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REVIEW ARTICLE

PHYSICS AVOIDANCE & COOPERATIVE SEMANTICS: INFERENTIALISM AND MARK WILSON'S ENGAGEMENT WITH NATURALISM QUA APPLIED MATHEMATICS

Ekin Erkan

§, INTRODUCTORY REMARKS

Within the final synoptic terms of the Sellarsian scientific worldview not only is science considered a value-laden activity but so, too, are media instrumentally entangled as part of our *practical reality*. Consider how, via the Einsteinian perspective, mass exists not as it was conceived of in Newton's theory—i.e., those masses no longer exist. Nonetheless, the *references* involved *do succeed*, wherein such masses *do exist* in their functional counterparts (insofar as they are predicated upon generic similarity. Without a blind causal theory of reference but, instead, wielding an epistemic view construed around similarity, both are generically similar as mass-concepts. Just as we can make sense of the claim that Newtonian physics is approximately true but, strictly speaking, false, we can also make sense of a scientific scaffolding through rational comparisons. Such is the harmonization of the irreducible normative nature of persons and the ultimately physical nature of the universe, which *includes* persons-in-the-world. As it concerns our project, the puzzle we deal with does not run against the Sellarsian harmonization of the scientific and manifest image but, instead, engages with Sellars' scenography of conceptual change. Inverting the traditional logical empiricism of correspondence, Sellars defines observables ideally in terms of theoreticals. As we advance through our project, let us recall the Sellarsian apothegm that:

> "[s]cience may offer instruments, even prostheses, to improve and supplement our basic, biologically determined capacities; perhaps we shall even be able to augment those biologically determined capacities by genetic engineering, but at any point there will always be some basic, minimal observational capacities and a corresponding vocabulary, some minimal set of basic actions and a corresponding vocabulary, and those will, to a very large degree, determine the way it makes sense for us to parse the world into the objects we deal with and care about. For most of the objects we are concerned with, there will be no intelligible definitions in pure scientific language unladen with any reference to norms and practices."[1]

Just as Sellars is pellucid in demonstrating how correspondence rules ought not be treated as definitions of theoretical expressions in terms of their observation language expressions—underscoring the semantic autonomy of theoretical expressions that cannot be captured in observation language—so too will we examine semantics and applied mathematics-cum-physics correspondence rules as proposals for reconsidering our observational vocabulary. In doing so, we will closely engage with philosopher of science Mark Wilson's work in *Physics Avoidance*, prodding the nine essays that comprise this books into a unique trajectory so as to carve an analytic and rationalist theory of media vis-à-vis set theory, while accepting an inherent contrast between *norms of correctness* and *effective thinking routines*.

Mathematics' abilities to capture nature's unfolding processes within its own conceptual terms rests upon its capacities for supplying algorithms that can graphically engage in deduction numerically, bolstered by the hope of paralleling natural processes. Inter alia, Wilson's project shows that nature presents us with a multiplicity of manifolds that simply cannot be smoothly mapped (vide the problems of graphing a bird's flight over our curvaceous Earth, a problem which

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[1] Willem DeVries, *Wilfred Sellars*, Chesam, Acumen Publishing, 2005, p. 279.

we shall often return to). Thus, even our most basic/fundamental modes of effective mathematical reasoning falls short of the 'real extent' of natural processes. Mathematicians have developed sophisticated strategies that string together patchworks of numerical *approximation*, despite the algorithmic limitations upon our concrete reasoning capacities. There is, in turn, a trans-historical element to Wilson's pursuit in *Physics Avoidance*, one which is driven by a self-correcting (Sellarsian) scientificity—directed at knowledge, while constantly refining itself both methodologically and substantively. Kindling the critiques of twentieth-century thinkers such as Clifford Truesdell and Walter Noll on the essential idealization thesis of physics (i.e., that 'physics always idealizes') while simultaneously parsing a distinction that was conceived of with the nineteenth-century distinctions between rari-constant and multi-constant approaches to elasticity (associated with the derivational methods pursued by Navier and Cauchy, respectively), Wilson approaches limits and infinitesimals qua multi-scalar localization. Meticulously engaging with Wilson's rendering of the problem of the physical infinitesimal, we not only set out to complicate the historical discussion of matter—which has bedeviled the entire epoch of classical mechanics' reign—but also to hold a candle to a novel methodological means of approaching the philosophy of media. Henceforth, we shall seek to illuminate the developmental exigencies that have not only lacerated and left scars upon modern philosophy of science but also the conceptual consideration of scientific laws via counterfactual grounding.

