Conditional Integral of Phenomenological Velocity

"In the beginning, He said let there be light, and there was light."
Praise Jehovah.

Higher - dimensional calculus and integral transformation play crucial roles in advancing our understanding of complex systems in mathematics and theoretical physics . Integral transformations are instrumental in simplifying complex differential equations, enabling the resolution of multi - dimensional problems that arise in various scientific fields . This paper aims to delve into a specific higher - dimensional integral transformation defined by the axioms \(F[q, s, l, \alpha]\) and \(G[q, s, l, \beta, c]\) . We start by outlining the axioms which define the functions \(F\) and \(G\) . Specifically, Axiom 1 defines \(F\) as a function of four variables : \(q\), \(s\), \(l\), and \(\alpha\), whereas Axiom 2 defines \(G\) as a function that additionally includes variables \(\beta\) and \(c\) . Axiom 3 relates \(h\) and \(l\) via a sine function . The core of our investigation is the integral transformation expressed as a five - dimensional integral involving \(G\) and proving its equivalence to \(F\), provided a specific condition on \(c\) holds . We approach this problem by first deriving the expression for \(c\) through detailed differentiation of \(F\) and equating it to \(G\) . The derivation involves advanced calculus techniques and symbolic mathematics to solve the resulting equations . We then verify the derived expression for \(c\) by substituting it back into the relationship between \(F\) and \(G\), ensuring that the equality holds under integral transformation . Finally, to corroborate our findings, we employ visualizations through multidimensional contour plots to illustrate the relationship between the derived expressions . This provides an intuitive confirmation of the mathematical consistency and validity of the transformation . This paper contributes to the field by providing a nuanced and detailed examination of higher - dimensional integral transformations and their underlying mathematical structures . The results have potential implications for theoretical physics, particularly in areas involving complex systems and multi - dimensional analyses .

Axiom 1 : $F[q, s, l, \alpha] = \dfrac{\sqrt{-q^2 + 2 q s - s^2 + l^2 \alpha^2}}{\alpha}$

Axiom 2 : $G[q, s, l, \beta, c] =$

$$\frac{\sqrt{-c^2 (l \alpha)^2 + c^2 q^2 - 2 c^2 s q + c^2 s^2 + c^2 (l \alpha)^2 \mathrm{Sin}[\beta]^2}}{\sqrt{-1. (l \alpha)^2 + q^2 - 2. s q + s^2 + (l \alpha)^2 \mathrm{Sin}[\beta]^2}}$$

Axiom 3 : $h / l = \mathrm{Sin}[\beta]$

Theorem 1 : The integral of ,

$$\int\int\int\int\int G[q, s, l, \beta, c]\, dq\, ds\, dl\, d\beta = F[q, s, l, \alpha]$$

$$\text{if } c = \Bigg(l \sqrt{\Bigg(-4. - \frac{225. l^8 \alpha^8}{(1. q^2 - 2. q s + 1. s^2 - 1. l^2 \alpha^2)^4} -}$$

$$\frac{450. l^6 \alpha^6}{(1. q^2 - 2. q s + 1. s^2 - 1. l^2 \alpha^2)^3} - \frac{285. l^4 \alpha^4}{(1. q^2 - 2. q s + 1. s^2 - 1. l^2 \alpha^2)^2} -$$

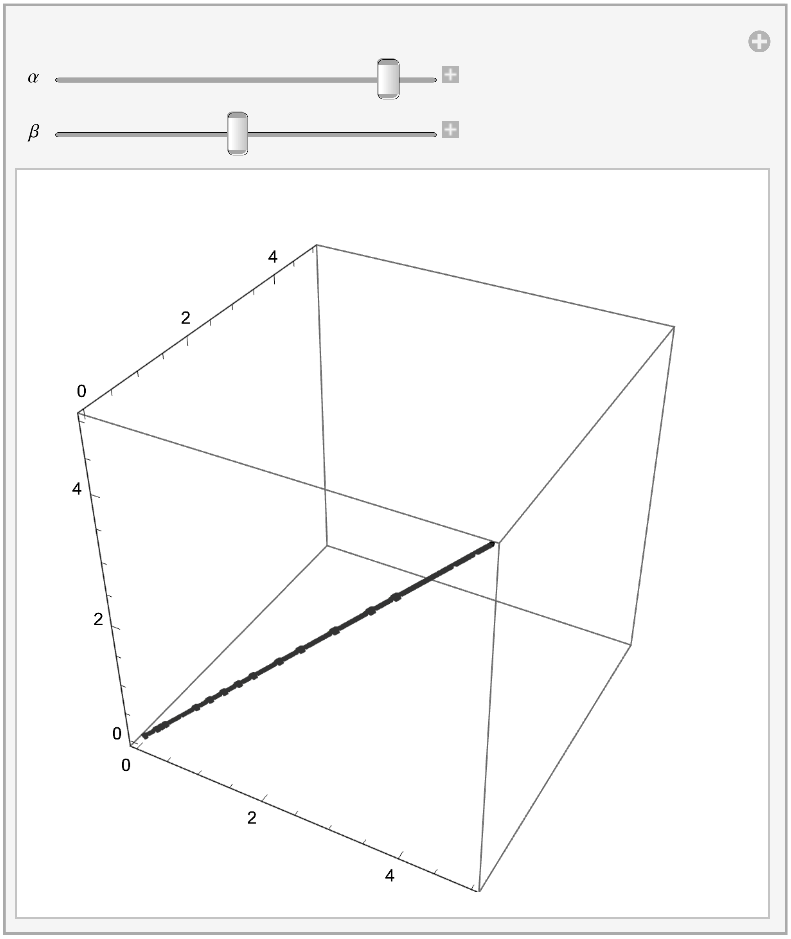
$$\frac{60.\,l^2\,\alpha^2}{1.\,q^2 - 2.\,q\,s + 1.\,s^2 - 1.\,l^2\,\alpha^2} - \frac{225.\,l^{10}\,\alpha^{10}\,\mathrm{Sin}[\beta]^2}{\left(1.\,q^2 - 2.\,q\,s + 1.\,s^2 - 1.\,l^2\,\alpha^2\right)^5} - \frac{450.\,l^8\,\alpha^8\,\mathrm{Sin}[\beta]^2}{\left(1.\,q^2 - 2.\,q\,s + 1.\,s^2 - 1.\,l^2\,\alpha^2\right)^4} - \frac{285.\,l^6\,\alpha^6\,\mathrm{Sin}[\beta]^2}{\left(1.\,q^2 - 2.\,q\,s + 1.\,s^2 - 1.\,l^2\,\alpha^2\right)^3} - \frac{60.\,l^4\,\alpha^4\,\mathrm{Sin}[\beta]^2}{\left(1.\,q^2 - 2.\,q\,s + 1.\,s^2 - 1.\,l^2\,\alpha^2\right)^2} - \frac{4.\,l^2\,\alpha^2\,\mathrm{Sin}[\beta]^2}{1.\,q^2 - 2.\,q\,s + 1.\,s^2 - 1.\,l^2\,\alpha^2}\Bigg)\Bigg) \Big/ \left(\left(1.\,q^2 - 2.\,q\,s + 1.\,s^2 - 1.\,l^2\,\alpha^2\right)\sqrt{q^2 - 2.\,q\,s + s^2 - 1.\,l^2\,\alpha^2 + l^2\,\alpha^2\,\mathrm{Sin}[\beta]^2}\right)$$

In[∘]:= **Manipulate[ContourPlot3D[**

$$\left(l\sqrt{\left(-4.\grave{} - \frac{225.\grave{}\,l^8\,\alpha^8}{\left(1.\grave{}\,q^2 - 2.\grave{}\,q\,s + 1.\grave{}\,s^2 - 1.\grave{}\,l^2\,\alpha^2\right)^4} - \frac{450.\grave{}\,l^6\,\alpha^6}{\left(1.\grave{}\,q^2 - 2.\grave{}\,q\,s + 1.\grave{}\,s^2 - 1.\grave{}\,l^2\,\alpha^2\right)^3} - \frac{285.\grave{}\,l^4\,\alpha^4}{\left(1.\grave{}\,q^2 - 2.\grave{}\,q\,s + 1.\grave{}\,s^2 - 1.\grave{}\,l^2\,\alpha^2\right)^2} - \frac{60.\grave{}\,l^2\,\alpha^2}{1.\grave{}\,q^2 - 2.\grave{}\,q\,s + 1.\grave{}\,s^2 - 1.\grave{}\,l^2\,\alpha^2} - \frac{225.\grave{}\,l^{10}\,\alpha^{10}\,\mathrm{Sin}[\beta]^2}{\left(1.\grave{}\,q^2 - 2.\grave{}\,q\,s + 1.\grave{}\,s^2 - 1.\grave{}\,l^2\,\alpha^2\right)^5} - \frac{450.\grave{}\,l^8\,\alpha^8\,\mathrm{Sin}[\beta]^2}{\left(1.\grave{}\,q^2 - 2.\grave{}\,q\,s + 1.\grave{}\,s^2 - 1.\grave{}\,l^2\,\alpha^2\right)^4} - \frac{285.\grave{}\,l^6\,\alpha^6\,\mathrm{Sin}[\beta]^2}{\left(1.\grave{}\,q^2 - 2.\grave{}\,q\,s + 1.\grave{}\,s^2 - 1.\grave{}\,l^2\,\alpha^2\right)^3} - \frac{60.\grave{}\,l^4\,\alpha^4\,\mathrm{Sin}[\beta]^2}{\left(1.\grave{}\,q^2 - 2.\grave{}\,q\,s + 1.\grave{}\,s^2 - 1.\grave{}\,l^2\,\alpha^2\right)^2} - \frac{4.\grave{}\,l^2\,\alpha^2\,\mathrm{Sin}[\beta]^2}{1.\grave{}\,q^2 - 2.\grave{}\,q\,s + 1.\grave{}\,s^2 - 1.\grave{}\,l^2\,\alpha^2}\right)}\right) \Big/ \left(\left(1.\grave{}\,q^2 - 2.\grave{}\,q\,s + 1.\grave{}\,s^2 - 1.\grave{}\,l^2\,\alpha^2\right)\sqrt{q^2 - 2.\grave{}\,q\,s + s^2 - 1.\grave{}\,l^2\,\alpha^2 + l^2\,\alpha^2\,\mathrm{Sin}[\beta]^2}\right),$$

{s, 0, 5}, {l, 0, 5}, {q, 0, 5}], {α,
0,
2 π}, {β,
0,
π / 2}]

Out[ ]=



Proof :

Take the derivative of F[q, s, l, α],

$$D\left[D\left[D\left[D\left[\frac{\sqrt{-q^2+2\,q\,s-s^2+l^2\,\alpha^2}}{\alpha},\,q\right],\,s\right],\,l\right],\,\alpha\right]$$

$$-\frac{15\,l^3\,(2\,q-2\,s)\,(-2\,q+2\,s)\,\alpha^2}{4\left(-q^2+2\,q\,s-s^2+l^2\,\alpha^2\right)^{7/2}}+\frac{3\,l\,(2\,q-2\,s)\,(-2\,q+2\,s)}{4\left(-q^2+2\,q\,s-s^2+l^2\,\alpha^2\right)^{5/2}}+\frac{3\,l^3\,\alpha^2}{\left(-q^2+2\,q\,s-s^2+l^2\,\alpha^2\right)^{5/2}}-\frac{l}{\left(-q^2+2\,q\,s-s^2+l^2\,\alpha^2\right)^{3/2}}$$

Equate it with G[q, s, l, β, c] :

$$D\left[D\left[D\left[D\left[\frac{\sqrt{-q^2+2\,q\,s-s^2+l^2\,\alpha^2}}{\alpha},\,q\right],\,s\right],\,l\right],\,\alpha\right]=\frac{\sqrt{-c^2\,(l\,\alpha)^2+c^2\,q^2-2\,c^2\,s\,q+c^2\,s^2+c^2\,(l\,\alpha)^2\,\mathrm{Sin}[\beta]^2}}{\sqrt{-1.\,(l\,\alpha)^2+q^2-2.\,s\,q+s^2+(l\,\alpha)^2\,\mathrm{Sin}[\beta]^2}}$$

Solve the equality for c :

In[∘]:=

$$\text{Solve}\left[-\frac{15\, l^3\,(2q-2s)\,(-2q+2s)\,\alpha^2}{4\left(-q^2+2qs-s^2+l^2\alpha^2\right)^{7/2}}+\frac{3\,l\,(2q-2s)\,(-2q+2s)}{4\left(-q^2+2qs-s^2+l^2\alpha^2\right)^{5/2}}+\frac{3\,l^3\,\alpha^2}{\left(-q^2+2qs-s^2+l^2\alpha^2\right)^{5/2}}-\frac{l}{\left(-q^2+2qs-s^2+l^2\alpha^2\right)^{3/2}}==\frac{\sqrt{-c^2\,(l\,\alpha)^2+c^2q^2-2c^2sq+c^2s^2+c^2\,(l\,\alpha)^2\,\text{Sin}[\beta]^2}}{\sqrt{-1.\,(l\,\alpha)^2+q^2-2.\,sq+s^2+(l\,\alpha)^2\,\text{Sin}[\beta]^2}},\ c\right]$$

Out[∘]=

$$\left\{\left\{c\to-\left(\left(1.\,l\sqrt{\left(-4.-\frac{225.\,l^8\alpha^8}{\left(1.\,q^2-2.\,qs+1.\,s^2-1.\,l^2\alpha^2\right)^4}-\frac{450.\,l^6\alpha^6}{\left(1.\,q^2-2.\,qs+1.\,s^2-1.\,l^2\alpha^2\right)^3}-\frac{285.\,l^4\alpha^4}{\left(1.\,q^2-2.\,qs+1.\,s^2-1.\,l^2\alpha^2\right)^2}-\frac{60.\,l^2\alpha^2}{1.\,q^2-2.\,qs+1.\,s^2-1.\,l^2\alpha^2}-\frac{225.\,l^{10}\alpha^{10}\,\text{Sin}[\beta]^2}{\left(1.\,q^2-2.\,qs+1.\,s^2-1.\,l^2\alpha^2\right)^5}-\frac{450.\,l^8\alpha^8\,\text{Sin}[\beta]^2}{\left(1.\,q^2-2.\,qs+1.\,s^2-1.\,l^2\alpha^2\right)^4}-\frac{285.\,l^6\alpha^6\,\text{Sin}[\beta]^2}{\left(1.\,q^2-2.\,qs+1.\,s^2-1.\,l^2\alpha^2\right)^3}-\frac{60.\,l^4\alpha^4\,\text{Sin}[\beta]^2}{\left(1.\,q^2-2.\,qs+1.\,s^2-1.\,l^2\alpha^2\right)^2}-\frac{4.\,l^2\alpha^2\,\text{Sin}[\beta]^2}{1.\,q^2-2.\,qs+1.\,s^2-1.\,l^2\alpha^2}\right)}\right)\Big/\left(\left(1.\,q^2-2.\,qs+1.\,s^2-1.\,l^2\alpha^2\right)\sqrt{q^2-2.\,qs+s^2-1.\,l^2\alpha^2+l^2\alpha^2\,\text{Sin}[\beta]^2}\right)\right)\right\},$$

$$\left\{c\to\left(l\sqrt{\left(-4.-\frac{225.\,l^8\alpha^8}{\left(1.\,q^2-2.\,qs+1.\,s^2-1.\,l^2\alpha^2\right)^4}-\frac{450.\,l^6\alpha^6}{\left(1.\,q^2-2.\,qs+1.\,s^2-1.\,l^2\alpha^2\right)^3}-\frac{285.\,l^4\alpha^4}{\left(1.\,q^2-2.\,qs+1.\,s^2-1.\,l^2\alpha^2\right)^2}-\frac{60.\,l^2\alpha^2}{1.\,q^2-2.\,qs+1.\,s^2-1.\,l^2\alpha^2}-\frac{225.\,l^{10}\alpha^{10}\,\text{Sin}[\beta]^2}{\left(1.\,q^2-2.\,qs+1.\,s^2-1.\,l^2\alpha^2\right)^5}-\right.}\right.$$

$$\frac{450.\, l^8 \alpha^8 \mathrm{Sin}[\beta]^2}{\left(1.\, q^2 - 2.\, q\, s + 1.\, s^2 - 1.\, l^2 \alpha^2\right)^4} - \frac{285.\, l^6 \alpha^6 \mathrm{Sin}[\beta]^2}{\left(1.\, q^2 - 2.\, q\, s + 1.\, s^2 - 1.\, l^2 \alpha^2\right)^3} - \frac{60.\, l^4 \alpha^4 \mathrm{Sin}[\beta]^2}{\left(1.\, q^2 - 2.\, q\, s + 1.\, s^2 - 1.\, l^2 \alpha^2\right)^2} - \frac{4.\, l^2 \alpha^2 \mathrm{Sin}[\beta]^2}{1.\, q^2 - 2.\, q\, s + 1.\, s^2 - 1.\, l^2 \alpha^2}\Bigg)\Bigg) \Big/ \left(\left(1.\, q^2 - 2.\, q\, s + 1.\, s^2 - 1.\, l^2 \alpha^2\right) \sqrt{q^2 - 2.\, q\, s + s^2 - 1.\, l^2 \alpha^2 + l^2 \alpha^2 \mathrm{Sin}[\beta]^2}\right)\}\}$$

Plug c back into the original equality to check the solution :

