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"This is a unique book in its scope, approach, and method. A novel physical and philosophical interpretation of sheaf theory sheds new light on the quantum measurement problem, entanglement, locality, and truth. A new systematic and rigorous relational realistic paradigm for natural philosophy has emerged, rooted in the same principles as Abstract (Modern) Differential Geometry, which transmutes the above into a full-fledged dynamical theory."
—**Anastasios Mallios**, professor emeritus of mathematics, University of Athens

If there is a central conceptual framework that has reliably borne the weight of modern physics as it ascends into the twenty-first century, it is the framework of quantum mechanics. Because of its enduring stability in experimental application, physics has reached heights that not only inspire wonder but arguably exceed the limits of intuitive vision, if not intuitive comprehension. For many physicists and philosophers, however, the currently fashionable tendency toward exotic interpretation of the theoretical formalism is recognized not as a mark of ascent for the tower of physics, but rather as an indicator of sway—one that must be dampened rather than encouraged if practical progress is to continue.

In this unique volume, designed to be comprehensible to both specialists and non-specialists, Michael Epperson and Elias Zafiris chart out a path forward by identifying the central deficiency in most interpretations of quantum mechanics: that in its conventional, metrical depiction of extension, inherited from the Enlightenment, objects are characterized as fundamental to relations—that is, relations presuppose objects but objects do not presuppose relations. The authors, by contrast, argue that quantum mechanics exemplifies the fact that physical extensiveness is fundamentally topological rather than metrical, with its proper logico-mathematical framework being category theoretic rather than set theoretic. By this thesis, extensiveness fundamentally entails not only relations of objects but also relations of relations. The fundamental quanta of quantum physics are thus properly defined as *units of logico-physical relation* rather than merely units of physical relata as is the current convention. The conventional notion of a history as "a story about fundamental objects" is thereby reversed, such that the classical "objects" become the story by which we understand physical systems that are fundamentally histories of quantum events.

MICHAEL EPPERSON is director of the Center for Philosophy and the Natural Sciences in the College of Natural Sciences and Mathematics at California State University, Sacramento, where he is research professor and principal investigator.

ELIAS ZAFIRIS is research professor in theoretical and mathematical physics at the Institute of Mathematics at the University of Athens and visiting professor of logic at the Institute of Philosophy, Eötvös Loránd University in Budapest.

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EPPERSON
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FOUNDATIONS OF RELATIONAL REALISM

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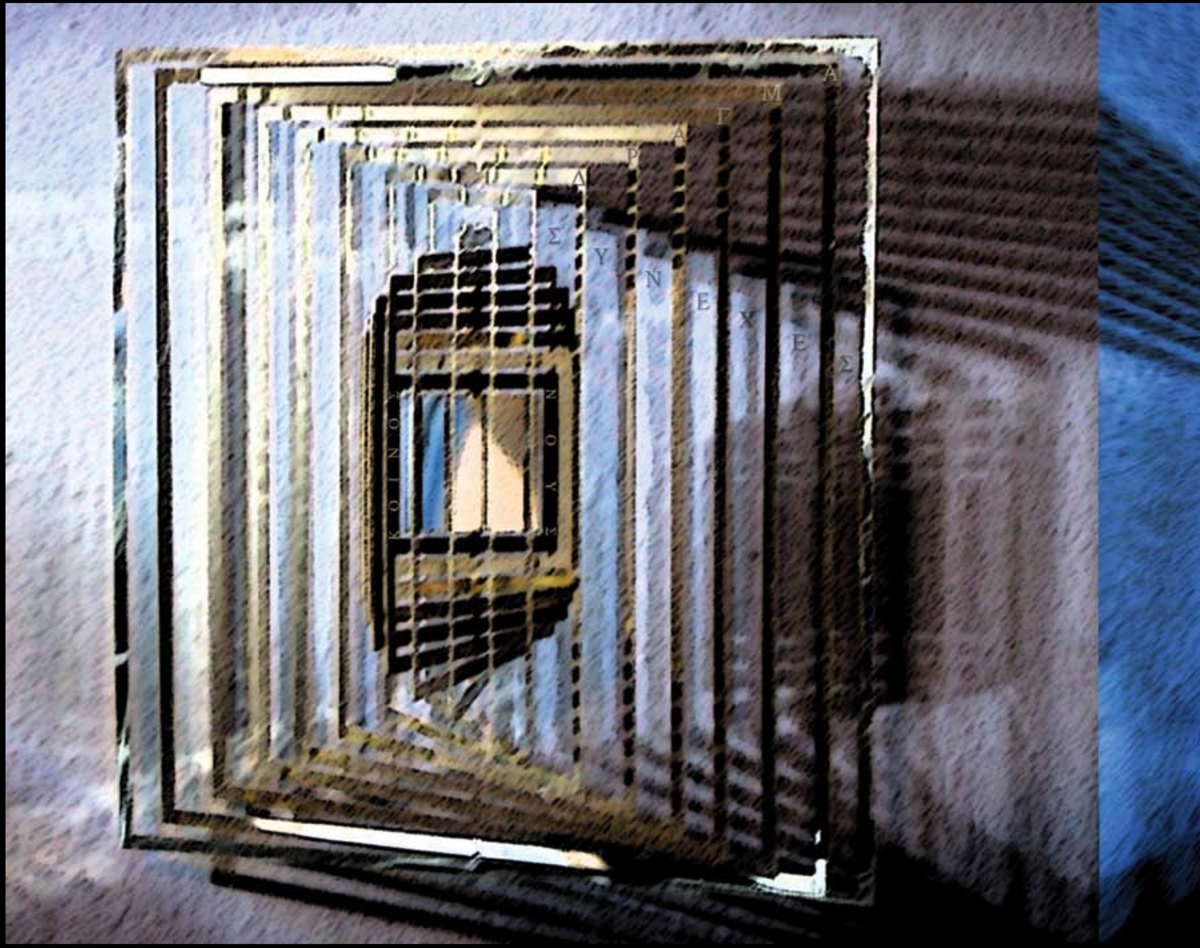
A TOPOLOGICAL APPROACH TO QUANTUM MECHANICS AND THE PHILOSOPHY OF NATURE



MICHAEL EPPERSON AND ELIAS ZAFIRIS

Foundations of Relational Realism

A Topological Approach to Quantum Mechanics and the Philosophy of Nature

Michael Epperson and Elias Zafiris

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Elias Zafiris

PREFACE

If there is a central conceptual framework that has reliably borne the weight of modern physics as it ascends into the twenty-first century, it is the framework of quantum mechanics. Because of its enduring stability in experimental application, physics has today reached heights that not only inspire wonder, but arguably exceed the limits of intuitive vision, if not intuitive comprehension. Indeed, it is the dizzying aspect of the ascent that many physicists and philosophers have taken as quantum theory's emblematic mark of achievement; the theoretical fruits of its complex formalism are, by this inclination, selectively interpreted as nothing less than the scientific discovery of a heretofore concealed and counterintuitive physical reality—a strange new world spanned by the extremes of quantum singularity on one end, and multiverse on the other.

For a great many other physicists and philosophers, however, this currently fashionable tendency toward exotic interpretation of the theoretical formalism is recognized not as a mark of ascent for the tower of physics, but rather as an indicator of sway—one that must be dampened rather than encouraged if practical progress is to continue. Indeed, it is the great irony of modern physics that a foundation so stable and reliable as quantum mechanics has proven so vulnerable to the attractor of exotic interpretation. One reason is that after over a century of development, the key conceptual and interpretive problems remain unsettled, even in the wake of evolutionary improvements in technology and experimental methodology. Among these now infamous and interrelated problems are: [1] the problem of measurement—i.e., the dependency of measurement objectivity upon measurement contextuality; [2] quantum nonlocality—e.g., a nonstandard, nonlocal conditional probability such as $P\ (B|A)$ (the probability of event B given event A) when these events are space-like separated, such that event *A* somehow effects a faster-than-light probability conditionalization of event *B*. (It should be emphasized that classical conditional probabilities are not the same as quantum conditional probabilities, though they are sufficiently analogous to invoke the simpler, classical notation here); [3] the problem of coherently integrating the notions of discreteness and continuity in fundamental physical theories. This is most notoriously exemplified in the problem of integrating 'quantum' and 'classical' physical theories—i.e., 'classical' in the sense of presupposing a physical continuum, with no quantization paradigm for the evaluation of observables. Newtonian mechanics, as well as Einstein's special and general theories of relativity, are both considered classical in this sense.

In the present volume, we demonstrate how all three of these difficulties can be properly understood as interrelated aspects of a single problem: The absence in quantum mechanics of a formal means of depicting local-global relations in an extensive continuum. As mentioned above, while this problem is most popularly instantiated as the incompatibility of quantum mechanics and the general theory of relativity (an aspect of problem [3] above), we argue that its proper solution lies first in recognizing the centrality of local-global relations in *all three* of the aforementioned problems; and second, in recognizing that the overall genesis of difficulty is the presumption of a fundamentally metrical theory of extension grounded in a set-theoretic conceptual framework, the latter being incapable of adequately representing the essential logical and algebraic structural features of quantum mechanics. To be sure, the set theoretic framework has proven extremely fruitful for formal physics throughout the twentieth century; but its adequacy as a conceptual foundation for modern physics is belied not only by the incompatibility of quantum theory and general relativity, but more deeply by the fact that it renders physics vulnerable to paradoxes, singularities, and infinities.

In attending to these incoherent structures, physics has two possible routes forward: It can either incorporate them into its models by proposing their exotic *physical* instantiation—e.g., the paradoxical violation of the principle of non-contradiction in the form of 'physical superpositions' of actual system states in quantum mechanics, or the notion of 'physical singularities' in modern cosmology, etc.; or it can identify and overcome the root source of these vulnerabilities, rendering them epistemic artifacts akin to the multitude of others so explicated throughout the history of science.

In the present volume, we proceed to chart out a pathway for this latter course by first identifying the central deficiency of the conventional metrical, set-theoretic notion of extension: *That it characterizes objects as fundamental to relations*—i.e., such that relations presuppose objects but objects do not presuppose relations. Prior to quantum mechanics, this deficiency went mostly unnoticed; but since it is a signature feature of quantum mechanics that it definitively proscribes specifying the existence of objects *in abstraction* from their relations (viz., system states in abstraction from measurement), the attempt to depict quantum mechanical extensiveness as fundamentally metrical via a set-theoretic formalism—again, such that objects are more fundamental than relations—is, we believe, doomed from the beginning.

The solution we propose begins with the following thesis: Quantum mechanics exemplifies the fact that physical extensiveness is fundamentally topological rather than metrical, with its proper logico-mathematical framework being category theoretic rather than set theoretic. By this thesis, extensiveness fundamentally entails not only relations of objects, but also relations of relations; thus fundamental quanta are properly defined as 'units of logico-physical relation' rather than merely 'units of physical relata.' Objects are, in this way, always understood *as* relata, and likewise relations are always understood objectively. Objects and relations, in other words, are coherently defined as mutually implicative.

In this unique two-part volume, designed to be comprehensible to both specialists and non-specialists, we substantiate this thesis by demonstrating that a revised decoherent histories interpretation of quantum mechanics, structured within a category-theoretic topological formalism, provides a coherent and consistent conceptual framework by which local quantum events can be globally internally related both causally and logically. Further, this framework allows for a quantum mechanical description of spatiotemporal extension that is highly compatible philosophically with the mereotopological model proposed by Alfred North Whitehead, refining and enhancing the latter by elevating it from a set-theoretic basis to a category-theoretic one.

As a philosophical enterprise, we propose this conceptual framework as a speculative ontological program that includes a rigorous mathematical formalism. This provides a uniquely powerful approach to solving the three critical problems of quantum mechanics discussed above, as well as others, by integrating their solution within a coherent and intuitive ontological scheme that is both novel and applicable practically to the physical sciences.

The central thesis of the relational realist speculative philosophical program introduced in this volume is that the classical, conventional conception of the relationship between [a] ‘*physical object*’ *as ontological extant*, and [b] ‘*history of facts*’ *as epistemic construct* by which physical objects are characterized, must be reversed if quantum mechanics is to be coherently understood as an ontologically significant theory. That is, the classical conception of a history as essentially contextual and therefore primarily epistemic—a particular story expressing particular knowledge of fundamental physical objects—must be reconceived, such that physical objects are not merely *understood by* their fundamental histories, but rather *understood as* fundamental histories of quantum events. This requires a novel reconceptualization of ‘ontological’ and ‘contextual’ as mutually implicative features of every quantum event, wherein the latter is understood as the fundamental, concrete constituent by which the natural world is physically and logically describable.

This primacy of objective quantum events (alternatively, quantum ‘facts’) is, in the opinion of the authors, the critical starting point for any attempt to provide a viable ontological interpretation of the standard formal framework of quantum theory. In this framework, quantum events are identified as measurement outcomes referent to corresponding physical observables. The theory then provides the means of relating these events. In this respect, the conceptual complexity of any ontological interpretation of quantum theory stems from two factors: [1] The actualization of a measurement outcome event representing the state of a quantum system, though always globally objective, can only be predicted probabilistically and *contextually*—i.e., relative to a particular *local Boolean measurement context* of a selected observable (that is, a context wherein measurement outcomes can be expressed as mutually exclusive and exhaustive true / false propositions). It is always via such local Boolean contextuality that the universe, represented by a global state vector, is decomposed into 'system,' 'measuring apparatus,' and 'environment' with their respective state vectors. Equally important, the probability valuations can be affirmed only retrodictively, *after* a measured result (i.e., a novel fact / unique actualization of a potential outcome state) has been registered by the corresponding measuring apparatus. [2] The totality of events related to the behavior of a quantum system cannot be represented within the *same* local Boolean measurement context due to the property of non-commutativity of quantum observables.

These two factors together necessitate a thorough rethinking of our conceptual and mathematical representation of the notion of a physical continuum suited to the quantum domain of discourse. The first factor illustrates the prominent role that *potential* relations play in the process of quantum measurement: Quantum mechanics always entails the evolution of [a] *potential* outcome states that *cannot* be integrated in terms of classical, Boolean logic (e.g., Schrödinger's Cat is alive *and* dead) to [b] *probable* outcome states that *can* be integrated in terms of classical, Boolean logic (Schrödinger's Cat is alive with probability x, *or* dead with probability y; and since $x + y = 1$, one and only one of these unique outcomes *must* occur). Apart from a local Boolean measurement context, the evolution of potentia to probability cannot ensue; and as we will argue, this evolution is the central engine of quantum mechanics, without which locally contextualized events cannot be integrated into the globally objective, logically consistent histories ubiquitous to experience (empirical or otherwise).

The second factor likewise illustrates the contextual significance of the empirical requirement of preparation procedures for the evaluation of observables via quantum measurement: The determination, by these procedures, of the *initial actual state* of the system and measuring apparatus establishes the local contextualization definitive of their *final actual state* terminal of the measurement. This presupposed contextual correlation is typically understood as an instrumental desideratum rather than a philosophical one—that is, a desideratum satisfied via an ad hoc conceptual intervention arbitrarily imposed upon the standard formalism (viz., von Neumann's projection postulate). By contrast, we will demonstrate that this instrumental desideratum can instead be interpreted both philosophically and logico-mathematically as a *necessarily presupposed feature of quantum mechanics* when these contextual correlations are depicted topologically—viz., a topological localization of quantum observables with respect to local Boolean contexts.

It must be emphasized that the philosophical implications of these two factors cannot easily be separated from their practical implications. This fact has previously been explored extensively by the authors individually—Epperson in his work relating the logical, mathematical, and philosophical features of Alfred North Whitehead's work to the decoherent histories interpretations of quantum mechanics;[1] and Zafiris in his modeling of the topological features of quantum mechanics by way of category-sheaf theory.[2] The deep compatibility underlying these efforts has motivated the authors to seek an integrated philosophical and mathematical framework, which we have termed 'relational realism,'[3] that advances refined conceptions of internal relation and extensive connection in Whitehead's process theory, amenable to a topological reinterpretation, and synthesizes these with the framework of category-sheaf theoretic Boolean localization systems viewed from a process standpoint.

To glimpse the essence of this synthesis, the reader is invited to imagine a conceptual triangle, where on the top node sits quantum theory (formulated in the usual Hilbert space language or in the decoherent histories language) and on the two bottom nodes sit the philosophical structure of Whitehead's process theory on one side, and the mathematical structure of category-sheaf theory on the other. The correlation of the quantum node with the process-theoretic node, as it has been presented by Epperson, finds its conceptual correspondence to the correlation of the quantum node with the category-sheaf theoretic node, as it has been presented by Zafiris—a correspondence both in terms of terminology and interpretation. Thus, the conceptual triangle commutes as we circulate around it, allowing for not only a detailed correlation of the process-theoretic node with the sheaf-theoretic node, but one that also preserves the already established correlations between each of these and the quantum node.

The focus of the present volume is the demonstration of the applicability and ultimate relevance of this synthetic scheme toward a coherent and empirically adequate ontological interpretation of quantum theory, and in this regard, a subtle balance is kept between the philosophical aspects and the corresponding mathematical aspects as these converge. To that end, part I introduces the philosophical foundations of relational realism as instantiated in quantum mechanics, the latter presented in introductory fashion so as to be suitable for non-specialists in physics or mathematics. Part II, in turn, further elaborates this conceptual framework in formal mathematical language appropriate for specialists in physics and mathematics, such that philosophical scheme's technical applicability to modern physics can be rigorously demonstrated. Despite this structure, however, each part is properly understood only within the context of the overall synthesis.

It is important to note that the construction of this synthesis does not follow a strictly linear course. Instead, it unfolds in such a way so as to show, from a convergence of both philosophical process theoretic and mathematical category-sheaf theoretic views, the adequacy and indeed advantage of the relational realist framework in the development of a coherent ontological interpretation of quantum mechanics—one that can be understood as exemplifying, more broadly, a coherent and empirically adequate philosophy of nature.

To be sure, the conceptual breadth of both Whitehead's process theory and categorical sheaf theory far exceeds their restricted application to quantum theory as explored herein, and we consider this to be advantageous for our objectives. By employing these rich conceptual philosophical and mathematical frameworks, taking particular advantage of their systematic coherence, we hope to contribute to the formulation of a realistic ontological interpretation of quantum mechanics that avoids the paradoxes, inconsistencies, and counter-intuitiveness typical of many alternative interpretations.

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Notes

1. See, for example: Michael Epperson, *Quantum Mechanics and the Philosophy of Alfred North Whitehead* (New York: Fordham University Press, 2004); Michael Epperson, "Quantum Mechanics and Relational Realism: Logical Causality and Wave Function Collapse," *Process Studies* 38, no. 2 (2009): 340-66.

2. See, for example: Elias Zafiris, "Quantum Event Structures from the Perspective of Grothendieck Topoi," *Foundations of Physics* 34, no. 7 (2004): 1063-90; Elias Zafiris, "Generalized Topological Covering Systems on Quantum Events' Structures," *Journal of Physics A: Mathematical and General* 39, no. 6 (2006).

3. Relational realism and its topological interpretation of quantum mechanics ought not be confused with the 'relational quantum mechanics' program of Carlo Rovelli. [See, for example: C. Rovelli, "Relational quantum mechanics," *International Journal of Theoretical Physics* 35 (1996): 1637-78.] Many aspects of both programs are sufficiently compatible for fruitful conversation; however the underlying conceptual frameworks and philosophical implications of each are very different.

PART I

Philosophical Foundations
of
Quantum Relational Realism

Elementary quantum relational events, logical causality,
mereotopological extension, and the internal relation of
the local to the global in quantum mechanical predication

MICHAEL EPPERSON
A.M., Ph.D. (University of Chicago)

CHAPTER 1

Introduction

Relational Realism: A Program in Speculative Philosophy

> Speculative Philosophy is the endeavour to frame a coherent, logical, necessary system of general ideas in terms of which every element of our experience can be interpreted . . . It is the ideal of speculative philosophy that its fundamental notions shall not seem capable of abstraction from each other. In other words, it is presupposed that no entity can be conceived in complete abstraction from the system of the universe, and that it is the business of speculative philosophy to exhibit this truth.[1]
>
> *Alfred North Whitehead*

The chief mark of progress in the evolution of a philosophical worldview, whether its foundation be scientific, humanistic, theistic, or some integration of these, is the extent to which it is able to coherently accommodate diverse categories of inquiry without either arbitrary dispensation or internal contradiction. The precise manner in which a worldview deals with incommensurable categorical principles and the internal contradictions and paradoxes often borne of them is of first importance, both to the proper understanding of the particular worldview and its implications, and also to its evaluation in contrast to competing worldviews. Indeed, one could argue that the emblematic feature of any philosophical genre in the history of Western philosophy, if not all philosophy, is its method of attending to the conceptual incompatibilities within its scope.

By that general metric, there have been two dominant approaches[2] by which principles incommensurable when treated categorically—e.g., objectivity and subjectivity, necessity and contingency, conceptual and physical, infinite and finite, global and local, continuous and quantum, potential and actual—have been accommodated in Western thought:

[1] Their categorization as fundamentally *mutually implicative* at some deeper level of analysis:

By this method of 'dipolar' relation it is explicitly recognized that the conception of one principle necessarily requires reference to its counterpart principle. Thus, each relatum constitutive of dipolar conceptual pairs is always contextualized by both the other relatum *and* the relation as a whole, such that neither the relata (the parts) nor the relation (the whole) can be adequately or meaningfully defined apart from their mutual reference. It is impossible, therefore, to conceptualize one principle in a dipolar pair in abstraction from its counterpart principle. Neither principle can be conceived as 'more fundamental than,' or 'wholly derivative of' the other.

Mutually implicative fundamental principles always find their exemplification in both the conceptual and physical features of experience. One cannot, for example, define either positive or negative numbers apart from their mutual implication; nor can one characterize either pole of a magnet without necessary reference to both its counterpart and the two poles in relation—i.e., the magnet itself. Without this double reference, neither the *definiendum* nor the *definiens* relative to the definition of either pole can adequately signify its meaning; neither pole can be understood in complete abstraction from the other.

[2] Their categorization as fundamentally *mutually exclusive* at the 'deepest possible' level of analysis:

By this method of 'bipolar' relation it is implied (often erroneously) that the definition of one principle does not necessarily entail reference to its counterpart principle. Therefore it is possible to conceptualize one principle as fundamental to the other. The Platonic dualism separating conceptual and physical objects is perhaps the preeminent example in Western philosophy. Its later rehabilitation in the Cartesian dualism of thought (mind) and extension (matter) is, many would argue, a central feature of the modern Western worldview, given the popular dualistic characterization of mind and brain, or soul and body. Familiar examples of bipolar relation in physics include the classical attempt to characterize basic physical processes as either fundamentally continuous or quantum, fundamentally wavelike or particulate, etc.

1.1 Bipolar Dualism: Reducing Contrast to Mutually Exclusive Relata

When one considers the long lineage by which one might trace the ideas of the ancient Milesians through the ideas of Parmenides and Heraclitus, and onward through Plato and Aristotle, Descartes and the rationalists and Locke and the empiricists, all the way to Bohr and von Neumann, it is clear that the relation of contrasting categorical principles as mutually exclusive relata (Method 2 above) has increasingly dominated Western thought over the centuries. This domination has today become especially acute thanks to its conventional projection onto the hypothetico-deductive and reductive scientific method. Indeed, despite a variety of schematizations throughout the history of philosophy, the general thesis of mutually exclusive fundamental principles, both ontological and epistemic, has arguably become in the twenty-first century the defining characteristic of practically every prevailing scientifically informed worldview. One further sees that there have been two general modes by which mutually exclusive fundamental principles have been brought into relation by this method:

[2a] The qualification of mutually exclusive fundamental principles as *complementary* and *irreducible* characteristics of a more general, unified, and necessarily transcendent ontology that lies beyond the scope of rational systematization:

As exemplified by modern science, for example, Bohr's principle of complementarity describes the fundamental physical properties of reality in terms of conjugate pairs of properties. Epistemically, these pairs are mutually implicative since they are Fourier transform pairs; thus a more precise specification of 'particle position' necessarily entails a less precise specification of 'particle momentum.' But ontologically, the matter becomes more complicated, since the conception and definition of particle position does not require reference to the concept of momentum; whereas the conception and definition of momentum does require reference to the concept of position. As will be seen in the discussion of Method 2b, below, such incongruities often appear to justify the attempt to assimilate one concept to the other—one aspect of nature to another. The present case of position and momentum, for example, is an echo of Parmenides' attempt to assimilate the *appearance of change* to the *fact of permanence*, versus Heraclitus's attempt to assimilate the *appearance of permanence* to the *fact of change*. Similarly, position-momentum complementarity finds its reflection in one of the most profound problems in modern physics—reconciling quantum theory's depiction of nature as fundamentally discrete, and general relativity's depiction of nature as fundamentally continuous. Efforts to quantize the continuum via the quantization of gravity can, in this sense, be seen as an effort to assimilate or reduce the continuous in nature to the discrete.

A cleaner example of complementarity, closely related to that of position and momentum, yet (seemingly) immune to the above difficulty and its solution via assimilation, is wave-particle complementarity, where neither wave nor particle requires direct reference to its counterpart for its definition. Each concept is therefore independent of the other and, in terms of its ontological significance, incommensurable in complementary relation to its counterpart.

In answer to these and other ontological difficulties associated with the concept of complementarity, Bohr's solution was to circumvent them altogether by asserting that physical qualifications are *essentially epistemic*—that is, descriptive of our knowledge of reality rather than reality itself. Any ontological implications of these descriptions therefore lie outside the scope of physics. "In physics," Bohr writes, "our problem consists in the co-ordination of our experience of the external world . . ." such that "in our description of nature the purpose is not to disclose the real essence of phenomena but only to track down as far as possible relations between the multifold aspects of our experience."[3]

Thus the coherence of this method of reconciling mutually exclusive principles via complementary relation rests upon a similarly sharp separation of epistemology and ontology as mutually exclusive concepts themselves, such that a 'fact of knowledge' no longer implies 'knowledge of a fact' as it does when epistemology and ontology are characterized as mutually implicative. Realism, if it is to remain viable in physics and natural philosophy, is by this method recast as 'transcendent realism,' implying an inaccessible level of reality where complementary, mutually exclusive principles would ultimately find their proper unification.

[2b] The qualification of mutually exclusive categorical principles as higher order characteristics or modes of a deeper, fundamental ontology and epistemology that can be further generalized by the reduction or assimilation of one characteristic to the other.

The history of philosophy reveals a clear tendency toward this method; indeed, one could argue that its dominance—seen especially in the philosophy of nature during the early modern period—was the primary fuel for its later dominance in modern science and, via the latter, its increasing popularity in most modern Western worldviews. One can find its origins in much of pre-Socratic philosophy, perhaps most dramatically in the Eleatic School. As an admonishment against the relation of actuality and potentiality as mutually exclusive, fundamental aspects of reality, for example, Parmenides gave a convincing argument for the reduction of potentiality *to* actuality: Anything that "can be," he asserted, "must already be." There can be no "coming into being" because anything that comes into being has only two possible derivations: It either came *from* being, in which case it already exists; or it came from non-being—i.e., nothing—in which case it, too, is nothing, since nothing comes from nothing. Indeed, for Parmenides, one cannot even *imagine* a 'potential' actuality that was not already actual in some way, since imagining something that did not actually exist would literally amount to imagining 'no thing,' and thinking about nothing, Parmenides argues, is the same as not thinking.

Since by this reasoning potentiality is merely a sensory-epistemic abstraction from fundamental actuality, existence is therefore both eternal and unchanging—the Parmenidean 'One'; any perceived differentiation is thus only apparently real as disclosed via sensation, not actually real as disclosed by reason. Though this laid the groundwork for Platonic idealism as another type of philosophical reduction from mutually exclusive fundamental principles—ideal / thought / form vs. material / sensation / extension—it is important to note that Parmenides was not an idealist, but rather a monistic materialist. Thus one can find in many modern physical theories a number of Parmenidean reflections: Everett's 'relative state' interpretation of quantum mechanics[4] (better known as the 'many worlds' interpretation, or MWI, which is an extrapolation of Everett's original concept) makes a similar assimilation of potentiality to actuality, such that every possibility is understood as an actuality in some particular universe. Further, the division of reality into multiple, mutually exclusive universes is understood by the theory as a higher order abstraction beneath which lies a first order, unified 'multiverse'—in many ways akin to the Parmenidean 'One.'