Wilson argues that the standard categorizations of 'Theory T thinking'—logic-centered conceptions of scientific organization (canonized via logical empiricists in the mid-twentieth century)—dampens the understanding and appreciation of those strategic subtleties working within science. By 'Theory T thinking', we mean to describe the simplistic methodology in which mathematical science allegedly supplies 'processes' that parallel nature's own in a tidily isomorphic fashion, wherein 'Theory T's' feigned rigor and methodological dogmas advance inadequate discrimination that fails to distinguish between explanatory structures that are architecturally distinct. One of Wilson's main goals is to reverse such premature exclusions and, thus, early on Wilson returns to John Locke's original physical concerns regarding material science and the congeries of descriptive concern insofar as capturing varied phenomena (i.e., cohesion, elasticity, fracture, and the transmission of coherent work) encountered amongst ordinary solids like wood and steel are concerned. Of course, Wilson methodologically updates such a purview by appealing to multiscalar techniques of modern computing, drawing from Robert Batterman's work on the greediness of scales and Jim Woodward's insights on causation.

§I, "PRAGMATICS' PLACE AT THE TABLE"

Here, Wilson ushers in an investigative context with which to explore the varied explanatory architectures of compression through which we capture the physical world in tractable terms (recapitulating the central argument of *Wandering Significance*). The kind of contextual dependencies that were underscored by twentieth century 'common sense' philosophers such as J.L. Austin are active within properly operating physical science. Recent progresses in multiscalar modeling of complex materials such as steel or granite demonstrate how scientific 'success' can be achieved solely if the descriptive vocabularies utilized are subjected to a monitored set of contextual restrictions and *usages*, or linguistic labor.

Wilson finds himself in agreement with Jerry Fodor's account of the language of thought, whereby *thought concerning the world* is prior to *thought about how to change world*. Here, appeals to 'context' can not resolve this distinction. According to Fodor, the argument unfolds as such:

1) The tasks that require language for successful execution generally demand that articulated stretches of individual sentences get laid down in dialogue(s), which reveals the linguistic level at which pragmatic purpose displays itself (i.e., that 'meaning is use').

2) However, individual 'component sentences' within such dialogues must be understood before the purpose of the larger groupings to which they belong to can be adequately recognized.

3) Parsing such 'component sentences' rests upon a foundation of lexical grasp and grammatical recursion.

4) Thus, such recursive processes of semantic recognition transpire *before* the pragmatic purpose of a dialogue can be grasped, whereby the 'primary

meanings' of such component sentences are already determined.[2]

Wilson, however, seeks to liberate philosophy from Fodor's anti-pragmatic straitjacket, turning to modern engineering in order to evaluate modeling tasks whereby a collection of independent algorithms are controlled by and appeal to an external register that 'monitors' or ties together and overarches their localized purposes. With structured discourse as it pertains to vocabularies that utilize terms such as 'force' and 'pressure', physical significances are adjusted and undergo subtle reorientations in referential attachment, thus reducing computational complexity for the sake of systematization (this is a kind of 'avoidance'). These reorientations "can easily baffle the observer who has not appreciated the efficiencies offered by these contextualized adjustments in meaning", or the sensitivities to usage (Wilson 2018: 5). Threading effective language design with multiscalar modeling, Wilson illuminates the variegated aspects of classificatory reference that semantically reference necessitates vis-a-vis compartmentalized registration. The multiscalar methods that Wilson surveys demonstrate how "descriptive problems of an otherwise insurmountable complexity can be conquered by breaking the task at hand into a set of local investigations whose relationships to one another are carefully monitored by a set of markets that keep track of investigative intent" (4-5). We can therefore make a distinction between executing complicated sequential routines and appreciating their underlying strategic rationale, so as to create a parallel with how descriptive vocabularies adjust referential foci qua investigative context and developmental mechanisms.

If cognitive architecture involves tacit adjustments in thinking, what does this mean for the representational structure of the syntactic demands in question? In order to answer this query we must examine the premise of compressive schemas. Let us consider, for instance, two standard pictorial modes, TIFF and JPEG formats, wherein the JPEG image employs far less data points than the TIFF image. With TIFF images, we see how data is encoded on a pixel-by-pixel basis (pixels encoded independently of one another). In the JPEG image, every pixel's front-end registration governs a fixed span of back-end determinacy, as if the

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[2] Jerry A. Fodor, *LOT 2: The Language of Thought Revisited*, Oxford, Oxford University Press, 2008, p. 14.