In[∘]:=

$$c := \Bigg(l \sqrt{\Bigg(-4. - \frac{225.\, l^8 \alpha^8}{\left(1.\, q^2 - 2.\, q\, s + 1.\, s^2 - 1.\, l^2 \alpha^2\right)^4} - \frac{450.\, l^6 \alpha^6}{\left(1.\, q^2 - 2.\, q\, s + 1.\, s^2 - 1.\, l^2 \alpha^2\right)^3} - \frac{285.\, l^4 \alpha^4}{\left(1.\, q^2 - 2.\, q\, s + 1.\, s^2 - 1.\, l^2 \alpha^2\right)^2} - \frac{60.\, l^2 \alpha^2}{1.\, q^2 - 2.\, q\, s + 1.\, s^2 - 1.\, l^2 \alpha^2} - \frac{225.\, l^{10} \alpha^{10} \mathrm{Sin}[\beta]^2}{\left(1.\, q^2 - 2.\, q\, s + 1.\, s^2 - 1.\, l^2 \alpha^2\right)^5} - \frac{450.\, l^8 \alpha^8 \mathrm{Sin}[\beta]^2}{\left(1.\, q^2 - 2.\, q\, s + 1.\, s^2 - 1.\, l^2 \alpha^2\right)^4} - \frac{285.\, l^6 \alpha^6 \mathrm{Sin}[\beta]^2}{\left(1.\, q^2 - 2.\, q\, s + 1.\, s^2 - 1.\, l^2 \alpha^2\right)^3} - \frac{60.\, l^4 \alpha^4 \mathrm{Sin}[\beta]^2}{\left(1.\, q^2 - 2.\, q\, s + 1.\, s^2 - 1.\, l^2 \alpha^2\right)^2} - \frac{4.\, l^2 \alpha^2 \mathrm{Sin}[\beta]^2}{1.\, q^2 - 2.\, q\, s + 1.\, s^2 - 1.\, l^2 \alpha^2}\Bigg)\Bigg) \Big/ \left(\left(1.\, q^2 - 2.\, q\, s + 1.\, s^2 - 1.\, l^2 \alpha^2\right) \sqrt{q^2 - 2.\, q\, s + s^2 - 1.\, l^2 \alpha^2 + l^2 \alpha^2 \mathrm{Sin}[\beta]^2}\right)$$

In[∘]:=

$$\mathrm{Solve}\Bigg[-\frac{15\, l^3 (2\, q - 2\, s)(-2\, q + 2\, s)\, \alpha^2}{4\left(-q^2 + 2\, q\, s - s^2 + l^2 \alpha^2\right)^{7/2}} + \frac{3\, l\, (2\, q - 2\, s)(-2\, q + 2\, s)}{4\left(-q^2 + 2\, q\, s - s^2 + l^2 \alpha^2\right)^{5/2}} + \frac{3\, l^3 \alpha^2}{\left(-q^2 + 2\, q\, s - s^2 + l^2 \alpha^2\right)^{5/2}} - \frac{l}{\left(-q^2 + 2\, q\, s - s^2 + l^2 \alpha^2\right)^{3/2}} == \frac{\sqrt{-c^2 (l\, \alpha)^2 + c^2 q^2 - 2\, c^2 s\, q + c^2 s^2 + c^2 (l\, \alpha)^2 (h / l)^2}}{\sqrt{-1.\, (l\, \alpha)^2 + q^2 - 2.\, s\, q + s^2 + (l\, \alpha)^2 (h / l)^2}}, \alpha\Bigg]$$

Out[∘]= {{}}

In[∘]:=

$$\mathrm{Solve}\Bigg[-\frac{15\, l^3 (2\, q - 2\, s)(-2\, q + 2\, s)\, \alpha^2}{4\left(-q^2 + 2\, q\, s - s^2 + l^2 \alpha^2\right)^{7/2}} + \frac{3\, l\, (2\, q - 2\, s)(-2\, q + 2\, s)}{4\left(-q^2 + 2\, q\, s - s^2 + l^2 \alpha^2\right)^{5/2}} +$$

$$\frac{3\,l^3\,\alpha^2}{\left(-q^2+2\,q\,s-s^2+l^2\,\alpha^2\right)^{5/2}}-\frac{l}{\left(-q^2+2\,q\,s-s^2+l^2\,\alpha^2\right)^{3/2}}==$$
$$\frac{\sqrt{-c^2\,(l\,\alpha)^2+c^2\,q^2-2\,c^2\,s\,q+c^2\,s^2+c^2\,(l\,\alpha)^2\,(h/l)^2}}{\sqrt{-1.\,(l\,\alpha)^2+q^2-2.\,s\,q+s^2+(l\,\alpha)^2\,(h/l)^2}},\,l\Big]$$

Out[∘]= {{}}

In[∘]:= $$\text{Solve}\Big[-\frac{15\,l^3\,(2\,q-2\,s)\,(-2\,q+2\,s)\,\alpha^2}{4\left(-q^2+2\,q\,s-s^2+l^2\,\alpha^2\right)^{7/2}}+\frac{3\,l\,(2\,q-2\,s)\,(-2\,q+2\,s)}{4\left(-q^2+2\,q\,s-s^2+l^2\,\alpha^2\right)^{5/2}}+$$
$$\frac{3\,l^3\,\alpha^2}{\left(-q^2+2\,q\,s-s^2+l^2\,\alpha^2\right)^{5/2}}-\frac{l}{\left(-q^2+2\,q\,s-s^2+l^2\,\alpha^2\right)^{3/2}}==$$
$$\frac{\sqrt{-c^2\,(l\,\alpha)^2+c^2\,q^2-2\,c^2\,s\,q+c^2\,s^2+c^2\,(l\,\alpha)^2\,\text{Sin}[\beta]^2}}{\sqrt{-1.\,(l\,\alpha)^2+q^2-2.\,s\,q+s^2+(l\,\alpha)^2\,\text{Sin}[\beta]^2}},\,s\Big]$$

Out[∘]= {{}}

In[∘]:= $$\text{Solve}\Big[-\frac{15\,l^3\,(2\,q-2\,s)\,(-2\,q+2\,s)\,\alpha^2}{4\left(-q^2+2\,q\,s-s^2+l^2\,\alpha^2\right)^{7/2}}+\frac{3\,l\,(2\,q-2\,s)\,(-2\,q+2\,s)}{4\left(-q^2+2\,q\,s-s^2+l^2\,\alpha^2\right)^{5/2}}+$$
$$\frac{3\,l^3\,\alpha^2}{\left(-q^2+2\,q\,s-s^2+l^2\,\alpha^2\right)^{5/2}}-\frac{l}{\left(-q^2+2\,q\,s-s^2+l^2\,\alpha^2\right)^{3/2}}==$$
$$\frac{\sqrt{-c^2\,(l\,\alpha)^2+c^2\,q^2-2\,c^2\,s\,q+c^2\,s^2+c^2\,(l\,\alpha)^2\,\text{Sin}[\beta]^2}}{\sqrt{-1.\,(l\,\alpha)^2+q^2-2.\,s\,q+s^2+(l\,\alpha)^2\,\text{Sin}[\beta]^2}},\,q\Big]$$

Out[∘]= {{}}

Since everything cancels out, the conditional expression :

$$-\frac{15\,l^3\,(2\,q-2\,s)\,(-2\,q+2\,s)\,\alpha^2}{4\left(-q^2+2\,q\,s-s^2+l^2\,\alpha^2\right)^{7/2}}+\frac{3\,l\,(2\,q-2\,s)\,(-2\,q+2\,s)}{4\left(-q^2+2\,q\,s-s^2+l^2\,\alpha^2\right)^{5/2}}+$$
$$\frac{3\,l^3\,\alpha^2}{\left(-q^2+2\,q\,s-s^2+l^2\,\alpha^2\right)^{5/2}}-\frac{l}{\left(-q^2+2\,q\,s-s^2+l^2\,\alpha^2\right)^{3/2}}==$$
$$\frac{\sqrt{-c^2\,(l\,\alpha)^2+c^2\,q^2-2\,c^2\,s\,q+c^2\,s^2+c^2\,(l\,\alpha)^2\,\text{Sin}[\beta]^2}}{\sqrt{-1.\,(l\,\alpha)^2+q^2-2.\,s\,q+s^2+(l\,\alpha)^2\,\text{Sin}[\beta]^2}}\ \text{if}$$

$$c=\Bigg(l\sqrt{\Bigg(-4.-\frac{225.\,l^8\,\alpha^8}{\left(1.\,q^2-2.\,q\,s+1.\,s^2-1.\,l^2\,\alpha^2\right)^4}-}$$

$$\frac{450.\,l^6\alpha^6}{\left(1.\,q^2-2.\,q\,s+1.\,s^2-1.\,l^2\alpha^2\right)^3}-\frac{285.\,l^4\alpha^4}{\left(1.\,q^2-2.\,q\,s+1.\,s^2-1.\,l^2\alpha^2\right)^2}-\frac{60.\,l^2\alpha^2}{1.\,q^2-2.\,q\,s+1.\,s^2-1.\,l^2\alpha^2}-\frac{225.\,l^{10}\alpha^{10}\,\mathrm{Sin}[\beta]^2}{\left(1.\,q^2-2.\,q\,s+1.\,s^2-1.\,l^2\alpha^2\right)^5}-\frac{450.\,l^8\alpha^8\,\mathrm{Sin}[\beta]^2}{\left(1.\,q^2-2.\,q\,s+1.\,s^2-1.\,l^2\alpha^2\right)^4}-\frac{285.\,l^6\alpha^6\,\mathrm{Sin}[\beta]^2}{\left(1.\,q^2-2.\,q\,s+1.\,s^2-1.\,l^2\alpha^2\right)^3}-\frac{60.\,l^4\alpha^4\,\mathrm{Sin}[\beta]^2}{\left(1.\,q^2-2.\,q\,s+1.\,s^2-1.\,l^2\alpha^2\right)^2}-\frac{4.\,l^2\alpha^2\,\mathrm{Sin}[\beta]^2}{1.\,q^2-2.\,q\,s+1.\,s^2-1.\,l^2\alpha^2}\Bigg)\Bigg)\Bigg/$$

$$\left(\left(1.\,q^2-2.\,q\,s+1.\,s^2-1.\,l^2\alpha^2\right)\sqrt{q^2-2.\,q\,s+s^2-1.\,l^2\alpha^2+l^2\alpha^2\,\mathrm{Sin}[\beta]^2}\right)$$

must be true.

Since :

$$\iiiint\left(-\frac{15\,l^3\,(2\,q-2\,s)\,(-2\,q+2\,s)\,\alpha^2}{4\left(-q^2+2\,q\,s-s^2+l^2\alpha^2\right)^{7/2}}+\frac{3\,l\,(2\,q-2\,s)\,(-2\,q+2\,s)}{4\left(-q^2+2\,q\,s-s^2+l^2\alpha^2\right)^{5/2}}+\frac{3\,l^3\alpha^2}{\left(-q^2+2\,q\,s-s^2+l^2\alpha^2\right)^{5/2}}-\frac{l}{\left(-q^2+2\,q\,s-s^2+l^2\alpha^2\right)^{3/2}}\right)dq\,dl\,ds\,d\alpha=\frac{\sqrt{-q^2+2\,q\,s-s^2+l^2\alpha^2}}{\alpha},$$

Then,

$$\int\int\int\int \frac{\sqrt{-c^2\,(l\,\alpha)^2 + c^2\,q^2 - 2\,c^2\,s\,q + c^2\,s^2 + c^2\,(l\,\alpha)^2\,\mathrm{Sin}[\beta]^2}}{\sqrt{-1.`\,(l\,\alpha)^2 + q^2 - 2.`\,s\,q + s^2 + (l\,\alpha)^2\,\mathrm{Sin}[\beta]^2}}$$

$dq\,dl\,ds\,d\alpha\,d\beta$ must equal $\frac{\sqrt{-q^2 + 2\,q\,s - s^2 + l^2\,\alpha^2}}{\alpha}$ as well if,

$$c = \left(l\sqrt{\left(-4.` - \frac{225.`\,l^8\,\alpha^8}{\left(1.`\,q^2 - 2.`\,q\,s + 1.`\,s^2 - 1.`\,l^2\,\alpha^2\right)^4} - \frac{450.`\,l^6\,\alpha^6}{\left(1.`\,q^2 - 2.`\,q\,s + 1.`\,s^2 - 1.`\,l^2\,\alpha^2\right)^3} - \frac{285.`\,l^4\,\alpha^4}{\left(1.`\,q^2 - 2.`\,q\,s + 1.`\,s^2 - 1.`\,l^2\,\alpha^2\right)^2} - \frac{60.`\,l^2\,\alpha^2}{1.`\,q^2 - 2.`\,q\,s + 1.`\,s^2 - 1.`\,l^2\,\alpha^2} - \frac{225.`\,l^{10}\,\alpha^{10}\,\mathrm{Sin}[\beta]^2}{\left(1.`\,q^2 - 2.`\,q\,s + 1.`\,s^2 - 1.`\,l^2\,\alpha^2\right)^5} - \frac{450.`\,l^8\,\alpha^8\,\mathrm{Sin}[\beta]^2}{\left(1.`\,q^2 - 2.`\,q\,s + 1.`\,s^2 - 1.`\,l^2\,\alpha^2\right)^4} - \frac{285.`\,l^6\,\alpha^6\,\mathrm{Sin}[\beta]^2}{\left(1.`\,q^2 - 2.`\,q\,s + 1.`\,s^2 - 1.`\,l^2\,\alpha^2\right)^3} - \frac{60.`\,l^4\,\alpha^4\,\mathrm{Sin}[\beta]^2}{\left(1.`\,q^2 - 2.`\,q\,s + 1.`\,s^2 - 1.`\,l^2\,\alpha^2\right)^2} - \frac{4.`\,l^2\,\alpha^2\,\mathrm{Sin}[\beta]^2}{1.`\,q^2 - 2.`\,q\,s + 1.`\,s^2 - 1.`\,l^2\,\alpha^2}\right)}\right) \Big/ \left(\left(1.`\,q^2 - 2.`\,q\,s + 1.`\,s^2 - 1.`\,l^2\,\alpha^2\right)\sqrt{q^2 - 2.`\,q\,s + s^2 - 1.`\,l^2\,\alpha^2 + l^2\,\alpha^2\,\mathrm{Sin}[\beta]^2}\right)$$

Thus, it also stands to reason that :

$$\int\int\int\int \frac{\sqrt{-c^2\,(l\,\alpha)^2 + c^2\,q^2 - 2\,c^2\,s\,q + c^2\,s^2 + c^2\,(l\,\alpha)^2\,(h/l)^2}}{\sqrt{-1.\,(l\,\alpha)^2 + q^2 - 2.\,s\,q + s^2 + (l\,\alpha)^2\,(h/l)^2}}\,dq\,dl\,ds\,d\alpha\,dh ==$$

$$h \text{ if } c = \left(l\sqrt{\left(-4. - \frac{225.\,l^8\,\alpha^8}{\left(1.\,q^2 - 2.\,q\,s + 1.\,s^2 - 1.\,l^2\,\alpha^2\right)^4} - \frac{450.\,l^6\,\alpha^6}{\left(1.\,q^2 - 2.\,q\,s + 1.\,s^2 - 1.\,l^2\,\alpha^2\right)^3} - \frac{285.\,l^4\,\alpha^4}{\left(1.\,q^2 - 2.\,q\,s + 1.\,s^2 - 1.\,l^2\,\alpha^2\right)^2} - \frac{60.\,l^2\,\alpha^2}{1.\,q^2 - 2.\,q\,s + 1.\,s^2 - 1.\,l^2\,\alpha^2} - \frac{225.\,l^{10}\,\alpha^{10}\,\mathrm{Sin}[\beta]^2}{\left(1.\,q^2 - 2.\,q\,s + 1.\,s^2 - 1.\,l^2\,\alpha^2\right)^5} - \frac{450.\,l^8\,\alpha^8\,\mathrm{Sin}[\beta]^2}{\left(1.\,q^2 - 2.\,q\,s + 1.\,s^2 - 1.\,l^2\,\alpha^2\right)^4} - \frac{285.\,l^6\,\alpha^6\,\mathrm{Sin}[\beta]^2}{\left(1.\,q^2 - 2.\,q\,s + 1.\,s^2 - 1.\,l^2\,\alpha^2\right)^3} - \frac{60.\,l^4\,\alpha^4\,\mathrm{Sin}[\beta]^2}{\left(1.\,q^2 - 2.\,q\,s + 1.\,s^2 - 1.\,l^2\,\alpha^2\right)^2} - \frac{4.\,l^2\,\alpha^2\,\mathrm{Sin}[\beta]^2}{1.\,q^2 - 2.\,q\,s + 1.\,s^2 - 1.\,l^2\,\alpha^2}\right)}\right) \Big/ \left(\left(1.\,q^2 - 2.\,q\,s + 1.\,s^2 - 1.\,l^2\,\alpha^2\right)\sqrt{q^2 - 2.\,q\,s + s^2 - 1.\,l^2\,\alpha^2 + l^2\,\alpha^2\,\mathrm{Sin}[\beta]^2}\right)$$

Theorem 2 :

Furthermore :

From, $v = \frac{\sqrt{-c^2 (l\,\alpha)^2 + c^2 q^2 - 2 c^2 s\,q + c^2 s^2 + c^2 (l\,\alpha)^2 \text{Sin}[\beta]^2}}{\sqrt{-1.`\,(l\,\alpha)^2 + q^2 - 2.`\,s\,q + s^2 + (l\,\alpha)^2 \text{Sin}[\beta]^2}}$,