Similarly, the Bohm-Hiley non-local hidden variables interpretation of quantum mechanics[5] describes the universe as epistemically divisible but ontologically undivided. Quantum physics as an epistemic enterprise glimpses this ontological unity via its fundamental characterization of the universe as a unified, ‘implicate order’ of actualities—which would at first glance seem to disagree with the Parmenidean worldview, since order implies division. But in the Bohm-Hiley interpretation, past and future are symmetrically related, and therefore ontologically indivisible; any potential ‘coming to be’ in the future is thus already contained in what already exists and existed. Past, present, and future, though epistemically distinct within the context of our finite observational structures, are nevertheless ontologically unified in a quasi-Parmenidean ‘One.’

Bohm writes:

> If it were possible for consciousness somehow to reach a very deep level, for example, that of pre-space or beyond, then all “nows” would not only be similar—they would all be one and essentially the same. One could say that in its inward depths now is eternity . . . (But eternity means the depths of the implicate order, not the whole of the successive moments of time.)[6]

Further, for Parmenides, the reduction of potentiality to actuality implied related reductions of other mutually exclusive principle-pairs, such as creativity vs. discovery. As noted above, in Parmenidean philosophy every ‘imagining’ is ontologically reducible to thinking about existence. And similarly, within the depths of the veiled implicate order underlying the Bohm-Hiley undivided universe, all potentia are ontologically reducible to actuality; novel creativity is ontologically reducible to discovery of the already extant. The ground of any experience interpreted as creativity (instead of understood as discovery) is the epistemic restriction or contextualization of all experience within our necessarily finite observational structures.

Bohm writes:

> As long as we restrict ourselves to some finite structures of this kind, however extended and deep they may be, then there is no question of complete determinism. Each context has a certain ambiguity, which may, in part, be removed by combination with and inclusion within other contexts . . . If we were to remove all ambiguity and uncertainty, however, creativity would no longer be possible.[7]

1.2 Bipolar Dualism and the Scientific Method: The Example of Physical and Philosophical Cosmology

The success of the method of reduction and assimilation in science, at least as measured by the apparent ability to predict and control nature yielded by this method, has inevitably led to a host of physics-inspired philosophical and cosmological reductions such as those discussed above. The philosophical cosmologies borne of the 'many worlds' and 'implicate order' interpretations of quantum mechanics are just two of many modern examples of reductive philosophical cosmologies inspired by and grounded in scientific reductionism. Indeed, for science-dominant worldviews in general, the thesis of fundamental mutually exclusive principles, and their attempted interrelation via either of the two modes, 2A and 2B described above, has today become their defining characteristic. The following pairings, for example, can be considered respectively: classical mechanics vs. quantum mechanics; fundamentally continuous spatiotemporal extension in general relativity vs. fundamentally discrete spatiotemporal extension in quantum mechanics; wave-particle dualism; local-efficient causality vs. nonlocal-quantum causality; classical determinism vs. quantum indeterminism; objective vs. contextual; infinite vs. finite; potentiality vs. actuality; form vs. fact . . .

Even when one turns attention to worldviews whose foundations either exceed or altogether ignore the philosophical foundations of modern science—e.g., various theistic-dominant worldviews—the thesis of mutually exclusive fundamental principles is exemplified in related pairings, including: determinism vs. free will; necessity vs. contingency; perfection vs. imperfection; permanence vs. change; heteronomy vs. autonomy... Perhaps most infamously, within the rhetorical spectacle of the modern ‘science and religion debate,’ the opposition of cosmogonic creation and cosmological evolution finds breathless celebration in popular portrayals of quantum cosmology. The primary fuel by which this debate is sparked into conflict is the stubborn characterization of creation and evolution, as well as necessity and contingency, as mutually exclusive concepts whose only proper mode of coherent relation is the reduction or assimilation of one concept to the other.

Many quantum cosmologists, for example, argue that the universe contingently evolved into existence quantum mechanically as a primordial, initial extant (whether as a singularity or some other form), *ex nihilo*, via a quantum probability function—viz., a ‘universal wavefunction’ yielding an unconditional probability that ‘our’ universe would actualize amid the primordially extant distribution of ‘alternative’ potential universes constitutive of this wavefunction. Some of these theorists have gone still further, promulgating quantum cosmology as a scientific validation of the ideological claim that a primordial, necessary extant such as ‘God’ is inherently irrational. What is neglected in this argument, however, is that when quantum mechanics is characterized in this way, as a cosmogonic engine operative primordially, *in nihilo*, then the logical structure presupposed by quantum mechanics must *itself* be understood as a primordial, necessary extant—a presupposition which, of course, belies the qualification ‘*in nihilo*.’ In other words, when quantum mechanics, a logico-relational structure expressible in the language of mathematics, is depicted as somehow operative in abstraction from a universe of relata, then this structure itself is necessarily ontological and primordial. One cannot, of course, coherently embrace both this ontological notion of logic and the notion that logic is a purely epistemic epiphenomenon of the human mind. Yet this is precisely what many quantum cosmologists propose to do in their efforts to reduce all necessity to contingency.

Finally, one might also consider those worldviews in which neither science nor religion is seen as foundational. These might simply be termed ‘humanistic,’ where fundamental mutually exclusive principles are housed, in the most general sense, within the dualism of subjectivity and objectivity. This pairing has its more specific application within a variety of disciplines in the humanities: In literature, deconstruction vs. intentionalism; in philosophy, conceptual vs. physical; mind vs. matter; thought vs. extension; the order of logical implication vs. the order of causal relation; necessity vs. contingency, permanence vs. change, unity vs. diversity, and so on.

In all of these examples, one can find within the history of Western thought a wide variety of systematic attempts to bridge fundamental mutually exclusive conceptual relata by way of either the method of complementarity in the context of transcendent realism (Method 2A above) or by reduction and assimilation (Method 2B). In the early modern period, the overly-general division of philosophy into the rationalist vs. empiricist traditions, each with deep roots in classical philosophy, well reflects the popularity of these methods. Though a careful survey of the philosophy of this period falls outside the scope of this chapter, readers familiar with the history of philosophy will be able to experiment with the above two classifications—Methods 2A and 2B; for example, Spinoza attempted to assimilate the causal order to the logical order, the contingent to the necessary; Locke attempted to assimilate the logical order to the causal order—both examples of Method 2B. Kant’s Transcendental Philosophy can likewise be seen as an example of Method 2A, the method of transcendent realism (i.e., in the sense of his transcendental idealism, by which ‘things in themselves’ transcend the understanding). Fundamental complementarity, within this context, is evinced throughout his system—his discussion of ‘conditioned’ and ‘unconditioned’ knowledge in the Transcendental Dialectic, for example.

1.3 The Fallacy of Misplaced Concreteness

As is especially clear in the example of Kant, the chief deficiency of the categorization of incommensurable principles as fundamentally mutually exclusive at the 'deepest possible' level of analysis is its presumption that the bounds of reason have been reached. Fundamental complementarity marks this boundary in Method 2A, where the problem of mutually exclusive foundational principles is relieved only by reference to some ineffable, transcendent unification or implicate order that lies on the other side of the boundary—either beneath the veil of finite observational contexts or, even worse, beyond the supposed scope of reason itself. But this merely exchanges one incoherence for another: the incoherence of incommensurable categorical principles is traded for the incoherence of qualifying (i.e., knowing) the 'unknowable' as unknowable, via the reasoning of the unreasonable—or, in modern science, the misuse of the hypothetico-deductive method to either posit or validate a theory that is, in principle, unfalsifiable.[8]

Method 2B is equally problematic—the attempted reduction or assimilation of mutually exclusive fundamental relata, one to the other, so that one is re-defined as concrete and the other abstract—one ontologically significant, the other an epistemic derivative or artifact. Again, the method of reduction and assimilation rightly recognizes the need to bring into coherence mutually exclusive, incommensurable categorical principles; but it wrongly grasps for that coherence by arbitrarily restricting the speculative schematization of the experience of nature to certain preferred categories of thought, in exclusion of other categories that could just as reasonably be characterized as fundamental. By this method, nature is always *either* fundamentally physical *or* fundamentally conceptual; either fundamentally continuous or fundamentally quantum; either fundamentally finite or infinite; either fundamentally deterministic or indeterministic. When one considers the increasingly profuse inflations of physical cosmological models into metaphysical cosmologies, their stipulated significance is belied by the fact that one can casually assemble practically any combination of the above qualifications and find a correlate interpretation of quantum theory or string theory or some other physical cosmology that can accommodate it.

At the level of principles deemed fundamental to any given scientific theory, then, the method of purely reductive science breaks down. The reason is because theorists enamored of the notion of an 'ultimate unifying reduction' fail to recognize the ineluctable possibility of deeper levels of abstraction underlying any 'ultimately reduced' principle or 'ultimately unified' theory qualified as 'fundamentally concrete' by this method. It is a conceptual hazard that Whitehead famously termed the 'fallacy of misplaced concreteness,' and its pertinence to modern science has never been more important—especially for those worldviews which, in their ongoing construction, use science as bedrock, framework, and scaffold. For these worldviews, science is tasked not only with constructing a fundamental *description* of the universe; the target height of construction is nothing less than a fundamental *explanation* of the universe via sheer deductive reduction. This bounding leap from fundamental scientific description to fundamental scientific explanation is one that over the past several decades has perhaps been less careful than such a leap would warrant. And likewise, these increasingly routine efforts typically receive little critical attention despite the obvious logical obstacles that belie the reasonableness of even attempting such a feat.

Again, one need only consider the growing number of philosophically loaded discussions of various physical cosmologies, the most esoteric of which have been embraced by the popular media for an entertainment-driven education of the public. The tacit stipulation of these presentations is that the demonstrable ability of the scientific method to construct sound reductionist descriptions of nature *in itself* warrants its broader application to the task of explaining or 'accounting for' nature's very existence. What is neglected in this misapplication of the scientific method is the fact that a 'fundamental explanation' of nature constructed via the method of deduction, such as that instantiated in the hypothetico-deductive scientific method, is impossible. This is because the categorical first principles at the base of any deductive scheme are always necessarily presupposed. Science, for example, cannot 'explain' the logical order—i.e., account for its existence—since the language and methodology of science necessarily presuppose this order.

To clarify this point, let us return briefly to the topic of quantum cosmology introduced earlier: In their 1983 paper "The Wave Function of the Universe,"[9] Stephen Hawking and James Hartle propose a quantum mechanical explanation of the origin of the universe whereby the universe spontaneously creates itself *ex nihilo* as a quantum mechanical probability function. (Hawking later asserts the theological implications of this model by stating that it allows for the elimination of the notion of God as a primordial extant in any rational explanation of the origin of the universe.[10] It is a claim that has since been repeatedly echoed by other cosmologists, primarily in mass-market books and media, and as such is arguably one of the driving reasons quantum cosmology has captured such popular notoriety today.)

Elaborating upon the previous discussion, what is neglected by both this model and its characterization by Hawking and other advocates is the fact that a quantum mechanical wavefunction is more than just a 'generic' integration of undefined potential physical states that, when applied universally, yields an unconditional probability distribution of possible universes; the wavefunction, rather, is a fundamentally *relational* structure depicting *conditional* probabilities for the evolution of an actual final physical state—e.g., that of the nascent universe—not from 'nothing,' but from some actual *initial* physical state. The wavefunction, in other words, is an integration of potential final physical states that is always conditionally *contextualized* by some actual, initial physical state. Further, it is a *logically conditioned* integration—specifically a Boolean logical conditionalization—presupposing final states that are always valuated as probabilities, and thus always mutually exclusive and exhaustive. Quantum mechanics, then, whether applied to the description of microscopic systems in a laboratory, or to the description of the universe itself, always presupposes a logical order by which potential outcome states are ultimately reduced into mutually exclusive and exhaustive (i.e., logically coherent) probability outcomes. Indeed, it is only via a presupposition of Boolean logic that probability theory is possible at all; and likewise, it is only via probability theory that quantum mechanics is possible.

But again, more than just presupposing a logical order, quantum mechanics presupposes a correlation of this logical order with *some* causal order of actualities from which potential outcome states derive. Quantum mechanics always begins with actualities, in other words—an initial actual system state—not simply bare, uncontextualized, 'generic potentialities' *in nihilo*. Indeed, it is impossible to define a potential outcome state in quantum mechanics without contextual reference to both an initial actual system state and an anticipated actual outcome state, just as it is impossible to define *any* particular potentiality without contextual reference to some actuality. Assigning the name 'quantum vacuum' to an initial primordial actual state of the universe, thus attempting to characterize it as 'nothing' as is the convention in most quantum cosmologies, neglects the fact that this state is both actual and logically ordered; that is, the causal relations of this primordial state are defined quantum mechanically as an evolution from initial actual state to final actual state, and this definition presupposes the structure of logical implication (as well as other logical structures to be discussed later in this volume). Therefore, quantum mechanical *cosmologies* cannot be properly described as *ex nihilo cosmogonies*, productive of a 'randomly' generated universe from nothing; indeed, even randomness mathematically presupposes an underlying logical order for its definition.

The particular example of quantum cosmology can be seen as exemplifying a broader truth: that any attempt to construct a philosophical cosmology simply by clothing it as a 'purely scientific' physical cosmology, borne of and supposedly validated solely by the scientific method and thus purified of any philosophical presuppositions, is doomed to incoherence. This is because any deductive scheme of reasoning, including the hypothetico-deductive method of modern science, must begin with first principles that are necessarily presupposed, either implicitly or explicitly. The first principles at the foundation of any deductive scheme cannot themselves be deduced, since there is no more general principle by which such a deduction might proceed. Thus, again, science cannot ‘explain’ the logical order—i.e., account for its origins—since the language and methodology of science necessarily presuppose this order. In the above example, quantum mechanical cosmological models might offer valuable fundamental descriptions of the earliest stages of the evolution of the universe; but when extended to the task of accounting for the origin of the universe *ex nihilo*, the model’s reliance on a primordial logical order precludes its success.

Whitehead’s fallacy of misplaced concreteness, when applied to these increasingly popular inflations of physical cosmology to philosophical cosmology, is crucial because it reminds us that science can never ‘explain away’ that which it necessarily presupposes. The logical order underlying mathematics can never be deductively explained or reductively accounted for by a scientific description of the causal order because the logical order is necessarily presupposed *by* the method of scientific description; thus, fundamental reductionist scientific descriptions of nature can never attain the status of fundamental explanation, because there is always a deeper level of abstraction underlying any deductive or reductive scheme, scientific or otherwise.

One of the earliest and clearest illuminations of the deficiency of the reductive-deductive method when applied to the task of ‘fundamental’ or ‘complete’ explanation can be found in Plato’s *Theaetetus*. There, Plato challenges the idea that a sufficiently deep reduction can serve as a sturdy bridge across the chasm separating description and explanation—appearance and reality. In the dialogue, Socrates tells Theaetetus of a dream he once had wherein he had learned of a theory of explanation by which all things are described as complexes of simpler elements, themselves complexes of still simpler elements. This reduction continues until the simplest elements are apprehended, at which point a complete and true explanation of the initial object is achieved. It is only then, proposes Socrates, that one can be said to possess true knowledge—i.e., explanation—of the object. Theaetetus eagerly accepts this epistemology, but Socrates advises caution; he explains that in his dream, the most fundamental elements are incapable of description by this epistemology, given that they contain no simpler parts. Therefore, the most fundamental elements are themselves unknowable. How, asks Socrates, can the unknowable be the foundation and ultimate justification of knowledge?

Though Plato's admonition on the limits of a purely reductive-deductive epistemology would ultimately be drowned out by the celebration of the manifold profound achievements of the modern scientific method over two millennia later, the historic advances in physics and mathematics emblematic of the twentieth century would also bring with them a number of sharp reminders of these limits. The misapplication of quantum mechanics to the construction of a 'purely physical' scientific cosmogony, discussed previously, is one example; but Plato's admonition would also find its rehabilitation in the key contributions of Gödel and Russell to the philosophy of mathematics—contributions crucial to the construction of a coherent ontological interpretation of quantum theory. One sees this well reflected, for example, in Russell's paradox, which attends to the logical problem of predicating totalities. This difficulty, as will be discussed in detail in chapter 3, finds its relevance in the quantum mechanical predication of system states defined as totalities—e.g., the universe itself when considered quantum mechanically—where the separation of 'measured system,' 'measuring apparatus,' and 'external environment' is properly recognized as arbitrary. This recognition is conventionally accepted in quantum theory, and indeed is explicitly embraced in the case of quantum cosmology, but its underlying implication that all wavefunctions are therefore ultimately 'universal' requires careful exploration.

Another exemplification of Plato's admonition in the *Theaetetus* can be seen in Gödel's two incompleteness theorems, which together establish that no effectively methodical deductive theory capable of expression in the language of arithmetic can ever be both internally consistent *and* complete. The reason, evocative of Socrates' dream, is that any formal theory that is both internally consistent and allows for the deduction of arithmetical proofs will always presuppose some arithmetical statement that is both true and incapable of proof by the theory. The specific relevance of Gödel's theorems to quantum mechanics will be taken up in greater detail in chapters 2 and 3; but in the context of this introductory discussion, it is useful to consider their pertinence to the philosophical problem of relating the *order of causal relation* and the *order of logical implication*, as well as the problem of mutually exclusive categorical principles discussed earlier.

Consider, for example, the various self-referential paradoxes often associated with Gödel’s theorem, such as the Epimenides paradox, given in the ancient Cretan philosopher’s infamous utterance: Κρῆτες ἀεί ψεύσται (“Cretans, always liars”). There are two features of this paradox by which its relevance to quantum mechanics will be demonstrated throughout this volume: First, the statement implies both a causal order and a logical order; second, it entails the predication of a totality. As we will see, in the case of both the Epimenides paradox and quantum mechanics, the root cause of the incoherence is the presumption of a *closed totality*, within which causal relation and logical implication—and more broadly, physical relation and conceptual relation—are treated as mutually exclusive categories, as are ‘truth’ and ‘falsity.’ The solution we propose in this volume is to instead recognize these as mutually implicative categories within an open totality defined as a history-in-process. The ‘decoherent histories’ approach to quantum mechanics, as we will see, is a fundamental exemplification of the latter. And likewise, the philosophy of Whitehead, in the context of this understanding of totality, will provide a solid framework for the coherent relation of the physical and the conceptual, as well as the causal and the logical, as mutually implicative features of nature.

To call Whitehead’s philosophy revolutionary in this regard would be an understatement; for by the end of the early modern period, during which the foundations of the modern scientific method were forged, contingency via the order of causal relation and necessity via the order of logical implication, as well as other associated bipolar conceptual pairings, were generally treated by natural philosophers as incommensurable, mutually exclusive concepts. This tendency maintained its dominance throughout Whitehead’s time, and indeed to the present day. The various proposals for their coherent relation as mutually exclusive typically entailed, as discussed thus far, either their treatment as complementary epistemic features and co-ordinations of our experiences of an otherwise transcendent reality; or as second-order epistemic concepts that can be distilled into a single first-order principle via the reduction or assimilation of one concept to the other. Whether continental rationalist or British empiricist, the common goal was to bridge the two sides of the Platonic chasm separating 'what appears to be' from 'what is reasoned to be.' Despite the richness of detail to be found in the various philosophical systems proposed during this period, the tendency toward the method of mutually exclusive relation of the causal and logical orders—and more broadly, again, the physical and the conceptual, as well as the contingent and the necessary, respectively—resulted in two alternative mappings across Plato's chasm:

[1] appearance: sensory perception of contingency via causal relation
reality: rational conception of necessity via logical implication

[2] reality: sensory perception of contingency via causal relation
appearance: rational conception of necessity via logical implication

It is clear that both of these mappings *together*, without any secure bridging, have been inherited by most modern, scientifically informed worldviews, and the unavoidable incoherence generated by the attempted embrace of both mappings at once has proven to be a major impediment to the evolution of philosophically informed science in general. One sees, for example, the epistemic separation of logical implication and causal relation well reflected, respectively, in the increasingly dubious distinction between 'theoretical physics' (a.k.a. 'mathematical physics') and 'experimental physics.' For many, these are seen not just as subdisciplines, but rather as 'branches' of physics that are somehow epistemically separable. Of course, experiments cannot be published without the appropriate theoretical work by which data are analyzed. It is often the case, however, that experimentalists are unable to assess the significance of their own data until a theorist is brought into the team, often long after the experiment has been completed.

Likewise, the straying of theoretical physics from its grounding in falsifiable hypotheses and experiment has indeed led to tantalizing imaginative constructs like the various quantum, multiverse, and string theory cosmologies mentioned earlier. And while it is true that these have captured the popular imagination and inspired increased interest in science, it is equally true that when elegance of formalism too often trumps empirical applicability, the measure of scientific progress begins to derive from the measure of its appeal, rather than its appeal deriving from its progress.

Ironically, for many, the greatest appeal and elegance of theoretical physics comes from its application to the 'external' foundational questions that are, in principle, empirically inaccessible—i.e., those that exceed the universe itself. It is ironic because 'elegance' here implies a brute Pythagorean reduction of the world to bare mathematics—the reduction of all contingency to calculable necessity. But in the same way, one can argue that experimental physics entails its own, complementary reduction; for when applied via disciplines such as molecular biology and neurophysiology to 'internal' foundational questions—e.g., the relationship of mind to brain, of conception to perception, of 'personality' to the physical structure of DNA—it implies a purely physical causal closure—a reduction of the world to 'fundamental' particles and forces.

In summary of the current discussion, one can trace throughout the histories of both philosophy and modern science a tendency to characterize the causal and logical orders (as well as contingency / necessity, and physical / conceptual, respectively) as mutually exclusive foundational concepts that are relatable by either [1] complementarity, or [2] reductive assimilation of one concept to its counterpart. The method of complementary relation is, by its own admitted limitations, incapable of yielding anything more than an arbitrary explanation of either the logical or causal orders as complementary epistemic co-ordinations of our experiences of an ontologically transcendent reality; and the method of ontological reduction and assimilation, when it reaches the level of fundamental explanation, is doomed by the fallacy of misplaced concreteness.

1.4 Dipolar Duality: Contrasts of Mutually Implicative Relata

The paradoxes borne of relating incommensurable categorical principles like causal relation / logical implication, and physical / conceptual as mutually exclusive can be relieved by instead treating such principles as mutually implicative within some presupposed unifying context. The paradoxes are relieved, in other words, when one ceases to consider each principle individually and instead considers it in definitive relation to its counterpart within the context of a presupposed, underlying unity. This underlying context can be *rationally* presupposed, since these principles are, in fact, definitively mutually-referential; the question is simply whether one desires that this mutual reference take one of two possible forms: [1] reduction to incoherence by the arbitrary definition of one principle as substantial and the other as epistemic abstraction; or [2] elevation to coherence via mutual implication. By the first approach, Parmenides might exclaim, “Diversity is illusory. There is only unity! There is no change, only permanence!” while at the same time Heraclitus exclaims, “The only permanence *is* change!” and Protagoras, attempting to silence them both, exclaims, “There is no objectively true statement!” The incoherence of this method is evinced not only by the paradoxical character of these exclamations (the last one closely analogous to the Epimenides paradox discussed earlier); it is also evinced by the fact that unity cannot be defined without reference to diversity, nor diversity without reference to unity, and likewise for the concepts of permanence and change. This clearly belies the assertion of exclusive ontological primacy for either concept over its counterpart.

The second approach—elevation to coherence via mutual implication within some presupposed, speculative, unifying context—is, by contrast, far more promising. Since coherence is a definitive feature of reason itself and an explicit desideratum of the enterprise of logic, the presupposition of a context that provides such coherence is entirely reasonable. While this might seem to warrant a Baconian objection to the intrusion of a priori supposition into the methodology of science, the presupposition of unifying context is no more problematic than the scientific method's equally speculative presupposition of a unifying logical order by which locally testable theories can be presumed to hold universally. It is only when science attempts to 'account for' this order, while at the same time presupposing it, that scientific reason confronts itself with incoherence and paradox.

Thus, the method of relating fundamental cosmological principles, both physical and metaphysical, by mutual implication, requires a philosophical method that transcends the typical restrictions of philosophical deduction and induction, as well as their typical coupling within the hypothetico-deductive method of modern science. A clear example of such a method can be found in Whitehead's concept of 'speculative philosophy,' referenced in the epigraph of this chapter and explicated at length in *Process and Reality*,[11] his epic essay on systematic cosmology. By this method, the presupposed context by which fundamental principles find their mutual implication is not derivable by simple deduction or reduction. Rather, it is the product of imaginative generalization, continuously tested and refined against experience according to the following desiderata: [1] *coherence*, in that the categorical concepts of the speculative scheme are mutually implicative; [2] *logical structure*, such that all constitutive concepts are consistent, free from contradiction, and always exemplify, in every specific instance, the general logical framework. As will be discussed later in this volume, this desideratum when applied to quantum mechanics can be more concretely defined as the system of Boolean logic; [3] *empirical applicability*, such that the speculative scheme can be empirically exemplified in nature; [4] *empirical adequacy*, such that the universality of empirical applicability is reasonably presupposed. Taken together, the two empirical desiderata are, of course, the same desiderata by which scientific theories (e.g., quantum theory, the Standard Model, general relativity, etc.) are likewise presumed to hold universally even though they are testable only in restricted domains. Thus, as has been well evinced throughout the evolution of modern science, empirical adequacy might be lost by one theory because of its failure of applicability in some newly discovered domain, and then regained anew by the theory's refinement or replacement.

In the enterprise of speculative philosophy, as in the enterprise of science, these four desiderata, the rational pair and the empirical pair, are the only metrics by which the success of a particular scheme can be measured. There is never any final arrival at, or ultimate reductive-deductive proof of, the categorical principles constitutive of the scheme's presupposed unifying context; nor is there an arbitrary claim that this context is wholly transcendent and thus exempt from deeper levels of analysis and understanding. There is instead an asymptotic approach toward this context's explication, such that progress is always both measurable via its empirical exemplification and, at the same time, unending.

Process and Reality itself is a clear (and for many, unsurpassed) example of a robust, thoroughly systematized cosmology built upon a foundation of mutually implicative categorical principles via the method of speculative philosophy. The philosophical program of relational realism introduced in the present volume takes as its conceptual foundation the Whiteheadian speculative cosmological scheme, refining and expanding it toward an empirically adequate accommodation of modern quantum theory. To this end, the program of relational realism also entails a refined mathematical formalism, presented in detail in part II. This category-sheaf theoretic formalism not only solves many key conceptual difficulties in Whitehead's attempted formalism; it also addresses many infamous conceptual difficulties in the interpretation of quantum theory, including the latter's integration with relativity theory and, more broadly, clarification of the ontological function of logic in quantum causality.[12]

For Whitehead, the underlying context by which the causal and logical orders are united in dipolar fashion, via mutual implication, is the dipolar quantum 'actual occasion' or 'final real thing'—the imaginatively generalized fundamental unit of reality. But rather than being solely a unit of being, which would expose the actual occasion to the argument of Parmenides, it is also, and more fundamentally, a unit of becoming—a unit of process, or actualization of potentiality, that entails both a causal-physical pole and a logical-conceptual pole. The Platonic chasm separating conceptual and physical (as well as logical and causal) is thus easily bridged in this dipolar scheme because each pole is implicative of the other *within the contextual unity of the actual occasion as a whole.* When actualized, the occasion has its causal efficacy upon subsequent actualizations; and likewise, its process of actualization is conditioned by both [1] its causal physical relations with the actualities 'physically' antecedent to it (prior in time) in terms of spatiotemporal extensiveness—i.e., within its backward lightcone, per the restrictions of relativity theory; and [2] its logical relations with those actualities 'logically' antecedent to it (i.e., prior in implicative order), per the restrictions of the Principle of Non-Contradiction (PNC), among other logical restrictions.