individual pixels of the TIFF had dissolved, forming an assemblage based on large scale hierarchies. With compressive schemas, on the other hand, there is an enactive scaffold of exploiting contextual registers. This begin with a broad metric Q1, followed up by finer grained metrics Q2 that rely upon the response to Q1, and so on through a nested array of further queries Q3, Q4, etc. Such interdependencies unfold within a segregated front-end register (Q1, Q2, Q3) etc., followed by an enumeration of their respective answers (A1, A2, A3,..). Such representational tactics are termed *multiple register schemes*, where syntactic complexity is reduced through scope restrictions vis-a-vis policies of contextual localization. Wilson remarks that "[p]resent-day philosophy of language could become more supple if its practitioners more warmly appreciated the substantive reductions in syntactic complexity achievable through various policies of contextual localization" (9). A conception of computational pragmatics as such is privy to responding registrations of linguistic capacities with respect to data and reasoning qua compression.

Such 'straightforward' reasoning procedures—a data sieve of sorts—transpire in modern engineering through the employment of particularized reasoning architectures in multiscalar design. This problem more thoroughly unfolds in Wilson's fifth essay, "The Greediness of Scales" but beforehand we must elaborate upon a critical term, the Representative Volume Element (RVE). The RVE denotes the descriptive depiction linked to a set of target-events in terms of the characteristic size-scale of an object during which those events unfold. The RVE level characteristics for steel demonstrate the central problem lurking behind the tyranny of scales problem:

> "[a]s we inspect ever smaller hunks of metal, we initially find that, starting at a macroscopic RVE length of one meter, steel obeys the same rules for stretching and compression through a large number of smaller scalar choices. However, when we eventually reach a characteristic length of about .1 millimeter, this regular scaling behavior fails, and the material responds to pushes and pulls in a different manner. Below this critical characteristic length, a complex hierarchy of varying behaviors comes into view as we inspect our metal at ever smaller RVE levels" (11).

As every RVE level interacts with the others in variegated and intervening ways, the attempt to capture the behavior of any piece of steel through a bottom-up molecular method generates descriptive overload. With a steel bar, for instance, there are induced dislocation movements that 'pile up' as we increase our RVE—the molecular bonds below become less protected, and our steel turns brittle. Such a change in response behavior after repeated loadings and unloading is called a *hysteresis effect*. The hysteresis effect presented a challenge for French theoretical physicist Pierre Duhem, who, toiling away at the indeterminacy of experimental criteria (c.f., the Duhem–Quine thesis) attempted to develop a thermomechanical framework that could model complex materials in a single-level fashion. Thus unfolded the "tyranny of scales," which describes the obstacle to computation wherein the range of size scales that we must tie together when assembling a composite story becomes impossible if we work in bottom-up fashion, beginning form the molecular scale and moving upwards. Multiscalar techniques extract new conditions, refined and localized estimates, from previous local estimates calculated within the RVE modeling that corresponds to a higher scale length. Thus, multiscalar modeling's upshot is that it presents an internally linked equilibrium through *homogenization*; beginning with the macroscopic scale and moving downwards, the multiscalar scheme decomposes its modeling efforts into small processes that involve sub-models linked to specific RVE size scales. "These submodels concentrate entirely upon the locally dominant behaviors normally witnessed at the appropriate RVE level", allocating the task of tracking smaller disturbances to other sub-models and thus dividing descriptive labor (12-13).

That which is 'dominant' varies from one RVE scale length to the other. If we take the case of hysteresis in a steel rail, those occasionally intruding minor effects on one level reflect alterations that comprise dominating events at the RVE scale level of the dislocations. Similarly, with anholonomous behavior, or systems that consist of implicitly dependent parameters, values that prolong locally in determinate fashion prove to be multi-valued when considered on a global basis (e.g., the Riemannian manifold has a metric fundamentally distinct from the metrics of a Euclidean space). As such, we must contend with the matter of analytic continuation and prolonged use-application as it relates to a meromorphic function where, grappling with a new patch of application, the same physical characteristics are not retained from a referential base.[3] With the multiscalar scheme, each sub-model performs its unique computational duties in a tractable manner involving descriptive parameters, assembling a hierarchy of separate sub-model computations. With this layered process, corrective interscalar adjustments are introduced that reassess initial estimates as lower RVE sub-models dictate. The corrective process of *homogenization*, neither purely bottom-up nor top-down in its descriptive policies, continues running through the successive stages (macroscopic estimation → microscopic correction) as needed until an overall descriptive accord is reached. That is, at the macroscopic level, multiscalar modeling employs isotropic modeling to predict the local shearing and stress environment of a small RVE block and, focusing on macroscopic environmental pressure, moves to the microscopic level. Here it employs a laminate modeling so as to verify that the RVE will continue to homogenize to the same upper-scale parameters in the assigned stress environment. This anisotropic process continues, sending the corrected parameters back to the upper-scale model until a self-consistent 'modeling assignment' is reached, or the scheme is stabilized (15).