$c = \frac{1.`\ v\sqrt{1.`\,q^2 - 2.`\,q\,s + 1.`\,s^2 - 1.`\,l^2\alpha^2 + 1.`\,l^2\alpha^2 \text{Sin}[\beta]^2}}{\sqrt{q^2 - 2.`\,q\,s + s^2 - 1.`\,l^2\alpha^2 + l^2\alpha^2 \text{Sin}[\beta]^2}}$, and

In[∘]:= c :=

$$\left(l\sqrt{\left(-4.` - \frac{225.`\,l^8\alpha^8}{(1.`\,q^2 - 2.`\,q\,s + 1.`\,s^2 - 1.`\,l^2\alpha^2)^4} - \frac{450.`\,l^6\alpha^6}{(1.`\,q^2 - 2.`\,q\,s + 1.`\,s^2 - 1.`\,l^2\alpha^2)^3} - \frac{285.`\,l^4\alpha^4}{(1.`\,q^2 - 2.`\,q\,s + 1.`\,s^2 - 1.`\,l^2\alpha^2)^2} - \frac{60.`\,l^2\alpha^2}{1.`\,q^2 - 2.`\,q\,s + 1.`\,s^2 - 1.`\,l^2\alpha^2} - \frac{225.`\,l^{10}\alpha^{10}\text{Sin}[\beta]^2}{(1.`\,q^2 - 2.`\,q\,s + 1.`\,s^2 - 1.`\,l^2\alpha^2)^5} - \frac{450.`\,l^8\alpha^8\text{Sin}[\beta]^2}{(1.`\,q^2 - 2.`\,q\,s + 1.`\,s^2 - 1.`\,l^2\alpha^2)^4} - \frac{285.`\,l^6\alpha^6\text{Sin}[\beta]^2}{(1.`\,q^2 - 2.`\,q\,s + 1.`\,s^2 - 1.`\,l^2\alpha^2)^3} - \frac{60.`\,l^4\alpha^4\text{Sin}[\beta]^2}{(1.`\,q^2 - 2.`\,q\,s + 1.`\,s^2 - 1.`\,l^2\alpha^2)^2} - \frac{4.`\,l^2\alpha^2\text{Sin}[\beta]^2}{1.`\,q^2 - 2.`\,q\,s + 1.`\,s^2 - 1.`\,l^2\alpha^2}\right)}\right) \Big/ \left((1.`\,q^2 - 2.`\,q\,s + 1.`\,s^2 - 1.`\,l^2\alpha^2)\sqrt{q^2 - 2.`\,q\,s + s^2 - 1.`\,l^2\alpha^2 + l^2\alpha^2\text{Sin}[\beta]^2}\right)$$

v does not have to equal c and,

In[∘]:= Simplify

$$\text{Simplify}\left[\left(1.`\,\frac{\sqrt{-c^2 (l\,\alpha)^2 + c^2 q^2 - 2 c^2 s\,q + c^2 s^2 + c^2 (l\,\alpha)^2 \text{Sin}[\beta]^2}}{\sqrt{-1.`\,(l\,\alpha)^2 + q^2 - 2.`\,s\,q + s^2 + (l\,\alpha)^2 \text{Sin}[\beta]^2}}\sqrt{1.`\,q^2 - 2.`\,q\,s + 1.`\,s^2 - 1.`\,l^2\alpha^2 + 1.`\,l^2\alpha^2\text{Sin}[\beta]^2}\right) \Big/ \left(\sqrt{q^2 - 2.`\,q\,s + s^2 - 1.`\,l^2\alpha^2 + l^2\alpha^2\text{Sin}[\beta]^2}\right)\right]$$

Out[∘]=

$$\frac{1.\ \sqrt{c^2\left(q^2 - 2\,q\,s + s^2 - l^2\alpha^2 + l^2\alpha^2\text{Sin}[\beta]^2\right)}\ \sqrt{1.\,q^2 - 2.\,q\,s + 1.\,s^2 - 1.\,l^2\alpha^2 + 1.\,l^2\alpha^2\text{Sin}[\beta]^2}}{q^2 - 2.\,q\,s + s^2 - 1.\,l^2\alpha^2 + l^2\alpha^2\text{Sin}[\beta]^2}$$

```
In[∘]:= Solve[(1.` Sqrt[-c^2 (l α)^2 + c^2 q^2 - 2 c^2 s q + c^2 s^2 + c^2 (l α)^2 Sin[β]^2]/
      Sqrt[-1.` (l α)^2 + q^2 - 2.` s q + s^2 + (l α)^2 Sin[β]^2]
      Sqrt[1.` q^2 - 2.` q s + 1.` s^2 - 1.` l^2 α^2 + 1.` l^2 α^2 Sin[β]^2])/
    (Sqrt[q^2 - 2.` q s + s^2 - 1.` l^2 α^2 + l^2 α^2 Sin[β]^2]) ==
   (l Sqrt[(-4.` - 225.` l^8 α^8/(1.` q^2 - 2.` q s + 1.` s^2 - 1.` l^2 α^2)^4 -
        450.` l^6 α^6/(1.` q^2 - 2.` q s + 1.` s^2 - 1.` l^2 α^2)^3 -
        285.` l^4 α^4/(1.` q^2 - 2.` q s + 1.` s^2 - 1.` l^2 α^2)^2 -
        60.` l^2 α^2/(1.` q^2 - 2.` q s + 1.` s^2 - 1.` l^2 α^2) -
        225.` l^10 α^10 Sin[β]^2/(1.` q^2 - 2.` q s + 1.` s^2 - 1.` l^2 α^2)^5 -
        450.` l^8 α^8 Sin[β]^2/(1.` q^2 - 2.` q s + 1.` s^2 - 1.` l^2 α^2)^4 -
        285.` l^6 α^6 Sin[β]^2/(1.` q^2 - 2.` q s + 1.` s^2 - 1.` l^2 α^2)^3 -
        60.` l^4 α^4 Sin[β]^2/(1.` q^2 - 2.` q s + 1.` s^2 - 1.` l^2 α^2)^2 -
        4.` l^2 α^2 Sin[β]^2/(1.` q^2 - 2.` q s + 1.` s^2 - 1.` l^2 α^2))])/
    ((1.` q^2 - 2.` q s + 1.` s^2 - 1.` l^2 α^2) Sqrt[q^2 - 2.` q s + s^2 - 1.` l^2 α^2 + l^2 α^2 Sin[β]^2]), l]

Out[∘]= {{}}
```

In[∘]:=

$$\text{Solve}\left[\left(1.\grave{}\ \frac{\sqrt{-c^2\,(l\,\alpha)^2+c^2\,q^2-2\,c^2\,s\,q+c^2\,s^2+c^2\,(l\,\alpha)^2\,\text{Sin}[\beta]^2}}{\sqrt{-1.\grave{}\ (l\,\alpha)^2+q^2-2.\grave{}\ s\,q+s^2+(l\,\alpha)^2\,\text{Sin}[\beta]^2}}\ \sqrt{1.\grave{}\ q^2-2.\grave{}\ q\,s+1.\grave{}\ s^2-1.\grave{}\ l^2\,\alpha^2+1.\grave{}\ l^2\,\alpha^2\,\text{Sin}[\beta]^2}\right)\Big/\left(\sqrt{q^2-2.\grave{}\ q\,s+s^2-1.\grave{}\ l^2\,\alpha^2+l^2\,\alpha^2\,\text{Sin}[\beta]^2}\right) ==\right.$$

$$\left(l\sqrt{\left(-4.\grave{}-\frac{225.\grave{}\ l^8\,\alpha^8}{\left(1.\grave{}\ q^2-2.\grave{}\ q\,s+1.\grave{}\ s^2-1.\grave{}\ l^2\,\alpha^2\right)^4}-\frac{450.\grave{}\ l^6\,\alpha^6}{\left(1.\grave{}\ q^2-2.\grave{}\ q\,s+1.\grave{}\ s^2-1.\grave{}\ l^2\,\alpha^2\right)^3}-\frac{285.\grave{}\ l^4\,\alpha^4}{\left(1.\grave{}\ q^2-2.\grave{}\ q\,s+1.\grave{}\ s^2-1.\grave{}\ l^2\,\alpha^2\right)^2}-\frac{60.\grave{}\ l^2\,\alpha^2}{1.\grave{}\ q^2-2.\grave{}\ q\,s+1.\grave{}\ s^2-1.\grave{}\ l^2\,\alpha^2}-\frac{225.\grave{}\ l^{10}\,\alpha^{10}\,\text{Sin}[\beta]^2}{\left(1.\grave{}\ q^2-2.\grave{}\ q\,s+1.\grave{}\ s^2-1.\grave{}\ l^2\,\alpha^2\right)^5}-\frac{450.\grave{}\ l^8\,\alpha^8\,\text{Sin}[\beta]^2}{\left(1.\grave{}\ q^2-2.\grave{}\ q\,s+1.\grave{}\ s^2-1.\grave{}\ l^2\,\alpha^2\right)^4}-\frac{285.\grave{}\ l^6\,\alpha^6\,\text{Sin}[\beta]^2}{\left(1.\grave{}\ q^2-2.\grave{}\ q\,s+1.\grave{}\ s^2-1.\grave{}\ l^2\,\alpha^2\right)^3}-\frac{60.\grave{}\ l^4\,\alpha^4\,\text{Sin}[\beta]^2}{\left(1.\grave{}\ q^2-2.\grave{}\ q\,s+1.\grave{}\ s^2-1.\grave{}\ l^2\,\alpha^2\right)^2}-\frac{4.\grave{}\ l^2\,\alpha^2\,\text{Sin}[\beta]^2}{1.\grave{}\ q^2-2.\grave{}\ q\,s+1.\grave{}\ s^2-1.\grave{}\ l^2\,\alpha^2}\right)}\right)\Big/$$

$$\left.\left(\left(1.\grave{}\ q^2-2.\grave{}\ q\,s+1.\grave{}\ s^2-1.\grave{}\ l^2\,\alpha^2\right)\sqrt{q^2-2.\grave{}\ q\,s+s^2-1.\grave{}\ l^2\,\alpha^2+l^2\,\alpha^2\,\text{Sin}[\beta]^2}\right),\ q\right]$$

Out[∘]= { { } }

In[∘]:=

$$\text{Solve}\Bigg[\left(1.\grave{}\ \frac{\sqrt{-c^2\,(l\,\alpha)^2+c^2\,q^2-2\,c^2\,s\,q+c^2\,s^2+c^2\,(l\,\alpha)^2\,\text{Sin}[\beta]^2}}{\sqrt{-1.\grave{}\ (l\,\alpha)^2+q^2-2.\grave{}\ s\,q+s^2+(l\,\alpha)^2\,\text{Sin}[\beta]^2}}\sqrt{1.\grave{}\ q^2-2.\grave{}\ q\,s+1.\grave{}\ s^2-1.\grave{}\ l^2\,\alpha^2+1.\grave{}\ l^2\,\alpha^2\,\text{Sin}[\beta]^2}\right)\Big/\left(\sqrt{q^2-2.\grave{}\ q\,s+s^2-1.\grave{}\ l^2\,\alpha^2+l^2\,\alpha^2\,\text{Sin}[\beta]^2}\right)==$$

$$\Bigg(l\sqrt{\Bigg(-4.\grave{}-\frac{225.\grave{}\ l^8\,\alpha^8}{\left(1.\grave{}\ q^2-2.\grave{}\ q\,s+1.\grave{}\ s^2-1.\grave{}\ l^2\,\alpha^2\right)^4}-\frac{450.\grave{}\ l^6\,\alpha^6}{\left(1.\grave{}\ q^2-2.\grave{}\ q\,s+1.\grave{}\ s^2-1.\grave{}\ l^2\,\alpha^2\right)^3}-\frac{285.\grave{}\ l^4\,\alpha^4}{\left(1.\grave{}\ q^2-2.\grave{}\ q\,s+1.\grave{}\ s^2-1.\grave{}\ l^2\,\alpha^2\right)^2}-\frac{60.\grave{}\ l^2\,\alpha^2}{1.\grave{}\ q^2-2.\grave{}\ q\,s+1.\grave{}\ s^2-1.\grave{}\ l^2\,\alpha^2}-\frac{225.\grave{}\ l^{10}\,\alpha^{10}\,\text{Sin}[\beta]^2}{\left(1.\grave{}\ q^2-2.\grave{}\ q\,s+1.\grave{}\ s^2-1.\grave{}\ l^2\,\alpha^2\right)^5}-\frac{450.\grave{}\ l^8\,\alpha^8\,\text{Sin}[\beta]^2}{\left(1.\grave{}\ q^2-2.\grave{}\ q\,s+1.\grave{}\ s^2-1.\grave{}\ l^2\,\alpha^2\right)^4}-\frac{285.\grave{}\ l^6\,\alpha^6\,\text{Sin}[\beta]^2}{\left(1.\grave{}\ q^2-2.\grave{}\ q\,s+1.\grave{}\ s^2-1.\grave{}\ l^2\,\alpha^2\right)^3}-\frac{60.\grave{}\ l^4\,\alpha^4\,\text{Sin}[\beta]^2}{\left(1.\grave{}\ q^2-2.\grave{}\ q\,s+1.\grave{}\ s^2-1.\grave{}\ l^2\,\alpha^2\right)^2}-\frac{4.\grave{}\ l^2\,\alpha^2\,\text{Sin}[\beta]^2}{1.\grave{}\ q^2-2.\grave{}\ q\,s+1.\grave{}\ s^2-1.\grave{}\ l^2\,\alpha^2}\Bigg)}\Bigg)\Big/$$

$$\left(\left(1.\grave{}\ q^2-2.\grave{}\ q\,s+1.\grave{}\ s^2-1.\grave{}\ l^2\,\alpha^2\right)\sqrt{q^2-2.\grave{}\ q\,s+s^2-1.\grave{}\ l^2\,\alpha^2+l^2\,\alpha^2\,\text{Sin}[\beta]^2}\right),\ \alpha\Bigg]$$

Out[∘]= { { } }

In[•]:=
$$\text{Solve}\Big[\Big(1.`\ \frac{\sqrt{-c^2\ (l\,\alpha)^2 + c^2\,q^2 - 2\,c^2\,s\,q + c^2\,s^2 + c^2\ (l\,\alpha)^2\ (h/l)^2}}{\sqrt{-1.`\ (l\,\alpha)^2 + q^2 - 2.`\ s\,q + s^2 + (l\,\alpha)^2\ (h/l)^2}}\ \sqrt{1.`\ q^2 - 2.`\ q\,s + 1.`\ s^2 - 1.`\ l^2\,\alpha^2 + 1.`\ l^2\,\alpha^2\ (h/l)^2}\Big)\Big/ \Big(\sqrt{q^2 - 2.`\ q\,s + s^2 - 1.`\ l^2\,\alpha^2 + l^2\,\alpha^2\ (h/l)^2}\Big) == \Big(l\,\sqrt{\Big(-4.` - \frac{225.`\ l^8\,\alpha^8}{(1.`\ q^2 - 2.`\ q\,s + 1.`\ s^2 - 1.`\ l^2\,\alpha^2)^4} - \frac{450.`\ l^6\,\alpha^6}{(1.`\ q^2 - 2.`\ q\,s + 1.`\ s^2 - 1.`\ l^2\,\alpha^2)^3} - \frac{285.`\ l^4\,\alpha^4}{(1.`\ q^2 - 2.`\ q\,s + 1.`\ s^2 - 1.`\ l^2\,\alpha^2)^2} - \frac{60.`\ l^2\,\alpha^2}{1.`\ q^2 - 2.`\ q\,s + 1.`\ s^2 - 1.`\ l^2\,\alpha^2} - \frac{225.`\ l^{10}\,\alpha^{10}\ (h/l)^2}{(1.`\ q^2 - 2.`\ q\,s + 1.`\ s^2 - 1.`\ l^2\,\alpha^2)^5} - \frac{450.`\ l^8\,\alpha^8\ (h/l)^2}{(1.`\ q^2 - 2.`\ q\,s + 1.`\ s^2 - 1.`\ l^2\,\alpha^2)^4} - \frac{285.`\ l^6\,\alpha^6\ (h/l)^2}{(1.`\ q^2 - 2.`\ q\,s + 1.`\ s^2 - 1.`\ l^2\,\alpha^2)^3} - \frac{60.`\ l^4\,\alpha^4\ (h/l)^2}{(1.`\ q^2 - 2.`\ q\,s + 1.`\ s^2 - 1.`\ l^2\,\alpha^2)^2} - \frac{4.`\ l^2\,\alpha^2\ (h/l)^2}{1.`\ q^2 - 2.`\ q\,s + 1.`\ s^2 - 1.`\ l^2\,\alpha^2}\Big)}\Big)\Big/ \Big((1.`\ q^2 - 2.`\ q\,s + 1.`\ s^2 - 1.`\ l^2\,\alpha^2)\ \sqrt{q^2 - 2.`\ q\,s + s^2 - 1.`\ l^2\,\alpha^2 + l^2\,\alpha^2\ (h/l)^2}\Big),\ l\Big]$$