Each actual occasion is thus fundamentally a quantum *unit of relation.* In the process of actualization, the becoming occasion is *internally related* to its entire dative world of antecedently actualized occasions, such that the potentia contextualized by these relations are both causally and logically ordered. By this mode of internal relation, then, every actual occasion, though it is a discrete, quantum unit of relation, entails a vast integration of potential relations between that occasion-in-process and the actual occasions of its dative world. (Thus, another philosophically problematic conceptual pair, 'unity' and 'diversity,' is also brought into coherence via mutual implication. "The many," writes Whitehead, "become one, and are increased by one. In their natures, entities are disjunctively 'many' in process of passage into conjunctive unity."[13]) These integrations are always logically conditioned, such that upon actualization, the novel occasion is internally related to the world in a manner free of violations of PNC, or any other such violation of the logical order.

Writ large, the universe is thus described as an integrated system of serial routes (histories) of actual occasions—that is, a global history of coherently integrated local histories. This is precisely analogous to the concept of local and global system histories in the decoherent histories interpretations of quantum mechanics introduced later in this volume.[14] Coherence at all scales derives from the fact that the internal relations of every actualization constitutive of each local history are logically conditioned, such that the orders of causal relation and logical implication are always correlated. Relations *among* histories therefore also evince this correlation—even those that are spacelike separated and thus incapable of causal physical ordering—i.e., ordering within a 4D spatiotemporal extensive framework. Indeed, it is by the physical-logical dipolarity of the actual occasion in relation to its dative world that the inability to *physically* or *causally* order spacelike separated occasions per the restrictions of relativistic spacetime (i.e., the impossibility of defining a total order in relativity theory) does not in any way imply that these spacelike separated occasions similarly lack a logical order. This is clearly evinced by the fact that partial ordering—asymmetry and transitivity—*does* hold in relativity theory, as it must, lest the theory collapse into incoherence. In other words, the relativistic proscription against specifying a *total* physical-causal order among spacelike separated occasions does not negate the *necessary presupposition* of a unifying, logical ordering of these occasions if the theory is to maintain its coherence. In the relational realist philosophical program, this coherence is grounded in the speculative presupposition that the causal and logical orders are mutually implicative via the unifying context of the dipolar actual occasion.

In the conventional mechanistic-materialist worldview, a history is understood to be a story about the objective world—an abstract scheme of epistemic relations among facts descriptive of concretely ‘material’ objects. In the relational realist worldview, a history is understood to be a concrete scheme of ontological relations among facts constitutive of abstractly ‘material’ objects. In both classical and relational realist histories, coherence is maintained when every novel fact is asymmetrically internally related to the totality of its antecedent facts; but in the case of classical histories, this is merely a conceptual, epistemic desideratum, and one impossible to achieve categorically. Contradictions always abound, indicating error and inspiring revision. By contrast, in the case of the ontological histories of relational realism, facts are not merely conceptual, logical objects; they are dipolar—objective both causally / physically and logically / conceptually. Coherence is categorical, and (if the speculative scheme is successful) impossible to avoid—at least by the metrics of empirical science, which thus far has given no evidence whatsoever of incoherent relations among the constituents of nature—no physical exemplification of conceptual paradox.

To be sure, such exemplifications have been conceived theoretically and in the exacting language of mathematics, and these attempts will be a topic of discussion in chapter 2—traveling ‘backwards’ through time and ‘undoing’ the facts of the past via so-called ‘retro-causality,’ physical ‘bilocation,’ and other such physical violations of PNC. To date, however, there has not been a single experiment in the sciences decisively demonstrating a *physical* instantiation of these or any other concept that would obviate the presupposed structural logical coherence of the world. Indeed, apart from this categorical logical structure, the scientific method would not only lose its applicability to the task of coherently describing nature; it would literally lose itself—and thereby, paradoxically, the very means by which it would claim success in these dubious efforts.

In this regard, the utility of the relational realist speculative philosophical scheme is perhaps best evinced by its application to the task of coherently interpreting scientific theories that are conventionally accepted as fundamental, yet are nevertheless profoundly resistant to coherent integration. The incompatibility of quantum theory and general relativity, as mentioned earlier, is a particularly problematic example. Its solution has proven unusually elusive, with little definitive progress made after decades of effort. If the various string theoretic and quantum cosmologies have fallen short, it is arguably because of their shared reliance upon the centuries-old physico-reductive, mechanistic-materialistic approach to relating the causal and logical orders as mutually exclusive features of reality. And likewise, a speculative philosophical relation of the causal and logical orders as mutually implicative features of reality lays the groundwork for an alternative and arguably improved set of physical theories—theories framed within a systematic ontology as robustly informed and confirmed by modern physics as the old mechanistic-materialistic approach, but without the underlying conceptual incoherence.

Notes

1. Alfred North Whitehead, *Process and Reality: An Essay in Cosmology, Corrected Edition*, ed. D. Griffin and D. Sherburne (New York: Free Press, 1978), 3.

2. Michael Epperson, “Relational Realism: The Evolution of Ontology to Praxiology in the Philosophy of Nature,” *World Futures* 65, no. 1 (2009), 19-41. The present chapter is a revision and expansion of the ideas presented in this article.

3. Niels Bohr, *Atomic Physics and Human Knowledge* (New York: Wiley, 1958), 18.

4. Hugh Everett, “‘Relative State’ Formulation of Quantum Mechanics,” *Reviews of Modern Physics* 29 (1957): 454-62.

5. David Bohm, “A Suggested Interpretation of the Quantum Theory in Terms of ‘Hidden’ Variables.” *Physical Review* 85 (1952):166-93; David Bohm, “Time, the Implicate Order, and Pre-space,” in *Physics and the Ultimate Significance of Time*, ed. David R. Griffin (New York: State University of New York Press, 1986), 177-208; David Bohm and B. J. Hiley, *The Undivided Universe: An Ontological Interpretation of Quantum Theory* (London: Routledge, 1993).

6. Bohm, “Time, the Implicate Order,” 199.

7. Bohm, “Time, the Implicate Order,” 198.

8. Again, the ‘many worlds’ interpretation of quantum mechanics is but one of the more infamous examples.

9. James Hartle and Stephen Hawking, “The Wave Function of the Universe,” *Physical Review D* 28 (1983): 2960-75.

10. R. Weber, *Dialogues with Scientists and Sages: The Search for Unity* (London: Routledge, 1986), 212.

11. Whitehead, *Process and Reality*, 3-17.

12. Evinced, for example, by recent experiments exploring the phenomenon of phase decoherence in condensed matter physics. This topic will be explored in more detail in subsequent chapters.

13. Whitehead, *Process and Reality*, 21.

14. See also, Michael Epperson, *Quantum Mechanics and the Philosophy of Alfred North Whitehead* (New York: Fordham University Press, 2004).

END OF CHAPTER SAMPLE

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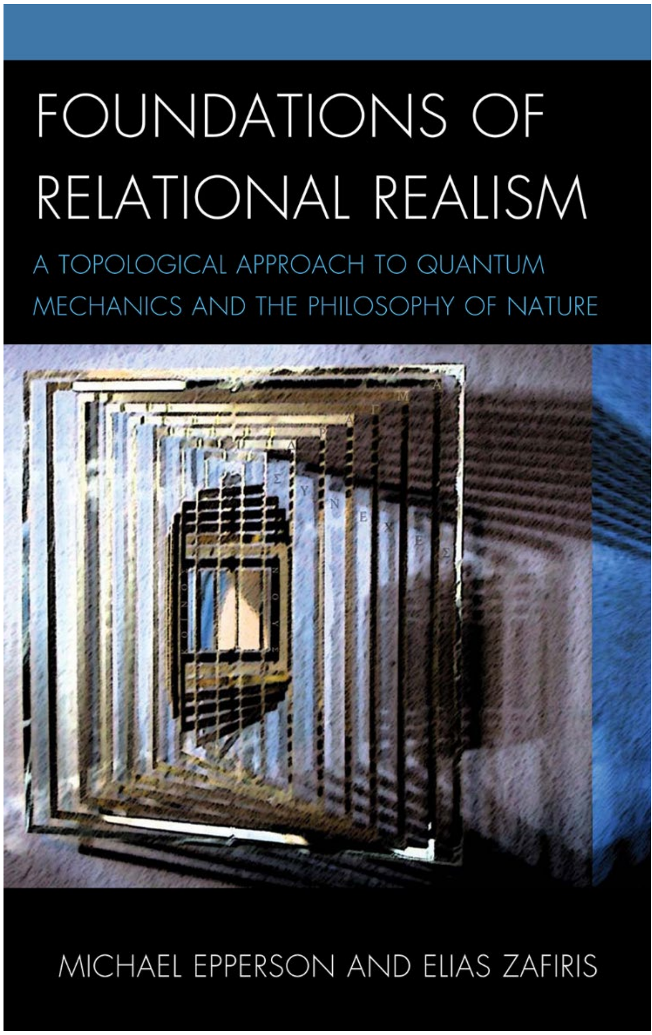

FOUNDATIONS OF RELATIONAL REALISM

A TOPOLOGICAL APPROACH TO QUANTUM MECHANICS AND THE PHILOSOPHY OF NATURE

MICHAEL EPPERSON AND ELIAS ZAFIRIS

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TIMOTHY E. EASTMAN, PH.D.
Senior Physicist (ret)
Sciences and Exploration Directorate
NASA Goddard Space Flight Center

MICHAEL EPPERSON
Ph.D. Philosophy
University of Chicago

ELIAS ZAFIRIS
Ph.D. Theoretical Physics
Imperial College
University of London

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CHAPTER 2

Substance and Logic in Quantum Mechanics

This chapter is intended to provide a general overview of the conceptual framework presented over the course of part I. As such, it gives an introductory discussion of concepts that will be revisited in greater detail throughout the remainder of the book.

The notion of an ontologically significant scientific theory—that is, one that aims toward an understanding of nature rather than just the nature of understanding—has undergone an evolution in the history of Western philosophy that has been punctuated by repeated cycles of approbation-as-profound and dismissal-as-vacuous, with each trend taking as its justification the same issue: The necessary presupposition, in *any* ontological interpretation of a scientific theory, of some particular framework of categorical principles. Whether generated by deduction, induction, imagination, or some combination of these, history is replete with examples whereby the profundity of first principles evolves first to dogmatic acceptance, then inevitably to rejection on the grounds of experimental disconfirmation. The first principles and their associated methodologies of inquiry are then modified or replaced, again via some combination of deduction, induction, and imagination, and the cycle repeats. The shift from Aristotelian hylomorphic physics in the medieval period to classical mechanistic-materialistic physics in the early-modern period is but one example, and the grounding of this shift in the specific issue of first principles is well represented in key writings of this period; the introduction to Book I of Bacon's *New Organon* and Boyle's "Of the Excellency and Grounds of the Corpuscular or Mechanical Philosophy" are but two examples. The most recent iteration of this cycle is the shift from classical mechanistic-materialism to quantum physics, the aftershocks of which have yet to subside.

After centuries of these tectonic shifts, one could argue that it is for lack of sure footing that in today's academy, the very notion of the systematic development of a coherent ontological interpretation of fundamental physical theories is little more than a cottage industry in most university philosophy departments. But the mere existence of presupposed foundational categorical principles and their recurrent displacement, abandonment, and replacement throughout the history of philosophy does not in itself warrant the wholesale repudiation of scientific realism by way of sheer reduction of ontological concepts to merely epistemic concepts as has been fashionable over the past century. One can trace the roots of such reduction back to Protagoras, and likewise Plato's riposte to the Protagorean doctrine maintains its relevance: The statement "it is always the case that there is no objectively true statement, only subjectively true statements" is self-refuting and therefore paradoxical, as is the statement "there exists no ontologically significant statement, only epistemologically significant statements." The fundamental incoherence of such claims is grounded in the fact that epistemic assertions of this kind—and indeed of any kind—themselves presuppose and exemplify an underlying ontological commitment. As suggested in the previous chapter, Plato's argument is ultimately grounded in, and can be extended further to include, the recognition that the mutually implicative relationship between epistemology and ontology wholly obviates the possibility of simply reducing or assimilating one mode of inquiry to the other.

One could perhaps counter that the Platonic refutation lacks self-sufficiency, since it relies upon a presupposed scheme of first principles themselves taken to be objectively true—namely, those of classical propositional logic which include, among others, the Principle of Identity

(PI), the Principle of Non-Contradiction (PNC), and the principle of the excluded middle (PEM). But of course the Protagorean refutation of philosophical realism is equally reliant upon this same scheme, even as it is undone by it; and the same can be said of the great variety of analogous present-day attempted refutations of scientific realism. Indeed, not only does the modern scientific method itself presuppose these first principles of classical logic, but so does any philosophical evaluation of the *meaningfulness* of scientific theories. Put most broadly, the possibility of any epistemically meaningful claim and any ontological claim of meaning presupposes an underlying logical structure of first principles including PI, PNC, and PEM. Indeed, this structure is a necessary presupposition of any coherent and consistent framework of conceptual relations of any kind. As discussed in the previous chapter, even in the case of mathematics, the most objective and universal scheme of conceptual relations yet conceived, Gödel demonstrated with his two incompleteness theorems the requirement of presupposed first principles of relation—namely, relations among the natural numbers that are necessarily true yet unprovable via any consistent mathematical axiomatic scheme. And likewise, the consistency of any such scheme itself requires reference to the same necessarily presupposed principles of relation. The Epimenides Paradox, as discussed earlier, provides a heuristically useful analog: The Cretan's claim, "Cretans, always liars" cannot be true, but nor can it be false. Thus it is clear that PNC [$\neg (P \wedge \neg P)$] is not merely a feature of mathematical conceptual relations; it is, rather, a necessary presupposition for the possibility of *any* scheme of conceptual relations that is coherent and consistent. Apart from such a scheme, the conception and evolution of the modern scientific method would have been impossible.

2.1 'Liberating' Science from Boolean Logic: A Sisyphean Adventure

Despite the centrality of a necessarily presupposed logical framework to both the scientific method and, more broadly, to the conventional Western worldview of scientific realism, one sees two puzzling trends that have been gaining traction over the past several decades: First, the attempt by some physicists to apply the scientific method to the task of 'accounting for' the Boolean logical structure *presupposed by* the scientific method. As discussed in the previous chapter, it is an ambition well exemplified by quantum cosmological models purporting to account scientifically for the origin of the universe and its logico-mathematical structure *ex nihilo*.

Second, and more puzzling, is the attempt by some quantum theorists to apply the scientific method to the task of invalidating the Boolean logical structure presupposed by the scientific method. It has become increasingly fashionable, for example, to claim that PNC and Boolean logic in general are routinely violated in quantum physics and that actual observable physical instantiations of these violations—i.e., 'physical paradoxes'—have been experimentally confirmed even at the macroscopic level. One such experiment recently published in the journal *Nature*,[1] for example, involved coupling a macroscopic mechanical oscillator (a 60 micron long silicon 'paddle') to a quantum system such that superpositions of *potential* system states violating PNC in the quantum system were interpreted by the paper's authors, as well as the journal's editors, as producing *actual* 'macroscopic superpositions' violating PNC in the mechanical oscillator. In a companion article describing the experiment, *Nature* reported that the experimenters

> used the weird rules of quantum mechanics to simultaneously set the paddle moving while leaving it standing still . . . Through a series of careful measurements, they were able to show that the paddle was both vibrating and not vibrating simultaneously . . . The experiment shows

> that the principles of quantum mechanics can apply to everyday objects as well as atomic-scale particles.[2]

The exotic notion of quantum measurements generating macroscopic, ‘physical superpositions’ of observable, alternative *actual* states rather than merely alternative *potential* states (one cannot, of course, ‘observe’ potentia, despite the fact that one can represent them mathematically) is as old as the Schrödinger’s Cat thought experiment. The latter, it is often forgotten, was conceived by Schrödinger not as a celebration of the violation of PNC in quantum theory, or an endorsement of the idea that such violations are physically instantiated; it was intended, rather, as an admonishment that any interpretation of quantum theory implying ‘observable’ superpositions—viz., observable physical violations of PNC—must be considered deficient. “It is typical of these cases,” he wrote, “that an indeterminacy originally restricted to the atomic domain becomes transformed into macroscopic indeterminacy.” But this indeterminacy, Schrödinger argued, “can then be resolved by direct observation. That prevents us from so naively accepting as valid a ‘blurred model’ for representing reality.”[3]

The key phrase for Schrödinger, here, is “by direct observation” and it provides the fundamental principle by which any experiment involving quantum mechanical measurement should be interpreted. One could, for example, improperly interpret the interference fringe of the familiar double-slit experiment[4] as a kind of ‘macroscopic superposition’ such that a single photon is envisioned as ‘actually going through’ both slits at the same time. By such an interpretation, the interference fringe itself is thus depicted as evincing a fundamental violation of PNC. But this interpretation is easily confuted by the fact that in observing an interference fringe, one is not ‘directly observing’ a macroscopic superposition; the interference fringe in a double-slit experiment is generated as a sequence of individual, discrete measurement events, wherein each photon impinges upon the detector at a particular point. After a sufficiently large succession of these discrete events, the interference fringe begins to manifest as a spatial relationship among these points.

Thus the form of the interference fringe, and by analogy, the interference effect described in the quantum oscillator experiment discussed above, is not properly understood as a macroscopic 'object-superposition' violating PNC but rather as a statistical relation among discrete quantum measurement outcome events, each of which satisfies PNC. In this regard, the quantum superposition associated with each individual photon is properly understood as a superposition of *potential* outcome states prior to the unique actualization of one of these states. And though such superpositions of potential outcome states (e.g., the 'pure state' in quantum theory) do violate PNC (potential passage through Slit 1 *and* potential passage through Slit 2), it is a fact of quantum mechanics that these superpositions of potential outcome states *always* evolve to become a discrete, actual measurement outcome, registered as a discrete impingement of the photon at the detector. Only after an ensemble of such detections has been recorded can an interference fringe be defined—again, evincing the fact that the latter is only statistically and retrodictively representative of the superpositions of potentia ingredient in each discrete actualized photon detection. This fact is especially crucial when interpreting so-called 'double-slit quantum eraser' experiments,[5] by which many theorists purport to 'erase,' via a kind of 'retro-causality,' these discrete actualizations of potentia, and with them the path information contained in the outcome states recorded by these measurements. These experiments will be discussed further in chapter 4.1.

With respect to the oscillator experiment discussed above, one should likewise interpret the superposition of oscillator states not as a superposition of 'contradictory actual states' that is somehow directly observable and persistent over a period of time as was implied in the *Nature* article, but rather as a superposition of potential outcome states by which actual initial states and actual final states of the device are statistically related. These potential relations are calculable predictively, but 'measurable' only retrodictively—that is, *after* an ensemble of discrete, actual outcome states has been registered.

The crucial point here is that *apart from the statistical analysis of a succession of actual, discrete outcome states, the superposition of states cannot be measured.* More crucially: Even though a time interval associated with a superposition is definable by calculation—i.e., a 'coherence time' during which the coherent superposition of potential states is interpreted by many theorists as 'persisting' (in the oscillator experiment, for example, this time was calculated to be 17 *ns*)—it is nevertheless impossible to directly observe a superposition of potential states *during* this defined time interval. Again, the very notion of 'observing potentia' is in itself problematic conceptually—a problem that is easily resolved by acknowledging that both theoretically and in scientific practice, as well as at the level of common intuition, *potential* states are always defined and measured via analysis of the *relation between actual states*—that is, the initial and final actual states that bound this relation. In quantum experiments such as the one discussed here, these relations take the form of oscillations ('Rabi oscillations', or 'Rabi cycles' named after physicist Isidor Isaac Rabi), such that a sequence of these oscillations is understood as a sequence of these relations. Thus, the 'coherence time' pertaining to the intervals 'between' the actual states constitutive of such a sequence, though it can be analytically *defined by calculation*, is always *observed by measurement* only retrodictively and statistically—that is, always after a sequence or history of discrete actual states has been registered and recorded.

This admonition is often neglected, in large part because of a failure to explicitly acknowledge that time is not itself an 'observable' in standard quantum theory—i.e., it is not properly expressible as a self-adjoint operator.[6] The technical meaning of this statement will be explored later in this volume, but for the current discussion its significance can be understood thus: While one can use quantum theory to compute the probability that an observable like an electron's spin polarization will be 'up' or 'down' upon arrival at a particular detector, one cannot use standard quantum theory to determine *when* an electron will arrive at the detector (this is commonly referred to as the quantum 'time of arrival' problem). In this way, and reflective of the literal sense of 'observable,' time is no more an observable in quantum physics than it is in classical physics, being neither an object itself, nor definable as a ‘quality’ intrinsic to the object measured. It is, rather, a *relational inference*, quantified via an *external parameter* by which an initial actual state of a measured system is metrically related to a subsequent actual state.

In summary, it is a fundamental feature of quantum mechanics that the object of observation is always a system in an actual state, and never a superposition of potential states. One cannot ‘directly observe’ potentiality, but rather only infer it as a calculable relation between actual initial and final relata—an assertion that is both intuitively comprehensible and experimentally demonstrable; yet the increasingly popular conflation of ‘calculation of the potential’ and ‘measurement of the actual’ in experimental reporting persists, even to the point of becoming the standard interpretation of these experiments as disseminated by the popular media.

Still more puzzling, this popularized interpretation directly contradicts fundamental concepts in quantum mechanics that are otherwise unquestionably regnant in the more familiar scenarios such as the double-slit interference experiment. Likewise, even in the most basic college physics course, it is well understood that quantum mechanics proscribes the possibility of evaluating any observable associated with an electron ‘in between’ its quantum states, or even conceptualizing the electron ‘moving as an actual object through the space between’ the spatial configurations associated with these states. Relative to this interstitial space, it is understood that the electron is only definable as a superposition of potential states (one of which will be actualized upon measurement), and thus not directly observable; it is not, as depicted in the analogous quantum oscillator experiment, a superposition of actual states that is somehow directly observable.

Put most simply, the depiction of a potential system state, or a superposition of potential states, as an observable physical extant, rather than as a relational conjunction of actual states, is fundamentally incoherent. It is akin to depicting ‘causality’ as an observable physical extant in abstraction from the actual states qualified as ‘cause’ and ‘effect.’ In both cases, the incoherence derives from the attempt to assimilate the idea of

[a] using the quantum *formalism* to *calculate* the probabilities of various potential states becoming actualized, into the idea of [b] using quantum *mechanics* to *measure* these potential states via observation. Philosophically, this attempted assimilation of 'observation of what is' to 'calculation of what is reasoned to be' is the modern scientific epitome of the attempted assimilation of ontology to epistemology discussed earlier in this chapter. It is a misplaced concretizing of calculation that neglects the fact that even in quantum mechanics, calculated superpositions of potential outcome states necessarily presuppose discrete, observable, actual initial and final system states and their logical relation, and it is only via the latter that predictive calculations are confirmed retrodictively. The electron, in other words, is always *observed as actualized*, in either one state or another, in satisfaction of PNC, and never observed as potentialized—i.e., as a superposition of potential states in violation of PNC. In this way, superpositions are properly understood as relations of successive actual states, initial and final, via an appropriate measurement interaction.

In quantum mechanics, this relation is fundamentally describable as an evolution of: [1] a pure state of *potential* outcome states (these are not mutually exclusive and can violate PNC), to [2] a mixed state of *probable* outcome states that are [a] mutually exclusive (satisfying PNC, i.e., '*at most* one outcome state will be actual upon measurement') *and* [b] exhaustive, in that the probability valuations, by definition, must sum to unity (satisfying PEM, i.e., '*at least* one outcome state will be actual upon measurement). Though this logically conditioned evolution of potential to probable outcome states is a fundamental feature of quantum mechanics (Von Neumann formalized the mathematics of this evolution as his 'Process 1'[7]), it is important to emphasize that quantum theory *presupposes* this evolution. The theory does not, in other words, entail any physical dynamical mechanism that 'generates' the evolution of potentiality to probability. Likewise, the theory also presupposes the evolution of probable outcome states to unique actual outcome state, in satisfaction of PEM. These are both aspects of the 'problem of state reduction' in quantum mechanics, which many theorists have cited as evidence of the incompleteness of quantum theory—that it presupposes [1] the existence of the actualities it measures, and [2] that its measurements will satisfy PNC and PEM.

As has been argued thus far, and will continue to be argued throughout the remainder of this volume, neither of these presuppositions can be reasonably assessed as a theoretical deficiency, since both are necessarily presupposed by the scientific method itself—the method by which quantum theory was initially conceived and validated, and by which it continues to be developed and implemented. In the case of the quantum oscillator experiment discussed above, it is the failure to explicitly recognize these presuppositions, coupled with the recently fashionable bent toward seeking profundity and mystique over intuitive coherence in the experimental application of quantum theory, that leads to unfortunate interpretive claims such as "the experimenters used the weird rules of quantum mechanics to simultaneously set the paddle moving while leaving it standing still . . . vibrating and not vibrating simultaneously . . ."[8]

Beyond being a necessary presupposition of both the scientific method and the mathematical language by which the method is formalized and applied to experiments, and by which the data of experiments are evaluated and interpreted, the presupposition of PNC is foundational to the possibility of coherence and consistency within any conceivable domain of experience. Thus, interpretive claims such as the one cited above seem to advocate a fundamental schism of critical reasoning, where PNC and Boolean logic in general are depicted as epistemic constructs borne of our classically restricted experiences of the world; and the world itself—the actual object of science disclosed most fundamentally via quantum theory—is at the same time depicted as routinely and categorically violating these constructs, even though it is these very constructs that allow science to proclaim this schism in the first place through experiments such as the one cited above.

Quantum theorist Roland Omnès describes the problem thus:

> There is indeed a crisis, for unlike the flourishing situation in the history of knowledge, the philosophical reflection about science has lost its

> way—or stagnates. The fashionable authors see only uncertainties, paradigms without enduring principles, an absence of method, and a presence of erratic revolutions, precisely when we should be trumpeting the success of a science whose extent and consistency are unprecedented... Beyond the shadow of a doubt, the origin of this crisis is to be found in an event that no one has fully recognized in all its significance: the irresistible irruption of the formal approach in some fundamental sciences such as logic, mathematics, and physics. As a consequence, these disciplines have become practically impenetrable, which explains the capitulation or the adventurousness of so many commentators, not to mention the disarray of the honest man or woman who wonders what those who should understand these subjects are talking about.[9]

2.2 Abandoning the Logic of One World for the Logic of Many Worlds: An Unlikely Liberation

There are three central principles of quantum theory that have contributed heavily to the 'impenetrability' and 'adventurousness' Omnès forewarns against in the above quotation, and which taken together have made it increasingly fashionable to dismiss as vacuous the entire enterprise of constructing a persuasive ontological interpretation:

[1] Objective Indeterminacy: Quantum indeterminacy displaces the classical physical first principle of 'objective determinism as a necessary implication of mathematical objectivity'—that mathematical necessity at the conceptual level implies deterministic contingency at the physical level. Instead, in quantum theory, mathematical probability implies indeterminacy at the physical level.