Compared to the repetition of 'storage and retrieval' successive approximations not only does this provide a particularly rich gradient insofar as localized investigative context is concerned but, also, vis-à-vis differential equations, the RVE sub-models in question demonstrate how the same interior region can be described in syntactically inconsistent ways. On a macroscopic RVE scale, a region of a rock can be described as uniformly granite whereas a laminate model may contradict this description. As Wilson remarks, "multiscalar tactics practice a division of linguistic labor in which no participating party proves

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[3] In his earlier work, Wilson examines questions such as: "[f]rom what source does that rigid 'patch *p* determining patch *q*" character of an analytic function spring? Answer: from the way that such quantities grow to cover their full domains through a step-by-step process of *analytic continuation*"; "[h]ow do we reach complex numbers that lie beyond the dominion of our first exploratory series? […] This step-by-step process for pushing functional meaning from one local domain into another through appeal to overlapping series is called *analytic continuation*." Working through complexity and prolongation, Wilson demonstrates how "[m]any analytic 'functions' manifest a twisted personality that refuses to spread out uniformly across the complex plane." See: Mark Wilson, *Wandering Significance*, New York, Oxford University Press, 2006, pp. 312-319.

'more correct' than any other" (16).[4] Multiscalar schemes' instructive advantages are correlated to the fact that their component investigate stages are linked to physical scale size in a direct and palpable way, allowing for one to ascertain linked contextual shifts.

Semantically speaking, with multiscalar modeling descriptive content is encoded within the architecture of the enveloping computational scheme—as data is shuttled from a local registration to another—rather than this descriptive content being captured within any particular component sub-model/one pertinent scale-size. RVE divisions as such correspond to descriptive opportunities, by which we mean "physical circumstances whose dominant ranges of variation can be adequately captured in a smallish number of descriptive parameters" that are feasibly calculable and, consequently, "offered within nature itself" (17). To be more specific, this means that a multiscalar reasoning scheme is successful solely if the component parts, or the specific RVE levels cited within the model, capture the special descriptive opportunities that are endowed by the hierarchal layers of 'dominant behavior', thus mimicking the complexities of a target material that can be identified empirically (e.g., through a microscope). When parsing such questions about nature, we need to anchor our 'syntax' to physical locales which supply the best opportunities to compute 'outwards' to extrapolate the rest of the nearby terrain. An integrated architecture can thus be mereological, where the integrated parts prove greater than the sum of its descriptive parts. Multiscalar modeling illuminates descriptive information about a target system whose contents do not otherwise appear in the "localized statements generated within its sundry sub-models" (19).

Thus, complexity is reduced by concentrating upon dominant effects within every sub-model, as each RVE sub-model is assigned the duty of capturing solely the central physical processes witnessed at a characteristic scale length, leaving minor effects to other RVE choices. Correction missives are integrated from companion sub-models that circumscribe other choices of RVE scale. This is a "concentrate-upon-dominant-behaviors stratagem" (19) where the full scheme

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[4] It remains the ambition of a future philosophical endeavor to mend the partition of descriptive tractability, or in this case the descriptive practice concerning an integrated networks of localized tasks qua contradiction, as a paraconsistent (metaphysical) logic compatible with Dialetheism.

defers minor effects' tracking to the sub-model level where these alterations initially appear as *significant effects*.

Consider, for instance, the collision of billiard balls. With the standard treatments of Wren, Huygens and Newton, the billiard balls' shapes do not vary throughout collision-interaction—instead, such treatments focus on codifying energy loss vis-à-vis coefficients of restitution. With the advent of high-speed photography and small-scale computing, contemporary continuum mechanics allows us to understand that the billiard balls actually *compress and expand* when they collide, with these changes occurring very fleetingly. Thus, high-speed photography prompts a 'cut out' by considering the billiard balls in specific terms vide Representative Time Elements (RTE)—as such, we are able to concern ourselves with the internal details particular to temporal intervals during which such compressions transpire, omitting other 'faster time' RTE emergences (20-21). Thus, RTE terms offer us a particular scalar tyranny. This shows how, in principle, we simply *can not* describe material in single level molecular terms, which solely exist as ideals—or, to co-opt the academic philosophical parlance that Wilson cautions against, 'in principle'—and not actualities.

