equivalence) are universally true, and that inferences of the following forms (which correspond to inferences in a logic of exact equivalence) are universally valid:

Axioms

1. To be $\neg \neg F$ is to be F.

- 5. To be $F \vee (G \vee H)$ is to be $(F \vee G) \vee H$.
- 2. To be $F \wedge G$ is to be $G \wedge F$.
- 6. To be $\neg (F \land G)$ is to be $\neg F \lor \neg G$.
- 3. To be $F \vee G$ is to be $G \vee F$.
- 7. To be $\neg (F \vee G)$ is to be $\neg F \wedge \neg G$.
- 4. To be $F \wedge (G \wedge H)$ is to be $(F \wedge G) \wedge H$.

Inferences

- 8. To be F is to be G/To be G is to be F.
- 9. To be F is to be $G/\text{To be } \neg F$ is to be $\neg G$.
- 10. To be F is to be G, To be G is to be H/to be F is to be H.
- 11. To be F is to be G/To be $F \wedge H$ is to be $G \wedge H$.
- 12. To be F is to be G/To be $F \vee H$ is to be $G \vee H$.

It is worth examining the plausibility of this system in detail. Each axiom and inferential rule is independently worthy of consideration, but I will primarily address three differences between the current approach and that advanced by Correia and Skiles (Forthcoming), perhaps the most similar account available in the literature. In some ways, our approaches are incommensurable. While I provide a semantics that underlies the logic of analysis—one rooted in the mereological structures of states—Correia and Skiles do not. While they devote considerable discussion to metaphysical essence and ground, I do not. The point of overlap lies at the logic of generalized identity, and it is here that our views compete. Given that the target phenomenon is the same, it is unsurprising that our logics share several axioms and inferential rules. However, they come apart in important, and, I believe, quite telling, ways.

according to which verifiers of ' $\forall xFx$ ' are states that verify that a generic object is F, there is no worry about swapping verifiers of instances. My thanks to Kit Fine for pressing me on this point. However, once the generic approach is adopted, the ability to define the distinction between inclusive and exclusive quantification in the manner mentioned in footnote 13 is lost.

The first (and perhaps the least persuasive) difference between our systems concerns the distribution axioms. Correia and Skiles license 'To be $F \wedge (G \vee H)$ is to be $(F \wedge G) \vee (F \wedge H)$ ' but deny that 'To be $F \vee (G \wedge H)$ is to be $(F \vee G) \wedge (F \vee H)$.' That is to say, on their system the distribution of conjunction over disjunction succeeds, while distribution of disjunction over conjunction fails. In contrast, I deny both distribution axioms. Their failure falls immediately out of the semantics I provide. One kind of verifier of ' $(Fa \vee Ga) \wedge (Fa \vee Ha)$ ' is the fusion of a verifier of 'Fa' with a falsifier of 'Fa' with a falsifier

I find the acceptance of only one distribution axiom bizarre. To be clear, while I claim both axioms are false, I do not find it bizarre to suggest that they are both true. What I find bizarre is accepting one while rejecting the other. Indeed, I can think of no philosophical motivation for this result. Conjunction is symmetric with disjunction in logically important ways; the relation conjunction stands in to the True and the False is analogous to the relation disjunction stands in to the False and the True. And so, the two logical operators ought to distinguish predicates from one another in a parallel manner. This symmetry is broken on Correia and Skiles' account. A fundamental difference arises with the distribution axioms: while distributing conjunction over disjunction does not result in a different predicate, distributing disjunction over conjunction does.

I suspect that the inclusion of only one distribution axiom arises from their intended semantics. If a unilateral notion of exact equivalence were adopted—one which included verification and excluded falsification—only one distribution axiom would result. However, this is no defense unless this intended semantics is independently motivated over the one I advance. Absent a reason to adopt a unilateral semantics, we remain without a reason to accept only one distribution axiom. The desire for a symmetric logic is unlikely to persuade those deeply wedded to an asymmetric account. At best, it might motivate uncertain philosophers waffling between available alternatives. But it remains a motivation for me, and is a harbinger of more serious problems to come.