Out[•]= $\{\{\}\}$

In[∘]:=

$$\text{Solve}\Big[\Big(1.\grave{}\ \frac{\sqrt{-c^2\ (l\ \alpha)^2 + c^2\ q^2 - 2\ c^2\ s\ q + c^2\ s^2 + c^2\ (l\ \alpha)^2\ (h/l)^2}}{\sqrt{-1.\grave{}\ (l\ \alpha)^2 + q^2 - 2.\grave{}\ s\ q + s^2 + (l\ \alpha)^2\ (h/l)^2}} \sqrt{1.\grave{}\ q^2 - 2.\grave{}\ q\ s + 1.\grave{}\ s^2 - 1.\grave{}\ l^2\ \alpha^2 + 1.\grave{}\ l^2\ \alpha^2\ (h/l)^2}\Big) \Big/ \Big(\sqrt{q^2 - 2.\grave{}\ q\ s + s^2 - 1.\grave{}\ l^2\ \alpha^2 + l^2\ \alpha^2\ (h/l)^2}\Big) ==$$

$$\Big(l \sqrt{\Big(-4.\grave{} - \frac{225.\grave{}\ l^8\ \alpha^8}{(1.\grave{}\ q^2 - 2.\grave{}\ q\ s + 1.\grave{}\ s^2 - 1.\grave{}\ l^2\ \alpha^2)^4} - \frac{450.\grave{}\ l^6\ \alpha^6}{(1.\grave{}\ q^2 - 2.\grave{}\ q\ s + 1.\grave{}\ s^2 - 1.\grave{}\ l^2\ \alpha^2)^3} - \frac{285.\grave{}\ l^4\ \alpha^4}{(1.\grave{}\ q^2 - 2.\grave{}\ q\ s + 1.\grave{}\ s^2 - 1.\grave{}\ l^2\ \alpha^2)^2} - \frac{60.\grave{}\ l^2\ \alpha^2}{1.\grave{}\ q^2 - 2.\grave{}\ q\ s + 1.\grave{}\ s^2 - 1.\grave{}\ l^2\ \alpha^2} - \frac{225.\grave{}\ l^{10}\ \alpha^{10}\ (h/l)^2}{(1.\grave{}\ q^2 - 2.\grave{}\ q\ s + 1.\grave{}\ s^2 - 1.\grave{}\ l^2\ \alpha^2)^5} - \frac{450.\grave{}\ l^8\ \alpha^8\ (h/l)^2}{(1.\grave{}\ q^2 - 2.\grave{}\ q\ s + 1.\grave{}\ s^2 - 1.\grave{}\ l^2\ \alpha^2)^4} - \frac{285.\grave{}\ l^6\ \alpha^6\ (h/l)^2}{(1.\grave{}\ q^2 - 2.\grave{}\ q\ s + 1.\grave{}\ s^2 - 1.\grave{}\ l^2\ \alpha^2)^3} - \frac{60.\grave{}\ l^4\ \alpha^4\ (h/l)^2}{(1.\grave{}\ q^2 - 2.\grave{}\ q\ s + 1.\grave{}\ s^2 - 1.\grave{}\ l^2\ \alpha^2)^2} - \frac{4.\grave{}\ l^2\ \alpha^2\ (h/l)^2}{1.\grave{}\ q^2 - 2.\grave{}\ q\ s + 1.\grave{}\ s^2 - 1.\grave{}\ l^2\ \alpha^2}\Big)}\Big) \Big/ \Big((1.\grave{}\ q^2 - 2.\grave{}\ q\ s + 1.\grave{}\ s^2 - 1.\grave{}\ l^2\ \alpha^2) \sqrt{q^2 - 2.\grave{}\ q\ s + s^2 - 1.\grave{}\ l^2\ \alpha^2 + l^2\ \alpha^2\ (h/l)^2}\Big), h\Big]$$

Out[∘]= { { } }

Theorem 3 :

$$\left(-\frac{1}{l^2}1.\grave{}\ \left(-1.\grave{}\ q^6\,\text{Cos}[\beta]^2+6.\grave{}\ q^5\,s\,\text{Cos}[\beta]^2-15.\grave{}\ q^4\,s^2\,\text{Cos}[\beta]^2+20.\grave{}\ q^3\,s^3\,\text{Cos}[\beta]^2-15.\grave{}\ q^2\,s^4\,\text{Cos}[\beta]^2+6.\grave{}\ q\,s^5\,\text{Cos}[\beta]^2-1.\grave{}\ s^6\,\text{Cos}[\beta]^2+3.\grave{}\ l^2\,q^4\,\alpha^2\,\text{Cos}[\beta]^2-12.\grave{}\ l^2\,q^3\,s\,\alpha^2\,\text{Cos}[\beta]^2+18.\grave{}\ l^2\,q^2\,s^2\,\alpha^2\,\text{Cos}[\beta]^2-12.\grave{}\ l^2\,q\,s^3\,\alpha^2\,\text{Cos}[\beta]^2+3.\grave{}\ l^2\,s^4\,\alpha^2\,\text{Cos}[\beta]^2-3.\grave{}\ l^4\,q^2\,\alpha^4\,\text{Cos}[\beta]^2+6.\grave{}\ l^4\,q\,s\,\alpha^4\,\text{Cos}[\beta]^2-3.\grave{}\ l^4\,s^2\,\alpha^4\,\text{Cos}[\beta]^2+1.\grave{}\ l^6\,\alpha^6\,\text{Cos}[\beta]^2-1.\grave{}\ l^2\,q^4\,\alpha^2\,\text{Cos}[\beta]^2\,\text{Sin}[\beta]^2+4.\grave{}\ l^2\,q^3\,s\,\alpha^2\,\text{Cos}[\beta]^2\,\text{Sin}[\beta]^2-6.\grave{}\ l^2\,q^2\,s^2\,\alpha^2\,\text{Cos}[\beta]^2\,\text{Sin}[\beta]^2+4.\grave{}\ l^2\,q\,s^3\,\alpha^2\,\text{Cos}[\beta]^2\,\text{Sin}[\beta]^2-1.\grave{}\ l^2\,s^4\,\alpha^2\,\text{Cos}[\beta]^2\,\text{Sin}[\beta]^2+2.\grave{}\ l^4\,q^2\,\alpha^4\,\text{Cos}[\beta]^2\,\text{Sin}[\beta]^2-4.\grave{}\ l^4\,q\,s\,\alpha^4\,\text{Cos}[\beta]^2\,\text{Sin}[\beta]^2+2.\grave{}\ l^4\,s^2\,\alpha^4\,\text{Cos}[\beta]^2\,\text{Sin}[\beta]^2-1.\grave{}\ l^6\,\alpha^6\,\text{Cos}[\beta]^2\,\text{Sin}[\beta]^2\right)\right)\hat{}2$$

$$\left(\left(1.\grave{}\ q^2-2.\grave{}\ q\,s+1.\grave{}\ s^2-1.\grave{}\ l^2\,\alpha^2\right)\hat{}2\,\left(q^2-2.\grave{}\ q\,s+s^2-1.\grave{}\ l^2\,\alpha^2+l^2\,\alpha^2\,\text{Sin}[\beta]^2\right)\right)==$$

$$\left(l\sqrt{\left(-4.\grave{}-\frac{225.\grave{}\ l^8\,\alpha^8}{\left(1.\grave{}\ q^2-2.\grave{}\ q\,s+1.\grave{}\ s^2-1.\grave{}\ l^2\,\alpha^2\right)^4}-\frac{450.\grave{}\ l^6\,\alpha^6}{\left(1.\grave{}\ q^2-2.\grave{}\ q\,s+1.\grave{}\ s^2-1.\grave{}\ l^2\,\alpha^2\right)^3}-\frac{285.\grave{}\ l^4\,\alpha^4}{\left(1.\grave{}\ q^2-2.\grave{}\ q\,s+1.\grave{}\ s^2-1.\grave{}\ l^2\,\alpha^2\right)^2}-\frac{60.\grave{}\ l^2\,\alpha^2}{1.\grave{}\ q^2-2.\grave{}\ q\,s+1.\grave{}\ s^2-1.\grave{}\ l^2\,\alpha^2}-\frac{225.\grave{}\ l^{10}\,\alpha^{10}\,\text{Sin}[\beta]^2}{\left(1.\grave{}\ q^2-2.\grave{}\ q\,s+1.\grave{}\ s^2-1.\grave{}\ l^2\,\alpha^2\right)^5}-\frac{450.\grave{}\ l^8\,\alpha^8\,\text{Sin}[\beta]^2}{\left(1.\grave{}\ q^2-2.\grave{}\ q\,s+1.\grave{}\ s^2-1.\grave{}\ l^2\,\alpha^2\right)^4}-\frac{285.\grave{}\ l^6\,\alpha^6\,\text{Sin}[\beta]^2}{\left(1.\grave{}\ q^2-2.\grave{}\ q\,s+1.\grave{}\ s^2-1.\grave{}\ l^2\,\alpha^2\right)^3}-\frac{60.\grave{}\ l^4\,\alpha^4\,\text{Sin}[\beta]^2}{\left(1.\grave{}\ q^2-2.\grave{}\ q\,s+1.\grave{}\ s^2-1.\grave{}\ l^2\,\alpha^2\right)^2}-\frac{4.\grave{}\ l^2\,\alpha^2\,\text{Sin}[\beta]^2}{1.\grave{}\ q^2-2.\grave{}\ q\,s+1.\grave{}\ s^2-1.\grave{}\ l^2\,\alpha^2}\right)}\right)$$

Proof :

Further formal investigations yield the following visualizations :

$$
c = \frac{1.` \, v \sqrt{1.` \, q^2 - 2.` \, q\, s + 1.` \, s^2 - 1.` \, l^2 \alpha^2 + 1.` \, l^2 \alpha^2 \text{Sin}[\beta]^2}}{\sqrt{q^2 - 2.` \, q\, s + s^2 - 1.` \, l^2 \alpha^2 + l^2 \alpha^2 \text{Sin}[\beta]^2}} =
$$

$$
\left(l \sqrt{\left(-4.` - \frac{225.` \, l^8 \alpha^8}{\left(1.` \, q^2 - 2.` \, q\, s + 1.` \, s^2 - 1.` \, l^2 \alpha^2\right)^4} - \frac{450.` \, l^6 \alpha^6}{\left(1.` \, q^2 - 2.` \, q\, s + 1.` \, s^2 - 1.` \, l^2 \alpha^2\right)^3} - \frac{285.` \, l^4 \alpha^4}{\left(1.` \, q^2 - 2.` \, q\, s + 1.` \, s^2 - 1.` \, l^2 \alpha^2\right)^2} - \frac{60.` \, l^2 \alpha^2}{1.` \, q^2 - 2.` \, q\, s + 1.` \, s^2 - 1.` \, l^2 \alpha^2} - \frac{225.` \, l^{10} \alpha^{10} \text{Sin}[\beta]^2}{\left(1.` \, q^2 - 2.` \, q\, s + 1.` \, s^2 - 1.` \, l^2 \alpha^2\right)^5} - \frac{450.` \, l^8 \alpha^8 \text{Sin}[\beta]^2}{\left(1.` \, q^2 - 2.` \, q\, s + 1.` \, s^2 - 1.` \, l^2 \alpha^2\right)^4} - \frac{285.` \, l^6 \alpha^6 \text{Sin}[\beta]^2}{\left(1.` \, q^2 - 2.` \, q\, s + 1.` \, s^2 - 1.` \, l^2 \alpha^2\right)^3} - \frac{60.` \, l^4 \alpha^4 \text{Sin}[\beta]^2}{\left(1.` \, q^2 - 2.` \, q\, s + 1.` \, s^2 - 1.` \, l^2 \alpha^2\right)^2} - \frac{4.` \, l^2 \alpha^2 \text{Sin}[\beta]^2}{1.` \, q^2 - 2.` \, q\, s + 1.` \, s^2 - 1.` \, l^2 \alpha^2} \right)} \right) \Big/ \left(\left(1.` \, q^2 - 2.` \, q\, s + 1.` \, s^2 - 1.` \, l^2 \alpha^2\right) \sqrt{q^2 - 2.` \, q\, s + s^2 - 1.` \, l^2 \alpha^2 + l^2 \alpha^2 \text{Sin}[\beta]^2} \right)
$$

In[∘]:=
```
Solve[(1.` v Sqrt[1.` q^2 - 2.` q s + 1.` s^2 - 1.` l^2 α^2 + 1.` l^2 α^2 Sin[β]^2]) / Sqrt[q^2 - 2.` q s + s^2 - 1.` l^2 α^2 + l^2 α^2 Sin[β]^2] ==
 (l Sqrt[(-4.` - (225.` l^8 α^8)/(1.` q^2 - 2.` q s + 1.` s^2 - 1.` l^2 α^2)^4 -
     (450.` l^6 α^6)/(1.` q^2 - 2.` q s + 1.` s^2 - 1.` l^2 α^2)^3 - (285.` l^4 α^4)/(1.` q^2 - 2.` q s + 1.` s^2 - 1.` l^2 α^2)^2 -
     (60.` l^2 α^2)/(1.` q^2 - 2.` q s + 1.` s^2 - 1.` l^2 α^2) - (225.` l^10 α^10 Sin[β]^2)/(1.` q^2 - 2.` q s + 1.` s^2 - 1.` l^2 α^2)^5 -
     (450.` l^8 α^8 Sin[β]^2)/(1.` q^2 - 2.` q s + 1.` s^2 - 1.` l^2 α^2)^4 - (285.` l^6 α^6 Sin[β]^2)/(1.` q^2 - 2.` q s + 1.` s^2 - 1.` l^2 α^2)^3 -
     (60.` l^4 α^4 Sin[β]^2)/(1.` q^2 - 2.` q s + 1.` s^2 - 1.` l^2 α^2)^2 - (4.` l^2 α^2 Sin[β]^2)/(1.` q^2 - 2.` q s + 1.` s^2 - 1.` l^2 α^2))]) /
  ((1.` q^2 - 2.` q s + 1.` s^2 - 1.` l^2 α^2) Sqrt[q^2 - 2.` q s + s^2 - 1.` l^2 α^2 + l^2 α^2 Sin[β]^2]), v]
```

Out[∘]=
```
{{v → (1. l Sqrt[(-4. - (225. l^8 α^8)/(1. q^2 - 2. q s + 1. s^2 - 1. l^2 α^2)^4 -
     (450. l^6 α^6)/(1. q^2 - 2. q s + 1. s^2 - 1. l^2 α^2)^3 - (285. l^4 α^4)/(1. q^2 - 2. q s + 1. s^2 - 1. l^2 α^2)^2 -
     (60. l^2 α^2)/(1. q^2 - 2. q s + 1. s^2 - 1. l^2 α^2) - (225. l^10 α^10 Sin[β]^2)/(1. q^2 - 2. q s + 1. s^2 - 1. l^2 α^2)^5 -
     (450. l^8 α^8 Sin[β]^2)/(1. q^2 - 2. q s + 1. s^2 - 1. l^2 α^2)^4 - (285. l^6 α^6 Sin[β]^2)/(1. q^2 - 2. q s + 1. s^2 - 1. l^2 α^2)^3 -
     (60. l^4 α^4 Sin[β]^2)/(1. q^2 - 2. q s + 1. s^2 - 1. l^2 α^2)^2 - (4. l^2 α^2 Sin[β]^2)/(1. q^2 - 2. q s + 1. s^2 - 1. l^2 α^2))]) /
  ((1. q^2 - 2. q s + 1. s^2 - 1. l^2 α^2) Sqrt[1. q^2 - 2. q s + 1. s^2 - 1. l^2 α^2 + 1. l^2 α^2 Sin[β]^2])}}
```

$$v\,\hat{}\,2 = \left(-\frac{15\,l^3\,(2\,q-2\,s)\,(-2\,q+2\,s)\,\alpha^2}{4\,\left(-q^2+2\,q\,s-s^2+l^2\,\alpha^2\right)^{7/2}}+\frac{3\,l\,(2\,q-2\,s)\,(-2\,q+2\,s)}{4\,\left(-q^2+2\,q\,s-s^2+l^2\,\alpha^2\right)^{5/2}}+\frac{3\,l^3\,\alpha^2}{\left(-q^2+2\,q\,s-s^2+l^2\,\alpha^2\right)^{5/2}}-\frac{l}{\left(-q^2+2\,q\,s-s^2+l^2\,\alpha^2\right)^{3/2}}\right)\hat{}\,2 = \left(1.\grave{}\,l\left(-4.\grave{}-\frac{225.\grave{}\,l^8\,\alpha^8}{\left(1.\grave{}\,q^2-2.\grave{}\,q\,s+1.\grave{}\,s^2-1.\grave{}\,l^2\,\alpha^2\right)^4}-\frac{450.\grave{}\,l^6\,\alpha^6}{\left(1.\grave{}\,q^2-2.\grave{}\,q\,s+1.\grave{}\,s^2-1.\grave{}\,l^2\,\alpha^2\right)^3}-\frac{285.\grave{}\,l^4\,\alpha^4}{\left(1.\grave{}\,q^2-2.\grave{}\,q\,s+1.\grave{}\,s^2-1.\grave{}\,l^2\,\alpha^2\right)^2}-\frac{60.\grave{}\,l^2\,\alpha^2}{1.\grave{}\,q^2-2.\grave{}\,q\,s+1.\grave{}\,s^2-1.\grave{}\,l^2\,\alpha^2}-\frac{225.\grave{}\,l^{10}\,\alpha^{10}\,\mathrm{Sin}[\beta]^2}{\left(1.\grave{}\,q^2-2.\grave{}\,q\,s+1.\grave{}\,s^2-1.\grave{}\,l^2\,\alpha^2\right)^5}-\frac{450.\grave{}\,l^8\,\alpha^8\,\mathrm{Sin}[\beta]^2}{\left(1.\grave{}\,q^2-2.\grave{}\,q\,s+1.\grave{}\,s^2-1.\grave{}\,l^2\,\alpha^2\right)^4}-\frac{285.\grave{}\,l^6\,\alpha^6\,\mathrm{Sin}[\beta]^2}{\left(1.\grave{}\,q^2-2.\grave{}\,q\,s+1.\grave{}\,s^2-1.\grave{}\,l^2\,\alpha^2\right)^3}-\frac{60.\grave{}\,l^4\,\alpha^4\,\mathrm{Sin}[\beta]^2}{\left(1.\grave{}\,q^2-2.\grave{}\,q\,s+1.\grave{}\,s^2-1.\grave{}\,l^2\,\alpha^2\right)^2}-\frac{4.\grave{}\,l^2\,\alpha^2\,\mathrm{Sin}[\beta]^2}{1.\grave{}\,q^2-2.\grave{}\,q\,s+1.\grave{}\,s^2-1.\grave{}\,l^2\,\alpha^2}\right)\right)\Big/\left(\left(1.\grave{}\,q^2-2.\grave{}\,q\,s+1.\grave{}\,s^2-1.\grave{}\,l^2\,\alpha^2\right)\hat{}\,2\,\left(1.\grave{}\,q^2-2.\grave{}\,q\,s+1.\grave{}\,s^2-1.\grave{}\,l^2\,\alpha^2+1.\grave{}\,l^2\,\alpha^2\,\mathrm{Sin}[\beta]^2\right)\right);$$

