Sand Drawings as Mathematics

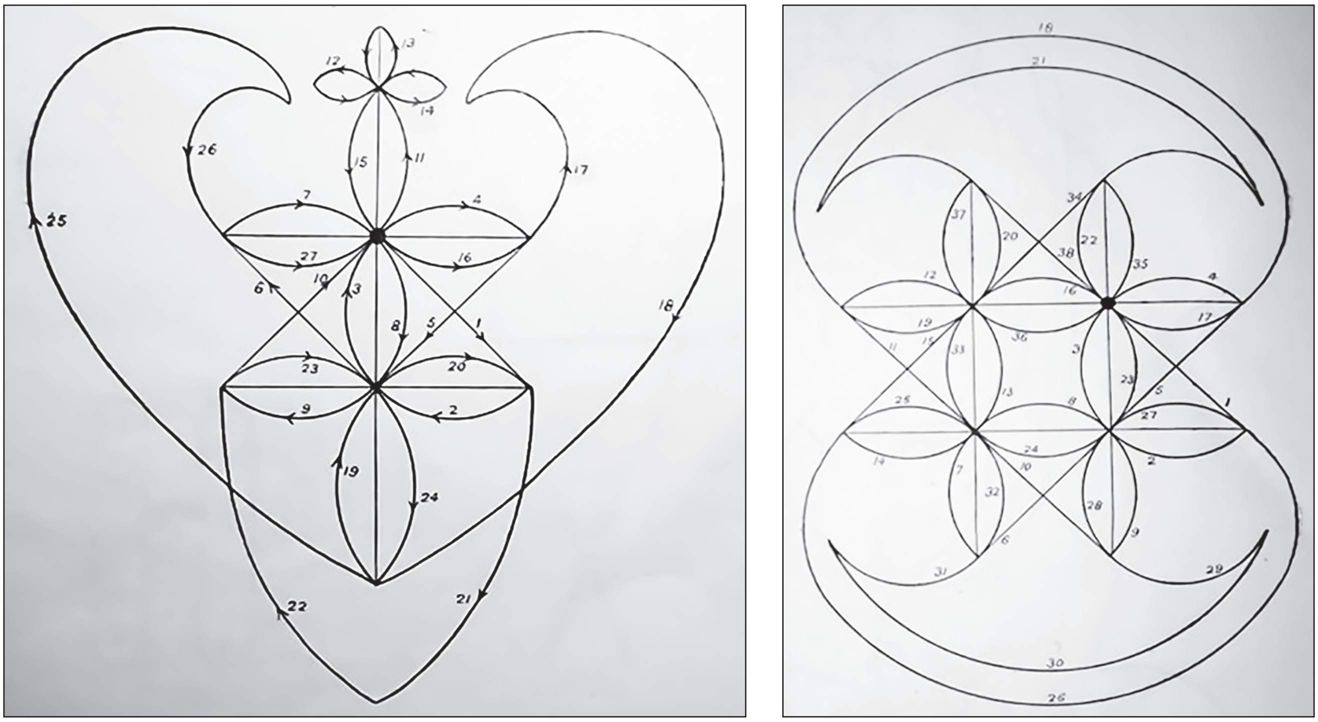
By **Andrew English**

During 1926 and 1927 the brilliant young Cambridge anthropologist Bernard Deacon (1903–1927) was on the island of Malekula in the New Hebrides (now Vanuatu) making an ethnographical study. Reporting on the progress of his research to A. C. Haddon (1855–1940), the recently retired Reader in Ethnology at Cambridge, he wrote enthusiastically of 'the remarkable geometrical figures' he had seen being drawn by native men, of which he had already 'collected some forty-five' (Deacon, 1934b, p. 129). Two such figures (pp. 155 & 156) were 'The Squid' (*Nooit Treverep*) and 'The Oyster' (*Netundong*).