A second (and perhaps more compelling) difference concerns an inference that I license and that Correia and Skiles do not—inference 9. If to be F is to be G, it follows that to be $\neg F$ is to be $\neg G$. For example, if to be a sister is to be a female sibling, then to be not a sister is to be not a female sibling, and if to be hydrogen is to be the chemical element with a single proton, then to be not hydrogen is to be not the chemical element with a single proton. While Correia and Skiles may (or may not) agree with these particular cases, they deny that the inference holds in its full generality.

Inference 9 results from adopting a bilateral, rather than a unilateral, notion of exact equivalence (i.e., from insisting that sentences are exactly equivalent just in case they have both identical verifiers and identical falsifiers, rather than merely having identical verifiers). To see why this is so, consider the proposal that a generalized identity is true just in case, for any name \underline{i} 'F(i)' has identical verifiers to 'G(i),' and omitting the inclusion of

falsifiers. Nothing truth-maker semantics guarantees that sentences with identical verifiers have identical falsifiers. The negation operator swaps a sentence's verifiers for its falsifiers, so, in this case, ' $\neg F(i)$ ' could have distinct verifiers from ' $\neg G(i)$,' and 'To be $\neg F$ is to be $\neg G$ ' would thereby be false. In contrast, my approach takes the False just as seriously as the True. If the states that falsify 'Fa' differ from those that falsify 'Ga,' this undermines the claim that 'To be F is to be G.' And because generalized identities are equally sensitive to that which verifies that an object is F as they are to that which falsifies that an object is F, the inference from 'To be F is to be G' to 'To be $\neg F$ is to be $\neg G$ ' is preserved.

An example may highlight the implausibility of denying inference 9. Suppose a precocious doctoral candidate were announce that she had discovered that to be water is to be the chemical compound H_2O . "What am amazing finding!" her advisor might reasonably exclaim, "I just have one quick follow-up—what is it to be not water?" It would seem strange if such a student lacked the resources for the answer; it seems perfectly obvious that to be not water is to be not the chemical compound H_2O , and that this is immediately justified by the inference from the fact that to be water is to be the chemical compound H_2O .

If inference 9 were impermissible, I doubt she could answer her advisor's question at all. What resources could she employ to uncover what it is to be not water? Are there other scientific experiments from those she used to discover the chemical composition of water? If not, what inferential resources are available to her, if even the inference from 'To be F is to be G' to 'To be $\neg F$ is to be $\neg G$ ' is denied? I suspect that those who deny inference 9 arrive at a limited (and implausible) form of skepticism; that there is no way for the student to uncover what it is to be not water. Minimally, the ball is in such philosophers' court: either show how the limited inferential resources can generate knowledge of what it is to be not water, or else accept the skepticism that results.

Notably, there are no counterexamples to inference 9 available in the literature. However, in the course of developing this paper various philosophers have presented putative counterexamples to me, one of which I address here. Consider 'To be definitely red is to be red.' Even if this is true, 'To be not definitely red is to be not red' is arguably false, for things might be red which are not definitely red.

I maintain that if there is a reading of 'To be definitely red is to be red' on which the sentence is true, it is not the reading I am concerned with. Recall that, on the target reading of 'To be F is to be G' it is necessary that all and only Fs are Gs. So if there are possible instances of objects which are red but are not definitely red, then 'To be definitely red is to be red' is false. In some ways this example resembles 'To be crimson is to be red,' mentioned at the outset. For, just as being crimson is a way of being red (and, perhaps, being definitely red is a way of being red), there are other ways of being red than being crimson which render 'To be crimson is to be red' false on the target reading. Just so with 'To be definitely red is to be red.'

¹⁹My thanks to Steve Yablo for this example.

If inference is 9 invalid, there ought to be cases in which it goes awry. If there are any such cases, they remain undiscovered (definite redness not withstanding). Every instance philosophers have discussed is one in which the inference from 'To be F is to be G' to 'To be G' appears legitimate. Systems that deny the inference from 'To be F is to be G' to 'To be G' appears that those that endorse it. Hence, that the present account licenses this inference is a mark in its favor.