In[•]:=
```
Solve[(1.` l^2 (-4.` - 225.` l^8 α^8/(1.` q^2 - 2.` q s + 1.` s^2 - 1.` l^2 α^2)^4 -
   450.` l^6 α^6/(1.` q^2 - 2.` q s + 1.` s^2 - 1.` l^2 α^2)^3 - 285.` l^4 α^4/(1.` q^2 - 2.` q s + 1.` s^2 - 1.` l^2 α^2)^2 -
   60.` l^2 α^2/(1.` q^2 - 2.` q s + 1.` s^2 - 1.` l^2 α^2) - 225.` l^10 α^10 Sin[β]^2/(1.` q^2 - 2.` q s + 1.` s^2 - 1.` l^2 α^2)^5 -
   450.` l^8 α^8 Sin[β]^2/(1.` q^2 - 2.` q s + 1.` s^2 - 1.` l^2 α^2)^4 - 285.` l^6 α^6 Sin[β]^2/(1.` q^2 - 2.` q s + 1.` s^2 - 1.` l^2 α^2)^3 -
   60.` l^4 α^4 Sin[β]^2/(1.` q^2 - 2.` q s + 1.` s^2 - 1.` l^2 α^2)^2 - 4.` l^2 α^2 Sin[β]^2/(1.` q^2 - 2.` q s + 1.` s^2 - 1.` l^2 α^2))) ==
  (-15 l^3 (2 q - 2 s) (-2 q + 2 s) α^2/(4 (-q^2 + 2 q s - s^2 + l^2 α^2)^(7/2)) + 3 l (2 q - 2 s) (-2 q + 2 s)/(4 (-q^2 + 2 q s - s^2 + l^2 α^2)^(5/2)) + 3 l^3 α^2/(-q^2 + 2 q s - s^2 + l^2 α^2)^(5/2) -
   l/(-q^2 + 2 q s - s^2 + l^2 α^2)^(3/2))^2 (1.` q^2 - 2.` q s + 1.` s^2 - 1.` l^2 α^2)^2
  (1.` q^2 - 2.` q s + 1.` s^2 - 1.` l^2 α^2 + 1.` l^2 α^2 Sin[β]^2), Reals]
```

… Solve: Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

Out[•]= { { } }

In[•]:=
```
Simplify[l^2 (c) == (D[D[D[D[Sqrt[-q^2 + 2 q s - s^2 + l^2 α^2]/α, q], s], l], α])^2
  (1.` q^2 - 2.` q s + 1.` s^2 - 1.` l^2 α^2)^2
  (1.` q^2 - 2.` q s + 1.` s^2 - 1.` l^2 α^2 + 1.` l^2 α^2 Sin[β]^2)]
```

In[•]:=
```
Simplify[(l^2 (c)) / ((1.` q^2 - 2.` q s + 1.` s^2 - 1.` l^2 α^2)^2
   (1.` q^2 - 2.` q s + 1.` s^2 - 1.` l^2 α^2 + 1.` l^2 α^2 Sin[β]^2))]
```

Out[•]=

$$\frac{c\, l^2}{\left(1.\, q^2 - 2.\, q\, s + 1.\, s^2 - 1.\, l^2 \alpha^2\right)^2 \left(1.\, q^2 - 2.\, q\, s + 1.\, s^2 - 1.\, l^2 \alpha^2 + 1.\, l^2 \alpha^2 \operatorname{Sin}[\beta]^2\right)}$$

In[∘]:= Solve$\left[\text{Sin}[\beta] == \dfrac{\sqrt{-(q-s-w)\ \sqrt{1-\frac{v^2}{c^2}}}\ \sqrt{(q-s+w)\Big/\ \sqrt{1-\frac{v^2}{c^2}}}}{w}, v\right]$

Out[∘]= $\Big\{\Big\{v \to -\Big(\Big(1.\ \sqrt{}\ \big(8.98755\times10^{16}\ q^2 - 1.79751\times10^{17}\ q\ s + 8.98755\times10^{16}\ s^2 - 8.98755\times10^{16}\ w^2 + 8.98755\times10^{16}\ w^2\ \text{Sin}[\beta]^2\big)\Big)\Big/\Big(\sqrt{q^2 - 2.\ q\ s + s^2 - 1.\ w^2 + w^2\ \text{Sin}[\beta]^2}\Big)\Big)\Big\},$
$\Big\{v \to \Big(\sqrt{}\ \big(8.98755\times10^{16}\ q^2 - 1.79751\times10^{17}\ q\ s + 8.98755\times10^{16}\ s^2 - 8.98755\times10^{16}\ w^2 + 8.98755\times10^{16}\ w^2\ \text{Sin}[\beta]^2\big)\Big)\Big/\Big(\sqrt{q^2 - 2.\ q\ s + s^2 - 1.\ w^2 + w^2\ \text{Sin}[\beta]^2}\Big)\Big\}\Big\}$

Since $D\left[D\left[D\left[D\left[\dfrac{\sqrt{-q^2+2\,q\,s-s^2+l^2\,\alpha^2}}{\alpha}, q\right], s\right], l\right], \alpha\right] =$

$$v = \frac{\sqrt{-c^2\ (l\,\alpha)^2 + c^2\,q^2 - 2\,c^2\,s\,q + c^2\,s^2 + c^2\ (l\,\alpha)^2\ (h/l)^2}}{\sqrt{-1.`\ (l\,\alpha)^2 + q^2 - 2.`\ s\,q + s^2 + (l\,\alpha)^2\ (h/l)^2}} =$$

$$\frac{\sqrt{c^2\,q^2 - 2\,c^2\,q\,s + c^2\,s^2 - c^2\,w^2 + c^2\,w^2\,\text{Sin}[\beta]^2}}{\sqrt{q^2 - 2.`\ q\,s + s^2 - 1.`\ w^2 + w^2\,\text{Sin}[\beta]^2}}$$

Solve$\Big[\sqrt{}\ \Big(\big(c\,l^2\big)\Big/\Big(\big(1.`\ q^2 - 2.`\ q\,s + 1.`\ s^2 - 1.`\ l^2\,\alpha^2\big)^2\ \big(1.`\ q^2 - 2.`\ q\,s + 1.`\ s^2 - 1.`\ l^2\,\alpha^2 + 1.`\ l^2\,\alpha^2\,\text{Sin}[\beta]^2\big)\Big)\Big) ==$

$D\left[D\left[D\left[D\left[\dfrac{\sqrt{-q^2+2\,q\,s-s^2+l^2\,\alpha^2}}{\alpha}, q\right], s\right], l\right], \alpha\right], c\Big] = D[D[l\ \text{Sin}[\beta], l], \beta] = \text{Cos}[\beta]$

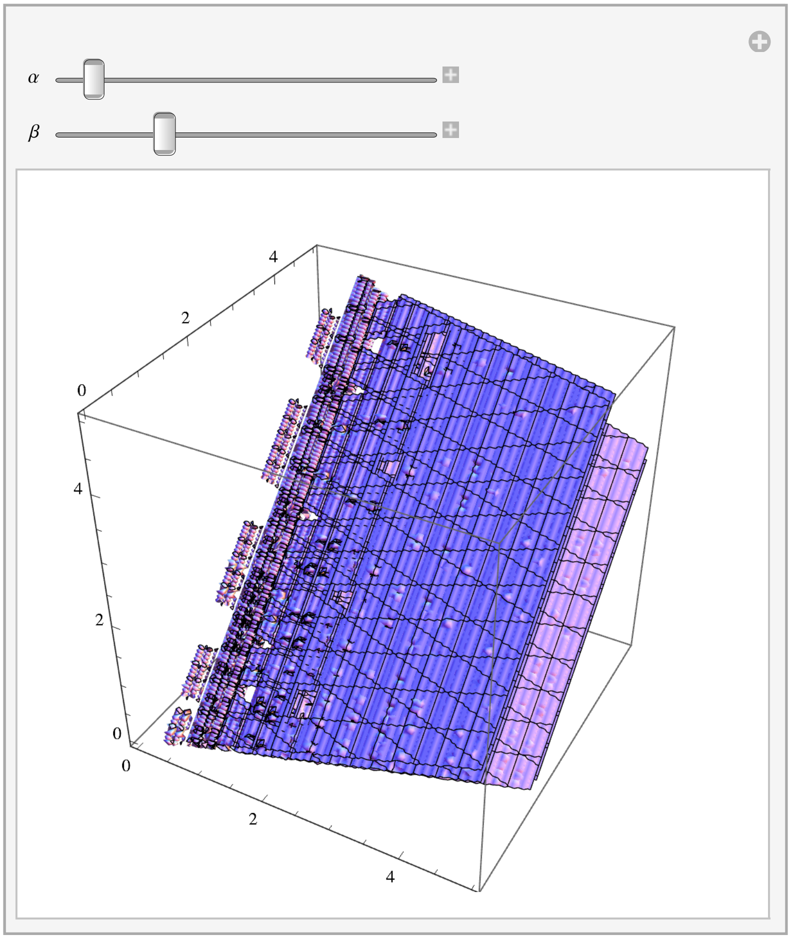
In[∘]:= Solve$\Big[\sqrt{}\ \Big(\big(c\,l^2\big)\Big/\Big(\big(1.`\ q^2 - 2.`\ q\,s + 1.`\ s^2 - 1.`\ l^2\,\alpha^2\big)^2\ \big(1.`\ q^2 - 2.`\ q\,s + 1.`\ s^2 - 1.`\ l^2\,\alpha^2 + 1.`\ l^2\,\alpha^2\,\text{Sin}[\beta]^2\big)\Big)\Big) == \text{Cos}[\beta], c\Big]$

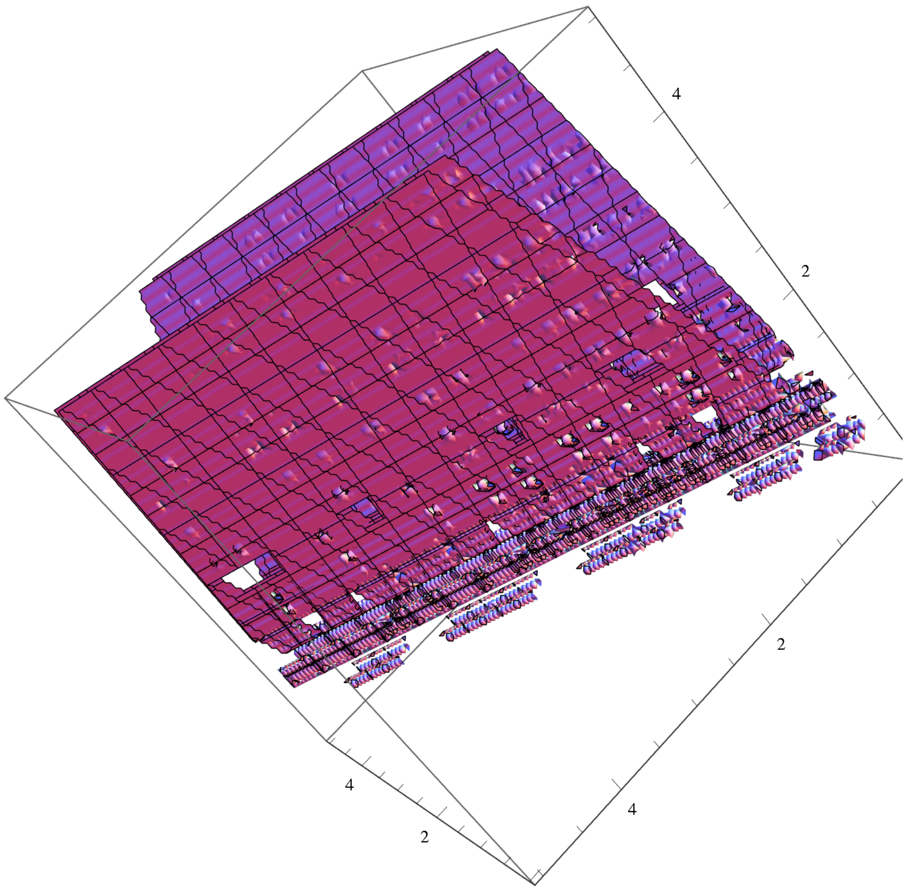
Out[∘]= $\Big\{\Big\{c \to -\dfrac{1}{l^2}\ 1.\ \big(-1.\ q^6\,\text{Cos}[\beta]^2 + 6.\ q^5\,s\,\text{Cos}[\beta]^2 - 15.\ q^4\,s^2\,\text{Cos}[\beta]^2 + 20.\ q^3\,s^3\,\text{Cos}[\beta]^2 -$
$15.\ q^2\,s^4\,\text{Cos}[\beta]^2 + 6.\ q\,s^5\,\text{Cos}[\beta]^2 - 1.\ s^6\,\text{Cos}[\beta]^2 + 3.\ l^2\,q^4\,\alpha^2\,\text{Cos}[\beta]^2 -$
$12.\ l^2\,q^3\,s\,\alpha^2\,\text{Cos}[\beta]^2 + 18.\ l^2\,q^2\,s^2\,\alpha^2\,\text{Cos}[\beta]^2 - 12.\ l^2\,q\,s^3\,\alpha^2\,\text{Cos}[\beta]^2 +$
$3.\ l^2\,s^4\,\alpha^2\,\text{Cos}[\beta]^2 - 3.\ l^4\,q^2\,\alpha^4\,\text{Cos}[\beta]^2 + 6.\ l^4\,q\,s\,\alpha^4\,\text{Cos}[\beta]^2 - 3.\ l^4\,s^2\,\alpha^4\,\text{Cos}[\beta]^2 +$
$1.\ l^6\,\alpha^6\,\text{Cos}[\beta]^2 - 1.\ l^2\,q^4\,\alpha^2\,\text{Cos}[\beta]^2\,\text{Sin}[\beta]^2 + 4.\ l^2\,q^3\,s\,\alpha^2\,\text{Cos}[\beta]^2\,\text{Sin}[\beta]^2 -$
$6.\ l^2\,q^2\,s^2\,\alpha^2\,\text{Cos}[\beta]^2\,\text{Sin}[\beta]^2 + 4.\ l^2\,q\,s^3\,\alpha^2\,\text{Cos}[\beta]^2\,\text{Sin}[\beta]^2 -$
$1.\ l^2\,s^4\,\alpha^2\,\text{Cos}[\beta]^2\,\text{Sin}[\beta]^2 + 2.\ l^4\,q^2\,\alpha^4\,\text{Cos}[\beta]^2\,\text{Sin}[\beta]^2 - 4.\ l^4\,q\,s\,\alpha^4$
$\text{Cos}[\beta]^2\,\text{Sin}[\beta]^2 + 2.\ l^4\,s^2\,\alpha^4\,\text{Cos}[\beta]^2\,\text{Sin}[\beta]^2 - 1.\ l^6\,\alpha^6\,\text{Cos}[\beta]^2\,\text{Sin}[\beta]^2\big)\Big\}\Big\}$