[2] Objective Local Contextuality: The local context dependence of quantum measurement is evinced by the fact that [a] probable outcome states of measured systems are definable only according to the Boolean-logical contextual measurement basis of a particular chosen detector. In the conventional Hilbert space formalism, this is exemplified by the requirement that the measurement basis be orthonormal, such that potential

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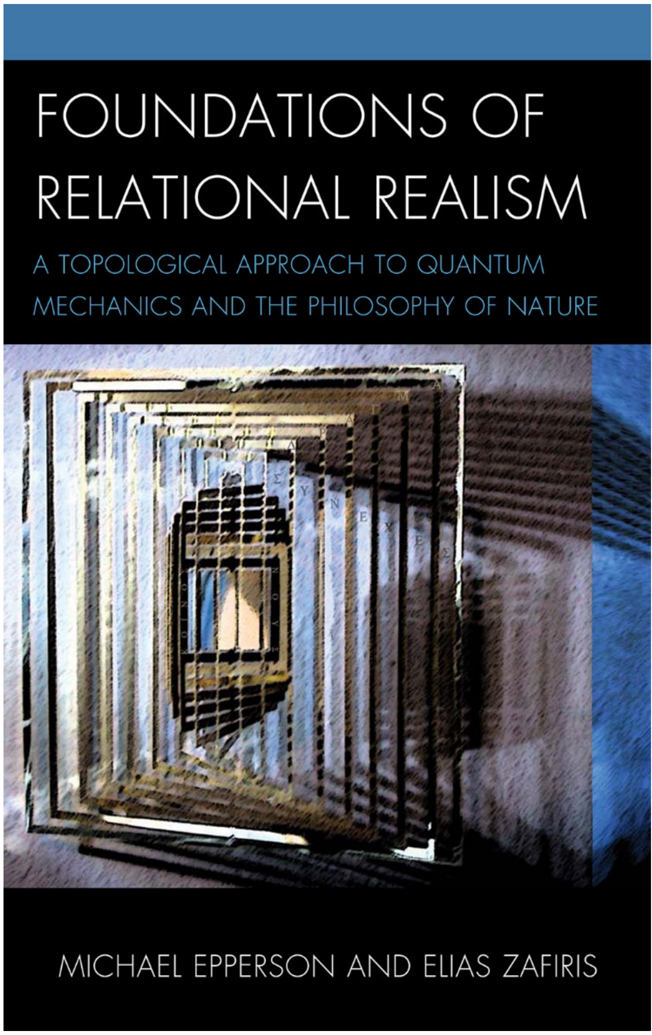

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Sciences and Exploration Directorate
NASA Goddard Space Flight Center

MICHAEL EPPERSON
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CHAPTER 3

Predication in Quantum Mechanics

If there is a common thread woven through the great variety of ontological interpretations of quantum mechanics, it is the belief that discrete measurement events are the fundamental objects of the theory, such that the possibility of specifying the state of a physical system continuously—i.e., 'between' these events—is, in principle, excluded. A quantum measurement event is the quantum mechanical evaluation of an observable, and this entails the assignment of a truth value F or T, via projection operators with eigenvalues 0 or 1, respectively, to each potential measurement value (e.g., particle spin 'up' or 'down') within some well-defined range of values. As discussed in chapter 2, these values are always defined in terms of a local Boolean measurement context and its associated orthonormal measurement basis. By the presupposition that system states are defined by eigenvalues,[1] measurement can thus be characterized as the actualization of some potential outcome state of the measured system, with each potential outcome state entailing an alternative maximal scheme of truth values for its constituent observables. Quantum observables, in other words, are always properly understood as potential facts, and an evaluated observable is likewise properly understood as an *actualized* potential fact.

In this way, as discussed previously, a fact yielded via quantum measurement is understood as generated consequent of measurement, such that measurement events are ontologically significant; they are facts constitutive of object systems and not merely constitutive of our knowledge of object systems. Classical measurement, by contrast, is understood as merely revelatory of facts—that is, revelatory of already extant truth values pertaining to the evaluated observables. As a result, classical observables are co-determinate and commutative, such that classical system states can be completely specified and represented by a global Boolean algebra—the essence of a Newtonian block universe model.

But unlike classical states, quantum mechanical system states, because they are specified as synthetically predicative facts generated by measurement rather than analytically predicative facts revealed by measurement, can never be *completely* specified—only maximally specified. This is evinced by the fact that the representative algebra of a quantum system is non-commutative and only partially Boolean globally; for the generation of a novel fact *consequent* of measurement clearly precludes the possibility of defining a measured system as a *complete* specification of a co-determinate set of facts. And likewise, there is no structure-preserving map on the global partial Boolean algebra that distinguishes between [a] facts constitutive of the system undergoing measurement and [b] facts generated *by* measurement. Against the argument that this is merely an epistemic limitation ensuing from the incompleteness of quantum theory, such that the disclosure of hidden variables would render such a mapping and thereby a complete state specification as in the case of classical mechanics, Kochen and Specker[2] have proved that the partial Boolean algebra of projection operators characteristic of quantum mechanics cannot, in general, be embedded into a global Boolean algebra.[3]

Closely associated with this difference between quantum and classical measurement are two others: [1] the relationship between the observer and the observed in quantum mechanics is considered to be ontologically significant, such that the eigenstates of a measured system always correlate with the particular Boolean measurement context (e.g., the preferred orthonormal measurement basis and representative Boolean subalgebra) of the chosen measuring apparatus. Thus the objectivity of facts that are generated by measurement is at least in some sense conditioned by the subjective context of the measurement—i.e., the facts constitutive of the measuring apparatus by which the preferred basis is defined; [2] the relationship between the observer and the *unobserved* in quantum mechanics is considered to be ontologically significant, as evinced by the phenomenon of quantum decoherence discussed in chapter 2.7, such that the probability valuations of the eigenstates of the measured system are logically (and in some cases physically) conditioned by facts 'environmental' to the measured system. This is because, in the words of Heisenberg, the description of the measuring apparatus

> contains all the uncertainties concerning the microscopic structure of the device which we know from thermodynamics, and since the device is connected with the rest of the world, it contains in fact the uncertainties of the microscopic structure of the whole world[4] . . . The transition from the 'possible' to the 'actual' takes place as soon as the interaction of the object with the measuring device, and thereby with the rest of the world, has come into play."[5]

Thus the 'objectivity' of a particular actualized local observable is conditioned by unmeasured global observables that exceed the local context of the measurement. But as noted in the previous chapter, unlike classical local system-environmental relations, the nonlocal relations of quantum physics do not necessarily entail a transfer of energy,[6] and are therefore not fully describable in the conventional sense of classical efficient causality. Thus, the causal conditioning of potential local measurement outcomes by global actualities—i.e., facts environmental to the local measurement context—is not primarily a conditioning via efficient causal relation, but rather a conditioning via logical correlation. In this way, unlike the classical conception of the latter as a purely epistemic epiphenomenon—that is, logic as a function of mentality—the ontological interpretation of quantum mechanics depicts logical correlations as a physically significant feature of nature itself.

As introduced in chapter 2, this notion of non-efficient causal conditioning by logical correlation can be expressed in terms of relations among the various maximal and nonmaximal Boolean subalgebras by which composite quantum systems can be defined. Jeffrey Bub, for example, has shown[7] that prior to a measurement interaction, a composite system $S_1 + S_2$ can be defined as a global equivalence class $\mathcal{B}$ of maximal Boolean subalgebras in the partial Boolean algebra of properties of $S_1 + S_2$. A local measurement M_2 at S_2 selects a local nonmaximal Boolean subalgebra $\mathcal{B}_2$ (that is, it locally contextualizes S_2 in terms of the preferred orthonormal measurement basis of M_2). This *local* selection of $\mathcal{B}_2$ produces a non-efficient-causal conditioning, or "revision," to use Bub's term, of the *global* equivalence class $\mathcal{B}$ of maximal Boolean subalgebras in the partial Boolean algebra of properties of $S_1 + S_2$, to a new global equivalence class $\mathcal{B}'$. This logical conditioning of $\mathcal{B}$, that is, its revision to $\mathcal{B}'$, thus entails a revision of all constituent subalgebras of $\mathcal{B}$, including both $\mathcal{B}_1$ (the local measurement context of S_1) *and* $\mathcal{B}_2$ (the local measurement context of S_2) even if S_1 and S_2 are spatially well separated, as in an EPR-type arrangement. In other words, any local contextualization of S_1 by a measurement M_1 (i.e., any possible nonmaximal Boolean subalgebra $\mathcal{B}_1$) must now be logically compatible with $\mathcal{B}_2$–that is, it must be a Boolean subalgebra yielded by the union of $\mathcal{B}_2$ and the set of elements in $\mathcal{B}_1$ that are compatible with $\mathcal{B}_2$.[8]

This logical conditioning, in concert with the efficient causal mechanisms of physics, together exemplify a fundamental *logical causality* in nature. As clearly demonstrated by quantum mechanics, the unspecified actual facts of the world, even if spatially well-separated from a particular local measurement context, always have a non-negligible, logical conditioning influence upon what is physically possible within that local context.

3.1 EPR and Decoherence

As introduced in chapter 2, there have been, over the past several decades, two key types of experimental exploration of logical causality in quantum measurement: experiments investigating quantum decoherence and experiments investigating non-local correlations among subsystems of a composite quantum system. The latter type, already introduced in the previous section, is well represented by the EPR-type experimental arrangement[9] discussed in chapter 2.8, where two measured local subsystems S_1 and S_2 of a composite system $S_1 + S_2$ are spatially well-separated—i.e., mutually environmental—such that any transfer of energy between the subsystems (energy intended to account for the causal conditioning of one subsystem's measurement outcome by that of the other) would need to be superluminal, in violation of relativity theory. To date, no such energy transfer has been detected, though several non-local, superluminal efficient causal mechanisms have been proposed over the decades.[10]

With respect to logical causality in quantum decoherence, the latter can be generally understood as a sub-process of every quantum measurement interaction, entailing [1] the individuation of potential measurement outcomes that are initially coherently superposed in the pure state (e.g., Schrödinger's cat as alive AND dead), and [2] the logically conditioned evolution of those individual *potential* outcome states that satisfy both PNC and PEM, to *probable* outcome states that are mutually exclusive and exhaustive (e.g., such that Schrödinger's cat is *either* alive XOR[11] dead). Together, these now individuated / decoherent states comprise the probable measurement outcomes of the mixed state. This evolution from the pure state to the mixed state is achieved via a *negative selection* of those potential outcome states whose conjunction violates the logical principle of non-contradiction (PNC)—i.e., those potential outcome states that 'logically interfere' with each other. These are formally represented by an 'interference term' corresponding to the off-diagonal terms of the density matrix; and likewise, the negative selection mechanism is represented as a mathematical self-cancellation of these off-diagonal terms, leaving only the diagonal terms, which do satisfy PNC.[12]

Thus, as discussed in chapter 2, the mutually exclusive nature of the probable outcomes of the mixed state reflects the satisfaction of PNC—i.e., 'at most one outcome state'; and likewise the exhaustive nature of the mixed state reflects the satisfaction of the logical principle of the excluded middle (PEM)—i.e., 'at least one outcome state.' The intrinsic presupposition of PNC and PEM together in any ontological interpretation of quantum mechanics is thus reflected in the fact that each outcome state is valuated as a probability and not merely a potentiality once decoherence occurs; that is to say, all probable outcome states are mutually exclusive per PNC, and exhaustive per PEM, such that together they must sum to unity. And as will be further explored presently, the function of probabilities in quantum mechanics presupposes not just PNC and PEM, but also the laws of Boolean logic[13] by which the Boolean algebras discussed in the previous section are constructed; for the mixed state is always defined according to the local Boolean measurement context corresponding to a particular observable.

The logical conditioning of potential outcome states into probable outcome states via local Boolean measurement contextualization is the central import of decoherence—and arguably of quantum mechanics in general. The result is the enabling of the distributive law and the probability sum rules that follow from it, which, because of quantum interference, are not operative in quantum systems apart from decoherence. Typically, as was discussed in chapter 2, quantum interference and its undermining of the distributive law[14] has been considered the hallmark feature of quantum mechanics because of the classically counterintuitive implications, captured most infamously by the popular Schrödinger's cat scenario. However, while superpositions of interfering potentia are certainly integral to quantum measurement, the fact that they *always* evolve via decoherence to become non-interfering probability distributions would seem to be even more emblematic of quantum mechanics than superposition itself—i.e., superposition in abstraction from the process of quantum measurement as a whole—a dubious abstraction given that apart from this process, superposition has no meaning. Thus, one could argue that it is not the superposition of potentia but rather the Boolean logical conditioning of potentia, productive of probability valuations such that PNC and PEM are always ultimately satisfied, that is quantum theory's signature feature—i.e., the evolution of potentiality to probability.

The selective evolution of a coherent superposition of potential outcome states, most of which are mutually logically inconsistent in terms of PNC, to a decoherent distribution of logically consistent probable outcome states, is achieved by incorporating the facts constitutive of the measured system's environment[15] into the quantum measurement formalism—that is, making explicit the system-environment relations that have traditionally been treated as implicit and irrelevant in quantum theory. Philosophically, this is justifiable simply because it explicates what is already implicit and presupposed given quantum theory's conventional assessment as a universal theory. But it is also justifiable practically in that it enables one to account for decoherence as a conceptual implication of the standard formalism: Rendering explicit the system-environment relations typically ignored as implicit introduces manifold degrees of freedom into the superposition of potential outcome states. This, in turn, allows for the integration of their respective maximal Boolean subalgebras (in the partial Boolean algebra of properties of the global system) into equivalence classes large enough to accommodate the negative selection process described above, with each class indexed to a different potential outcome state of the locally contextualized measuring apparatus.

As discussed earlier, these alternative potential outcome states are always defined according to the particular orthonormal measurement basis of the local measuring apparatus—that is, according to the Boolean subalgebra by which the local measurement context is defined. In this way, the preferred basis functions as a local Boolean context by which the potentia of the global quantum system are logically integrated—that is, by which the equivalence class of Boolean subalgebras in the partial Boolean algebra defining the global system is revised by a local measurement.

Again, it is the magnitude of environmental degrees of freedom within each equivalence class that produces sufficient logical-mathematical cancellation among potential system-detector-environment outcome states that interfere (i.e., that violate PNC) within each class. A traceover of the environmental degrees of freedom thus eliminates these logically incoherent states represented by the off-diagonal terms in the density matrix. The elimination of these terms effects the reduction of the pure state, with its interfering superposition of potential measurement outcomes, to the mixed state and its logically conditioned, decoherent matrix of mutually exclusive and exhaustive probable outcome states.

As is the case with EPR-type nonlocality experiments, where this nonlocal logical conditioning obviates the need for any proposed superluminal efficient-causal mechanism to account for the quantum correlations among spatially-well-separated systems, recent experiments on precessional decoherence in spin bath dynamics,[16] as discussed in chapter 2.7, likewise entail no energy transfer between the bath and the measured system, unlike previous oscillator bath and other environmental decoherence models.[17] Thus the once conventional description of decoherence-generating system-environment interactions purely as a function of energy exchange—i.e., via an effective Hamiltonian—is arguably incomplete.[18] In light of these experiments, it is clear that the decoherence of global potentia into individuated, locally contextualized probabilities is properly understood not as an efficient casual mechanism, as it is often portrayed, but rather as a logical integration of potential outcome states such that a subset of these can evolve to become probable outcome states. This conditioning is formalized as a revision of the equivalence classes of maximal Boolean subalgebras in the partial Boolean algebra defining the global system, 'logically revised' by the nonmaximal Boolean algebras representing locally contextualized measurements.

3.2 The Problem of Self Reference in Quantum Systems

In summary of the discussion so far, the evolution of classical to quantum mechanics has carried with it a parallel evolution of the concept of measurement. Classical mechanics depicts measurement as revelatory of objective, context-independent, co-determinate facts such that even

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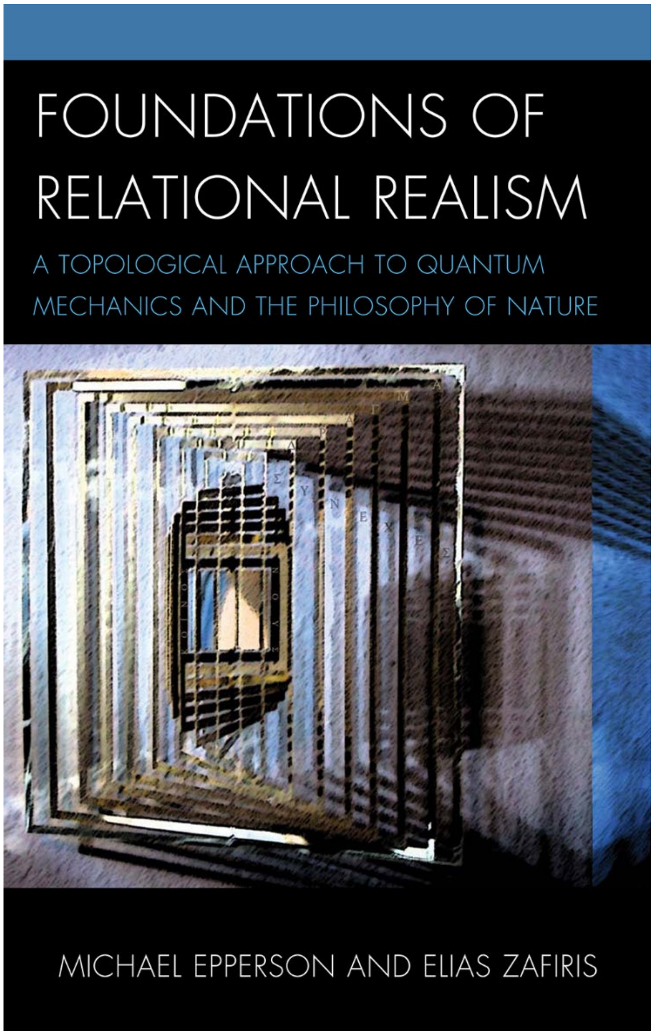

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TIMOTHY E. EASTMAN, PH.D.
Senior Physicist (ret)
Sciences and Exploration Directorate
NASA Goddard Space Flight Center

MICHAEL EPPERSON
Ph.D. Philosophy
University of Chicago

ELIAS ZAFIRIS
Ph.D. Theoretical Physics
Imperial College
University of London

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CHAPTER 4

Logical Causality in Quantum Mechanics: A Relational Realist Ontology

The process of the quantum mechanical actualization of a potential outcome state (i.e., the predication of an observable) is affected by both [1] the potential actuality's causal-physical relations with the dative actualities 'physically' antecedent to it (prior in time) in terms of metrical, spatiotemporal extensiveness—i.e., within its backward light cone, per the restrictions of relativity theory; and [2] its logical internal relations with those actualities 'logically' antecedent to it (prior in order), per the restrictions of PNC and Boolean material implication, among other logical restrictions. (As discussed in the previous chapter, these are, respectively, the Whiteheadian 'coordinate' and 'genetic' analyses of a quantum actual occasion.[1]) Unlike the causal-physical relations, the logical internal relations are not relativistically restricted, and include both local and nonlocal data. For example, with respect to quantum decoherence, these relations include the unmeasured degrees of freedom environmental to the measured system, which may or may not fall outside the observable's backward light cone. Both kinds of relation, physical and logical, are unified by mutual implication within each predicative fact / quantum actual occasion via its dipolar logico-physical structure, such that all predication is reflective of logical causality.

4.1 Internal Relation and Logical Implication in Quantum Mechanics

The internal relation of local potential predicative fact to global dative actuality, which is the basis of the asymmetrical nature of this scheme, is reflected in the asymmetrical orders of syntactic, Boolean material implication, and semantic, logical implication (i.e., entailment). And with respect to its application to the ontological interpretation of quantum mechanics, this metaphysical and logical asymmetry is crucial. As introduced in chapter 2.4, asymmetrical internal relation, given in the expression $p \Rightarrow q$ (read, '*p only if q'*) can be understood as Boolean material implication $p \rightarrow q$ (read 'if p then q') that is always true semantically—i.e., as a matter of *fact*, not form.

Material Implication		
p	*q*	→
T	T	T
T	F	F
F	T	T
F	F	T

Internal Relation		
p	*Q*	⇒
T	T	T
F	T	T
F	F	T

As applied to the discussion of Boolean local measurement contextuality, in the truth table above depicting asymmetrical internal relation, Q thus refers to the logical context by which p can be evaluated. This is denoted by using the capital Q in the case of internal relation, versus the lowercase q in the case of material implication.

It should be emphasized here that it is because material implication is purely syntactic, i.e., as a truth-functional, that it is representable as a truth table, by which the truth or falsity of the proposition derived simply as a matter of mapping *T* and *F* as values, whether or not these values are *evaluations* of actual facts. Asymmetrical internal relation, by contrast, is syntactic *and* semantic—that is, relating both form *and* fact—and thus its depiction as a truth table here is purely heuristic. It ought not, in other words, be taken to imply that internal relation is reducible to a simple truth-functional.

With respect to its reflection in the relational realist ontology, asymmetrical internal relations, where *p* necessarily implies *Q*, presuppose a global 'objectivity' of the implicate *Q* at the pre-theoretic level that is not presupposed in the material conditional; yet at the same time, it preserves an indeterminacy in the local implicans *p* (which, as a predicative function of the argument in *Q*, must be of the next order above *Q*). This is because the truth value of the potential predicative fact *p* is not *determined* by the global objective facts constitutive of *Q*; rather, the latter determine the *conditions* to which all predication must conform. For example, if *p* = 'Socrates is in Athens' and *Q* = 'Socrates is in Greece,' Socrates being in Greece does not determine his being in Athens, though it does condition the possibility of the latter. (Conversely, his being in Athens does determine his being in Greece—thus the asymmetry of internal relation via logical implication.) This is analogous to the case in quantum mechanics, such that in the spin ½ system,

$$|\Psi\rangle = \alpha|\psi\uparrow\rangle\,|\phi\uparrow\rangle\,|e\uparrow\rangle + \beta|\psi\downarrow\rangle\,|\phi\downarrow\rangle\,|e\downarrow\rangle$$

$|\Psi\rangle$ is a vector of unit length, and thus representative of the actual (though indeterminate) state of the composite global system and its facts prior to the measurement outcome. $|\Psi\rangle$, for example, thus represents the *implicate* in the statement $|\psi\uparrow\rangle\,|\phi\uparrow\rangle\,|e\uparrow\rangle \Rightarrow |\Psi\rangle$ (read, $|\psi\uparrow\rangle\,|\phi\uparrow\rangle\,|e\uparrow\rangle$ *only if* $|\Psi\rangle$). These facts subsumed by $|\Psi\rangle$ condition, but do not determine, the novel predicative outcome fact (in this case, either eigenstate $|\psi\uparrow\rangle\,|\phi\uparrow\rangle\,|e\uparrow\rangle$ or $|\psi\downarrow\rangle\,|\phi\downarrow\rangle\,|e\downarrow\rangle$) generated by measurement. This indeterminacy is reflected, for example, in the fact that each eigenstate (i.e., each *implicans* of the internal relation) is always valuated as a probability, via the complex coefficients α and β, respectively—$\alpha|\psi\uparrow\rangle\,|\phi\uparrow\rangle\,|e\uparrow\rangle$

and $\beta|\psi\downarrow\rangle\,|\phi\downarrow\rangle\,|e\downarrow\rangle$—with $|\alpha|^2 + |\beta|^2 = 1$. Again, valuation as a probability, which necessarily satisfies PEM, rather than as merely a potentiality, which does not necessarily satisfy PEM, is only possible because the evaluation is relative to a local Boolean context (e.g., that given by the chosen detector, whose pointer reads either exclusively 'up' or 'down'—that is, $\phi\uparrow \veebar \phi\downarrow$—represented in the above expression as $\alpha|\phi\uparrow\rangle + \beta|\phi\downarrow\rangle$). Further, this local context is itself only definable in quantum mechanics via reference to a global context $|\Psi\rangle$ (represented by Q in the above example).

In other words, though the novel predicative fact p is not determined by the argument Q, it is *contextually dependent* upon Q for its definition. In the same way that Athens cannot be defined without implicit or explicit reference to Greece, no local system can be defined without implicit or explicit reference to a larger system subsuming it. In quantum mechanics, the latter must be a closed system, and interpreted ontologically, the only closed system is the universe itself—i.e., the totality represented by $|\Psi\rangle$. And in $|\Psi\rangle = \alpha|\psi\uparrow\rangle\,|\phi\uparrow\rangle\,|e\uparrow\rangle + \beta|\psi\downarrow\rangle\,|\phi\downarrow\rangle\,|e\downarrow\rangle$, the probability valuated, predicative outcome states are defined as projections of $|\Psi\rangle$ upon each eigenstate. Thus in quantum mechanics, each locally contextualized predicative fact is, by internal relation, dependent upon the global totality for its definition.

Via this conception of logically conditioned, asymmetrical internal relation, then, the ontological interpretation of quantum mechanics described herein posits that every becoming actual occasion/propositional predicative fact is always internally related to a global, objective actual world—an actual system of facts with objective truth values. It is in this way that the objective facts of the actual world serve to condition the local possibilities internally relative to that actual world. Thus every eigenstate p predicative of a local observable with *potential* truth value T or F makes necessary reference to a global *actual* world Q, such that p is internally related to Q, where the implicans p is of the next logical order above that of the implicate Q.

As related to the discussion in chapter 3, Q is always locally Boolean contextualized in quantum mechanics, represented by an equivalence class of maximal Boolean subalgebras in the partial Boolean algebra of observables defining the global system. Recall how a local measurement M_2 on a subsystem S_2 of a global system $S_1 + S_2$ will condition the local measurement outcomes at S_1 via a revision of this equivalence class of Boolean subalgebras; it is only in this way that the facts of Q condition, via *restriction of the local by the global*, the possibilities for p. In the above example of Socrates' location, Q represents the selection of a Boolean context for p among all the potentia referent to the global totality (i.e., $|\Psi\rangle$). But in quantum mechanics, it is not the case that all possible local contexts can be related *to each other* in terms of a global, fully Boolean associative order of inclusion. In the above example, for any two local Boolean contexts $p \Rightarrow Q$ and $r \Rightarrow S$, there are three possible logical internal relations:

[1] $Q \Rightarrow S$ (deductive)
[2] $S \Rightarrow Q$ (deductive)
[3] $(p \Rightarrow Q) \wedge (r \Rightarrow S) \rightarrow \exists(Q \otimes S)$ (inductive)

The third, inductive type is an essential feature of the topological category-sheaf theoretic approach to quantum mechanics, and will be further explored presently, as well as more formally in part II of this volume. For now it is sufficient to note that it depicts a global totality-of-contexts as a *maximally* Boolean overlap among all local contexts, each of which is representable as a local nonmaximal Boolean subalgebra.

As was explored in chapters 2 and 3, the utility of this conceptual framework becomes especially apparent when applied toward the coherent interpretation of quantum phenomena such as EPR-type nonlocality. Against many popular interpretations of the latter, the relational realist interpretation depicts EPR nonlocality as a non-metrical, topologically formalized logical conditioning of potentia; this is in sharp contrast to other interpretations that depict EPR nonlocality as an efficient causal influence requiring a superluminal physical-dynamical mechanism, or as evidence of 'retro-causality' requiring the abandonment of temporal asymmetry and its presupposed correlation with logical asymmetry. In particular, recent experiments in quantum optics have been interpreted as evincing such 'retro-causality,' including the so-called 'delayed choice, double-slit quantum eraser' experiments mentioned in chapter 2.1. In the context of the current discussion of nonlocal logical causality, a brief exploration of quantum eraser experiments is worth exploring here.

As introduced in chapter 2.1, these experiments exploring quantum superposition and entanglement are based on the Young double-slit experiment of 1803, which was originally devised to explore the dual wave-particle nature of light. As we would characterize it today, the experimental arrangement essentially entails photons propagating through either of two parallel slits prior to impinging upon a detector. When either slit is closed, the photons are well localized at the detector, thus exhibiting their classically particulate character; and when both slits are open, the photons form an interference fringe at the detector, thus exhibiting their 'quantum superpositional' wave-like character.

A straightforward example of a double-slit quantum eraser experiment, based on the above, is that of Walborn et al.[2] Many theorists (as well as many science journalists[3]) have interpreted such experiments as 'erasing,' via a kind of 'retro-causality,'[4] individual actualized measurements, and with them the Slit 1 / Slit 2 path information contained in the outcome states recorded by these measurements. This path information is included in the state specification by 'labeling' each path in terms of a combination of linear and circular polarization of the photons particular to each path. Thus, in the same way that electron spin direction served as a Boolean measurement context in the EPR example, linear and circular polarization serve as Boolean contexts in the quantum eraser example. At emission, the photons are initially split into two entangled beams, a 'signal' beam *S* directed at the double-slits, and an 'idler' beam *P* whose linear polarization will be manipulated in order to 'erase' the path information of the signal beam. In front of each slit, a quarter wave plate 'labels' a photon's passage by circularly polarizing it either left or right without otherwise disturbing it as it heads for the detector.