How do different scales 'communicate' with one another? That is, how do events that manifest on one characteristic scale process the lower RVE-level events? This process, of *homogenization*, consists of both coherent change and signal noise, with corrective messaging depending on how elastically dominant parameter patterns have been informed at higher RVE scales by lower 'grain-level' submodels.[5] The question then arises of how to syntactically model such corrective messages and homogenization methods. One such filter is the *central limit method* of extracting Gaussian modeling parameters of *mean* and *variance*, but, veering towards macroscale modeling, such stochastic processing does not treat material as properly laminate. Instead, it produces raw data generated within an individual scale of RVE sub-modeling. The multiscalar method, on the other

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[5] "If our laminate sub-model flexes in a manner that is consistent with our upper scale E and G expectations, fine; no correction in our upper-scale reasoning is required. But if our sub-model recrystallizes within its new stress requirement, our upper-scale calculations must take this revised datum into consideration. So the corrective messages we send across scales should constitute reports on whether normal dominant behavior patterns have broken down or not" (22).

hand, allows for various scales to be held within a latticework of asymptotic limits, or in a collective 'union', whereby attributions are dependent upon one another.

The linguistic register of context-sensitive dependency and concerns relegated to devices of language management are also central when articulating structural subdivisions. Terms that articulate the investigative architectures of structural subdivisions are what Wilson terms "devices of language management". Such terms are not concerned with imperatives such as what is 'true' or 'false'—indeed, we do not assume that data established or stresses computed within a laminate sub-level can be "entered freely into an upper-scale treatment without undergoing significant homogenization filtering" (25). Instead, we are concerned with the practical endeavor of how truth-values are obtained vis-à-vis enveloping schemes of placement, and, thus, are interested in that which is appropriate, or inappropriate, to the reasoning tasks at hand. Understanding 'use' as equipollent to reasoning as practical purposiveness means that it can precede semantics, which are *adequate attachments to exterior circumstances*. Here, Wilson dissents with asservations that retain investigative heuristics with the virtues of efficient reasoning, noting that conventional divisions between semantics and pragmatics neglect the investigative strategy's molding. While scale-based isolation permits reduction(s) in descriptive vocabulary—e.g., a term such as 'force' can adjust its precise referential focus to suit the RVE unit within which it is presented—semantic adaptation causes substantial data compression and, consequently, inferential difficulties when terms migrate from an (inconstant and divergent) descriptive semblance.

Descriptive terms do not acquire referential connections to the physical world until their employment is situated within a localized investigative context—consider, for instance, the managerial positioning of terms like "true"/"truth"[6].

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[6] With Tarski's semantic concept of truth, we that the correspondence theory definition of truth is in conformity with the classical Aristotelian conception of truth, wherein there are enactive underlying pragmata that structure the truth-value of statements and "[t]o say of what is that it is not, or of what is not that it is, is false, while to say of what is that it is, or of what is not that it is not, is true". See: Aristotle, *Metaphysics*, trans. W. D. Ross, Sioux Falls, NuVision Publications, 2009 §1011b25. According to Tarski, both laws of contradiction and of excluded middle can be deduced from this formula. In Tarski's definition of truth of an interpreted sentence, a sentence A is true if and only if A is satisfied by every infinite sequence of objects. Such a formulation expresses a concept of absolute truth wherein the truth of a sentence is not restricted to a certain domain or circumstances. For Tarski, truth is not relative to circumstances. See:

With materials such as steel or rubber there are significant 'frozen disorders' at middle-level RVE scales (for instance, dislocations and grain boundaries). Such irregularities preclude these materials from satisfying the equilibrium requirements necessitated within elementary expositions. Were we to adhere to the terms of traditional correlational demand, we would remonstrate the assignment of temperature to these materials. Here, we must direct our attention to *applicational migrations*: of course, we do not have reason to question that terms such as 'temperature' capture critical characteristics of a macroscopic steel bar. Nonetheless, we do not possess a clear (iconic) picture of its referential content. Thus, our semantic lesson, as it involves inferential terms like 'force' and 'temperature', is directed towards common 'meanings' that allow unique utility in novel RVE environments qua behavioral-contextual circumscription(s).

Wilson's adaptive approach emphasizes how mastery of 'wandering words' like 'force' or 'use' precede firm referential semantics; only after applicational enclosures are set can they attach to moorings suited for novel modeling environments. Thus, Wilson does not agree with Fodor's anti-pragmatic approach to meaning, an a priori assurance which suppresses adaptive behaviors by compounding variegated facets of language-learning. Fodor and fellow anti-pragmatists such as Susan Stebbing[7] argue that semantic scenarios (inherently) anticipate altered adjustments, claiming that terms such as 'force' are "first assigned, strong, referentially determinate core meanings before the pragmatic influences of applicational context can begin their work" (30).