In[∘]:= c := 2.99792458\` * 10^8

```
In[∘]:= Manipulate[ContourPlot3D[
  (-1/l^2 1.` (-1.` q^6 Cos[β]^2 + 6.` q^5 s Cos[β]^2 - 15.` q^4 s^2 Cos[β]^2 + 20.` q^3 s^3 Cos[β]^2 -
        15.` q^2 s^4 Cos[β]^2 + 6.` q s^5 Cos[β]^2 - 1.` s^6 Cos[β]^2 + 3.` l^2 q^4 α^2 Cos[β]^2 -
        12.` l^2 q^3 s α^2 Cos[β]^2 + 18.` l^2 q^2 s^2 α^2 Cos[β]^2 - 12.` l^2 q s^3 α^2 Cos[β]^2 +
        3.` l^2 s^4 α^2 Cos[β]^2 - 3.` l^4 q^2 α^4 Cos[β]^2 + 6.` l^4 q s α^4 Cos[β]^2 -
        3.` l^4 s^2 α^4 Cos[β]^2 + 1.` l^6 α^6 Cos[β]^2 - 1.` l^2 q^4 α^2 Cos[β]^2 Sin[β]^2 +
        4.` l^2 q^3 s α^2 Cos[β]^2 Sin[β]^2 - 6.` l^2 q^2 s^2 α^2 Cos[β]^2 Sin[β]^2 +
        4.` l^2 q s^3 α^2 Cos[β]^2 Sin[β]^2 - 1.` l^2 s^4 α^2 Cos[β]^2 Sin[β]^2 +
        2.` l^4 q^2 α^4 Cos[β]^2 Sin[β]^2 - 4.` l^4 q s α^4 Cos[β]^2 Sin[β]^2 +
        2.` l^4 s^2 α^4 Cos[β]^2 Sin[β]^2 - 1.` l^6 α^6 Cos[β]^2 Sin[β]^2)) ^2
   ((1.` q^2 - 2.` q s + 1.` s^2 - 1.` l^2 α^2) ^2 (q^2 - 2.` q s + s^2 - 1.` l^2 α^2 + l^2 α^2 Sin[β]^2)) ==
  (l Sqrt[(-4.` - 225.` l^8 α^8/(1.` q^2 - 2.` q s + 1.` s^2 - 1.` l^2 α^2)^4 -
        450.` l^6 α^6/(1.` q^2 - 2.` q s + 1.` s^2 - 1.` l^2 α^2)^3 - 285.` l^4 α^4/(1.` q^2 - 2.` q s + 1.` s^2 - 1.` l^2 α^2)^2 -
        60.` l^2 α^2/(1.` q^2 - 2.` q s + 1.` s^2 - 1.` l^2 α^2) - 225.` l^10 α^10 Sin[β]^2/(1.` q^2 - 2.` q s + 1.` s^2 - 1.` l^2 α^2)^5 -
        450.` l^8 α^8 Sin[β]^2/(1.` q^2 - 2.` q s + 1.` s^2 - 1.` l^2 α^2)^4 - 285.` l^6 α^6 Sin[β]^2/(1.` q^2 - 2.` q s + 1.` s^2 - 1.` l^2 α^2)^3 -
        60.` l^4 α^4 Sin[β]^2/(1.` q^2 - 2.` q s + 1.` s^2 - 1.` l^2 α^2)^2 - 4.` l^2 α^2 Sin[β]^2/(1.` q^2 - 2.` q s + 1.` s^2 - 1.` l^2 α^2))]),
  {l, 0, 5}, {q, 0, 5}, {s, 0, 5}, PlotTheme → {"Classic",
   "ClassicLights"}], {α, 0, 2 π}, {β, 0, π / 2}]
```

Out[•]=

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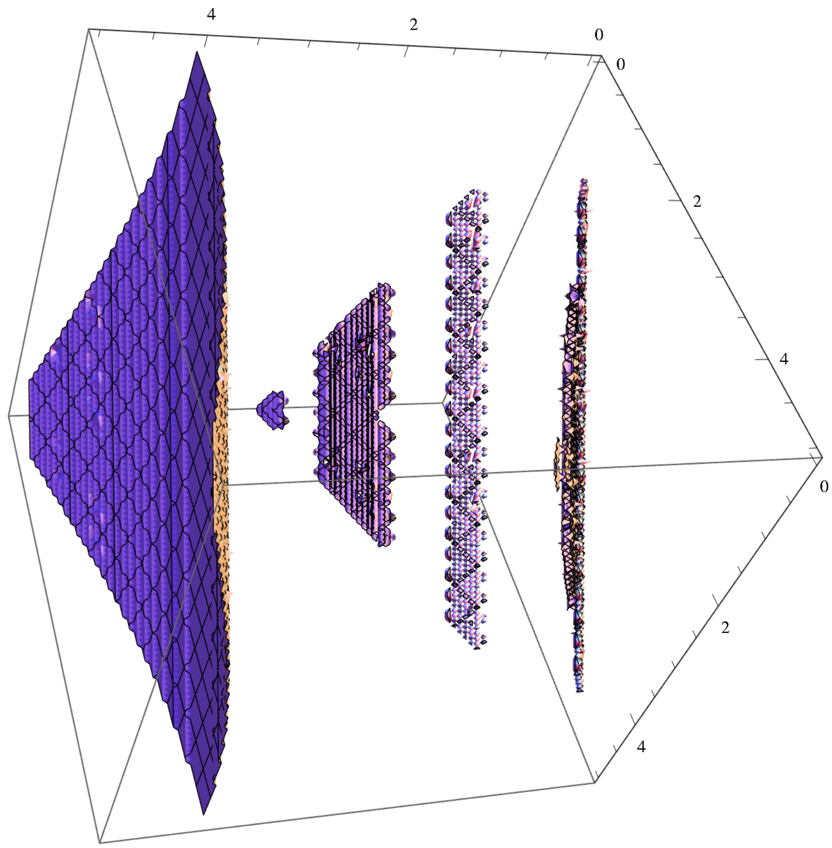
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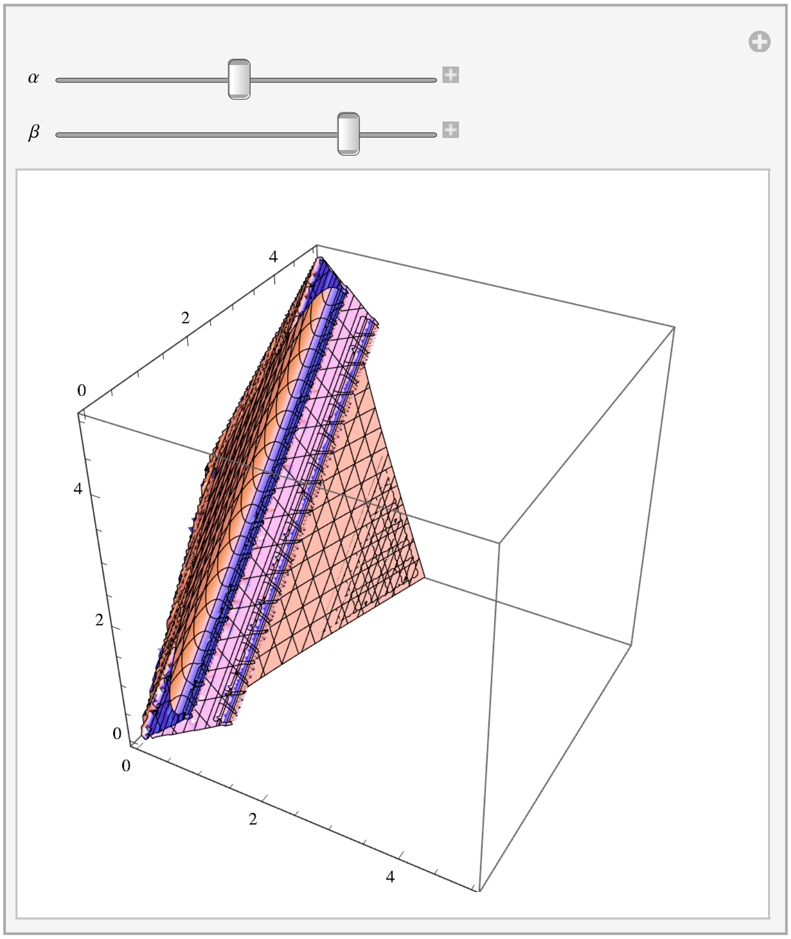

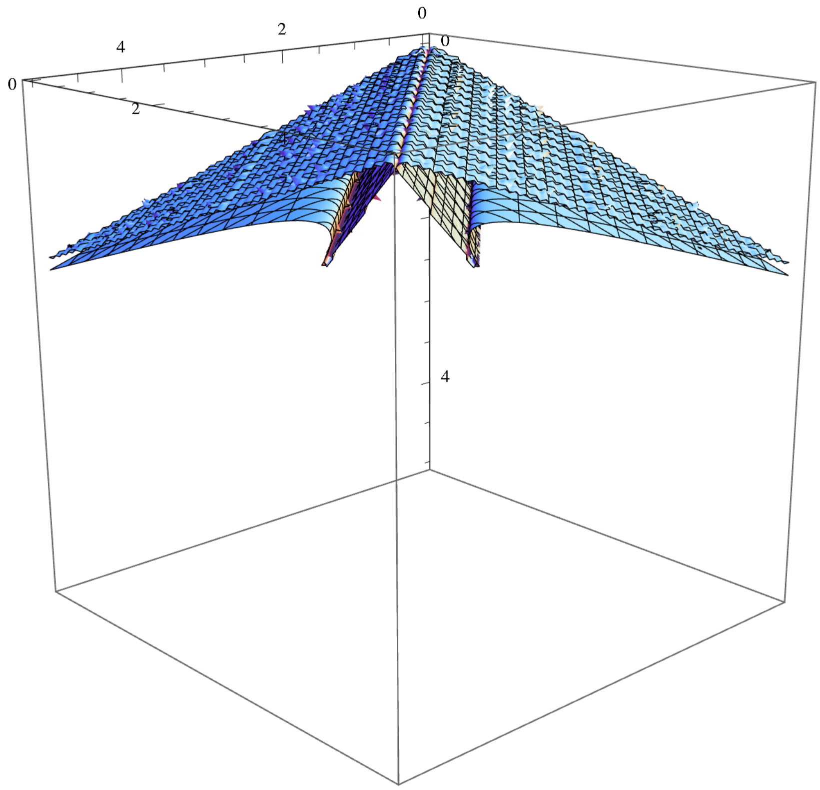
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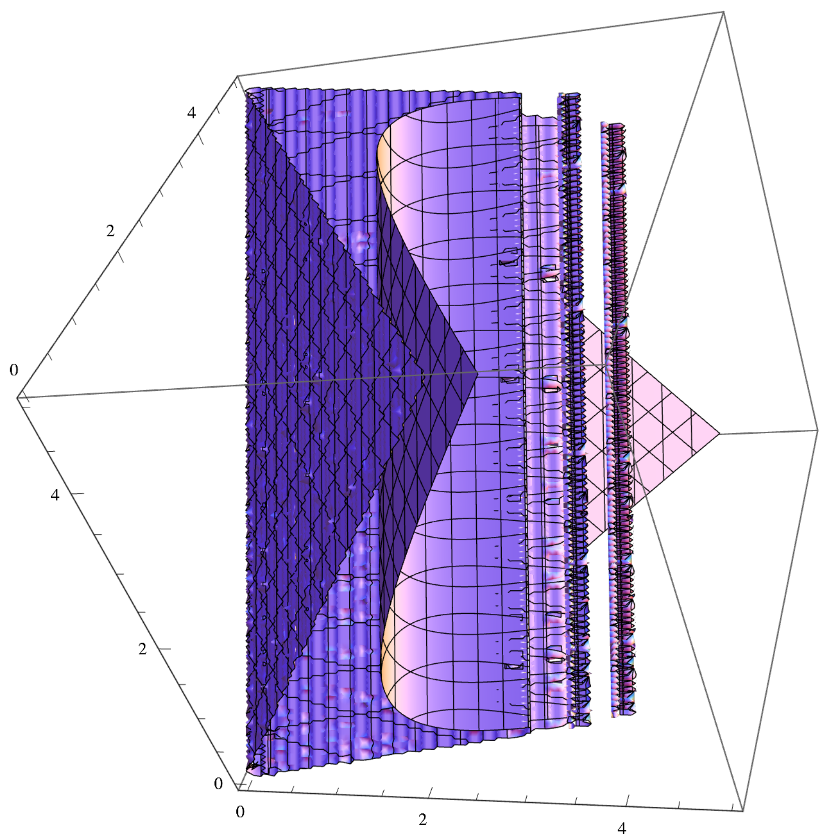

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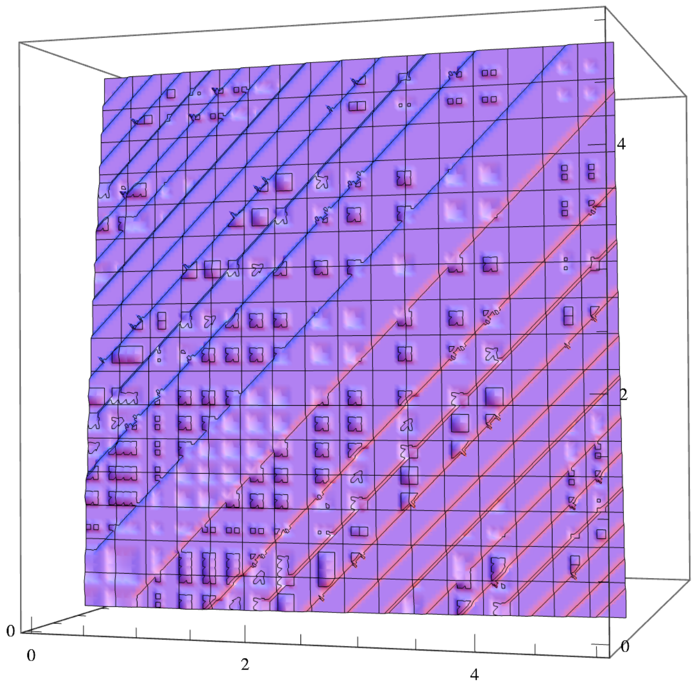


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        15.` q^2 s^4 Cos[β]^2 + 6.` q s^5 Cos[β]^2 - 1.` s^6 Cos[β]^2 + 3.` l^2 q^4 α^2 Cos[β]^2 -
        12.` l^2 q^3 s α^2 Cos[β]^2 + 18.` l^2 q^2 s^2 α^2 Cos[β]^2 - 12.` l^2 q s^3 α^2 Cos[β]^2 +
        3.` l^2 s^4 α^2 Cos[β]^2 - 3.` l^4 q^2 α^4 Cos[β]^2 + 6.` l^4 q s α^4 Cos[β]^2 -
        3.` l^4 s^2 α^4 Cos[β]^2 + 1.` l^6 α^6 Cos[β]^2 - 1.` l^2 q^4 α^2 Cos[β]^2 Sin[β]^2 +
        4.` l^2 q^3 s α^2 Cos[β]^2 Sin[β]^2 - 6.` l^2 q^2 s^2 α^2 Cos[β]^2 Sin[β]^2 +
        4.` l^2 q s^3 α^2 Cos[β]^2 Sin[β]^2 - 1.` l^2 s^4 α^2 Cos[β]^2 Sin[β]^2 +
        2.` l^4 q^2 α^4 Cos[β]^2 Sin[β]^2 - 4.` l^4 q s α^4 Cos[β]^2 Sin[β]^2 +
        2.` l^4 s^2 α^4 Cos[β]^2 Sin[β]^2 - 1.` l^6 α^6 Cos[β]^2 Sin[β]^2)) ^2
    ((1.` q^2 - 2.` q s + 1.` s^2 - 1.` l^2 α^2) ^2 (q^2 - 2.` q s + s^2 - 1.` l^2 α^2 + l^2 α^2 Sin[β]^2)),
   {α, 0, 2 π}, {β, 0, π / 2}, PlotTheme →
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   0,
   5}]
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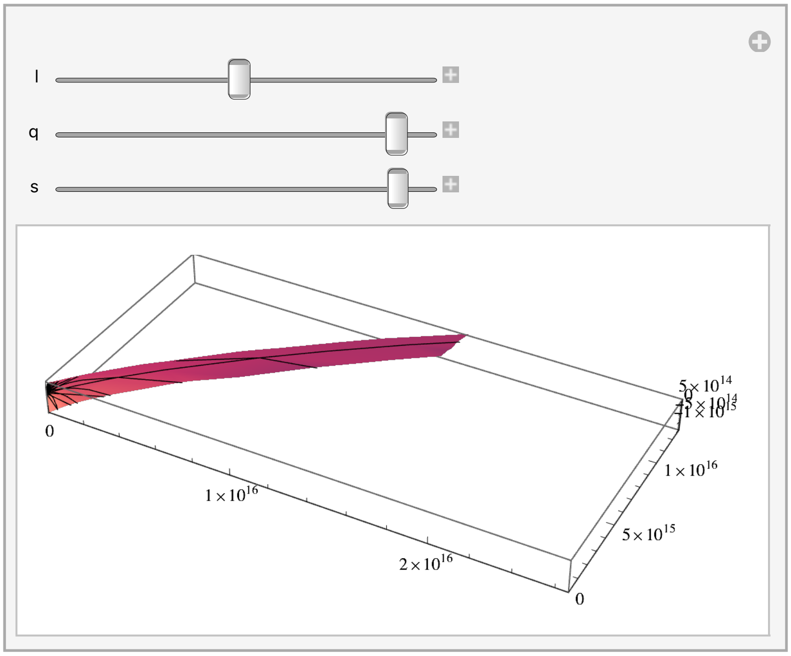
Out[9]=



Power: Infinite expression $\frac{1}{0}$ encountered.

Infinity: Indeterminate expression 0. ComplexInfinity encountered.

Power: Infinite expression $\frac{1}{0}$ encountered.

Infinity: Indeterminate expression 0. ComplexInfinity encountered.

Power: Infinite expression $\frac{1}{0}$ encountered.

General: Further output of Power::infy will be suppressed during this calculation.