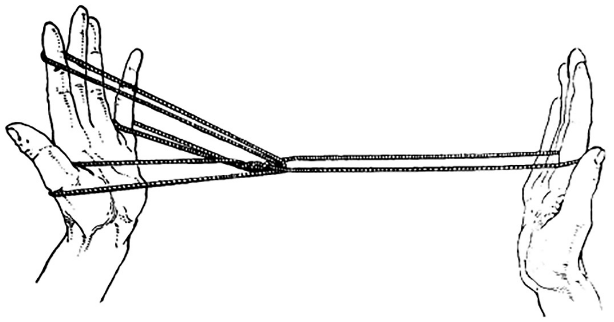
'The whole point of the art,' writes Deacon, 'is to execute the designs perfectly, smoothly, and continuously; to halt in the middle is regarded as an imperfection'.

> I have photographed and got native drawings of some of the designs, but to the native these are not interesting, what is interesting is my execution of designs which I have taken down and numbered. Each design is regarded rather as a kind of maze, the great thing is to move smoothly and continuously through it from starting point to starting point (Deacon, 1934b, pp. 129-130).

So, the resulting design did not much interest the islanders. It was the execution of the design that interested them: 'process and result' (Wittgenstein, 1922, § 6.1261). Haddon already had some knowledge of these sand drawings, because another anthropologist John Layard (1891–1974) had sent to him seven examples in 1915 from the 'Small Islands' fringing the north-east coast of Malekula (Deacon, 1934b, p. 143). Haddon had also learned at first hand what it is to 'collect' such things as sand drawings, for in 1898 he had collected numerous string figures in the islands of the Torres Straits, another Melanesian archipelago (Haddon, 1912, pp. 320–341). And to collect a string figure, a design woven on the fingers using a six-foot loop of string, is likewise to collect a *procedure* and not merely its perceptible end point. Here again 'process and result are equivalent' (Wittgenstein, 1978, I, § 82).



The Squid and the Oyster



Fish-spear (Rouse Ball, 1921, p. 30)

One of the first books on string figures *Cat's Cradles from Many Lands* (1911) was written by Haddon's daughter Kathleen, and when Trinity mathematician W. W. Rouse Ball lectured on string figures at London's Royal Institution in 1920 it was Kathleen who concluded the lecture by demonstrating various examples from around the world. The simple procedure for constructing Fish-spear (*Bauer*), which her father had collected in the island of Mer, is as follows, where 'proximal' and 'distal' mean roughly 'upper' and 'lower', or more accurately 'fingertipwards' and 'wristwards'. Begin with the palms facing each other, fingers pointing upwards, with only the thumbs and little fingers inside the loop of string.

> Take up with the right index the transverse string on the left palm from its proximal side, give it one twist and return. Pass the left index through the right index loop from the distal side, and take up the transverse string of the right hand from the proximal side and return through the loop. Drop the thumb and little-finger loops of the right hand and draw the hands apart (Haddon, 1911, p. 7).

The resulting figure is a string trident.

Haddon's translation of string figure procedures into this 'unambiguous nomenclature' (Rouse Ball, 1921, p. 15), worked out in collaboration with his colleague W. H. R. Rivers (1864–1922) while on expedition, can be seen in the context of later developments (see Vandendriessche, 2015) as a significant step towards an equivalent 'sign-technique' (Wittgenstein, 1978, IV, § 19) for string figures. Deacon's numbered and arrowed diagrams are more obviously on the way to becoming a mathematical equivalent for the Malekulan sand drawing procedures. The possibility of such rigorous translation between sign-techniques seems distinctive of mathematics. We need only recall how pen and ink arithmetic replaced abacus arithmetic in Renaissance England and elsewhere in Europe.