At emission, the linear polarization of both beams is determined, and because they are correlated, direct measurement of the idler beam *P*'s linear polarization indirectly yields, via implicative internal relation, the signal beam *S*'s polarization. Thus, $S_{\mathrm{y}} \Rightarrow P_x$ and $S_{\mathrm{x}} \Rightarrow P_{\mathrm{y}}$. Finally, because of the relationship between linear and circular polarization in this arrangement, it is also a matter of implicative internal relation that depending on the linear polarization of *S*, passage through the quarter wave plates labeling Slit 1 (SL1) and Slit 2 (SL2) will result in either left or right circular polarization of *S* after passage through the slits. Thus, $S_{left}\,(\mathrm{SL1}) \wedge S_{right}\,(\mathrm{SL2}) \Rightarrow S_{\mathrm{x}}$, and $S_{right}\,(\mathrm{SL1}) \wedge S_{left}\,(\mathrm{SL2}) \Rightarrow S_{\mathrm{y}}$. When this arrangement is in place, the signal photon paths are effectively labeled via logical entailment, and detection events are well-localized, with no interference fringe.

The 'eraser' procedure simply involves placing an additional linear polarizer in the idler beam path, oriented so that *both x* and *y* polarized photons will pass through, thus 'erasing' the linear polarization information for the idler beam, which also erases, by logical entailment, the linear polarization information for the signal beam. This, in turn, 'erases' the logically implicative circular polarization labeling by the quarter wave plates at Slit 1 and Slit 2, described above. In this way, erasing the linear polarization information from the idler beam erases the path labeling mechanism for the signal beam, and causes the characteristic interference fringe to appear at the detector.

Finally, it is purported to be significant that the path of the idler beam is longer than the path of the signal beam, such that if the linear polarizer is placed at the end of the idler beam, then for any particular photon pair, the idler polarizer's 'erasure' of path information will occur *after* its counterpart signal photon has already reached its detector.[5] As a result, the characteristic interference fringe reappears as it does with regular erasure, but in this case, *after* the path-labeled signal photon has already been detected. Again, this is often described as 'retro-causal' or 'delayed' erasure, implying that already actualized quantum facts (i.e., facts entailing path information) have somehow been 'erased' from reality.

At once exotic and esoteric, this interpretation has proven understandably tantalizing to both specialists and popular audiences alike. The reason is that the purported disconnection of the asymmetrical order of causal relation and the asymmetrical order of logical implication fatally undermines the foundational principle by which nature is coherently accessible to human reason in general, and to the scientific method in particular: the categorical correlation of these asymmetrical orders—i.e., the presupposition of logical causality. Thus, as was argued in chapter 2, any purported *scientific* invalidation of this presupposition amounts to nothing less than a scientific invalidation of the scientific method itself—which, of course, makes the application of the latter to such an endeavor paradoxical at best.

For this reason alone, one could argue that the only truly coherent scientific interpretations of these double slit, 'quantum eraser' experiments are those that make explicit their reliance on logical causality as a categorical presupposition, on the grounds that the method of science itself is ineluctably rooted in this same presupposition. The relational realist ontological interpretation is one such candidate, and its application to the quantum eraser experiment outlined above is fully consistent with its application to the EPR-type quantum nonlocality experiments previously described, and presented formally in chapter 9.4: The local Boolean measurement contexts of the experimental arrangement are identified, in this case as:

[1] $P_x \veebar P_\mathrm{y}$

[2] $S_x \veebar S_\mathrm{y}$

[3] $S_{left}(\mathrm{SL1}) \veebar S_{right}(\mathrm{SL1})$

[4] $S_{left}(\mathrm{SL2}) \veebar S_{right}(\mathrm{SL2})$

These measurement contexts, representable as Boolean subalgebras, are mereotopologically internally related such that in the logical order of evaluation (i.e., the order of detection events), every contextualized ob-

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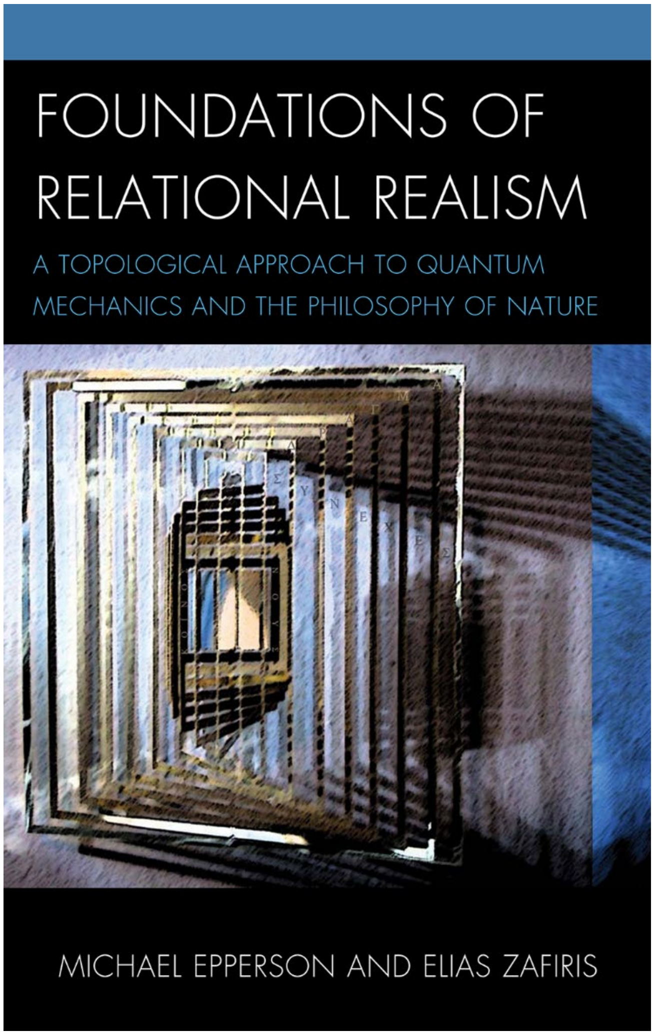

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TIMOTHY E. EASTMAN, PH.D.
Senior Physicist (ret)
Sciences and Exploration Directorate
NASA Goddard Space Flight Center

MICHAEL EPPERSON
Ph.D. Philosophy
University of Chicago

ELIAS ZAFIRIS
Ph.D. Theoretical Physics
Imperial College
University of London

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CHAPTER 5

Integrating Logical Relation and Extensive Relation: Mereotopology and Quantum Mechanics

The internal relational framework by which the discrete, logical, ‘genetic’ relations among quantum events are correlated with their continuous physical ‘coordinate’ relations is presented by Whitehead in part IV of *Process and Reality:* It is his mereotopological scheme of ‘extensive connection,’ whose fundamental features are the relations among locally defined actual occasions and their internally related global regions. In earlier works, this was a set-theoretic mereological scheme, whose fundamental units were ‘extensive wholes’ and ‘extensive parts’ rather than the internal relation morphisms connecting actual occasions to their regions, and their regions to other regions via internal relation to the global totality. “This defect of starting-point,” wrote Whitehead, “revenged itself in the fact that the ‘method of extensive abstraction’ developed in those works was unable to define a ‘point’ without the intervention of the theory of ‘duration.’”[1] His earlier works, in other words, defined ‘point’ as the discretization of an ‘actual’ continuum. But defining a point as an extensive element of the continuous real line implies that the point, too, must be continuously divisible extensively (i.e., in terms of its coordinate analysis); thus the actual occasion, when represented coordinately as a point, loses its quantum character.

By contrast, Whitehead's later argument in *Process and Reality* is more firmly anchored to his commitment to the Aristotelian notion of "*infinitum actu non datur*": There is no 'actual' super-denumerable infinite—only infinite *potentia* for relations among actual occasions. In the relational realist scheme, for example, the apparent infinite divisibility of a finite length is properly understood as an infinite number of potential relations among the denumerable actual occasions constitutive of that length; in this way, 'length' is properly understood as a higher order, metrical abstraction derived from these underlying discrete relations. The potential relations are infinite ontologically, not merely epistemically, because predication is synthetic,[2] such that the actualization of any potential relation yields a novel actual relatum—and thus novel potential relations. This is easily depicted via the real number line, a finite representation of the infinite, wherein the totality of relations among numbers implies augmentation by either infinite extrapolation or interpolation.

Likewise, recall the mathematical analogy suggested in chapter 2: Consider the relationship between $\mathbb{R}$ (the set of real numbers—a continuum and thus uncountable), $\mathbb{Q}$ (the set of rational numbers—discrete and thus countable), and $\mathbb{Z}$ (the set of integers—also discrete and countable). According to classical intuition, we usually take the 'foundational' set to be the continuum $\mathbb{R}$; and in extending this mathematical intuition to nature more broadly, we likewise consider an 'extensive continuum' as a fundamental spatiotemporal background against which all physical relations, including quantum events, might be related metrically via coordinate division. The mathematical analog representative of such relations (e.g., as ratios) would be $\mathbb{Q}$, defined as a discretized subset of $\mathbb{R}$ (i.e., we quantize the continuum $\mathbb{R}$). Likewise, the mathematical analog of the quantum objects related might be $\mathbb{Z}$, defined as a subset of $\mathbb{Q}$.

But one could just as intuitively begin with $\mathbb{Z}$ as the fundamental background of discrete quantum units and proceed in the other direction, deriving $\mathbb{Q}$ as a set of discrete relations (i.e., ratios) among these discrete units of $\mathbb{Z}$, and likewise deriving $\mathbb{R}$ as a set of relations among the units of $\mathbb{Q}$ (viz., the Cauchy sequence), with *relations of relations*, etc. By proceeding in this direction, one could extend the mathematical representation even further: Since each quantum actual occasion is both unique and internally relatable to the totality of occasions constitutive of its dative world, one could mathematically represent actual occasions via the prime numbers—each unique and indivisible (i.e., quantum), but *relatable* via their shared property of belonging to $\mathbb{Z}$.

Given that the conceptual foundation of Whitehead's ontological scheme is the quantum actual occasion, the latter order, from $\mathbb{Z}$ as fundamental to $\mathbb{R}$ as a higher-order derivation, would seem to be the more appropriate in defining extensive connection among quantum *res verae*. Whitehead writes, for example:

> The concrescence presupposes its basic region, and not the region its concrescence. Thus the subjective unity of the concrescence is irrelevant to the divisibility of the region. In dividing the region we are ignoring the subjective unity which is inconsistent with such division. But the region is, after all, divisible, although in genetic growth it is undivided.[3]

The key impediment confronting Whitehead in his attempt to formalize the conceptual relationships described in the above quote—quantum actual occasions internally related genetically (via a logically asymmetrical structure) but extensively related coordinately (via a continuous, symmetrical structure)—is his presupposition that extensive, regional divisibility is at once fundamentally set-theoretic and continuous. However, since quantum actual occasions are, by definition, discrete units, it is clear that the *regional contexts* by which they are defined *must also be discrete* if internal relation and extensive connection are to be coherently integrated. The idea of internal relations among fundamentally discrete regions cannot be adequately formalized via the language of set theory, the conventional language available to Whitehead in his time, because it is grounded in a continuum (i.e., the real line); however, as introduced in chapter 3, structure-preserving internal relations among discrete regions *can* be adequately formalized topologically in the language of category theory, as will be explored presently.

By making the structure-preserving internal relations among occasions *and* their regions the fundamental objects of his scheme of extensive connection, rather than just the related quantum objects themselves, Whitehead attempted to construct a rudimentary, quasi-set-theoretic framework of extensive connection in part IV of *Process and Reality*; but because of the inherent limitations of set theory, this framework—an abstractive shift from a sheerly set-theoretic structure, but still firmly grounded in the latter—lacked the formal rigor necessary for serious application to the fundamental physics of his time.[4]

Less than two decades later, Samuel Eilenberg and Saunders Mac Lane would develop their category theoretic approach to algebraic topology in a similar abstractive shift of focus from the purely set-theoretic framework, whose fundamental units are the elements of sets, to the category-theoretic framework, whose fundamental units are the structure-preserving relations (morphisms) among discrete objects that are *themselves* relational structures. Indeed, one finds in category theory, and in particular as pertaining to sheaf theory (to be discussed presently), a rigorous formalism that would seem to satisfy all the demands Whitehead made of his 'theory of extensive connection' in part IV of *Process and Reality*, but was not able to fully satisfy via the set theoretic conceptual framework he had earlier developed with Russell.

For example, Russell and Whitehead's original method of bypassing the paradoxes of set-theoretic self-reference, as discussed in chapter 3, was the Theory of Logical Types; but as part IV of *Process and Reality* well demonstrates, the set-theoretic framework of Logical Types was difficult to apply to the extensive features of physical relations. By contrast, the category-sheaf theoretic formalism, as a mereotopological schematization of quantum mechanics,[5] provides a uniquely appropriate and formally rigorous method of translating the asymmetrical logical and mereological features of quantum mechanical relations ('genetic division') to the extensive features of these relations ('coordinate division') without difficulty. Self-reference is avoided in this scheme analogous to the way that it is avoided in the Theory of Logical Types—that is, via a hierarchical structure of logical contexts internally related asymmetrically. But again, unlike the quasi-set-theoretic framework of extensive connection given in part IV, category-sheaf theoretic regional contexts are fundamentally topological, and therefore discrete, rather than continuous.

Several key aspects of the sheaf-theoretic structure were introduced in chapters 3 and 4—namely, those aspects by which the logical integration of potential relations to probable relations could be defined *apart* from the extensive features of these relations, such that even when relata are spacelike separated (i.e., nonlocally related, as in the EPR experimental arrangement), the logical integration of potential relations to probable relations is unimpeded. Again, this can only be accounted for via the categorical presupposition that all local contexts of related data are Boolean, such that each context is representable in quantum mechanics as a Boolean subalgebra. Thus, the totality of local contexts, to the extent that they are compatible for integration into an induced, approximated global context, can be formalized as a mereotopological framework of nested (i.e., internally related) contextual inclusions. It is through this framework that the internal relation of the local novel actual occasion / predicative quantum measurement outcome to the global dative world is logically coordinated such that the latter can be formalized topologically. Thus the sheaf-theoretic Boolean localization scheme has both logical and ontological significance in the relational realist philosophy, via the concepts of logical implication and internal relation, respectively.

In exploring the extensive aspects of the logical concept of 'local Boolean context' discussed above, it is clear that in any ontological interpretation of quantum mechanics, these aspects are fundamentally topological rather than metrical; for the concept of 'local context' in quantum mechanics has no ontological meaning within the metrical framework of the relativistic spacetime continuum, where 'local' refers to points connected by continuously divisible spatiotemporal intervals, not discrete regions connected by discrete logical internal relations. In a mereotopological continuum, however, one can represent a discrete 'local context,' in precisely this latter way, as an open cover. A scheme of internally related local contexts can then be represented topologically via a scheme of open covers of an open set. But as applied to quantum mechanics, there is an additional level of abstraction required, in that the open set must be understood as representing a Boolean algebra. This further abstraction is also required in order to accommodate the fact that potential relations in quantum mechanics, defined over a framework of inclusively related local contexts, are always indexed to the local Boolean context / Boolean subalgebra of the measuring device—i.e., the orthonormal basis of the detector. Apart from this restriction, potential outcomes cannot evolve to become probable outcomes.

Classical topological structures like open sets and covering systems of open sets are heuristically useful in exploring the extensive features of quantum mechanical relations in a manner consistent with its logical relations. In this way, an open set is an intuitive representation of a local context, and a scheme of inclusively related open covers of the open set is an intuitive representation of a global scheme of internally related local contexts. However, to the extent that [a] local contexts in quantum mechanics are represented by Boolean subalgebras, and [b] equivalence classes of these subalgebras require reference to an indexical local Boolean context in order for potential relations to evolve to become probable relations, the more abstractive sheaf-theoretic scheme of mereotopological extension is required to fully capture these logico-extensive aspects of quantum mechanical relation.

In part II of this volume, this scheme will be presented in detail, both in terms of its technical formalism and its application to quantum mechanics from the standpoint of philosophical foundations of relational realism presented in part I. In preface to that presentation, the remainder of this chapter will be devoted to exploring some key formal connections between the category-sheaf theoretic framework of mereotopologically extensive relations and the relational realist interpretation of quantum mechanics. In order to mediate between these two frameworks, the mathematical concepts used will be restricted to those of classical topology as a heuristic representation of the more technical Grothendieck topological concepts introduced in part II.

END OF CHAPTER SAMPLE

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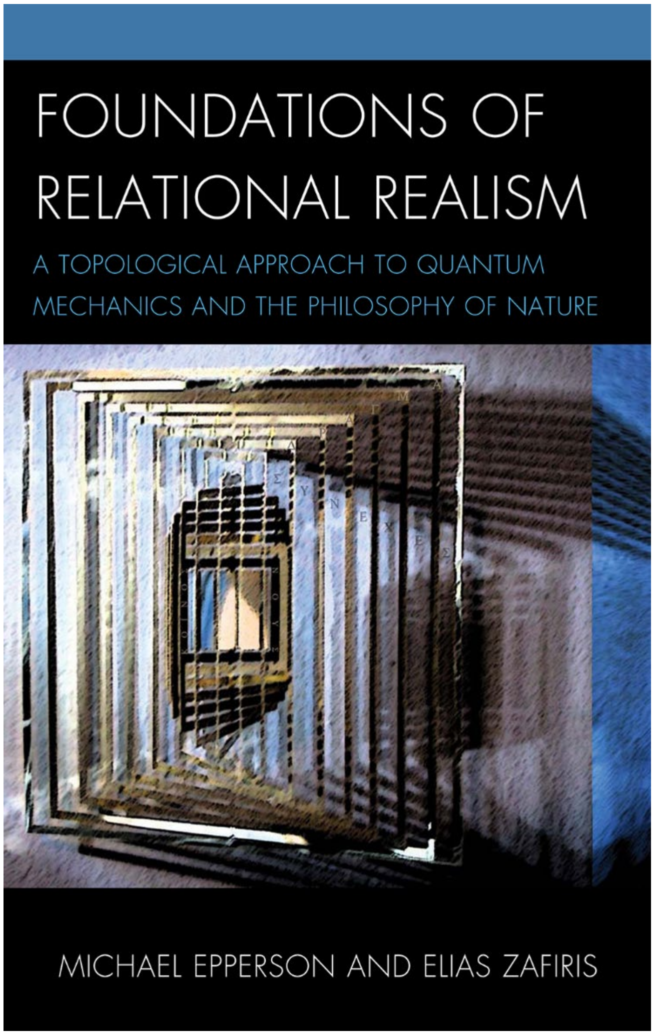

FOUNDATIONS OF RELATIONAL REALISM

A TOPOLOGICAL APPROACH TO QUANTUM MECHANICS AND THE PHILOSOPHY OF NATURE

MICHAEL EPPERSON AND ELIAS ZAFIRIS

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Sciences and Exploration Directorate
NASA Goddard Space Flight Center

MICHAEL EPPERSON
Ph.D. Philosophy
University of Chicago

ELIAS ZAFIRIS
Ph.D. Theoretical Physics
Imperial College
University of London

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INTERLUDE

As applied to the ontological interpretation of quantum mechanics, the convergence of category-sheaf theory and the relational realist philosophy, with its grounding in the theoretical perspectives offered by Whitehead's process theory, is based on the conception of a mereotopological physical continuum of events in a process of continuous ontological formation via extensive connection. This is achieved by explicitly considering potentiality as ontologically significant—that is, in terms of potential predicative facts (i.e., observables) whose actualization is an ontological process rather than merely an epistemic analysis. This process entails a logically conditioned integration of potentia via a selection mechanism by which certain potential predicative facts evolve to become probable predicative facts.

In the category-sheaf theoretic formalism of relational realism, this mechanism is described by the functionality of a uniform fibration of the global totality of quantum events and their relations in terms of local Boolean reference frames. The crucial requirement of sheaf theory in this conception of the quantum event continuum is that the selection mechanism by which certain potential relations evolve to become probable relations is ultimately local and describable in a precise topological sense. With respect to quantum theory this means that the selection of an observable to be measured, by means of a corresponding preparation procedure, locally instantiates a physical context—a local Boolean reference frame for the individuation of measurement outcome events. This is conceived as analogous to the subjective standpoint of a Whiteheadian actual occasion undergoing concrescence, and its associated mereotopological, internal relational selection mechanism by which potential relations are logically integrated to become probable relations—that is, mutually exclusive and exhaustive, valuated subjective forms that satisfy the principle of non-contradiction and the principle of the excluded middle.

While it is true that the notion of a local Boolean reference frame has no practical utility in the usual metrical point-spacetime continuum of classical and relativistic physics, it is absolutely essential when depicting physical systems in a topological continuum where the distinction between local and global is made explicit. In the usual metrical continuum, for example, 'local' refers merely to a point and not to a region. In a topological continuum, however, regions are the focus, and points are derived from them.

The definition of 'local topological region' in quantum mechanics is far from trivial. In classical topological theory, for example, a region is easily defined as an open set, and while this is heuristically useful for an introductory understanding of quantum mechanical topology, it is not formally adequate. This is because the local evolution of potentiality to probability in quantum mechanics (the reduction of the pure state to the mixed state) is enacted via a Boolean logic constraint—i.e., a constraint that subsumes the logical function of the preparation procedure for the measurement of some observable. That is to say, the local selection of a Boolean reference frame is instantiated via the usual preparation measurement, always required in quantum mechanics, by which the 'selected-to-be-measured' observable is initially defined. It is this Boolean contextualization via the standard preparation procedure that enables the probabilistic inference of the actualization of an outcome event contextualized *as* this measured observable. Thus, the topological notion of a local region in this regard—i.e., the measured object's local Boolean contextualization—cannot be adequately represented simply via an open set in a classical topological continuum; rather, it must be represented topologically as a Boolean algebra. By using Grothendieck topology to this end, which generalizes the notion of a topological space, it is possible to use a sheaf-theoretic topological conceptual framework to progress from [a] local or partial information over a local Boolean algebra to [b] a global non-Boolean (i.e., partially Boolean) algebra of events, via the definition of local contexts as Boolean algebras.

Proceeding in this direction, having defined the concept of a local topological region as a local Boolean measurement context in quantum mechanics, one must likewise define the concept of a 'quantum event': In the framework of quantum theory, events are identified as measurement outcomes referent to corresponding observables, and the theory provides the means of correlating these events. In this respect, the conceptual complexity of any ontological interpretation of quantum theory stems from two factors: First, the existence of an event referring to a quantum system can be inferred only probabilistically with respect to the local Boolean measurement context of a selected observable, and moreover, can be affirmed only after a measured result has been registered by a corresponding measuring device. Second, if we agree that a quantum event corresponds to an evaluation of an observable with respect to a measuring device as above, then the totality of events related to the behavior of a quantum system cannot be actualized within the same local Boolean measurement context due to the property of global non-commutativity of quantum observables. Each quantum event, then, is a novel actualized fact conditioned by the selection constraints of the local context by which it is individuated.

Quantum-theoretically this means that locally, each event is a Boolean event. This is important because locally in the quantum continuum, all the rules of Boolean logic for logical inferences are operative. As discussed extensively in part I, this is due to the categorical specification of every local context as a local Boolean algebra which, in turn, allows for the local reduction of potentiality to probability in quantum theory. More significantly, this local Boolean reduction obeys the rules of restriction in the definition of a presheaf for nested sequences of Boolean algebras. This is exactly what is meant by 'uniformity' in the continuum, and how this is modeled in terms of a uniform fibration. In the same vein of ideas, the notion of a Boolean algebra thought of as a 'local context' refers to a variable-elastic topological concept; it is, in other words, not rigid like a point.

In order to correlate the above with the relational realist philosophical notion of an extensive event continuum discussed in chapter 5, it is instructive to stress the following: In the Whiteheadian, relational realist scheme, the notion of 'continuum' captures the concept of all potential extensive relations. In the sheaf-theoretic scheme, this is captured by means of a uniform fibration—that is, by means of a presheaf of observables defined over a category of local contexts. The local contexts thus provide the base category by which potential extensive relations can be defined as 'relations among local sections.' These potential extensive relations, in other words, are induced by the relations among local contexts. Thus, one must distinguish between: [a] a horizontal dimension, which contains the base objects (local contexts identified with local Boolean algebras) and [b] a vertical dimension, which contains the potential extensive relations (in terms of local predicative facts or local sections, or local observables). In a pictorial sense, then, the vertical dimension is the display space of potential relations induced by the structure of a uniform fibration over the base category of local contexts. Thus global observables are locally graded with respect to local contexts—i.e., structured in terms of distinctive levels of potential relations forming a category.

In this way, local observables can be related only by means of the relations induced from their underlying local contexts. The notion of a continuum, then, is captured by the totality of potential relations defined over the underlying category of local contexts according to the requirements of a uniform fibration. Thus, a quantum event—i.e., a quantum measurement outcome or 'actual occasion' in Whitehead's terminology—is a local, Boolean context-dependent, true-false evaluation of a section. In other words, a section, which represents a local quantum observable contextualized according to its underlying local Boolean algebra, is defined via a morphism of that Boolean algebra to the bivalent logical (true / false) evaluation. Thus, the evaluation of a local section is a basic formalization of the concrescence of an actual occasion. As discussed in part I, according to the Kochen-Specker theorem, one cannot evaluate simultaneously all observables on a single two-valued Boolean algebra.

This is possible only locally—that is, only with respect to some local Boolean algebra corresponding to some selected observable defined by the standard preparation procedure in quantum measurement.

A natural question, then, is, “How do we manage to reconcile all these varied forms of ‘local’ and avoid inconsistencies?” First, we proceed by obviating the metrical spacetime point continuum as the conventionally assumed fundamental order, recognizing it instead as a higher order abstraction which we can recover when appropriate at the limit where all local elastic variability is contracted to rigid points. There, we can subsequently impose a Euclidean or Lorentzian or Riemannian metric inducing the well-known causality conditions. Second, we introduce in its place a category-theoretic, mereotopological continuum in a process of ontological formation (rather than predefined existence as in the case of the metrical continuum) apprehended locally via the Boolean contexts covering it. Third, we invoke a process of gluing topologically from the local level to the global level.

The proper understanding of the topological term ‘gluing’ is crucial, here, because this is the key step to progress mereotopologically from a local level to a higher local level (keeping in mind that the notion of local is variable or elastic) and, ultimately, to an inductive approximation of the global level. Gluing of sections, as introduced in chapter 5, takes place over partially compatible local Boolean contexts, such that gluing is essentially a compatibility relation between local sections defined over overlapping local Boolean contexts. Thus gluing is the means of extending a local section over a local context to a higher level local context, and so on, towards an inductive approximation of the global level.

With respect to classical physics, wherein a single local Boolean context is also considered global, the evaluation of a section (observable) would simply represent a measurement outcome—i.e., an actual occasion—such that the latter could be reduced analytically to that formal structure. Thus, classically we might think of a becoming actual occasion as the process of evaluating a section, and likewise think of the result of the evaluation—the measurement outcome—as the actualized occasion. But quantum theoretically, this cannot be achieved via analytical reduction the way it can be classically, because the global cannot be defined as a simple composite of local contexts (again, per the Kochen-Specker theorem). Quantum mechanically, we are limited to local relations. So how is it possible to extend this procedure from the local to the global in the quantum case? The answer, provided by the sheaf-theoretic scheme, is by forming Boolean germs—that is, equivalence classes of compatible sections over compatible underlying Boolean contexts. The notion of a germ of sections, in other words, is defined by means of inducing the same contextual information if sections are restricted to an overlap of local Boolean contexts. The germ thus cannot be defined in abstraction from its local contextuality—i.e., wherein local sections agree; likewise, at the same time, the local context associated with a germ is never considered apart from the local information content contextualized.