Infinity: Indeterminate expression 0. ComplexInfinity encountered.

General: Further output of Infinity::indet will be suppressed during this calculation.

In[∘]:=
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        15.` q^2 s^4 Cos[β]^2 + 6.` q s^5 Cos[β]^2 - 1.` s^6 Cos[β]^2 + 3.` l^2 q^4 α^2 Cos[β]^2 -
        12.` l^2 q^3 s α^2 Cos[β]^2 + 18.` l^2 q^2 s^2 α^2 Cos[β]^2 - 12.` l^2 q s^3 α^2 Cos[β]^2 +
        3.` l^2 s^4 α^2 Cos[β]^2 - 3.` l^4 q^2 α^4 Cos[β]^2 + 6.` l^4 q s α^4 Cos[β]^2 -
        3.` l^4 s^2 α^4 Cos[β]^2 + 1.` l^6 α^6 Cos[β]^2 - 1.` l^2 q^4 α^2 Cos[β]^2 Sin[β]^2 +
        4.` l^2 q^3 s α^2 Cos[β]^2 Sin[β]^2 - 6.` l^2 q^2 s^2 α^2 Cos[β]^2 Sin[β]^2 +
        4.` l^2 q s^3 α^2 Cos[β]^2 Sin[β]^2 - 1.` l^2 s^4 α^2 Cos[β]^2 Sin[β]^2 +
        2.` l^4 q^2 α^4 Cos[β]^2 Sin[β]^2 - 4.` l^4 q s α^4 Cos[β]^2 Sin[β]^2 +
        2.` l^4 s^2 α^4 Cos[β]^2 Sin[β]^2 - 1.` l^6 α^6 Cos[β]^2 Sin[β]^2)) ^2
    ((1.` q^2 - 2.` q s + 1.` s^2 - 1.` l^2 α^2) ^2 (q^2 - 2.` q s + s^2 -
```

```
1.` l^2 α^2 + l^2 α^2 Sin[β]^2)))/
(((l √((-4.` - (225.` l^8 α^8)/(1.` q^2 - 2.` q s + 1.` s^2 - 1.` l^2 α^2)^4 -
    (450.` l^6 α^6)/(1.` q^2 - 2.` q s + 1.` s^2 - 1.` l^2 α^2)^3 - (285.` l^4 α^4)/(1.` q^2 - 2.` q s + 1.` s^2 - 1.` l^2 α^2)^2 -
    (60.` l^2 α^2)/(1.` q^2 - 2.` q s + 1.` s^2 - 1.` l^2 α^2) - (225.` l^10 α^10 Sin[β]^2)/(1.` q^2 - 2.` q s + 1.` s^2 - 1.` l^2 α^2)^5 -
    (450.` l^8 α^8 Sin[β]^2)/(1.` q^2 - 2.` q s + 1.` s^2 - 1.` l^2 α^2)^4 - (285.` l^6 α^6 Sin[β]^2)/(1.` q^2 - 2.` q s + 1.` s^2 - 1.` l^2 α^2)^3 -
    (60.` l^4 α^4 Sin[β]^2)/(1.` q^2 - 2.` q s + 1.` s^2 - 1.` l^2 α^2)^2 - (4.` l^2 α^2 Sin[β]^2)/(1.` q^2 - 2.` q s + 1.` s^2 - 1.` l^2 α^2)))) ==
  ((√(-q^2 + 2 q s - s^2 + l^2 α^2))/α)/ (l Sin[β]), {l,
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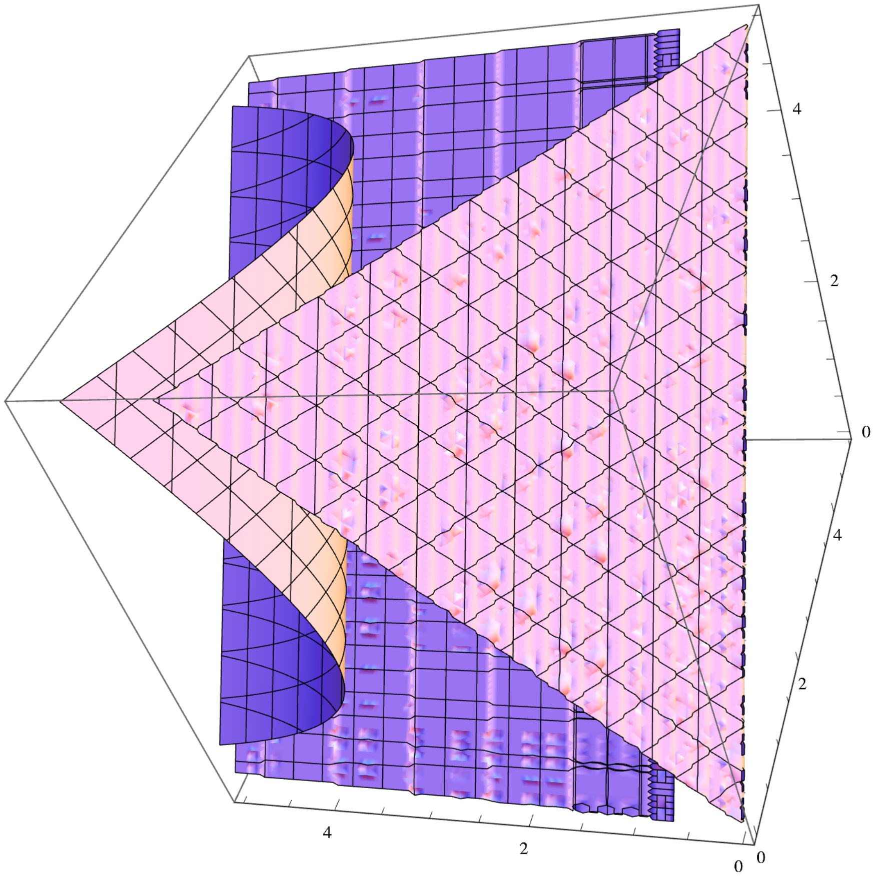
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```
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        15.` q^2 s^4 Cos[β]^2 + 6.` q s^5 Cos[β]^2 - 1.` s^6 Cos[β]^2 + 3.` l^2 q^4 α^2 Cos[β]^2 -
        12.` l^2 q^3 s α^2 Cos[β]^2 + 18.` l^2 q^2 s^2 α^2 Cos[β]^2 - 12.` l^2 q s^3 α^2 Cos[β]^2 +
        3.` l^2 s^4 α^2 Cos[β]^2 - 3.` l^4 q^2 α^4 Cos[β]^2 + 6.` l^4 q s α^4 Cos[β]^2 -
        3.` l^4 s^2 α^4 Cos[β]^2 + 1.` l^6 α^6 Cos[β]^2 - 1.` l^2 q^4 α^2 Cos[β]^2 Sin[β]^2 +
        4.` l^2 q^3 s α^2 Cos[β]^2 Sin[β]^2 - 6.` l^2 q^2 s^2 α^2 Cos[β]^2 Sin[β]^2 +
        4.` l^2 q s^3 α^2 Cos[β]^2 Sin[β]^2 - 1.` l^2 s^4 α^2 Cos[β]^2 Sin[β]^2 +
        2.` l^4 q^2 α^4 Cos[β]^2 Sin[β]^2 - 4.` l^4 q s α^4 Cos[β]^2 Sin[β]^2 +
        2.` l^4 s^2 α^4 Cos[β]^2 Sin[β]^2 - 1.` l^6 α^6 Cos[β]^2 Sin[β]^2)) ^2
   ((1.` q^2 - 2.` q s + 1.` s^2 - 1.` l^2 α^2) ^2 (q^2 - 2.` q s + s^2 - 1.` l^2 α^2 + l^2 α^2 Sin[β]^2)) ==
  (l Sqrt[(-4.` - 225.` l^8 α^8/(1.` q^2 - 2.` q s + 1.` s^2 - 1.` l^2 α^2)^4 -
        450.` l^6 α^6/(1.` q^2 - 2.` q s + 1.` s^2 - 1.` l^2 α^2)^3 - 285.` l^4 α^4/(1.` q^2 - 2.` q s + 1.` s^2 - 1.` l^2 α^2)^2 -
        60.` l^2 α^2/(1.` q^2 - 2.` q s + 1.` s^2 - 1.` l^2 α^2) - 225.` l^10 α^10 Sin[β]^2/(1.` q^2 - 2.` q s + 1.` s^2 - 1.` l^2 α^2)^5 -
        450.` l^8 α^8 Sin[β]^2/(1.` q^2 - 2.` q s + 1.` s^2 - 1.` l^2 α^2)^4 - 285.` l^6 α^6 Sin[β]^2/(1.` q^2 - 2.` q s + 1.` s^2 - 1.` l^2 α^2)^3 -
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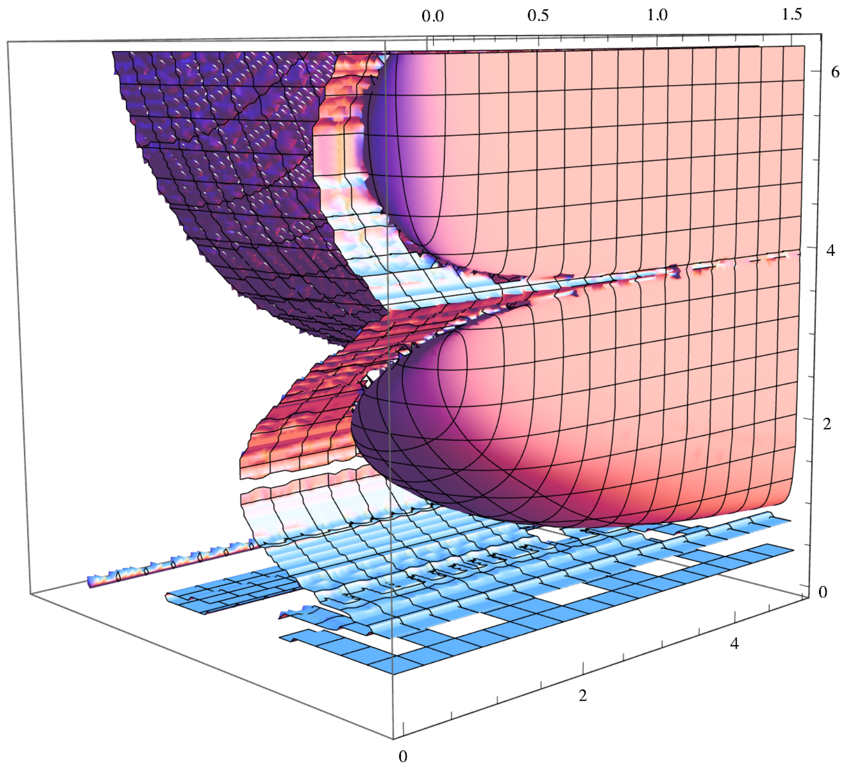

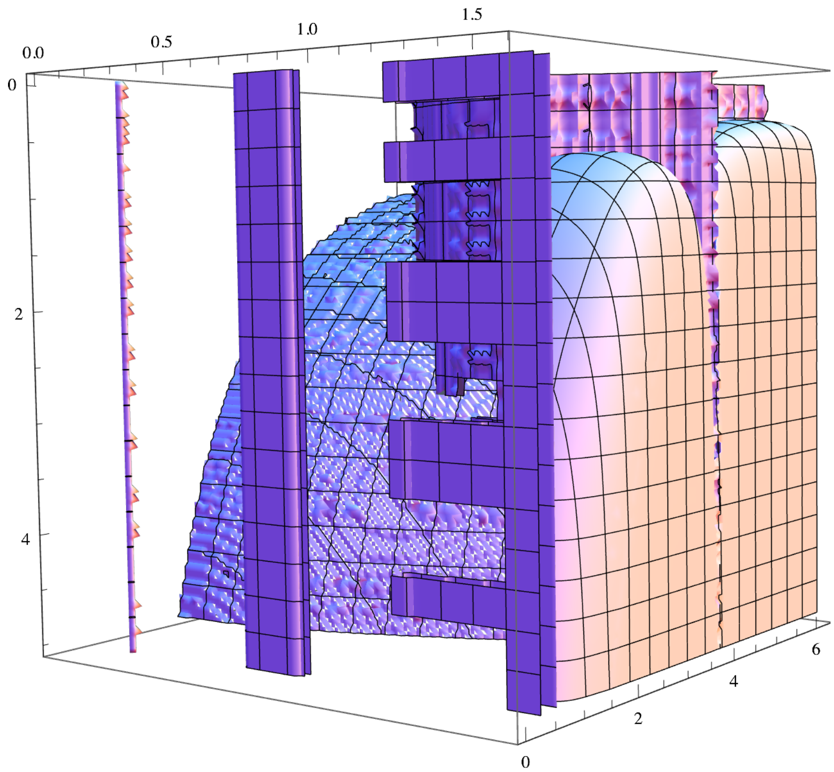

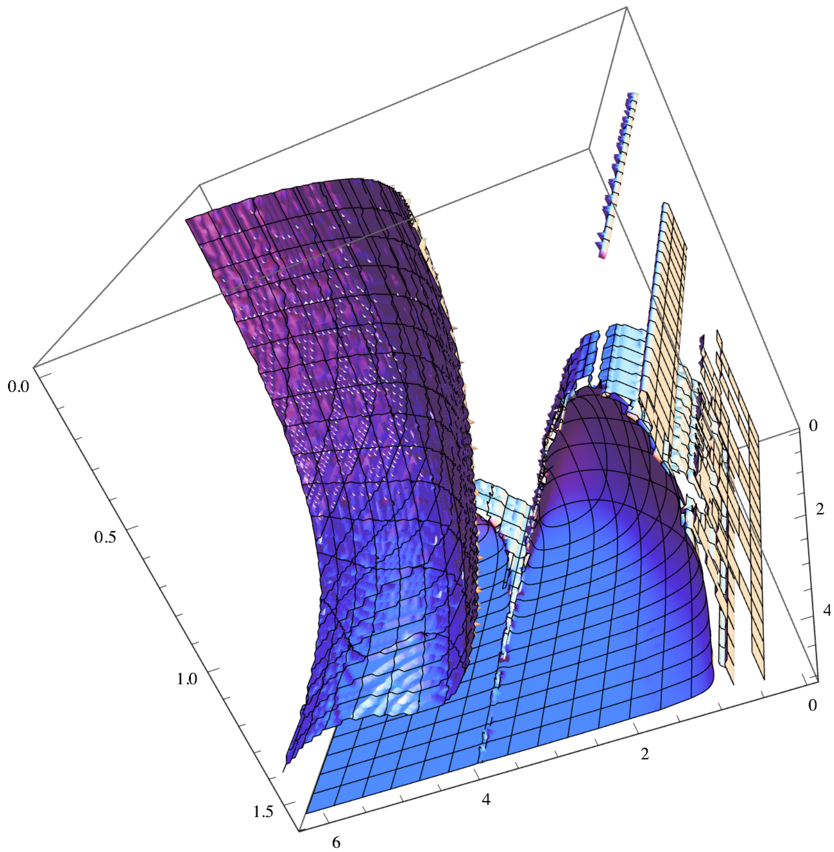
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2
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In[∘]:=
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        15.` q^2 s^4 Cos[β]^2 + 6.` q s^5 Cos[β]^2 - 1.` s^6 Cos[β]^2 + 3.` l^2 q^4 α^2 Cos[β]^2 -
        12.` l^2 q^3 s α^2 Cos[β]^2 + 18.` l^2 q^2 s^2 α^2 Cos[β]^2 - 12.` l^2 q s^3 α^2 Cos[β]^2 +
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        4.` l^2 q^3 s α^2 Cos[β]^2 Sin[β]^2 - 6.` l^2 q^2 s^2 α^2 Cos[β]^2 Sin[β]^2 +
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        2.` l^4 q^2 α^4 Cos[β]^2 Sin[β]^2 - 4.` l^4 q s α^4 Cos[β]^2 Sin[β]^2 +
        2.` l^4 s^2 α^4 Cos[β]^2 Sin[β]^2 - 1.` l^6 α^6 Cos[β]^2 Sin[β]^2)) ^2
   ((1.` q^2 - 2.` q s + 1.` s^2 - 1.` l^2 α^2) ^2 (q^2 - 2.` q s + s^2 - 1.` l^2 α^2 + l^2 α^2 Sin[β]^2)) ==
  (l Sqrt[(-4.` - (225.` l^8 α^8)/(1.` q^2 - 2.` q s + 1.` s^2 - 1.` l^2 α^2)^4 -
        (450.` l^6 α^6)/(1.` q^2 - 2.` q s + 1.` s^2 - 1.` l^2 α^2)^3 - (285.` l^4 α^4)/(1.` q^2 - 2.` q s + 1.` s^2 - 1.` l^2 α^2)^2 -
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        (60.` l^4 α^4 Sin[β]^2)/(1.` q^2 - 2.` q s + 1.` s^2 - 1.` l^2 α^2)^2 - (4.` l^2 α^2 Sin[β]^2)/(1.` q^2 - 2.` q s + 1.` s^2 - 1.` l^2 α^2))]),
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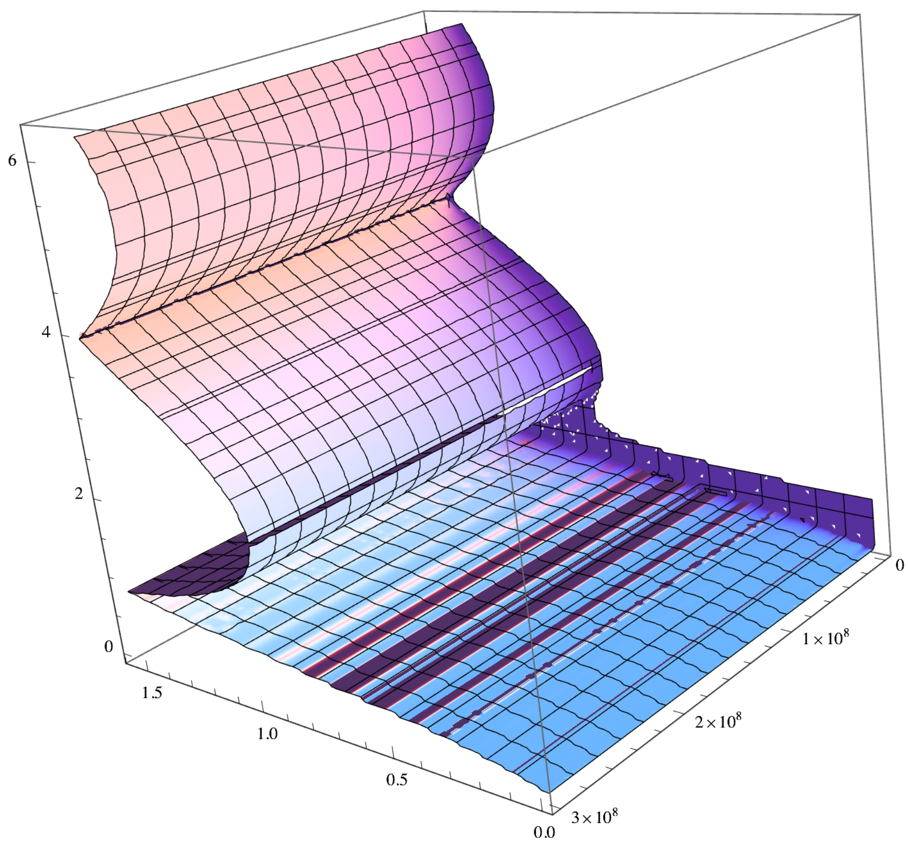
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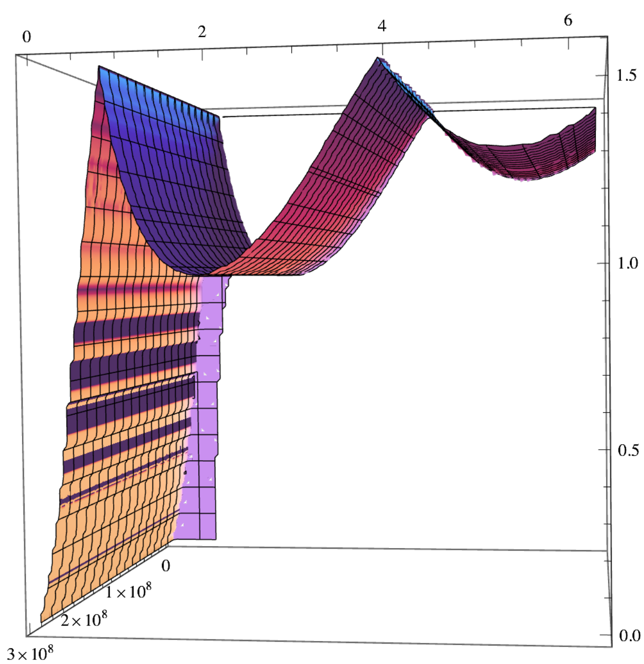
2×10^8
3×10^8