Terms such as 'force', 'temperature', or 'cause' are granted enlarged descriptive

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Alfred Tarski "The Concept of Truth in Formalized Languages" in *Logic, Semantics, Metamathematics*, Indianapolis, Hackett, 1983, pp. 152-278. With Gupta's Rule of Reason, the mode of truth-values construction is wholly determined by Tarski's semantic concept of truth, so that this mode of construction is nonarbitrary, despite the bootstrapping of predicates. However, bootstrapping may, in fact, involve arbitrary extension; Gupta observes that the two conditionals within the 'if and only if' of Tarski's formula represent ordinary material conditions but that they serve to lift and lower assertions from one contextual level to another—that is, bootstrapping our way through non-successor stages by instrumentalizing the outcome of previous stages to the extent that they gave as a stable verdict (i.e., Gupta's Rule of Reason). See: Anil Gupta and Nuel Belnap, *The Revision Theory of Truth*, Cambridge, MIT, 2003. Wilson's work on language management focuses on the inferential transitions transferred by the word 'true' with respect to multiscalar architecture.

[7] Lizzie Susan Stebbing, *Pragmatism and French Voluntarism*, Cambridge, UK, Cambridge University Press, 2013.

utilities when they are developed within local adaptations, whereby such arrangements are protected by 'homogenization barriers'. Within the multiscale model, structural portioning into segregated patching abets swift processes of adaption; it is precisely the 'ready reprogrammability' of any multiple-register language's format that facilitates the adaptive plasticity of its conceptual practice. In short, the descriptive focus of terms such as 'force', 'temperature', 'strain energy', etc. is contingent upon scale-level application; "arrangements facilitate the reassignment of old computational routines to novel applicational purposes" (30).[8]

Wilson's conception of the *specialized space of possibility* introduces the context of the 'controlled search', where laminate possibilities predetermine both spaces of assembly and companion searches. Any initial search is initially given its determinant 'shape' by equilibrium-state and finite-element computation(s) that use trial-stress distribution to search through a collection, or 'space', of possibilities before cross-checking these in light of their laminate underpinnings. After searching through a new 'space' of laminate possibilities, companion searches settle upon suitable fixed points. In the case of a granite/gneiss rock sample, multi-scalar analysis obtains lower-scale recrystallization possibilities that are kept in tandem with initial search space (preliminary assignment of stress to the entirety of a rock based upon a suitable macroscopic model with highly stressed sectors that allow for recrystallizations). What does this mean for practical possibilities within possible worlds? Much like Tarskian semantics, Saul Kripke's interpretation of modal language motivates the concept of a possible world, or 'cut-out', to give substance to the notion of alternative extensions and alternative domains of quantification. Multi-scalar considerations—particularly localized search spaces—qualify such capacious considerations.

Let us introduce a term from differential mathematics, the "boundary

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[8] There is a curious parallel between Wilson's cyclic description of our restricted linguistic capacities, "condemned to wobble between seasons of brash inferential extension and epochs of qualified retrenchment later on" (32) and André Leroi-Gourhan's notion of the evolutional *chaîne opératoire*; according to Gourhan, "[f]or each species a cycle is established between its technical ability (its body) and its ability to organize itself (its brain). Within this cycle, through economy of design, a way opens up toward increasingly pertinent selective adaptation". See: André Leroi-Gourhan, *Gesture and Speech*, Cambridge: MA, MIT Press: 1993, p. 60.

condition", which "describes data that have been compressed to fit onto a lower dimensional surface extending in a *time-like* direction into the future" (35). In the example of a collapsing bubble, the boundary conditions denote how surface tensions tax energy from the interior. The boundary condition illuminates the bricolage between interfacial conditions and interior modes of description, pulling the two within homogenized arrangement (much like the sub-models that are homogenized within multiscalar modeling).

Just as 'force' undergoes referential refocusing vis-à-vis scale change, considerations of 'dominant behavior' that rest upon interior/exterior data allocations are reassigned when RVE levels are adjusted. Therefore, the characteristic regional localities that correspond to descriptive vocabularies are linked to RVE scale (re)adjustments:

> "[w]hat was formerly described as a smooth and placid surface around a granite block is now approached as a jagged landscape in which significant layers of complicated surface chemistry are active [….] on a macroscopic level the indentability of a steel plate appears to be a localized property of its outer surface alone, but a lower-scale examination reveals that such traits depend vitally upon capacities for plastic flow that transpire many molecular layers away from the metal's nominal surface" (37).