Deacon died of blackwater fever on Malekula in March 1927. His field notes were written up and edited by his friend the anthropologist Camilla Wedgwood. 'The art of drawing these designs', reports Deacon, 'is handed down from generation to generation, and it requires considerable skill and practice. Not only are there very many designs to memorize – some of the older men executed twenty to thirty drawings one after the other – but many of them are extraordinarily complicated.' The handing down of such procedures evidently involves clear explanation, faithful copying and careful memorization. Our own techniques of arithmetical calculation and geometrical construction are handed down in the same 'perspicuous' fashion (Wittgenstein, 1978, III, § 1), though memorization is less crucial in our written tradition. 'A rectangular area', Deacon explains, 'is made level and smooth on the sand, or ashes are spread over the surface of the smoothed earth.'

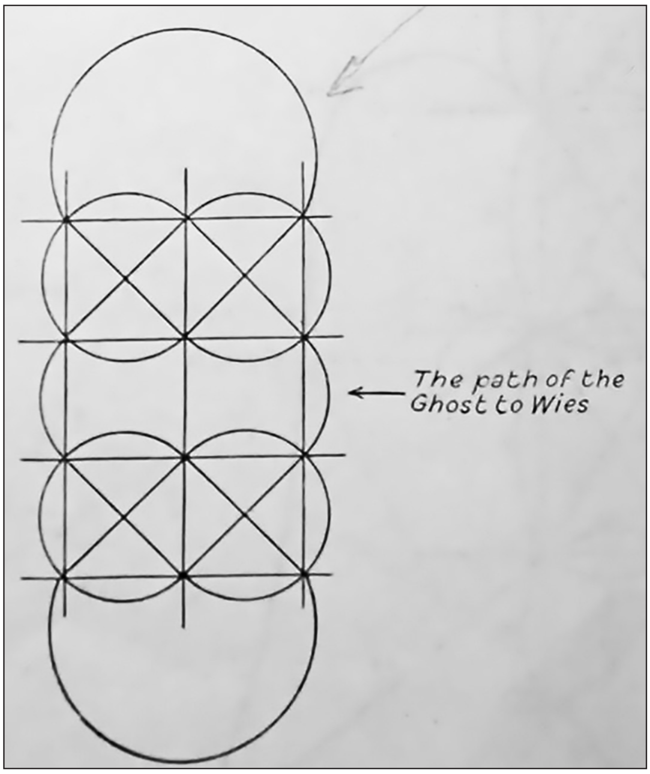
> A framework is then drawn, around which the design is traced. The framework may consist of a number of lines running at right angles to each other, thus producing a number of squares or oblongs, or the rectangles may be indicated by a dot or cross at their angles; or else a number of dots are arranged symmetrically.

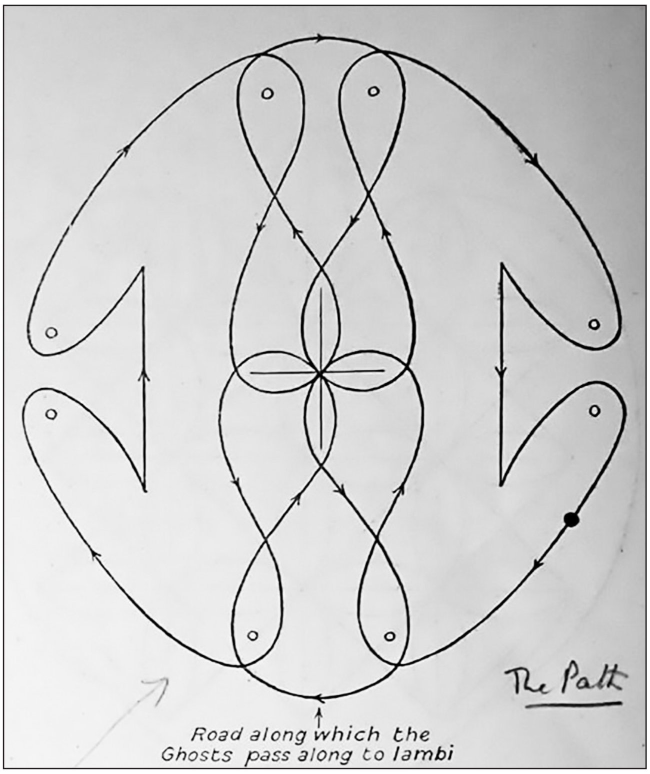
'Having finished his preparations', Deacon continues, 'the artist proceeds with his main task'.