For example, in an EPR experiment for spin ½ systems (see chapter 2.8 and chapter 9.4) we have two local Boolean algebras correlated and glued together into a higher level, which is manifested by their tensor product. The latter thus essentially represents equivalence classes of compatible local observable information contained in the separated Boolean algebras. The tensor product, in other words, incorporates the compatible correlated information of two local Boolean contexts overlapping partially due to the common origin of the two separated systems before the emission. It is this Boolean correlative context with respect to the global level that defines the nonlocal correlations that are so problematic from the perspective of a metrical spatiotemporal point continuum. But in a topological continuum it is still a legitimate context, formalized via the tensor product of the respective Boolean contexts. Again, this is not possible in a metrical point continuum. Thus, what we gain in the topologically defined continuum is the fact that the tensor product, when understood *as a local context itself*, can be glued further to an even higher level if we entangle it with the Boolean algebra of a third system correlated initially with the two former ones, and so on. What has been described in pictorial terms, then, is the mechanism of extensive connection in a topological continuum through the transition from a local context to a higher local context and so on towards the global, such that inversely, the reduction of the global into different levels of local contexts is consistent, compatible, uniform and non-paradoxical.

But how is this relational framework consistent with the notion of a quantum event? This is the crux of the distinction between the local and the global, and the necessity of extensive connection. As discussed in part I, and as will be explored further in part II, we argue that each quantum measurement event is a *novel* actualized fact, but one whose novelty is nevertheless conditioned by the selection constraints of the local Boolean context by which it is individuated. Thus, quantum measurement events as elementary units of relation are always individuated and actualized with respect to local Boolean contexts. But in that regard, how can we think of a locally contextualized quantum event as a globally relevant, ontologically significant constituent of nature, such that it can have the capacity to change the continuum globally? Again, the application of sheaf theory to quantum theory provides the answer: Globally, a quantum event is not specified solely by the evaluation of an observable, as is the case with a classical event, but rather is specified by the evaluation of a corresponding Boolean observable germ. That is to say, a quantum event is specified in a global sense by the equivalence class of all compatible Boolean contexts, conceived in all different local levels, 'nested' within each other by internal relation, with respect to which observable information can be glued. Thus a Boolean germ is an internal relational progression in the ontological formation of the quantum continuum; it is the relational structure of extensive connection from the local to the global.

It is instructive now to correlate concisely the above sheaf-theoretic viewpoint with the corresponding Whiteheadian, relational realist viewpoint explicated in part I. This correlation rests upon two central concepts asserting the ontological primacy of quantum events / facts / actual occasions: [1] The subjective aspect of each quantum actual occasion, whereby the latter determines its *own* subjective standpoint or local context, and thus its own logical integration of potential internal relations contextualized by this standpoint; [2] The objective ('superjective') aspect of each quantum actual occasion, whereby the latter, once actualized, serves as a datum to which subsequent actual occasions-in-process will be internally related. With respect to both the subjective and objective aspects of the actual occasion, as discussed in chapter 5.1, it is useful to think of a Boolean germ as representing the structure of contextualized internal relations between an actual occasion-in-process and its dative world—i.e., representing the internal constitution of an actual occasion in its process of concrescence. As discussed in chapter 4, this structure pertains to the supplementary phase of Whiteheadian concrescence, and presupposes a primary phase whereby the subject occasion's local context / subjective standpoint is initially defined via internal relation to a dative totality by way of a uniform fibration of this totality.

Indeed, the particular structure of connectivity definitive of quantum actual occasion / Whiteheadian 'concrescence' here is reflected in the Latin root of the word, which translates as 'growing together.' From a sheaf-theoretic viewpoint, this is exactly what a Boolean germ represents with respect to the actualization of a quantum measurement outcome: It integrates compatible information at higher and higher local Boolean levels via equivalence classes, and in this way it progresses from the local to the global. Most important, the integration is possible only through the uniform restriction of potential internal relations among quantum facts via their associated internally related local Boolean contexts. Thus, a global internal relational covering structure—viz., a Boolean localization scheme of the quantum continuum—is needed for a coherent and consistent understanding of how a locally ('subjectively') contextualized quantum event is objectively constitutive of a global totality of such events.

We can thus conceive of the generative progression of a Boolean germ as a process formative of a coherent and consistent global history of internally related quantum events, each event carrying with it its own local contextualization. Thus the locally contextualized actual occasion-in-process is internally related to its dative actual occasions *via their own local contextualizations*, such that it is not only the dative occasions that are internally related, but also their local contexts. It is an internal relation of the local to the global, in other words, wherein the global is always incorporated as a nested series of local, partially compatible Boolean contexts extending to higher and higher levels, thus inductively constitutive of the global-in-formation. This inductive characterization of the global via internal relation of locally contextualized data is defined category-theoretically as the inductive limit (introduced in part I and mathematically formalized in part II). Note that since the Boolean germ is a generative structure—i.e., it is serially increased with each quantum actual occasion—it cannot be conceived in the set-theoretic sense of an already extant, predefined totality; rather, it is a synthetic totality, continuously in extensive formation via a process of discrete, internally related, predicative actualizations.

On the basis of this conceptualization of the functional role of Boolean germs in the formation of the quantum continuum, the formation of an equivalence class constitutive of a Boolean germ always takes place relative to the indexical subjective standpoint / local context of a particular actual occasion-in-process. Equivalently put, the standpoint of an actual occasion-in-process instantiates the local context over which some germ of compatible sections (an equivalence class of sections) may be defined. In this sense, an actual occasion in the process of concrescence (i.e., a quantum actualization-in-process) can be understood sheaf-theoretically as the synthetic evaluation of a germ—that is, the actualization of some potential augmentation of the germ. In this way, the subjective and objective features of the quantum actual occasion are always cooperative. Its subjective aspect is reflected in its own local contextualization; and once actualized, its objective aspect is reflected in its role as a datum to which other actual occasions-in-process will be internally related. Thus, one can trace an actual occasion's genetic internal relational structure by defining germs of compatible sections over partially overlapping dative contexts. The compatibility of these sections is always relative to the indexical local context of the actual occasion-in-process; in this way, the latter is always considered the starting point of its own concrescence, constituted internally via the formation of a corresponding germ.

It is important to emphasize again that the formation of a germ involves [a] the localization of sections and [b] the formation of equivalence classes of compatible sections by which a local context of agreement among these sections (i.e., inter-contextual compatibility) is always indexed to the local context of the actual occasion-in-process. In this way, the latter with its own local contextualization always defines a germ of local potential predicative facts / local observables / local sections, one of which will become actualized (e.g., as a quantum measurement outcome.) Thus a germ describes the internal constitution of an actual occasion-in-process in terms of its *potential* internal relations with the dative world. Likewise, once actualized, the occasion is specified in terms of its *actual* internal relations with the dative world—i.e., a specification whereby the germ is augmented by the fact of the occasion's actualization.

In this way, the sheaf-theoretic explication of Whitehead's dictum "the many become one and are increased by one" is as follows: The many become one by compatible interconnection or gluing of their information content relative to the local 'indexical' context of a particular actual occasion-in-process. This interconnection entails the internal relation of the actual occasion-in-process to the global totality of already actualized, dative occasions in their own respective local, partially compatible contexts, thus forming a germ. Once the process of concrescence is complete and the occasion is actualized, the dative totality is thereby 'increased by one'; that is, the novel occasion, constitutive of a novel totality, will serve as a datum to which a subsequent actual occasion-in-process will be internally related according to the latter's own local context, and so on. The conceptual framework by which this process was elucidated in part I can be rigorously formalized in terms of the mathematical framework of sheaf theory, as will be demonstrated in part II.

In preface to this more technical discussion, it will be useful to emphasize two key aspects of the relational realist philosophical cosmology as it will be formalized sheaf theoretically: The first of these aspects refers to the issue of self-reference, discussed in chapter 3, wherein the same theoretical entity can assume more than one functional role within the same theory. An example of this is provided by the dipolar 'subject-superject' role characteristic of the Whiteheadian actual occasion. In the present account, self-reference is bypassed via the transition from a set-theoretic universe of discourse to a topological (or better, mereotopological) category-theoretic universe, which is structured into distinct, internally related, extensive local levels. In this way, it is possible to move from level to level keeping the adjective 'local' during the transition, because what is local is variable-elastic; that is, its structure is preserved under extensive connection via germs (equivalence classes of potential internal relations) and is thus coherently constitutive of the global—not as a predefined totality, but rather as a totality in process—i.e., the global in-formation. A reader trained in formal logic might simply say that self-reference is bypassed via making identity locally variable.

The second essential aspect refers to the novel idea of the mutually implicative internal relation of local and global. On the one hand, every local observable (potential fact) is internally related, via the congruence and compatibility relations induced by a Boolean localization scheme, to a global quantum event structure of actualities. Thus, as noted earlier, a global quantum event 'continuum' is not formalized as a completed, already extant set-theoretic structure; rather, it is constituted in a continuous mereotopological process of ontological formation via the actualization of potential facts conditioned by local Boolean contexts. Thus it is also the case that the global is internally related to the local. As will be elaborated throughout the course of part II, with respect to the functional role of Boolean germs as internally constitutive of becoming actual occasions, this process of formation via the mutually implicative internal relation of local and global is fully logically coherent and consistent in the sense of respecting the rules of transition from the local and partial to the global and total, and conversely.

PART II

Mathematical Foundations of Quantum Relational Realism

A sheaf-theoretic description
of internally related quantum event structures
in terms of Boolean covering systems

ELIAS ZAFIRIS
M.Sc., Ph.D. (Imperial College, University of London)

CHAPTER 6

Notion of Localization Processes

6.1 Localization in Physical Theories

The operational foundations of physics as a natural science are based on observation and measurement. The abstraction of the measurement process gives rise to the notion of *coordinatization* or *arithmetization*. In this sense the formation of algebraic structures of 'number-like' quantities of any particular operational form can be thought of as solutions to corresponding physical measurement problems. In the interpretation of physical theories, our empirical access to the world is coordinatized through the concept of observables. *Observables* denote physical quantities that, in principle, can be measured in the context of appropriate experimental arrangements. The crucial further assumption is that quantities admissible as measured results must be *real numbers*, since it is accepted that the resort to real numbers has the advantage of making our empirical access to the world secure. More concretely,the model adopted to represent the measured values of observables is the real line $\mathbb{R}$ and its powers, specified as a set-theoretic structure of points that are independent and possess the property of infinite distinguishability with absolute precision. In this way, the semantics of the *physical continuum* in the standard interpretation of physical systems theories is associated with the codomain of valuation of observables, that is, the set-theoretic real line model $\mathbb{R}$ and its powers. Due to this assumption, in any experiment performed by an observer, the propositions that can be made concerning an observable are of the type which asserts that the value of an observable lies in some *measurable set* of the real numbers. A proposition of this kind corresponds to an *event* as it is apprehended by an observer using his measuring device.

The stipulation of the set-theoretic real line model $\mathbb{R}$ as the semantic information carrier of the notion of physical 'continuum' has posed enormous interpretational problems for quantum theory, and in particular quantum measurement. The root of this problem is situated on the different notions of *localization* characterizing *classical* and *quantum* observables. Overlooking this subtle difference, referring to localization properties of observables in different physical regimes, is a source of paradoxes and misinterpretations.

First, it is important to clarify the meaning of the notion of localization. Physical observation and measurement presupposes, at the fundamental level, the existence of a *localization process* via which we may extract information related to the local behavior of a natural system. Only on the basis of a localization process does it become possible to discern events and assign an individuality to them. Generally, a localization process is co-implied by the preparation of suitable local reference contexts for measurement. These contexts identify concretely the kind of reference frames used for observation of events. The methodology of observation is effectuated by the functioning of event-registering measurement devices, which operate locally within the context of prepared reference frames. In this general setting, it is important to notice that registering an event, which has been observed in the context of a reference frame, is not always equivalent to conferring a numerical identity to it by means of a real value corresponding to a physical attribute. On the contrary, the latter is only a *limited case* of the localization process, when, in particular, it is assumed that all reference contexts can be contracted to points. This is exactly the crucial assumption underlying the employment of the set-theoretic structure of the real line as a model of 'physical continuum.' The consequences of this common assumption have posed enormous problems for the interpretation of quantum theories, which are characterized by different localization schemes for the individuation of events. More concretely, set-theoretic axiomatizations of quantum event structures tend to hide the intrinsic significance of *Boolean* localizing contexts in the formation of these structures.

The operational procedures followed in quantum measurement are based explicitly on the employment of appropriate Boolean environments. The construction of these contexts of observation is related to our specific measurement arrangements and can be metaphorically considered as Boolean pattern recognition mechanisms. In this way, we may argue that the real significance of a quantum event structure proves to be, not at the level of events, but at the level of the specific form of interlocking or gluing together overlapping Boolean localizing contexts of observation into a globally coherent whole. From a logical perspective, the *gluing mechanism* accompanying quantum observable localization schemes is effectively a novel form of *logical internal relation*, which is absent classically, but plays a fundamental role in conceptualizing quantum-theoretic predication in topological terms. This is crucial for modern physics because topology is uniqiuely appropriate to the depiction of the transition from the *local* to the *global*, and vice versa, without using any metrical notions. For example, the significance of localization and gluing is of paramount importance for making sense of some unusual *topological quantum effects*, like quantal phase factors (the Berry phase) and the Aharonov-Bohm effect, as well as for understanding *decoherence* in the framework of the consistent histories approach to quantum theory, via coarse-graining localization processes.[1]

A natural question arising in this setting, inspired by the various interpretational problems of quantum theory, is the following: Is there an alternative conception of the ‘physical continuum’ that does not rely on an assumed preexisting set-theoretic structure of points on the real line? This question properly challenges the hypothesis that our form of observation is tautological with set-theoretic real number representability. In order to realize the necessity of an evolution in the semantics of the ‘physical continuum’ we emphasize that we have to consider carefully the different notions of localization associated with physical observables in classical and quantum measurement processes, respectively. The elucidation of this difference induces a natural evolution in the semantics of the physical continuum from a set-theoretic to a sheaf-theoretic one, as will be explained presently.

6.2 Localization Schemes

Before discussing the functioning of localization processes or schemes related with physical observables, it is necessary to include a brief intuitive description of ideas related with the notion of a physical continuum. For this purpose, the following quotations from Henri Poincare[2] and Alfred North Whitehead,[3] respectively, are particularly helpful:

> We cannot say that our element is without extension, since we cannot distinguish it from neighboring elements and it is thus surrounded by a sort of haze. If the astronomical comparison may be allowed, our “elements” would be like nebulae, whereas the mathematical points would be like stars.

> The ultimate facts of nature are events. . . The constitutive character of nature is expressed by the “contingency of appearance” and the “uniform significance of events”. . . By extension we mean that quality by virtue of which one event may be part of another, or two events may have a common part. Nature is a continuum of events so that any two events are both parts of some larger event.

The primary conception of a ‘physical continuum’ constitutes an inexhaustible complex of overlapping and non-overlapping events. Such an *event ontology* lies at the basis of our physical theories and necessitates a careful analysis of its premises before the acceptance of a particular mathematical model as a representational device for its functioning. The consideration of the notion of ‘event’ as a primary concept in the comprehension of the ‘physical continuum’ poses immediately the following question: How are events to be *related* to each other? If the continuity of nature is to be ascribed to the relations among events, then the fundamental relation is *extension*. The relata in the relation of extension are the events, such that each event is part of a larger whole and each event encompasses smaller events. Extension is also inextricably tied with the assumption of *divisibility* of events signifying a *part-whole* or *local-global* type of relation. Thus, the crucial role in the conceptualization of event ontology is played by the topological notion of a localization scheme, inevitably endowed with an internal relational gluing structure, such that there exists a consistency in the *inverse processes* of *extending* or *inducing* observables from the local or partial to the global, and conversely, *restricting* or reducing observables from the global to the local. The internal relational gluing structure of a localization scheme expresses the fundamental idea that “the primary relationship of physical occasions is extensive connection.”[4]

It is important to emphasize, again, that only via a localization scheme does it become possible to discern events and assign an individuality to them. The formulation of the relations of extension and restriction via the physical and mathematical notion of a localization scheme elucidates the following excerpts from Whitehead’s *Process and Reality*, seen earlier in chapter 5:

> The real potentialities relative to all standpoints are coordinated as diverse determinations of one extensive continuum. This extensive continuum is one relational complex in which all potential objectifications find their niche. It underlies the whole world, past, present, and future. Considered in its full generality, apart from the additional conditions proper only to the cosmic epoch of electrons, protons, molecules, and star-systems, the properties of this continuum are very few and do not include the relationships of metrical geometry. An extensive continuum is a complex of entities united by the various allied relationships of whole to part, and of overlapping so as to possess common parts, and of contact, and of other relationships derived from

> these primary relationships. This extensive continuum expresses the solidarity of all possible standpoints throughout the whole process of the world. It is not a fact prior to the world; it is the first determination of order—that is, of real potentiality—arising out of the general character of the world. . . All actual entities are related according to the determinations of this continuum; and all possible actual entities in the future must exemplify these determinations in their relations within an already actual world.[5]

> Extension, apart from its spatialization and temporalization, is that general scheme of relationships providing the capacity that many objects can be welded into the real unity of one experience. Thus, an act of experience has an objective scheme of extensive order by reason of the double fact that its own perspective standpoint has extensive content, and that the other actual entities are objectified with the retention of their extensive relationships. These extensive relationships are more fundamental than their more special spatial and temporal relationships. Extension is the most general scheme of real potentiality, providing the background for all other organic relations.[6]

The general purpose of a localization scheme amounts to filtering the information contained in a global structure of partially ordered physical events, through an appropriate *algebraic structure of localizing frames*, which can be determined by a *homologous* operational physical procedure of measurements. In this way, the substantial constituents of a localization scheme should specify the kind of loci of variation (or equivalently, reference contexts) to be used for observation of events. These contexts should instantiate generalized *reference frames* such that reference to concrete events of the specified kind can be made possible with respect to them. The requirements accompanying their instantiation are the following: First, they should constitute expressions of event ontology respecting the inverse relations of extension and restriction. Second, they should *not* be based on an underlying structure of set-theoretic points on the real line. This equivalently means that localization schemes used for the individuation of events should not depend on the existence of points. Third, due to their determination by a homologous operational physical procedure of measurements the *internal relations* of these varying reference contexts should play a crucial role in the coherent action of a localization scheme, and thus should not be neglected in its initial specification. According to Whitehead:

> The extensive scheme is nothing else than the generic morphology of the internal relations which bind the actual occasions into a nexus, and which bind the prehensions of any one actual occasion into a unity, coordinately divisible.[7]

Interpreted mathematically, this essentially means that the reference contexts of a localization scheme should not only form a class, but they should form a *category*. More concretely, *the reference contexts together with their structural transformations should form a mathematical category*[8] of localizing frames. For example, a localization scheme, which is based on the process of coarse-graining, should be implemented by a partial ordering relation among the localizing frames, which is obviously a category, where the only possible arrow between any two localizing frames is an inclusion. It is necessary to emphasize that the kind of reference contexts or loci of variation, together with their structural internal relations, determines precisely the concrete categorical environment employed operationally by a homologous operational physical procedure of measurements. An instance of this is a category of open sets, ordered by inclusion, in a topological measurement space. Another instance, of particular significance in quantum theory, is a category of Boolean localizing algebraic contexts of quantum measurements, which may be considered as local Boolean subalgebras of a global quantum observable algebra.[9]

6.3 The Conceptual Framework of Category Theory

6.3.1 The Necessity of a Categorical Framework

Category theory provides a general theoretical framework for dealing with systems formalized through appropriate mathematical structures emphasizing their *mutual relations* and *transformations*. The central focus of the categorical way of rethinking basic notions can be described as a shift in the emphasis of what is considered to be fundamental for the formation of structures. In the set-theoretic mode of thinking, structures of any conceivable form are defined as sets of elements endowed with appropriate external relations. In the category-theoretic mode, the emphasis is placed on the transformations among the objects of a category devised to represent its internal relational structure by means of appropriate constraints on the collection of these transformations. In this sense, the notion of structure does not refer exclusively to a fixed universe of sets of predetermined elements, but acquires a *variable reference*.

The major foundational significance of category theory regarding our present enquiry on the problem of modeling the physical continuum, according to the requirements posed previously, is related to the following realization: The effect of casting a mathematical construct for modeling the physical continuum in category-theoretic language is equivalent to conferring a degree of ambiguity of reference on this construct. This is an appealing aspect for our purposes since it is exactly the doctrine of employing these constructs to depict a fixed, absolute universe of sets that we wish to overthrow. According to category theory the mathematical constructs used for our modeling purposes should be regarded as possessing meaning only in relation to local frameworks, thus replacing the notion of unique reference to the universe of sets with other legitimate and varying frameworks of interpretation.[10] This means, in turn, that a model of the physical continuum can be determined only in relation to a category and, most important, this category may vary.

We claim that it is exactly the referential ambiguity of the construct representing the physical continuum, interpreted in varying categorical environments, that is needed in order to accomplish the critical evolution away from the conventionally assumed preexisting structure of points representing a true 'physical continuum' of events—an evolution that is of paramount importance to modern fundamental physics. Of course, the ambiguity of reference after casting the construct in categorical language should be fixed by its applicability in concrete categorical environments. Thus, the instantiation of these reference contexts should, from a physical viewpoint, depend on the *operational procedures* devised for the individuation of events in the continuum. In this sense, the notion of a localization scheme becomes a fundamental ingredient in modeling the physical continuum because it depicts the appropriate categorical environment suited to express its meaning, in agreement with the operational procedures followed by distinct physical theories.

Apart from determining the appropriate categorical environments for the operational modeling of the physical continuum, localization schemes referring directly to events likewise play the equally important role of representing a *base structure of varying loci* of the categorical kind depicted—a base structure over which the *partial order* of the extended continuum of events should *fiber*. Notice that in the set-theoretic real line model the role of such a locus is played only by a point, since a point conferring numerical identity to an event is the unique idealized localization measure. Consequently, from a topological perspective, adopting this assumption of an *atomic topology* on the continuum set is responsible for the conception of observable events as sharply distinguishable in classical theories. Indeed, from such a perspective, the fibering of an event continuum over a base structure of varying loci, representing concrete localization schemes in a topological context, can provide for such distinguishability and yet account for possible variations in the topology as well.

It is instructive to make clear that the process of casting a concept in categorical language is achieved by the characterization of the concept in terms of an *arrow-theoretic* formulation, and not by a specification in terms of a set equipped with certain operations and satisfying axioms expressible as relations among its elements. In this sense, the adoption of the categorical syntax involves at least two important conceptual shifts away from typical thinking about mathematical constructs considered for the modeling of the physical continuum. The first shift concerns the primary significance of *structure-preserving mappings*, referred to as arrows in category theoretic language. This shift reflects an opposition to the usual prevailing significance of sets with structure, referred to as objects, correspondingly. The second shift concerns the meaning of a *universal construction*, which is made precise in the categorical formalism, and moreover constitutes the main objective of the latter in the syntactical terms of this theory.

6.3.2 Categorical Principles and Terminology

The basic categorical principles that we adopt for this application are summarized as follows:

[i] To each kind of mathematical structure used to represent systems, there corresponds a *category* whose objects have that structure, and whose morphisms preserve it.

[ii] To any natural construction on structures of one kind, yielding structures of another kind, there corresponds a *functor* from the category of the first specified kind to the category of the second. The implementation of this principle is associated with the fact that a construction is not merely a function from objects of one kind to objects of another kind, but must preserve the essential relationships among objects.

[iii] To each natural translation between two functors having identical domains and codomains there corresponds a *natural transformation*.

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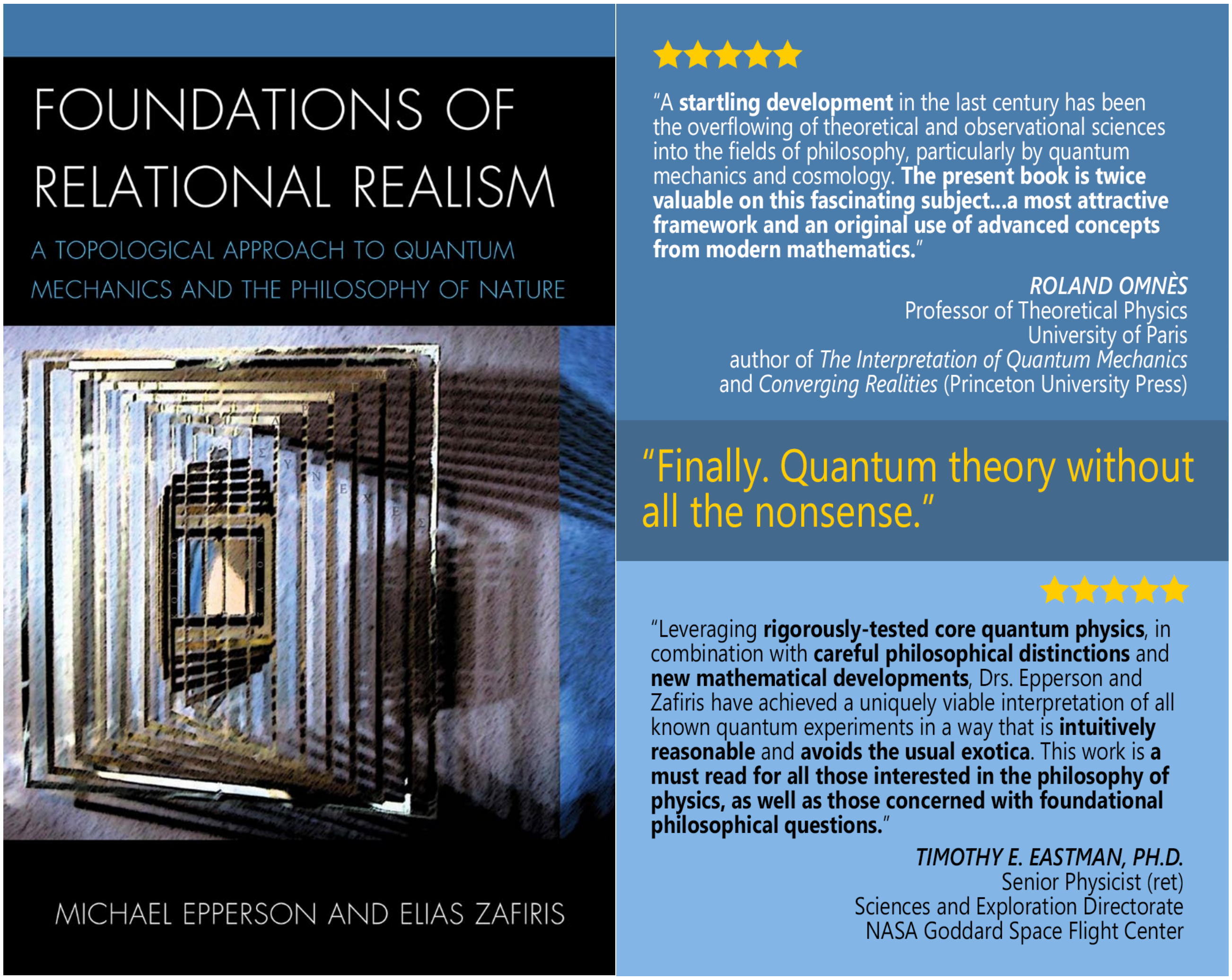

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Sciences and Exploration Directorate
NASA Goddard Space Flight Center

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Ph.D. Theoretical Physics
Imperial College
University of London

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that L has with all Boolean contexts B, formalized as a presheaf, incorporates the physical requirement of *uniformity* among events. By this we mean that for any two quantum events observed over the same Boolean domain of measurement, the structure of all Boolean logical reference frames that relate to the first cannot be distinguished in any possible way from the structure of Boolean frames relating to the second. According to the principle of uniformity, in other words, all the localized quantum events within any particular Boolean reference context should be *uniformly equivalent* to each other. The compatibility of the Boolean localization process of a quantum event algebra with the requirement of uniformity entails that the partial order of relations in a global structure of quantum events is induced by lifting an appropriate family of morphisms from the base category of local Boolean reference contexts to the fibers. Equivalently, *the partial order of relations among quantum events is induced by corresponding relations among their localizing Boolean reference contexts*.