§II, "PHYSICS AVOIDANCE"

Let us apprehend the time lag that a fluid undergoes before regaining its former volume by considering the die well within a tube of toothpaste that is pushed upon: there is a short period of time before the toothpaste 'remembers' the uncompressed volume that it occupied inside the toothpaste tube. Such tacit considerations provoke an alternative form of 'cause' and 'effect' investigation, which allots credence to (wave) disturbances that move through the interiors of continuous media. The notion of 'cause' generally focuses attention on evolutionary events of this stripe, and we can further consider such fluid-compression paradoxes that apply to solid objects which appear to be superficially rigid, as in the example of two billiard balls colliding. If, for example, during T_1, force is applied to the left of billiard ball A—with A adjacent to billiard ball B, which is to A's right—during T_2 billiard ball A stores strain energy through compression. Then, during T_3, billiard ball A employs energy storage to *re-expand*, performing compressive work upon billiard ball B. Volume change is required for this—each successive billiard ball must momentarily contract and, thereafter, *re-expand* so as to transmit a compression wave across its interior. In order to rationalize these causal activities within fluids and billiard balls, we need to revise the notion of internal pressure with directional subtleties (e.g., intermediate conceptions between fluidity and sustaining shear such as 'absolute pressure').[9]

Wilson discerns fundamental classes of explanatory architecture via applied mathematics' diagnostic tools. Running water is an event that rarely ever is encountered in a truly 'steady' state. By focusing upon steady-state conditions through *evolutionary flow* portraitures we can break down any temporally connected set of states and regard them as stochastic differential equations. By parsing computational pathways and time-effaced continua, we can bifurcate a system's behavior into approximate divisions: 1) *transitory response* (this records how a moving wall of water first invades a pipe after a spigot is opened); 2) *steady-state response* (which becomes dominant as initial effects of *transient response* fade). Given our fluid flow example, we can examine what shifts transpire with an evolutionary-manifold to base-manifold adjustment. Chiefly, the 'time' variable is purged from the original modeling equation(s) as a shift to steady-state methodology is undergone. That is, base-manifolds offer a representative picture that *factors out time*. By considering structural alterations in a control parameter, terms such as 'force' and 'cause' acclimate to the explanatory landscape into which they are cast. Within the evolutionary framework, such terms attach themselves to processes that affect temporal change within a developing flow. Such is the case regardless of if we shift the background explanatory landscape from an initial conditions problem to a control variable setting (53).

When we consider 'causal processes', we generally designate continuously acting evolutionary development (e.g., a sound wave moving through an iron bar) although this undoubtedly glosses over a gradient of evolutionary richness. This can perhaps be best captured with partial differential equations (PDEs), an endeavor that—although solidified circa 1750—was taken up earlier by Newton and Leibniz as they developed ordinary differential equations (ODEs) during the 1670s. Newton and Leibniz' development significantly advanced beyond *finite*

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[9] Clifford Truesdell, "The Creation and Unfolding of the concept of Stress" in *Essays in the History of Mechanics,* Berlin, Springer-Verlag, 1968.

difference terms where time is divided into small step sizes (Δ_{ti}) and approximated temporal intervals (Δ_{ti+1}). As in Euler's method of finite differences, which conflates 'causes' with 'effects' in describing infinitesimal relationships through approximate terms, we hereby exact a procrustean 'data compression'. Ordinary differential equations, ODEs, serve as a 'halfway point' to a suitable mathematical representation of causal process; ODEs contain only one single independent variable which, within the evolutionary setting, is time (t). To capture the richness of causal processes robustly, we also need to account for spatial variations (x, y, z) as independent variables as well. PDEs take us significantly further as they account, *at minimum*, for one spatial variable (x) as well as time (t).

There are a number of misunderstanding that have plagued the philosophical treatment of 'cause' since Bertrand Russell's formative 1912 paper, "On the Notion of 'Cause'". Here, Russell, significantly inspired by Ernst Mach's work on coherence and related complexity, makes the case that:

> "[w]e cannot say that every law which has held hitherto must hold in the future, because past facts which obey one law will also obey others, hitherto indistinguishable but diverging in future. Hence there must, at every moment, be laws hitherto unbroken which are now broken for the first time. What science does, in fact, is to select the simplest formula that will fit the facts."[10]

Russel's conclusion is premised upon the argument that the notion of 'cause' and 'effect' plays no significant role in science because 'effects' employ differential equations devoid of a pertinent 'before and after'. However, according to Wilson, Russell's critique glosses over the fact that finite differences (Δ_{ti} and Δ_{ti+1}) provide a naturalized syntax for communicating causal processes when one "doesn't have the vocabulary of the calculus available" (62). That is, Russell conflates distinct types of equational sets indiscriminately—Wilson illuminates that such considerations impinge upon the treatment of time (t).[11] For instance, the steady-

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[10] Bertrand Russell, "On the Notion of Cause'", *Proceedings of the Aristotelian Society, New Series* Vol. 13 (1912 - 1913), p. 23.