> With his forefinger he traces around the framework curves, circles, and ellipses. In theory the whole should be done in a single, continuous line which ends where it began; the finger should never be lifted from the ground, nor should any part of the line be traversed twice (Deacon, 1934b, pp. 132-133).

The Squid, drawn by the author,
Swansea Bay, June 2022

Readers of Rouse Ball's *Mathematical Recreations and Essays* (1892), a book once commonly chosen as a school prize in mathematics, would have recognized in Deacon's description the 'Unicursal Problems' in Rouse Ball's chapter of that title. There the mathematician also points out the applicability to mazes of Euler's familiar





Nahal and *Nevet Hor Lambi* (Deacon, 1934b, p. 162, Layard's pencil annotations)

results. But the Malekulans need not have been aware of the evenness of the 'nodes' in their most characteristic designs, such as The Squid and The Oyster, nor troubled by the designs in Rouse Ball having more than one pair of odd 'nodes', designs they would never actually draw (Nissen, 1988). The Squid, for example, is constructed 'unicursally', and so it can obviously be traced 'unicursally', that is, the whole of it can be described in one continuous movement without going over the same line twice. So, we may choose to look elsewhere for an understanding of Malekulan sand drawings *as mathematics*. (See English, 2022 for the same approach, applied in greater detail, to string figures.)

Layard saw in the great variety of Malekulan designs the unstereotyped stage of a once-significant ritual performance, a performance that had since become a pure game of skill. 'Indeed', says Layard, 'now that they are used as games of skill this tendency, as with cats' cradles, with which they are probably allied, now serves to stimulate the creative ingenuity of the natives, who, while certain designs, of course, become traditional and are copied over and over again, constantly invent new ones as an intellectual pastime' (Layard, 1942, p. 654). But to invent a new design, and then to name it and to have it accepted is to introduce a new mathematical concept. For the design becomes in this way a standard of correctness of its own procedure. Thus, if we see a mistake in the final figure, we assume that the artist *must* have made a mistake in the procedure, not merely a mistake in the performance, such as a hesitation, but a *mathematical* mistake, say the addition of a loop. Arithmetical calculations and geometrical constructions, for example Euclid's construction of the pentagon, are dealt with by us in the same way (Wittgenstein, 1978, IV, §§ 29-30).

Layard discerns the original significance of the Malekulan designs in two examples (shown above), both labelled by him 'The Path' in his own copy of Deacon's article: the first, *Nahal*, from Seniang and the second, *Nevet Hor Lambi*, from Lumbumbu (see Layard, 1942, chapter 25). The islanders of both districts shared a belief in the journey of the dead. In Seniang, for instance, the ghosts of the dead were said to pass along a road to the land of the dead, called *Wies*. Along this road they had to pass a rock inhabited by a dreaded ogress *Temes Savsap*. Traced in the sand before her is the figure *Nahal*, the route to *Wies* going through its middle.

> As the ghost approaches, *Temes Savsap* wipes out half of the tracing and tells the traveller that before he proceeds any further he must complete the diagram correctly. Most men during their life-time have learnt how to make this and other geometrical figures, and so, on their death, they are able to do as *Temes Savsap* tells them and pass safely on their way. But should a man be ignorant of how to complete the figure, *Temes Savsap* seizes him and devours him so that he can never reach the land of *Wies* (Deacon, 1934a, pp. 554-555).

Layard conjectured that The Path is a labyrinth that once represented the *wandering* route taken by ghosts to the land of the dead, and was thus a symbol of rebirth, something like 'the turf mazes which the village boys run round in our own country' (Layard, 1942, p. 653), usually adjacent to churches – yet, either way, an application, a use, in the land of the living would serve better to confirm this uncommonly beautiful 'sign-game' (Wittgenstein, 1978, V, § 2) as 'mathematics'.

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