8.5 Functorial Boolean-Quantum Internal Relation

The network of relationships defined by a quantum event algebra with Boolean logical frames, formalized categorically by the notion of a presheaf functor of Boolean frames inducing a Boolean localization scheme, is the semantic information carrier of a *mutually implicative internal relation* between the local Boolean level and the global quantum level. This *dipolar* mutually implicative internal relation is formulated in the category theoretic syntax in terms of a pair of adjoint functors between the category of presheaves of Boolean event algebras and the category of quantum event algebras, thus forming a categorical *adjunction*. This categorical adjunction formalizes the process-theoretic operation of the category of *transmutation* by relating internally and bidirectionally the local Boolean and global quantum levels of the event structure. More precisely, we formulate the following proposition:[7]

There exists a pair of adjoint functors $\mathbf{L} \dashv \mathbf{R}$ *as follows:*

$$\mathbf{L}: Sets^{\mathcal{B}^{op}} \rightleftarrows \mathcal{L}: \mathbf{R}$$

The Boolean frames-quantum adjunction consists of the functors $\mathbf{L}$ and $\mathbf{R}$, called left and right adjoints with respect to each other respectively, as well as the natural bijection:

$$Nat(\mathbf{P}, \mathbf{R}(L)) \cong Hom_{\mathcal{L}}(\mathbf{LP}, L)$$

The above bijective correspondence, interpreted functorially, says that the *Boolean realization functor* of a quantum categorical event structure $\mathcal{L}$, realized for each quantum event algebra L in $\mathcal{L}$ by its *functor of Boolean frames*, viz., by

$$\mathbf{R}(L): B \mapsto Hom_{\mathcal{L}}(B, L)$$

has a *left adjoint* functor $\mathbf{L}: Sets^{\mathcal{B}^{op}} \to \mathcal{L}$, which is defined for each presheaf of Boolean algebras $\mathbf{P}$ in $Sets^{\mathcal{B}^{op}}$ as the colimit (inductive limit):

$$\mathbf{L}(\mathbf{P}) = Colim\{\textstyle\int(\mathbf{P}, \mathcal{B}) \to \mathcal{B} \to \mathcal{L}\}$$

Thus, the following diagram commutes:

$$\begin{tikzcd}
\mathscr{B} \arrow[d, tail, "\mathbf{y}"'] \arrow[dr, "\mathbf{M}"] & \\
\mathbf{Sets}^{\mathscr{B}^{op}} \arrow[r, dotted, "\mathbf{L}"] & \mathscr{L}
\end{tikzcd}$$

The physical meaning of the adjunction between presheaves of Boolean logical frames and quantum event algebras is made transparent if we consider that the mutually inverse pair of internally related adjoint functors formalizes the *mutually implicative processes of encoding and decoding information* relevant to the structural form of their domain and codomain categories. If we think of $Sets^{\mathcal{B}^{op}}$ as the categorical universe of variable local Boolean frames modeled in $Sets$, and of $\mathcal{L}$ as the categorical universe of quantum event structures, then the functor $\mathbf{L}: Sets^{\mathcal{B}^{op}} \to \mathcal{L}$ signifies a *translational code* of information from the level of locally Boolean events to the level of global quantum events; and likewise, the Boolean realization functor $\mathbf{R}: \mathcal{L} \to Sets^{\mathcal{B}^{op}}$ signifies a translational code in the inverse direction.

In general, the content of the information cannot remain completely invariant with respect to translation from one categorical universe to another and conversely. However, there remain two alternatives for a variable set over local Boolean algebras $\mathbf{P}$ to exchange information with a quantum algebra L: *either* the content of information is exchanged in *quantum terms* with the inductive limit in the category of elements of $\mathbf{P}$ translating, represented as the quantum morphism $\mathbf{LP} \to L$; *or* the content of information is exchanged in *Boolean terms* with the functor of Boolean frames of L translating, represented correspondingly as the natural transformation $\mathbf{P} \to \mathbf{R}(L)$.

In the first case, from the perspective of L, information is being received in quantum terms while in the second, from the perspective of $\mathbf{P}$, information is being sent in Boolean terms. The natural bijection then corresponds to the assertion that these *two distinct ways of communicating are equivalent*. Thus, the fact that these two functors are adjoint expresses a relation of variation, regulated by *two poles*, with respect to the meaning of the information related to observation. We claim that the totality of the content of information included in quantum observable structures remains *invariant under Boolean encodings*, corresponding to local Boolean logical frames, *if and only if* the adjunctive correspondence can be appropriately restricted to an *equivalence* of the functorially correlated categories. This task can be accomplished by defining an appropriate Grothendieck topology on the base category of Boolean frames, which essentially paves the way for the understanding of a quantum event algebra as a *sheaf* of local Boolean algebras over an appropriately specified covering system of the former. This categorical equivalence is interpreted physically in terms of a *transmuted information invariance property* between the Boolean and the quantum species of structure, referring to the *bidirectional internal relation* between variable local Boolean algebras and global quantum ones.

For any presheaf $\mathbf{P}$ in $Sets^{\mathcal{B}^{op}}$, the *unit* natural transformation of the Boolean frames-quantum adjunction is defined as follows:

$$\delta_{\mathbf{P}} : \mathbf{P} \to \mathbf{RLP}$$

Likewise, for each quantum event algebra L in $\mathcal{L}$ the *counit* natural transformation is defined as follows:

$$\varepsilon_L : \mathbf{LR}(L) \to L$$

The representation of a quantum event algebra L in $\mathcal{L}$, in terms of the functor of Boolean frames $\mathbf{R}(L)$ of L, is *full and faithful*, if and only if the counit is a *quantum algebraic isomorphism*. In turn, the counit is a quantum algebraic isomorphism if and only if the right adjoint functor is full and faithful. In the latter case we characterize the Boolean shaping functor $\mathbf{M} : \mathcal{B} \to \mathcal{L}$ as a *proper* or *dense shaping functor*. It can be shown that the Boolean realization functor is full and faithful if it corresponds to a *functor of Boolean localizations* of L for an appropriate *Grothendieck topology*.[8] Thus, the counit is an isomorphism if it is restricted to some *Boolean localization system* of L.

From the above, we deduce that the representation of a quantum event algebra L in $\mathcal{L}$, in terms of the functor of Boolean frames $\mathbf{R}(L)$ of L, is full and faithful, if the Boolean frames-quantum adjunction is *restricted* to a Boolean localization system of L. As a corollary we obtain that a quantum categorical event structure $\mathcal{L}$ is a *reflection* of the topos of presheaves $Sets^{\mathcal{B}^{op}}$.

8.6 Quantum Localization: Sheaf of Boolean Reference Frames

According to the defining requirements of a localization scheme, the established fibred representation of quantum events, realized for each quantum event algebra by its functor of Boolean frames, should be also coherent under extension from the local to the global level and conversely. Thus, we should define an appropriate Grothendieck topology $\mathbf{J}$ on the base category of Boolean contexts $\mathcal{B}$, such that: [i] The Boolean reference frames acquire the semantics of *local frames* with respect to that Grothendieck topology $\mathbf{J}$ on $\mathcal{B}$, and [ii] The functor of Boolean frames for a quantum event algebra L in $\mathcal{L}$ becomes a *sheaf on the site* $(\mathcal{B},\mathbf{J})$ for that $\mathbf{J}$. For this purpose, we use the machinery of covering sieves of Boolean frames, defined as follows:

A B-sieve S on a Boolean reference context B in $\mathcal{B}$ is called a *covering sieve* of B, if all the arrows $s : C \to B$ belonging to the sieve S, taken together, form an *epimorphic family* in $\mathcal{L}$. This requirement may be, equivalently, expressed in terms of a map:

$$G_S : \coprod_{(s:C\to B)\in S} C \to B$$

being an *epimorphism* in $\mathcal{L}$. It can be proved that the specification of covering sieves on Boolean contexts B in $\mathcal{B}$, in terms of epimorphic families of arrows in $\mathcal{L}$, *does indeed define a categorical topological covering system (Grothendieck topology)* $\mathbf{J}$ on $\mathcal{B}$.[9] Thus, the function $\mathbf{J}$, which assigns to each Boolean reference context B in $\mathcal{B}$, a collection $\mathbf{J}(B)$ of covering B-sieves, being epimorphic families of arrows in $\mathcal{L}$, constitutes a Grothendieck topology $\mathbf{J}$ on $\mathcal{B}$. Then, we can show that the *functor of Boolean reference frames* $\mathbf{R}_L$ is a *sheaf for*

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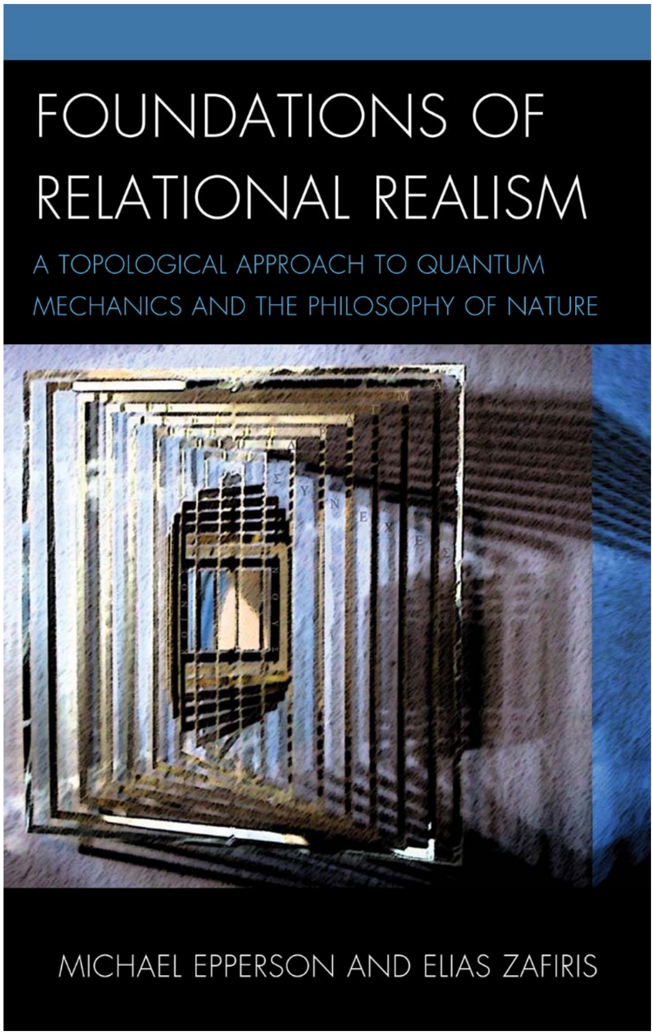

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CHAPTER 10

Quantum Localization in a Broader Conceptual Perspective

10.1 Boolean Covers and Physical Contexts

The association of a *local Boolean cover* in the mathematical descriptive language of sheaves, with a concrete *physical context* in the physical world, takes into account two fundamental distinctions inextricably connected with the quantum-theoretic formalism: The first of them refers to a distinction being made between an *event* and the *physical context* that constitutes a set of necessary and sufficient constraints for the occurrence of an event of the observed kind. To the event, there corresponds a formal descriptive proposition language. To the physical context there corresponds a context-description in a formal descriptive language assuming existence at the level of Boolean covers, followed by an appropriate terminology providing names for the characterization of the language of events occurring in that context. These latter descriptions can be said to belong to the constitutive level of the Boolean localization systems. The second refers to a distinction being made between *possessed* physical quantities, as those found in classical physics, and *dispositional* ones, as those found in quantum physics. The dispositional character of quantum observables is associated with the fact that they may only be specified via the measurement process, and more precisely, as *relationally* appearing with respect to theoretical or actual

Boolean preparatory environments, that is, Boolean logical frames. In the mathematical descriptive terminology this distinction is encoded in a *transition* from globally Boolean event structures to globally non-Boolean event structures, being *covered* by epimorphic families of local Boolean reference frames forming covering sieves. In this sense, creating a preparatory Boolean environment for a system to interact with a measuring device *does not* determine which event will take place, but it *does* determine the *kind* of event that will take place. It forces the outcome, whatever it is, to belong to a certain definite *Boolean algebra* of events for which the standard measurement conditions are invariant. Such a set of standard conditions for a definite kind of measurement is named a physical context and reflects a Boolean reference frame in the mathematical descriptive language.

10.2 Partial Congruence and Adjunction

From a general category-theoretic perspective the *relational realist* framework proposed for the interpretation of quantum event structures and their logical semantics is based on the existence of the Boolean frames-quantum adjunction. From this adjunction, characterized by means of the counit and unit natural transformations, we have constructed a *sheaf-theoretic representation* of quantum event algebras formulated with respect to Boolean localization systems, as well as a quantum subobject classifier, which plays the role of a *logical classifying object* in the quantum universe of discourse. In this way, the Boolean frames-quantum adjunction incorporates both the *representational semantics* of quantum event algebras as sheaves of local Boolean coefficients, and the *logical semantics* of the classifying Boolean germs concerning truth valuations with respect to Boolean localization systems.

Thus, from a physical viewpoint, the Boolean frames-quantum adjunction stands as a theoretical platform for *decoding* the global structural information contained in quantum algebras of events via *processes of localization* with respect to Boolean logical frames, realized as physical contexts for measurement of observables, and subsequent *processes of information classification* in terms of corresponding truth valuations. The functioning of this platform is based on the establishment of a *bidirectional* dependence between the Boolean and quantum structural levels being brought into *local* or *partial* congruence. Most significantly, this bidirectional dependence is implicated through the topos-theoretic universe of sheaves of sets over local Boolean frames of quantum events algebras forming Boolean localization systems, where these local Boolean frames play the role of local Boolean covers, effectuating in this way the specific form of the above partial congruence. Of course, the sheaf-theoretic requirements secure the *compatibility* of the locally Booleanized information in the overlapping regions of physical measurement contexts under extension from the local to the global and conversely.

Additionally the *global closure* of this bidirectional dependence is necessary to be constrained in order to obey certain conditions, such that its total constitutive information content, unfolded in the multitude of local Boolean reference frames, is both preserved and coherently organized in a logical manner. Remarkably, the necessary and sufficient conditions for both of these requirements, that is: [i] *preservation* of quantum information content in Boolean localization systems associated with compatible physical measurement contexts; and [ii] *logical classification* of quantum information by means of the classifying Boolean germ *true* (equivalence class) used for the valuation of quantum propositions, are supplied by the Boolean frames-quantum adjunctive correspondence itself, via the counit and unit natural transformations, respectively.

More concretely, regarding the first condition, we conclude that it is satisfied if the *counit* of the adjunction is an *isomorphism* for each quantum event algebra. In this case, there exists a full and faithful sheaf-theoretic representation of quantum events algebras in the descriptive terms of Boolean covering systems, characterized as Boolean localization systems of measurement. Regarding the second condition, we conclude that it is satisfied, given the *validity* of the first condition, if the *unit* of the adjunction is an *isomorphism* for the subobject functor. In this case respectively, the subobject functor is representable in the category of quantum event algebras by a *classifying object* in this category, called a *subobject classifier*. This classifying object, which is defined by an *inductive limit* operation, contains the information of equivalence classes of truth valuations with respect to all compatible Boolean frames belonging to a Boolean localization system of a quantum event algebra L in $\mathcal{L}$. Thus, it can be legitimately used for *truth valuations* of quantum propositions in exact correspondence with the use of the two-valued Boolean object, which is used for valuations of classical propositions. Moreover, the quantum subobject classifier provides the key logical device for the analysis of typical quantum measurement situations, providing a *criterion of truth* for a complete description of reality in the quantum regime with respect to a Boolean localization system.

10.3 Classical to Quantum: From Groups to Categories

It is instructive to draw an *analogy* between the notion of a reference frame as it is used in classical physics (presupposing the existence of a set-theoretic *spacetime* substratum) and the notion of a variable *Boolean localizing reference frame* in quantum physics. In classical or relativistic physics we always have to define first the notion of a reference frame (spatiotemporal reference frame) where we may describe a physical phenomenon or formulate a physical law. The consideration of a single reference frame is not enough. What we have to consider is a whole *class* of reference frames and their *transformations*. In classical theories the class of all reference frames together with their interrelations form a *group* of transformations. Then the physical laws involved should be *invariant* under the action of this group of transformations.

Now, a group of transformations is a very special kind of category having only one object. It is a natural requirement, in case we generalize to quantum theory for example, to obtain an analogous description of phenomena. For this purpose, we generalize from a spatiotemporal reference frame (*a metrical notion*) to an analogous localizing notion of reference frame (*a topological notion*), which admits a concrete physical interpretation and plays a similar role. This is the functionality of a Boolean reference frame in quantum theory. Thinking categorically means that we have to specify the transformations from a Boolean reference frame (corresponding to a quantum measurement situation) into another. Now the class of all Boolean reference frames and their transformations do not form a group but they form a *category*. So the quantum description of phenomena should be *invariant* under the action of the concrete category of all partially compatible Boolean reference frames. More concretely, the physical content of the topological sheaf-theoretic representation of quantum event algebras can be formulated in terms of an invariance principle. According to this, *the information content of a quantum event algebra is invariant under the category of gluing isomorphisms (technically called a groupoid) between overlapping local Boolean reference frames, along their intersections, in a Boolean localization system, preserving the quantum algebraic structure.* [1]

It is important to notice the obvious analogy to the metrical spacetime frame case. The requirement of *invariance* is the conceptual reason necessitating the introduction of the notion of a sheaf of Boolean logical frames, or equivalently the notion of a Boolean topological localization system of a quantum event algebra. We emphasize that the normative requirements for the localization of information in the quantum case *does not* refer to set-theoretic spacetime localization. Categorically thinking, localization has to do with the partiality of the information referring to a Boolean reference frame, interpreted in topological terms, and its consistent gluing effectuating a coherent *extension* or *induction* from the local to the global level.

It is instructive to note that in the special case of a group of transformations (that is the usual spacetime case with point-defined spatiotemporal reference frames) the notion of categorical localization is identical with the notion of spacetime localization. So, using the categorical framework we incorporate the classical spacetime localization case as a *very special case of the general localization notion* (the former is referring to a group of transformations, whereas the latter is referring to a category of transformations among the corresponding frames). Presheaves and sheaves (incorporating the gluing compatibility conditions) provide the conceptual and technical devices in order to express such a *generalization in the notion of localization*, which finds an important application in quantum theory (by introducing Boolean reference frames). Now the *gluing process* is a *vast generalization* of the usual process of requiring *inter-transformability* of our descriptions when we pass from some reference frame to another, if we take into account this more general localization idea. The benefit is that we obtain additionally a coherent *mereotopological* semantic methodology of extension from the local or partial into the global (considered in observable-theoretic terms as it is actually practiced via physical measurement) and conversely.

We claim that it is precisely the *sheaf-theoretic gluing process* which provides a rich and adequate mathematical model of Whitehead's scheme of *extensive connection*. The distinguishing feature of this claim is that the gluing process is of a purely *topological* nature and thus it captures the character of the relationship of extension idealized by Whitehead as being *prior to the metrical notions* of space and time:

> In this general description of the states of extension, nothing has been said about physical time or physical space, or of the more general notion of creative advance. These are notions which presuppose the more general relationship of extension. They express additional facts about the actual occasions. The extensiveness of space is really the spatialization of extension; and the extensiveness of time is really the temporalization of extension. Physical time expresses the reflection of genetic divisibility into coordinate divisibility. . .[2]

10.4 Topological Localization as Genetic Analysis

From a general philosophical standpoint, the general conceptual underpinning of the notion of sheaf-theoretic localization of observability is deeply related to the theoretical systems of Aristotle and Whitehead. According to Aristotle the *divisions* that we project into an observable continuum assume a *potential* existence, unless their *actuality* can be confirmed by property-indicating facts, viz., by experimental events. Thus, from an Aristotelian perspective, ontological *primacy* should be shifted from the level of states, to the level of observable and event information structures. This means that any complex systems theory (including quantum theory) should be viewed as a theory specifying a particular way of *correlating property-indicating events*, according to a schema of *potential division* of an observable continuum in terms of *local information carriers* (for instance Boolean reference frames), assuming its actuality *only* by the observed events and their correlations.

The process-theoretic system of Whitehead advocates this conception and poses additionally the natural condition that the correlations should conform to a *theory of congruence*, which should act as a normative requirement in order to make sense of the observed events and their correlations. Furthermore, a theory of congruence would manifest the potential division of an observable continuum as commonly presupposed in the study of natural phenomena, whereas its functional necessity and essential correctness would be affirmed only by means of the observed events (*actual occasions*).

From a logical perspective, observed events in quantum theory correspond to evaluations of potential facts (observables) with respect to some logical binary schemata (yes/no experiments subsumed by measuring devices), interpreted as property-indicating facts. Note that this is meaningful from a topological perspective only if it respects the normative constraints of the potential division of a global quantum event continuum with respect to local Boolean reference frames (preparation stage of a quantum measurement), as well as the rules of compatibility under extension from the local to the global and conversely. The *local-*

to-global bidirectional scheme corresponds to Whitehead's notion of *genetic analysis*, and is reflected in the modeling of the continuum related to the behavior of a quantum system via a *category of sheaves* of local information carriers (local Boolean reference frames). More precisely, the action of a presheaf corresponds to a potential division of the continuum according to the normative rules of a congruence theory of local Boolean observables, whereas the transformation of a presheaf into a sheaf (that is to a Boolean localization system) reflects the gluing and compatibility conditions determining the correlations of observed events under the transition from the local to the global, and conversely under the reduction of the global to the local.

Note that in full agreement with Whitehead, the genetic passage from phase to phase, viz., from each local Boolean observable frame to another, is *not* in physical time, meaning that the genetic process is *not* the temporal succession. *This is due to the dependence of each local phase in the genetic process on the global level, meaning that each such phase presupposes the global quantum level implicitly*. It is also important to notice that the conditioning of actualized events by such phases (considered as local Boolean reference frames) is indispensable for the existence of properties. In other words, every property is a possessed property, indicated by an actualized event, which is conditioned on the existence of a local phase, presupposing the global quantum level.

10.5 Boolean Localization and Decoherence

A significant application of the proposed Boolean localization scheme of quantum event structures concerns the elucidation of the phenomenon of quantum decoherence.[3] *Decoherence* is understood as the phenomenon according to which all the features of quantum observable behavior are suppressed, and instead a quantum system *behaves like* it can be described by classical probabilistic reasoning. Typical quantum features of this form constitute interference phases and the existence of off-diagonal elements in the density matrix description of a quantum system. Thus, decoherence is mathematically modeled either by the process of *diagonalization* of the density matrix with respect to a particular basis, called technically the *pointer basis*, or by the process of *minimization of interference phases* in a sufficiently *coarse-grained* resolution scale of quantum observable behavior. The essential aspect of all modeling approaches to decoherence is the explanation of the *emergence of classical behavior*, meaning the emergence of a classical statistical description referring to ensembles of equivalently prepared systems.

The first of the current modeling approaches to decoherence may be called *environment induced* decoherence, whereas the second may be called *coarse-graining* induced decoherence. These approaches are based on different conceptual foundations regarding the explanation of emergence of classical behavior. More specifically, environment-induced decoherence is an *extrinsic* approach in the sense that it requires the coupling action of an external to the system agent, called the *environment*, in order to explain the emergence of a classical statistical description with respect to some particular basis, that is the pointer basis. The crucial aspects of this approach may be summarized by stressing the *contingent* and extrinsic influence of some environment on a quantum system and the conception that the process of decoherence requires the passage of time, called *decoherence time*, within which energy and information are being dissipated to the environment. On the other side, coarse-graining induced decoherence is an *intrinsic* approach in the sense that classical behavior emerges intrinsically within the description of a quantum system by *coarsening* the scale of resolution of quantum observable behavior by means of *macroscopic* or gross or collective observables, called classical observables. Hence, decoherence is not conceived as a process requiring the coupling of a quantum system to some external environment, but as a process of resolving quantum observable behavior at sufficiently coarse-grained scales, expressed in terms of *macroscopic algebras of observables*, with respect to whom interference phases are being suppressed. The crucial aspects of the latter approach may be summarized by stressing the intrinsic character of emergence of a classical statistical description in terms of *commutative algebras of macroscopic observables* incorporated at sufficiently coarse-grained levels within a *global non-commutative algebra* of microscopic quantum observables.

We argue that the full implications of coarse-graining induced decoherence require a *topological* modeling approach based on the functional role of Boolean localization systems. More concretely, the phenomenon of decoherence should be understood as a consequence of covering the intrinsic structure of a global non-commutative (non-Boolean) algebra of quantum observables by covering sieves of local commutative (Boolean) algebras of macroscopic or classical observables corresponding to suitable coarse-grained resolution scales of quantum measurement and then localizing sheaf-theoretically at these scales.[4]

The crucial mathematical advantage of this topological approach to quantum decoherence is that *the phenomenon of decoherence can be understood as a process of topological localization of a global quantum (non-Boolean) algebra of observables at some local commutative (Boolean) algebra of classical observables associated with some macroscopic resolution scale*. Topologically speaking, this becomes possible via sieving the global information content of a quantum algebra of observables with respect to compatible covering families (covering sieves) of local commutative (Boolean) algebras of macroscopic observables assuming existence at various *coarse-grained* measurement levels. In this conceptual framework, it becomes possible to understand the notion of environment *not* as an external agent forcing the selection of some pointer basis with respect to whom a classical statistical or Boolean description emerges, but *internally as localizing* topologically a global algebra of fine-grained microscopic observables at some coarse-grained macroscopic level *sufficient* for emergence of classical behavior.

Most significantly, the topological approach not only models decoherence via the sheaf-theoretic mathematical technique of Boolean localization via coarse-graining sieves, but allows the conceptualization and modeling of *recoherence* from the local macroscopic to the global microscopic level of observable structure via sheaf-theoretic amalgamation of locally compatible observables, that is by gluing locally compatible observables together globally in the form of equivalence classes, which we have called *Boolean germs*.