0
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1.5
1.0
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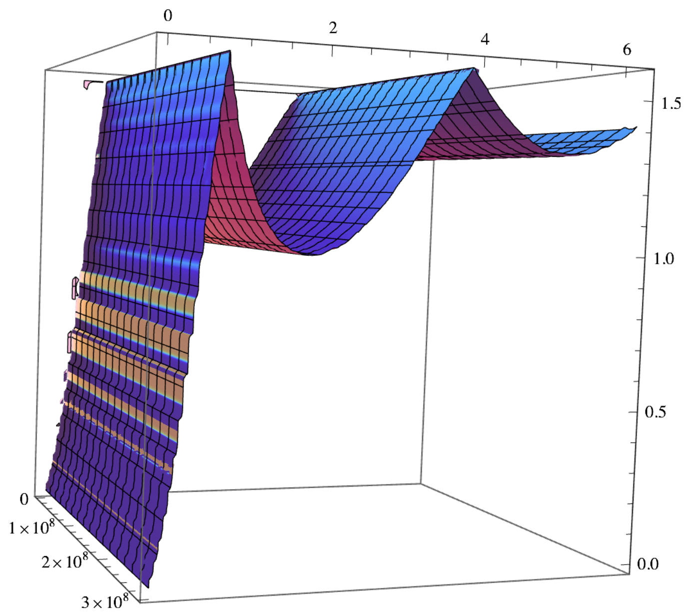
0
1×10^8
2×10^8
3×10^8



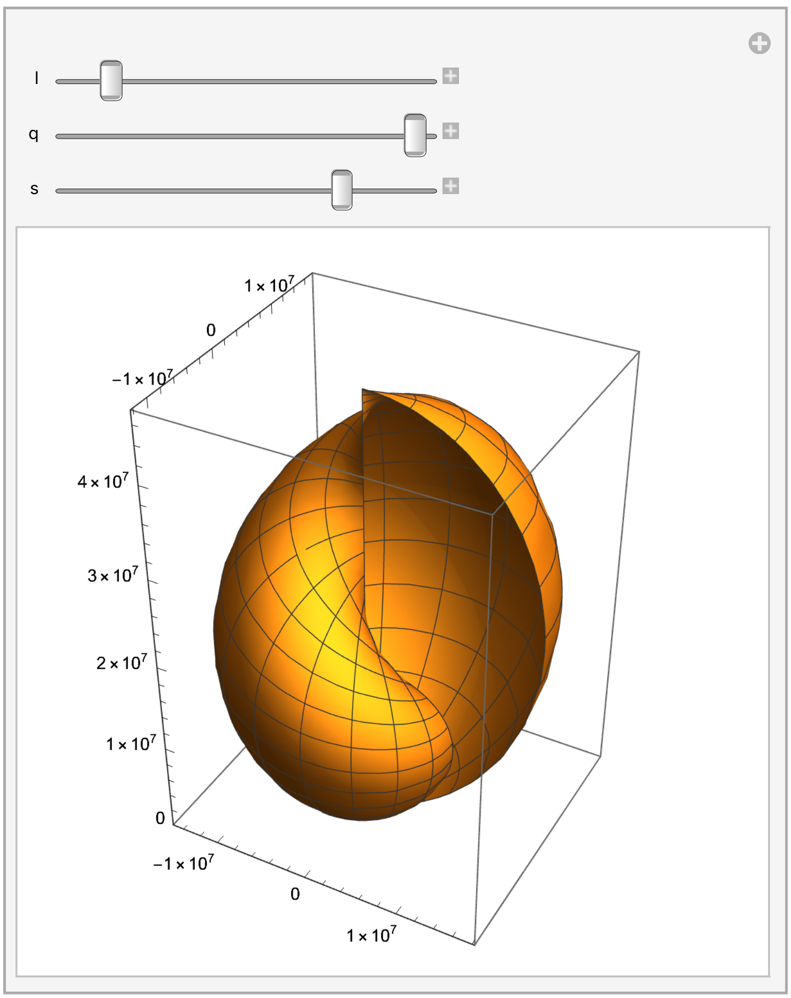
In[∘]:=
$$\frac{1}{l^2}\left(\sum_{i=1}^{6}(-1)^i\left(q^i-l^2\alpha^2 q^{i-4}\right)s^{6-i}\,\mathrm{Cos}[\beta]^2+\sum_{i=1}^{4}(-1)^i\left(3\,l^2 q^{i-2}\alpha^2 s^{4-i}\right)\mathrm{Cos}[\beta]^2-\sum_{i=1}^{2}(-1)^i\left(3\,l^4 q^{i-2}\alpha^4 s^{2-i}\right)\mathrm{Cos}[\beta]^2+l^6\alpha^6\,\mathrm{Cos}[\beta]^2\right)\left(\sum_{i=1}^{2}1^i\left(q^i-l^2\alpha^2 q^{i-2}\right)s^{2-i}+l^2\alpha^2\,\mathrm{Sin}[\beta]^2\right)^2$$

Out[∘]=
$$\frac{1}{l^2}\left(3\,l^2 q^2\alpha^2\,\mathrm{Cos}[\beta]^2-3\,l^2 q\,s\,\alpha^2\,\mathrm{Cos}[\beta]^2+3\,l^2 s^2\alpha^2\,\mathrm{Cos}[\beta]^2-\frac{3\,l^2 s^3\alpha^2\,\mathrm{Cos}[\beta]^2}{q}-3\,l^4\alpha^4\,\mathrm{Cos}[\beta]^2+\frac{3\,l^4 s\,\alpha^4\,\mathrm{Cos}[\beta]^2}{q}+l^6\alpha^6\,\mathrm{Cos}[\beta]^2+s^2\left(q^4-l^2\alpha^2\right)\mathrm{Cos}[\beta]^2-s^5\left(q-\frac{l^2\alpha^2}{q^3}\right)\mathrm{Cos}[\beta]^2+s^4\left(q^2-\frac{l^2\alpha^2}{q^2}\right)\mathrm{Cos}[\beta]^2-s^3\left(q^3-\frac{l^2\alpha^2}{q}\right)\mathrm{Cos}[\beta]^2-s\left(q^5-l^2 q\,\alpha^2\right)\mathrm{Cos}[\beta]^2+\left(q^6-l^2q^2\alpha^2\right)\mathrm{Cos}[\beta]^2\right)\left(q^2-l^2\alpha^2+s\left(q-\frac{l^2\alpha^2}{q}\right)+l^2\alpha^2\,\mathrm{Sin}[\beta]^2\right)^2$$

In[8]:=

$$\text{Manipulate}\Big[\text{SphericalPlot3D}\Big[\frac{1}{l^2}\Big(3\,l^2 q^2 \alpha^2 \text{Cos}[\beta]^2 - 3\,l^2 q\, s\, \alpha^2 \text{Cos}[\beta]^2 + 3\,l^2 s^2 \alpha^2 \text{Cos}[\beta]^2 - \frac{3\,l^2 s^3 \alpha^2 \text{Cos}[\beta]^2}{q} - 3\,l^4 \alpha^4 \text{Cos}[\beta]^2 + \frac{3\,l^4 s\, \alpha^4 \text{Cos}[\beta]^2}{q} + l^6 \alpha^6 \text{Cos}[\beta]^2 + s^2 \left(q^4 - l^2 \alpha^2\right) \text{Cos}[\beta]^2 - s^5 \left(q - \frac{l^2 \alpha^2}{q^3}\right) \text{Cos}[\beta]^2 + s^4 \left(q^2 - \frac{l^2 \alpha^2}{q^2}\right) \text{Cos}[\beta]^2 - s^3 \left(q^3 - \frac{l^2 \alpha^2}{q}\right) \text{Cos}[\beta]^2 - s\left(q^5 - l^2 q\, \alpha^2\right) \text{Cos}[\beta]^2 + \left(q^6 - l^2 q^2 \alpha^2\right) \text{Cos}[\beta]^2\Big) \left(q^2 - l^2 \alpha^2 + s\left(q - \frac{l^2 \alpha^2}{q}\right) + l^2 \alpha^2 \text{Sin}[\beta]^2\right)^2, \{\beta, 0, \pi/2\}, \{\alpha, 0, 2\pi\}\Big], \{l, 0, 5\}, \{q, 0, 5\}, \{s, 0, 5\}\Big]$$

Out[8]=



Power: Infinite expression $\frac{1}{0}$ encountered.

Power : Infinite expression $\frac{1}{0}$ encountered.

Infinity : Indeterminate expression $0\,\alpha^2\,\text{Cos}[\beta]^2$ ComplexInfinity encountered.

Power : Infinite expression $\frac{1}{0}$ encountered.

General : Further output of Power::infy will be suppressed during this calculation.

Infinity : Indeterminate expression $0\,\alpha^4\,\text{Cos}[\beta]^2$ ComplexInfinity encountered.

Infinity : Indeterminate expression $0\,\alpha^2$ ComplexInfinity encountered.

General : Further output of Infinity::indet will be suppressed during this calculation.

Power : Infinite expression $\frac{1}{0}$ encountered.

Infinity : Indeterminate expression $0.\,\alpha^2\,\text{Cos}[\beta]^2$ ComplexInfinity encountered.

Power : Infinite expression $\frac{1}{0}$ encountered.

Infinity : Indeterminate expression $0.\,\alpha^4\,\text{Cos}[\beta]^2$ ComplexInfinity encountered.

Power : Infinite expression $\frac{1}{0}$ encountered.

General : Further output of Power::infy will be suppressed during this calculation.

Infinity : Indeterminate expression $0\,\text{Cos}[\beta]^2$ ComplexInfinity encountered.

General : Further output of Infinity::indet will be suppressed during this calculation.

Future Research :

1. Generalized Transformations
Given the specific form of \(c\) that satisfies the relationship between \(F[q, s, l, \alpha]\) and the integral of \(G[q, s, l, \beta, c]\), we can investigate whether similar transformations hold for other functional forms . This exploration can lead to a more general theory of integral transformations in higher dimensions . ## ## Generalized Theorem : Consider a function \(H[q, s, l, \gamma]\) defined as : \[H[q, s, l, \gamma] = \sqrt {-q^2 + 2 q s - s^2 + l^2\gamma^2}\]
We can explore the possibility of transforming \(G[q, s, l, \beta, c]\) into \(H[q, s, l, \gamma]\) through a similar integral approach, potentially leading to a new condition on \(c\) .

2. Higher - Dimensional Analogues
The methods used in this proof can be extended to integrals in higher dimensions, i . e ., 6 - dimensional or more . The insights gained can aid in formulating and solving integrals in these higher - dimensional spaces .

Example : Consider the 6 - dimensional analog : \[\int\int\int\int\int\int G[q, s, l, \beta, c, \theta]\, dq\, ds\, dl\, d\beta\, d\theta\, dc = H[q, s, l, \gamma, \theta]\]
where the additional variable \(\theta\) introduces another layer of complexity akin to \(\beta\) .

3. Exploration of Functional Dependencies
Investigate how the specific choice of \(c\) influences the dependency structure between independent variables (\(q, s, l, \beta\)) . Identifying these dependencies can lead to new mathematical relationships or symmetries .

Study : Explore \(\partial c/ \partial q\), \(\partial c/ \partial s\), etc ., to see how small changes in \(q\) or \(s\) affect \(c\) . This can reveal deeper insights into the structure of \(c\) : \[\frac {\partial c} {\partial q}\quad\text {and}\quad\frac {\partial c} {\partial s}\]

4. Stability and Convergence Analysis
Investigate the stability and convergence of the integral and the functions involved . This can lead to new results in the convergence theory of multi - dimensional integrals, which have applications in numerical integration and computational mathematics .

Stability : Identify conditions under which the integral \(\int\int\int\int\int G\) converges and remains stable as the dimensions are scaled or altered .

5. Applications in Theoretical Physics
Apply the derived transformation to solve specific problems in theoretical physics, such as quantum field theory, where higher - dimensional integrals frequently occur .

Example Application : Consider a system described by a Lagrangian dependent on \(q, s, l, \alpha\) and \(\beta\) . The transformation can be used to simplify the Lagrangian by transforming an integral form into a simplified function \(F[q, s, l, \alpha]\) .

New Mathematical Results
To formalize some of these ideas, let' s derive one such generalization and its new mathematical result :

New Mathematical Result : ##

Proposition : Given the relationship and transformation established in the proof, we can generalize to higher - order transformations and functional dependencies . For any smooth function \(H[q, s, l, \gamma]\) similar in structure to \(F[q, s, l, \alpha]\), there exists a composite function \(K[q, s, l, \beta, c, \theta]\) and a corresponding \(c\) such that : \[\int\int\int\int\int K[q, s, l, \beta, c, \theta]\, dq\, ds\, dl\, d\beta\, d\theta = H[q, s, l, \gamma]\]
provided \(c\) satisfies the derived conditional structure .

Proof Sketch : 1. ** Define Higher - Dimensional Function : ** Let \(K[q, s, l, \beta, c, \theta]\) be an extension incorporating \(\theta\) and \(\gamma\) : \[K[q, s, l, \beta, c, \theta] = \sqrt {-c^2 (l\gamma)^2

+ c^2 q^2 - 2 c^2 s q + c^2 s^2 + c^2 (l\gamma)^2\sin (\beta)^2 + c^2\theta^2}\]

2. ** Transform and Integrate : ** Show the equivalence to : \[H[q, s, l, \gamma, \theta] = \sqrt {-q^2 + 2 q s - s^2 + l^2\gamma^2 + \theta^2}\]
under the specified integral transformation .

3. ** Derive General Condition on \(c\) : ** Using similar differentiation approach as in the original proof, derive the condition on \(c\) such that the integral holds true . This proposition opens avenues for exploring more generalized transformations and integral relationships in multi - dimensional calculus, contributing to the advancement of mathematical knowledge in this domain . By pursuing these directions, we can derive new insights, generalize existing results, and potentially discover novel mathematical structures and their applications .