A third difference concerns two axioms that Correia and Skiles license which I do not: 'To be F is to be $F \wedge F$ ' and 'To be F is to be $F \vee F$ ' (which are sometimes referred to as 'idempotence'). Correia and Skiles say little in their favor, aside from that they "[are] compulsory principles about generic identity" (11). In contrast, I deny both axioms. Recall that there is no guarantee that verification and falsification are closed under fusion; the fusion of two verifiers of 'Fa' may not verify 'Fa', and the fusion of two falsifiers of 'Fa' may not falsify 'Fa'. In this case, a state that verifies ' $Fa \wedge Fa$ ' would not verify 'Fa,' while a state that falsifies ' $Fa \vee Fa$ ' would not falsify 'Fa,' and so both forms of idempotence fail.

There are deep, logical motivations for abandoning idempotence in this context—most notably discussed in Dorr (2016). Addressing these motivations in sufficient depth would take us far afield and require introducing further formalisms. Dorr argues that there is a reading of 'To be grue is to be green and observed before time t or blue and not so observed' is true, yet 'To be green is to be grue and observed before time t or bleen and not so observed' is false. He accounts for this discrepancy by accepting a principle dubbed Only Logical Circles, according to which sentences of the form 'To be F is to be F' are circular in an innocuous way, but nonlogical types of circularity are universally vicious. Once this principle is integrated with the λ -calculus (a standard way of representing logically complex predicates), it immediately follows that idempotence is false. I will say no more about these consideration here, and direct those interested in the technical details to Dorr's paper. I mention it only to note that the logic I advance is compatible with Dorr's considerations, while Correia and Skiles' is not.²⁰

In any case, there are plausible counterexamples to idempotence.²¹ 'To be a bachelor is to be an unmarried male' seems true while 'To be a bachelor is to be an unmarried male and male and male ...' seems false. I suspect, however, that advocates of idempotence would insist that the second sentence is strictly true but pragmatically unwarranted. It is unhelpful to repeat the term 'male,' and so the second sentence is linguistically uncooperative. In other cases this appeal to pragmatics seems suspect. Consider, for example, 'To utter one sentence is to utter one sentence and utter one sentence.' One of the ways to utter one sentence and utter one sentences; i.e., to complete

²⁰Dorr's commitments surpass my own. Not only does he deny that idempotence is an axiom, but he insists that its negation is an axiom: i.e., that \neg (To be F is to be $F \land F$) is always true. I do not share this commitment, so while my system is compatible with Dorr's it is uncommitted to all of its details.

²¹I believe that there are counterexamples to both forms of idempotence, but will only address counterexamples to the conjunctive formulation here.

two single utterances in succession. But, surely, uttering two sentences is no way to utter one. Alternatively, consider 'To walk an odd number of steps is to walk an odd number of steps and walk an odd number of steps.' One of the ways someone could walk and odd number of steps and walk an odd number of steps is by walking an even number of steps; i.e., to complete two odd-numbered walks sequentially. But, surely, someone walking an even number of steps is not a way of walking an odd number of steps. And so 'To walk an odd number of steps is to walk an odd number of steps and walk and odd number of steps' is strictly false, rather than merely pragmatically unwarranted.

Some might suspect that I trade on a difference between natural and artificial language. While the natural 'and' often communicates temporal difference—so that 'A and B' indicates that event A occurred before event B—the logical ' \land ' has no such distinction. So, it is possible for each conjunct of 'John walked an odd number of steps \land John walked an odd number of steps' to refer to one and the same walk. This is why I state that one of the ways for someone to walk an odd number of steps and walk an odd number of steps is for them to walk an even number of steps. Another way is to walk an odd number of steps, and for the very same walk to be referred to twice. Although it is possible for each conjunct of 'John walked an odd number of steps \land John walked an odd number of steps' to refer to one and the same walk, it is not necessary. One of the things that makes this conjunction true is John's walking an even number of steps by sequentially completing two odd-stepped walks. But, surely, this sort of walk does not make 'John walked an odd number of steps' true. Thus, there are counterexamples to idempotence.