The conceptual motivating precursors of the topological sheaf-theoretic modeling of coarse-graining induced decoherence are, on the physical side, the *consistent or decoherent histories* approach to quantum theory[5] and on the mathematical side, the *topological representation theorems* of algebras of observables by *sections of a sheaf* over a topological space, or a category endowed with a Grothendieck-type of a topology, tracing their roots to the famous *Stone and Gelfand topological representation theorems* of Boolean and commutative algebras, respectively.[6]

On the physical state of affairs, the consistent histories approach to quantum theory focuses on the formulation of an appropriate criterion, subsumed in the formulation of a *complex-valued functional* for pairs of histories, the vanishing of which specifies the condition of *consistency*, that is it decides which histories decohere. Typically, *decoherent sets* of histories contain coarse-grained histories. More precisely, in quantum mechanics, propositions about the attributes of a system at a fixed moment of time are represented by sets of projection operators. The projection operators P_α enable a partition of the possible alternatives α a system may exhibit at each moment of time. They are exhaustive and exclusive,

$$\sum_\alpha P_\alpha = 1, \qquad P_\alpha P_\beta = \delta_{\alpha\beta}\, P_\alpha$$

A projector is said to be *fine-grained* if it is of the form $|\alpha\rangle\langle\alpha|$, where $\{|\alpha\rangle\}$ are a complete set of states. Otherwise it is *coarse-grained.* A *homogenous quantum-mechanical history* is characterized by a string of time-dependent projections, $P^1_{\alpha_1}(t_1), \cdots P^n_{\alpha_n}(t_n)$, together with an initial state ρ. The time-dependent projections are related to the time-independent ones by:

$$P^k_{\alpha_k}(t_k) = e^{iH(t_k - t_0)} P^k_{\alpha_k} e^{-iH(t_k - t_0)}$$

where H is the Hamiltonian. The *candidate probability* for these homogeneous histories is:

$$p(\alpha_1, \alpha_2, \cdots \alpha_n) = \mathrm{T}r\left(P^n_{\alpha_n}(t_n) \cdots P^1_{\alpha_1}(t_1) \rho P^1_{\alpha_1}(t_1) \cdots P^n_{\alpha_n}(t_n)\right)$$

It is straightforward to show that $p(\alpha_1, \alpha_2, \cdots \alpha_n)$ is both non-negative and normalized to unity when summed over $\alpha_1, \cdots \alpha_n$. However, it *does not* satisfy all the axioms of probability theory, and for that reason it is referred to as a candidate probability. For example, it does not satisfy the requirement of *additivity* on disjoint regions of sample space. More precisely, for each set of histories, one may construct coarser-grained histories by grouping the histories together. This may be achieved, for example, by *summing over* the projections at each moment of time, $\overline{P}_{\overline{\alpha}} = \sum_{\alpha \in \overline{\alpha}} P_\alpha$. The additivity requirement is then that the probabilities for each coarser-grained history should be the sum of the probabilities of the finer-grained histories of which it is comprised. Quantum-mechanical interference generally prevents this requirement from being satisfied. Histories of closed quantum systems *cannot* in general be assigned probabilities. There are, however, certain types of histories for which interference is negligible, and the candidate probabilities for histories do satisfy the sum rules. These histories may be found using the complex-valued *decoherence functional*:

$$D(\underline{\alpha}, \underline{\alpha}') = \mathrm{T}r\left(P^n_{\alpha_n}(t_n) \cdots P^1_{\alpha_1}(t_1) \rho P^1_{\alpha_{1'}}(t_1) \cdots P^n_{\alpha_{n'}}(t_n)\right)$$

Here $\underline{\alpha}$ denotes the string $\alpha_1, \alpha_2, \cdots \alpha_n$. Intuitively, the decoherence functional measures the *amount of interference* between pairs of histories. It may be shown that the additivity requirement is satisfied for all coarse-grainings if and only if the *real part* of the complex-valued decoherence functional vanishes, that is $\mathrm{R}eD(\underline{\alpha}, \underline{\alpha}') = 0$, for all distinct pairs of histories $\underline{\alpha}, \underline{\alpha}'$. Such sets of histories are said to be consistent, or weakly decoherent. The consistency condition is typically satisfied only for coarse–grained histories, and this then often leads to satisfaction of the stronger condition of decoherence $D(\underline{\alpha},\underline{\alpha}')=0$ for $\underline{\alpha}\neq\underline{\alpha}'$. The condition of decoherence is associated with the existence of so-called *generalized records*, corresponding to the idea that information about the variables to follow is stored in the variables ignored in the coarse-graining procedure.

We may state concisely the key idea of the consistent histories approach to quantum mechanics by the realization that we can have additive probabilities only in case we *restrict* our resolution scale within particular sets of histories, called *consistent sets*, which are typically sets of appropriately coarse-grained histories. The main problems of the consistent-histories approach to coarse-graining induced decoherence may be summarized as follows:

[i] It uses an *external time* parameter to both formulate the notion of history, and express the process of coarse-graining by summation of projection operators at each moment of this external time;

[ii] It turns out that there exist *many consistent sets* of histories which are mutually *incompatible* such that they cannot be combined to give a maximal consistent set, and at the same time there is *no criterion* of selecting uniquely some consistent set among them; and

[iii] The criterion of consistency of an arbitrary set of histories is *not correlated* with the requirement of classical behavior of a large class of observables. This is due to the fact that the difference between the algebraic structure of quantum observables (non-Boolean and non-commutative) in comparison to the algebraic structure of macroscopic observables at suitable coarse-grained scales (Boolean and commutative) is essentially *ignored* in this formalism.

In order to cope with these problems and formulate a more satisfactory account of coarse-graining induced decoherence, we need to topologize quantum algebras of observables in the spirit of Stone and Gelfand topological representation theorems of Boolean and commutative algebras respectively.[7]

The link between the consistent histories approach and the topological approach is obtained in two steps:

[i] By considering explicitly the *difference in the algebraic structure* between an algebra of microscopic observables and various algebras of macroscopic observables at suitable coarse-grained measurement scales with respect to the property of *commutativity*, and

[ii] By *topologizing* this difference in terms of a local-to-global sheaf-theoretic construction, where the information of a global non-commutative algebra of quantum observables may be *localized* by *restriction* or *reduction* to *compatible families* of local commutative algebras of classical macroscopic observables topologically *covering* the former, and inversely, recovering its content by the topological method of *gluing*.

At the propositional level, that is at the level of projection operators, as we have already pointed out, it is a common realization that existing set-theoretic lattice axiomatizations of quantum observable structures hide the intrinsic fundamental significance of Boolean commutative localization systems in the formation of these structures. On the other side, topologically, we can explicitly associate them with the action of appropriate Boolean coverings of global quantum algebras. Consequently, the algebraic representation of a quantum observable structure in the relational topological terms of localization systems of commutative algebras of macroscopic Boolean observables, effectuates a *semantic transition* from the axiomatic set-theoretic context of orthocomplemented partially ordered sets, *à la* Birkhoff and Von Neumann, to the categorical sheaf-theoretic context of Boolean information sieves, *à la* Grothendieck.

In order to understand the semantic transition of quantum observable algebras from the set-theoretic level to the sheaf-theoretic level, as well as the significance of this transition in relation to the conceptual and technical modeling of decoherence, it is instructive to explain briefly the philosophy behind Stone and Gelfand topological representation theorems of Boolean and commutative algebras, respectively, being the precursors of Grothendieck-type sheaf-theoretic formulations expressed in terms of topological covering families (covering sieves) on categories. These theorems state that an arbitrary commutative algebra of observables $\mathcal{A}$ of a certain type (for instance a Boolean algebra of observables in Stone's case) may be represented as the *algebra of continuous* $\mathbb{F}$*-valued functions*, where $\mathbb{F}$ is a particular *standard algebra* of the given type, on a *topological space* associated with $\mathcal{A}$ and constituting in this way its $\mathbb{F}$*- spectrum*. For example, in Stone's case, the standard algebra is the two-valued Boolean algebra $\{0,1\}$ and we make use of the discrete topology. The topological representation theorem is proved by the fact that a continuous function from the topological $\{0,1\}$-spectrum space X to $\{0,1\}$, that is $f : X \to \{0,1\}$ is completely determined by the clopen subset $f^{-1}(1)$ of X.

Now, the extension of this topological representation philosophy to larger classes of algebras of observables proves to be more complicated in the sense that a general algebra of observables $\mathcal{A}$ *cannot* be built up in this way from a single standard algebra like the two-valued Boolean algebra $\{0,1\}$ in Stone's case. This means that its spectrum is more complicated. An algebra of quantum observables falls into the latter category, and this is precisely the meaning of Kochen-Specker theorem, which essentially states that it is impossible to recover the global information content of a global quantum observable algebra using a single type of a Boolean measurement device globally. Of course, this can be done *locally at a certain coarse-grained resolution measurement scale*, but the global information content may be approximated only by employing simultaneously an appropriate *multitude of compatible Boolean devices* at different overlapping measurement scales *covering* its behavior completely. Mathematically, the inability to build up a global algebra of observables from a single standard algebra necessitates the introduction of *continuously variable* local standard algebras being capable of covering it completely. This pictorially means that we may identify the observables of $\mathcal{A}$ locally with *sections of a sheaf* defined on a spectrum X and taking values in one of a given class of standard local algebras, provided that the local algebra in which an observable takes values may vary continuously from point to point in the spectrum X.

The extrapolation of the sheaf-theoretic topological representation methodology, in the case of a quantum algebra of observables, sheds new light on the process of coarse-graining induced decoherence and the emergence of classical behavior. *In a nutshell it implies that quantum observables are not specified globally by their measured values, but they are specified by their Boolean observable germs*, viz., equivalence classes of locally compatible Boolean observables forming an inductive limit in a sheaf of macroscopic coarse-grained Boolean observable algebras covering the former at various resolution measurement scales. It is instructive to stress again that the concept of a categorical topology epitomizes the meaning of a *localization scheme* of a quantum algebra, whereas the covering sieves are interpreted as generalized measures of topological localization of quantum observables at various coarse-grained macroscopic (Boolean) levels. In the same vein of ideas the notion of a site generalizes the notion of a topological state space in categorical terms and constitutes the *relational variable topological background* for the effectuation of a localization scheme for coarse-graining the global information contained in a quantum algebra through localizing Boolean algebraic frames of macroscopic observables at various coarse-grained resolution scales. Thus, from the physical point of view, covering sieves of a global quantum algebra by local Boolean frames effectuate Boolean localization systems of the former. It is instructive to note that covering sieves of a quantum algebra L are actually subfunctors of the functor of Boolean frames of L, viz., *relational covering networks* of morphisms from local Boolean frames to L, expressing the generic morphology of the *internal mereotopological relations* constituting the process-theoretic scheme of extensive connection from the variable Boolean local to the quantum global level. These subfunctors are *partially ordered* by inclusion. The partial order relation among covering sieves of a quantum algebra L is interpreted as the *operation of coarse-graining* among Boolean localization systems. A Boolean localization system forces a transition in the semantics of quantum observables from the set-theoretic to the topological sheaf-theoretic level. The fact that the functor $\mathbf{R}_L$ of Boolean frames of L is a sheaf, means that there exists a gluing compatibility isomorphism of local Boolean frames on their overlap, meaning that these Boolean frames of L cover the same observable information content of a quantum algebra L in a *compatible* way.

The topological compatibility relation between local Boolean frames of a global quantum algebra sheds new light and forces a *strict criterion of consistency* in the process of coarse-graining as it is currently implemented in the context of the consistent histories approach. More concretely, as we have already mentioned, the major problem of this approach is that the criterion of consistency of an arbitrary set of histories is not correlated with the requirement of classical behavior of a large class of observables. This problem is due to the fact that the process of coarse-graining as it is currently implemented *does not have to satisfy any constraint*, it is just a summation over projection operators which describes the transition from a fine-grained description to some coarse-grained description. *We claim that an appropriate compatibility relation has to be imposed on the process of coarse-graining projection operators in order to obtain meaningful probabilities from the decoherence functional, and thus to explain the emergence of classical behavior at suitable coarse-grained macroscopic scales*. The Boolean localization scheme implies that the *coarse-graining process should respect the local Boolean structural rule as we pass from a local Boolean context to another*; that is, it should respect the morphology *of internal relation between local Boolean frames* covering a global quantum algebra.

In order to express this formally we need a compatibility condition which is given precisely by the *transition morphisms from a local Boolean frame to another local Boolean frame* as prescribed by the specification of the category of elements of the functor (sheaf) of Boolean frames of a quantum algebra. For example, referring to the same observable (for example, position) the proper coarse-graining procedure corresponds to a nesting of local Boolean frames, such that there exists compatibility on their overlaps preserving the local Boolean structural rule. In case we consider different observables and apply the coarse-graining process properly, we require partial compatibility on their overlapping coarse-grained Boolean frames (for instance, partial Boolean compatibility of coarse-grained position and momentum up to the limit of Heisenberg uncertainty relations) which is given by the topological gluing isomorphism. Then the formation of the inductive limit (which gives the transition from the local to the global) requires the formation of a set of equivalence classes (Boolean germs) of partially compatible Boolean observable information on all possible overlaps, glued topologically together in this way. This sheaf-theoretic methodology, based on the formation of Boolean localization systems of a quantum algebra, resolves completely the problem associated with coarse-graining in the consistent histories approach via the *idea of gluing on overlaps* such that the local Boolean structure rule is preserved via these transitions.

We may summarize the basic idea of the topological modeling approach to coarse-graining induced decoherence as follows: Although the global description of a quantum system should obey the laws of quantum logic and quantum probability, there exist coarse-grained descriptions at macroscopic levels, formulated for example in terms of *macroscopic hydrodynamic variables*, and expressed in terms of local Boolean subalgebras of a global non-Boolean quantum algebra, such that the attribution of properties by measurement valuations, effectuated by means of various distinct binary devices at these coarse-grained levels, takes place locally according to the laws of Boolean logic and classical probability theory. Essentially this means that *a local Boolean restriction or reduction of a quantum system with respect to a local Boolean context, or more generally with respect to a Boolean localization system, is conceived as a result of a decoherence process assuming validity with respect to an appropriate corresponding notion of coarse-graining subsumed by the function of covering sieves*. Only in this case, it is then meaningful to infer predictions about the behavior of the localized quantum system, via its local macroscopic reduction with respect to some local Boolean context in a Boolean localization system obtained by topological coarse-graining, using the methodology of classical probability theory.

The crucial point is that according to the general conceptual framework of topological localization, *decoherence makes sense only as an intrinsic, locally conceived, reductive process of the global-level description of a quantum system*. Now, recent experimental work seems to verify the above theoretical perspective.[8] More specifically, within the descriptive context of the environment-induced decoherence theories, it was assumed until now that the environment modeled by means of a bath of oscillators acts externally to a quantum system via an appropriate interaction coupling, and most importantly extends globally. These recent experiments show *on the contrary* that decoherence is caused by entities considered within a *localized environment*, and thus the global oscillator-bath picture is not appropriate to describe the phenomenon. More specifically, the role of such a locally conceived environment may be played by a nuclear spin bath, which gets internally entangled with a qubit (for example, a magnetically polarized molecule) and causing *intrinsic decoherence* without dissipation of energy.

10.6 Sheaf-Theoretic Quantum Relational Realism

The idea of a reality admitting *objective* existence is the starting point of our scientific investigations and the conceptual basis pertaining to the realistic interpretation of physics. We argue that our proposed sheaf-theoretic modeling scheme provides a *realistic* perspective on quantum structure, although *conceptually different* from the classical one. The classical view of reality presupposes that objects of our observation are simply individual entities in the world. Moreover an entity may be specified, both qualitatively and quantitatively, in a definite manner *independently* of procedures of observation, being capable of assuming individuality in isolation. According to the interpretative standards of classical realism, an objective description is one that determines the properties possessed by an independently real physical object, standing for an entity, by adopting a representation of that object as a physical system isolated from any observational interaction. Quantum theory does *not* conform to the descriptive ideals of classical realism. Thus, the proposed sheaf-theoretic localization scheme of quantum event algebras should *not* be judged by the descriptive ideals of the classical realism position, but instead should be considered as generating a generalization of the classical framework, mainly by initiating a *revision* of classical realism assumptions, concerning in particular the descriptive concepts involved in association with the objects they are used to describe. The basic claim is that the sheaf-theoretic descriptive framework makes good sense only on the basis of a *revised* realistic interpretation.

Our philosophical starting point is that a quantum event algebra reflects an *objective physical reality* at the quantum level having existence *independently* of some mind perceiving it. Evidently, such an interpretation viewpoint is compatible with the formalism of quantum theory if and only if a *non-Boolean* event structure is being manifested *globally*. Let us initially notice that we describe our *observations* using notions of validity adhering to *Boolean logic* as a consequence of the preparation of Boolean environments of measurement. This is due to the fact that only in such environments is it possible to *separate* elements and conceive of them as existing *in isolation* from the rest of the world. Indeed, the Boolean specification of environments of measurement incorporates the implicit assumption of a separative and compact underlying topology (Stone representation theorem).[9] Of course, such an assumption, in general, can be used as a methodological tool of enquiry, *only locally or partially*, and can never be claimed to assume global validity in the name of empirical findings, being themselves amenable to the specification of such global environments. The appropriate border in the domain of validity of such a Boolean description is decided by the relevant *abstractions* of a measurement situation conceived precisely as a *Boolean localizing context*.

It is instructive at this point to remind ourselves of Bohr's definition of the word *"phenomenon"* to refer exclusively to observations obtained under specific circumstances, including the account of the whole experiment. Thus, for methodological purposes, we may adopt Bohr's concept of phenomenon as a referent of the assignment of an observable quantity to a system, legitimately thought of as approximately isolated, *strictly* in the context of a local Boolean environment of measurement. Let us notice that this consideration is naturally required because of the globally non-Boolean event structure, necessitating a *multitude* of locally considered, isolated yet correlated manifestations with respect to distinct Boolean reference frames. In this sense, every Boolean filtering context corresponds to a particular *type* of observable phenomenon and every proposition of the universe of discourse *belongs to* at least one Boolean frame. Thus every proposition is in principle descriptive of a classically conceived observable, but *only under* the choice of a local physical context. It is also important to notice that the physical claim of partial descriptions embodies in an essential manner the claim that the description of a phenomenon, as above, necessitates a theoretical conception of the phenomenon as the referent of an interactive process structured in *two levels*:

The first level of this process is constitutive of the *generation* of a *localizing environment*, as a reality probing filter, endowed with an intentionally prepared mechanism of abstraction determined explicitly by the qualitative observable nature of the specific Boolean environment (Boolean reference frame).

The second level of this process is constitutive of the *actualization* of some observed event only after *interaction* with the relevant measuring apparatus attached as a binary code to the corresponding Boolean frame.

The conception of phenomena in this approach enforces reference to quantum systems as actually existing objects behind the phenomena, the latter being only their *local or partial* manifestations with respect to Boolean environments designed for that purpose. Evidently, each separate phenomenon *cannot* be regarded as a full representation of a quantum system. Only at the level of sheaves of Boolean frames, that is *Boolean localization systems*, constituting consistently *interconnected multiplicities* of phenomena in terms of Boolean observable germs (equivalence classes), can an *isomorphic representation* of the structure of events of a quantum system be reproduced by means of the *inductive limit* construction.

In the light of the above, it is legitimate to say that the sheaf-theoretic localization scheme replaces the classical static monolithic realist view with a form of *relational realism*, admitting *multiplicities in logical space*, but not some "actualized" space of existing objects, as an expression of qualitative structured *potential continuous variation* in the observable universe of discourse, which are simultaneously strictly conditioned to obey *collectively* the *global constraint* of global uniqueness of a sheaf. Hence, this explicitly *rules out* any interpretation in the classical realist sense of a one-to-one correspondence between concepts used to describe a phenomenal object and the presumed properties of an independent reality. Still, a revised realistic understanding of the description of quantum systems is retained.

Consequently, by virtue of the sheaf-theoretic scheme of interpretation, the *classical realist assumption* that knowledge of an object is achieved by forming a representation of that object as a substance possessing properties is *rejected* and subsequently *replaced* by the possibility of formulating local or partially compatible contextual Boolean descriptions with respect to Boolean localization systems. The systems are then *grounded* on the same actually existing quantum object, where the sameness is precisely determined by the possibility of *uniquely gluing* these descriptions globally via the synthetic ontological process of sheaf-theoretic local-to-global formation. In this way, the revised realist position justifies the usage of the adjective *complex* when we refer to quantum systems. Moreover, they are distinguished from simple systems by applying precisely *the norm* of this revised realist position. So, whereas classical systems are simple because they conform to the descriptive ideals of classical realism, quantum systems are *complex* because they conform to the more complex local-global requirements of revised relational realism.

In this way, we may argue that the quantum level of reality can be conceived only through a *relational sheaf-theoretic perspective*. It has now emerged that relations occupy substantial and remarkable territory in our description of nature, one which was once occupied exclusively by properties. An actually existing quantum system is *not* described through isolated properties, but only through its *relations* with localizing physical contexts interconnected together according to the *sheaf-theoretic* specifications of Boolean localization systems. More concretely, the topological covering process of a quantum event algebra associated with a Boolean localization system leads naturally to a contextual description of quantum events with respect to local Boolean reference frames of measurement. Stated more precisely, *quantum events are represented by evaluations of equivalence classes of Boolean coefficients (Boolean germs) with respect to local compatible Boolean contexts belonging to a Boolean localization system*. Thus, the global information content of a quantum event algebra remains *invariant* by the action of some topological covering system, if and only if that system forms a *Boolean localization system*. *Hence, the significance of a quantum event algebra is lifted from the orthoposet axiomatization at the level of events, to the sheaf-theoretic gluing conditions at the level of Boolean localization systems*. Eventually, the former axiomatization is fully and faithfully recaptured at the level of equivalence classes in these localization systems.

We argue that the relational sheaf-theoretic perspective on quantum reality provides a *viable mathematical model* to describe and understand Whitehead's conception of an *extensive event continuum*. Extension is fundamentally tied with the assumption of *potential divisibility* signifying a part-whole or local-global type of relation. In this way, the crucial role in the conceptualization of quantum event ontology is being played by the notion of a Boolean localization scheme, modeled in the *functorial/relational* terms of *sheaves*. The topological internal gluing relations characterizing sheaves force a consistency in the action of the inverse processes of extending from the local (Boolean) to the global (quantum) level and conversely reducing from the global to the local. Thus, *the real potentialities relative to all standpoints (local Boolean logical frames) are coordinated as diverse determinations of one* *extensive continuum, being a relational complex united by the sheaf-theoretic part-whole or local-global mereotopological relations*. Most importantly, the sheaf-theoretic extensive scheme provides the *generic morphology of the internal relations which bind the actual occasions into a nexus*. More concretely, the qualification of a uniform fibration expressing the potentialities relative to all Boolean frames, viz., of a presheaf of Boolean frames as a sheaf requires the explicit consideration of covering sieves. The covering sieves are functorial objects acting as the semantic carriers of networks of internal relations among Boolean frames with respect to which the *process-theoretic scheme of extensive connection* in the continuum is consistently realized. Notice that the sheaf-theoretic scheme of extensive connection is based on the *ontological primacy of events* or facts or actual occasions. This is the case because each actual occasion *self-determines its own subjective standpoint* (Boolean logical frame), and simultaneously instantiates a *bundle of potential relations* referring to this standpoint. It is crucial that all these potential relations, viz., all the relations among local observables, are *induced* by the internal relations among their underlying standpoints. Now, the idea of an actual occasion conceived in the sense of an actualized *concrescence*, via the evaluation of some Boolean germ expressing its internal constitution in this process, serves as a *datum* for subsequent becoming actual occasions in the extensive connection of the continuum and so on. Consequently, a global quantum event continuum is *not* conceptualized as a completed a priori existing set-theoretic structure but it is constituted in a *continuous* topological process of extension from the local to the global by *actualization* of new potential facts with respect to local Boolean frames.

The framework of category theory in its function to express the process of extensive connection in the quantum event continuum via the notion of Boolean localization systems, that is in the topological terms of sheaf theory, is indispensable in order to understand the revised relational realist scheme in connection to Whitehead's process theory. The most important aspect of its function in this context is the *conceptualization of the quantum event continuum via an inductive limit* *categorical construction in a sheaf of germs of local Boolean observable relations*. This is significant because the formulation of the mathematical procedure by which the *approximation to the global quantum level* is consistently achieved depends on the properties of the relation of *extension of Boolean germs* (equivalence classes), and most distinctively, this is implemented by *universal* means. This is the case because the inductive limit is described categorically via a *universal mapping property* and thus functions relationally as a *potentiality for actuality*.

Thus, from a generalized viewpoint, the congruence of the proposed sheaf-theoretic quantum relational realist position with Whitehead's process-theoretic scheme traces back to the semantic convergence between the category-theoretic notion of a *universal* (like the inductive limit) and Whitehead's notion of an *eternal object* or *pure potential for the specific determination of fact*. The theoretical basis of this semantic convergence stems from the fact that both of them act as *agents in objectification* under the operation of the *Category of Transmutation* which, in category-theoretic terms, is subsumed by the action of pairs of *adjoint functors*. In particular, keeping in focus quantum theory, the *mutually implicative dipolar functorial internal relation* between the multiplicity of local Boolean frames and the global quantum is expressed in terms of a pair of adjoint functors between the category of sheaves of Boolean event algebras and the category of quantum event algebras. As we have argued, this categorical adjunction formalizes the process-theoretic operation of the category of transmutation by relating internally and bidirectionally the variable and local Boolean with the quantum global level of structure.

A remarkable application of this scheme is the theoretical explanation of *coarse-graining induced decoherence* and the *emergence* of classical behavior. In this way, coarse-graining induced decoherence is understood and modeled as an *intrinsic* reductive global-to-local process, in the sense that classical behavior emerges intrinsically from the global quantum level via coarsening the scale of resolution of quantum observable behavior by means of macroscopic Boolean observables locally according to the sheaf-theoretic topological compatibility relation.

Notes

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5. Omnès, *The Interpretation of Quantum Mechanics*. See also: J. J. Halliwell, “A review of the decoherent histories approach to quantum mechanics.” In “Fundamental Problems in Quantum Theory - A Conference Held in Honor of Professor John A. Wheeler,” *Annals of the New York Academy of Sciences*, Vol. 755 (1995): 726-40.
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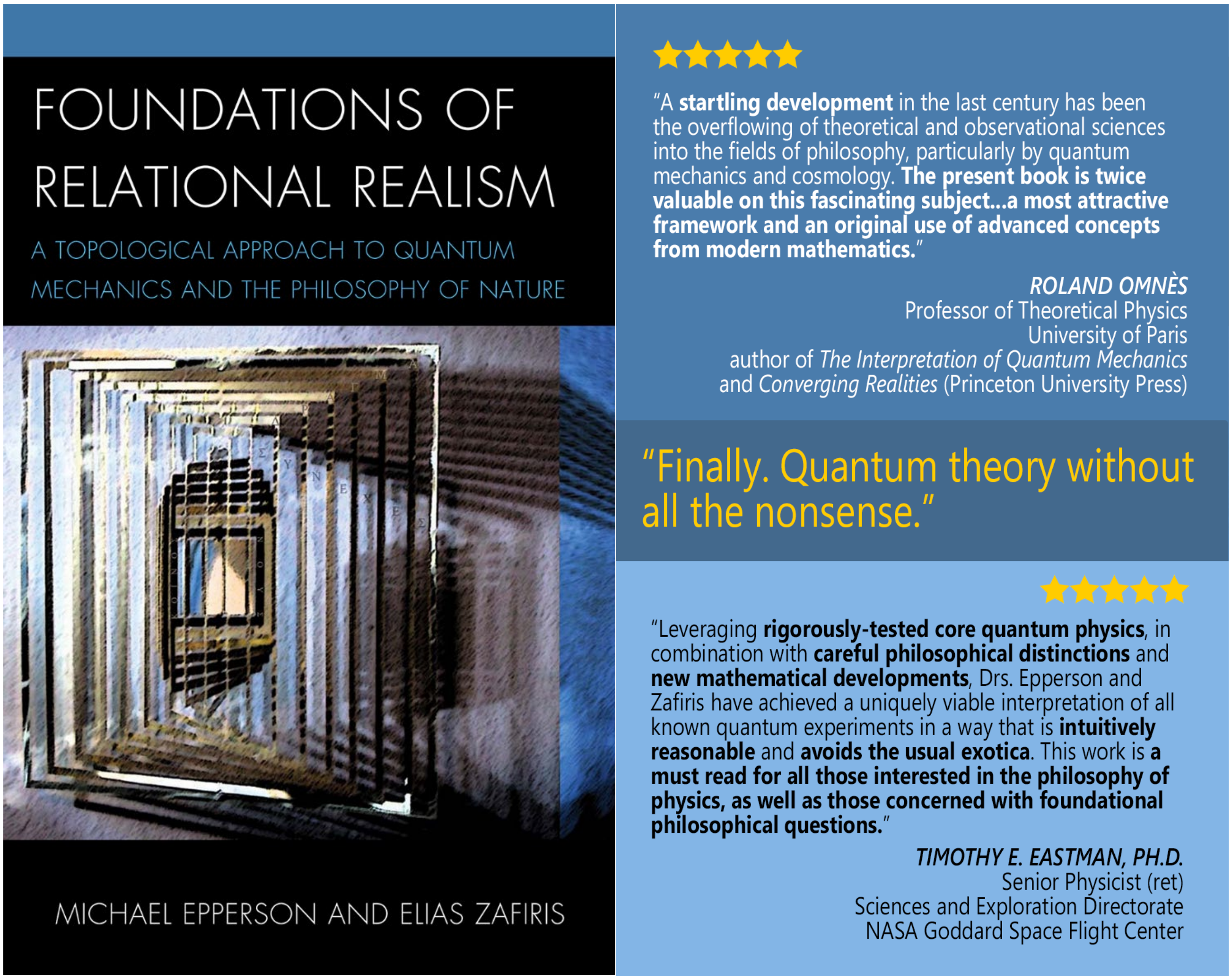

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ELIAS ZAFIRIS
Ph.D. Theoretical Physics
Imperial College
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