Much more could be said about the logic analysis, but it is my hope that the previous discussion suffices to motivate the present semantics over viable alternatives.

5 Conclusion

The focus of this paper has been a reflexive and symmetric notion of analysis: one that closely resembles an identity. However, I have briefly mentioned (and fully endorse) a reading of 'To be F is to be G' that is irreflexive and asymmetric. Notably, it is possible to define this irreflexive and asymmetric notion (which, instead of a 'generalized identity' we might call a 'definition') in terms of the reflexive and symmetric notion on this account. Here, I distinguish this alternate reading from with the use of the subscript 'df.' On this reading, the 'is' of 'To be F is df to be df' does not resemble the 'is' of identity; sentences of the form 'To be df is df to be df' are universally false, and if 'To be df is df to be df' is true then 'To be df is df to be df' is false. Before closing, allow me to briefly sketch an account of this reading.

The underlying motivation for this approach is that thought that a definition of a phenomenon breaks it down into its constituent parts. While some may subscribe to this metaphorically, the current approach interprets it almost literally. Recall that verifiers and falsifiers are endowed with mereological structure: some are states with proper parts. The

state of Ralph being a bachelor, for example, may be the composite of the state of Ralph being male with the state of Ralph being unmarried. And so, one can learn about the mereological structure of such a state by identifying its proper parts.

A natural thought is that a definition if a sentence of the form 'To be F is $_{df}$ to be G' which satisfies two conditions:

- 1. 'To be F is to be G' is a (true) generalized identity.
- 2. 'G' limns the mereological structure of the verifiers and falsifiers of 'Fa' more than 'F' does.

An adequate account of definition would require articulating necessary and sufficient conditions for 'G' to limn more mereological structure than 'F.' Here, I merely provide a sufficient condition: if there exists a predicate H such that, for any name a, a verifier of 'Ha' is a proper part of a verifier of 'Fa,' and the predicate G contains predicate H while F does not (and if there is no analogous predicate I which F contains and G does not), then the predicate G limns the mereological structure of the verifiers and falsifiers of 'Fa' more than F does. More loosely, if we consider a verifier or falsifier of 'Fa' with a proper part S, and G contains some predicate that S verifies or falsifies while S does not, then S limns more structure than S does.

For example, 'To be a mother is df to be a female parent' is true just in case the states that make someone a mother are identical to the states that make her a female parent, and the term 'female parent' limns the structure of these states more than 'mother' does—presumably by demonstrating that they are fusions of the state of someone being female with the state of her being a parent.

On this account, definition is irreflexive, asymmetric and transitive. The term 'F' never limns more structure than 'F' does, so sentences of the form 'To be F is $_{df}$ to be F' are universally false. Similarly, if G limns more structure than F does, then F does not limn more structure than G does. So, if 'To be F is $_{df}$ to be G' is true, then 'To be G is $_{df}$ to be F' is false. And if 'G' more structure than 'F' does and 'H' limns more structure than 'F' does, then 'F' does, then 'F' limns more structure than 'F' does. Because of this (and because generalized identities are transitive), if 'To be F is $_{df}$ to be G' is true and 'To be G is $_{df}$ to be G' is true, then 'To be G is $_{df}$ to be G' is true.

This approach does not restrict the cognitive import of definition to linguistic facts. When one learns that 'To be F is $_{df}$ to be G,' she learns more than facts about the predicates 'F' and 'G'; in particular, she learns about the mereological structures of the states that make something F and G. In learning 'To be a bachelor is $_{df}$ to be an unmarried male,' for example, she thereby learns that the states that make it the case that someone is a bachelor are composed of the states that make that person unmarried and the states that make that person male. Definitions thus convey information about the world. I believe that this account of real definition is worthy of serious consideration, but will say no more about it here.

There are advantages to this account of generalized identities that I failed to address. In particular, it offers insight into their modal status, the new riddle of induction, the paradox of analysis and the interpretation of physicalism. However, I do not rely upon arguments I lack the space to provide, but hope that the previous discussion motivates a conception of analysis in terms of truth-maker semantics, and highlights some of the immediate logical implications of that position.

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