[11] Wilson's internal mathematical closure is certainly indebted to Descartes' relationism, wherein any 'position' within physics refers solely to the *internal relationships of parts* and not to any background container space or space-time such as in Newton's absolutist conception. "Normally, this solitary factor is settled by an outside agency turning the crank, in which case time enters our picture as a control space intrusion" (96).

state strategy for modeling fluid flow gains significant computational advantages by suppressing the *temporal transients* which trouble how fluid flow behaves in 'real life' circumstances. Indeed, such altered differential equations provide a replacement modeling, but not as an attempt to capture genuine causal processes. The intentional suppression of direct registration in temporal processes allots for distinct richness which we lose in the syntactic indicators present within ongoing evolutionary developments.[12] Alan Turing well understood the advantages to this 'suppression' when he took up the "discrete-state machine" model as the relevant description for the brain's functional operation(s), opposing it to "continuous-state machines".[13] Today, the set of core predictive analytic techniques involved with machine learning, such as logistic regression, Naive Bayes, k-nearest neighbors, decision trees or more recent variations can also be stratified from linear analysis to a 'steady-state' vector space of 'suppression' that allows binary classification (i.e., from lines of best fit).

By disregarding *temporal transients*, the altered differential equations in steady-state analysis come into prominence, marked by *altered signatures*. Notably, equations for 'steady flow' are, generally, of *elliptic signature*, which disregard time-terms and produce equilibrium states in comparison to fully evolutionary sets which are *hyperbolic*.[14] Differences in the signatures of differential equations' operators indicate that relevant equations play unique explanatory roles within science. Formally speaking, what we have just described is Euler's approximation

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[12] Nancy Cartwright and those following her, such as Mathias Frisch, have argued that causal notions should be invoked as a doctrinal *supplement* to the canonical laws of physics that Russell bases his rejection upon. According to Frisch, causality-as-a-supplementary-condition provides a means of discarding *unphysical* solutions (i.e., "causal asymmetry"), such as ripples traveling *inwards* towards a focus (i.e., "circularly converging waves") rather than outwards, as in a pond. For Wilson, to invoke such a 'supplementation' is mistaken, as "the basic relationships of causal evolutionary development are already marked within *the mathematics of hyperbolic equations"* and "[o]ptical circumstances are rarely suited to true evolutionary initial/boundary value modelings" (93; emphasis added). See: Mathias Frisch, *Inconsistency, Asymmetry, and Non-locality*, New York, Oxford University Press, 2005, pp. 289-342.

[13] Alan M. Turing, "Computing machinery and intelligence", *Mind* vol. 59, 1950, pp. 433-460.

[14] *Elliptic signatures* directly delineate a constant field of velocity vectors, which do not mark passing time in the direct manner of an evolutionary modeling ('time' does not appear as an independent variable in these equations). The distinctions between *elliptic* and *hyperbolic* signatures can be drawn back to Paul du Bois-Reymond, although it is Jacques Hadamard's distinctions in which we are presented with the relevant explanatory condensation. See: Jacques Hadamard, *Lectures on Cauchy's Problem in Linear Partial Differential Equations*, London, Forgotten Books, 2018.

method, a kind of the physics modeling where we begin with a purely evolutionary paradigm and reach an equilibrium-focused shift by treating an *initial/boundary value problem* as a *pure boundary value problem*, thus altering the explanatory landscape.[15] Focusing on an interior membrane, such approximations exact an 'avoidance' (from which Wilson's project gains its moniker)—that is, we examine the standard and differential equation for an *interior* membrane while conceding to the fact that we are not pursuing the physical processes responsible for *boundary* region behaviors.[16]

Iterative computational techniques, or 'shooting methods',[17] are used to resolve pure boundary value problems. With a single spatial variable (i.e., employing Euler's reduced equation) we are obliged to reduce a three-dimensional topology to a computational path that responds to the top boundary (72). Given their PDE requirements, equations of elliptic signature—such as the 'time-terms-dropped-for-equilibrium' or 'drumhead equation'—solely accept analytic functions as solutions (viz., solutions that have been coordinated and are reproducible—that is, solutions that are universalizable).

Such adjustments—transmogrifying temporal tracking events for their eventual inevitable equilibriums—creates a novel *manipulation time* that does not represent 'natural endogenous time'. As it concerns the case study of the *manipulation time* capturing the driving rate when we increase the weight on a boundary (as it applies to our strut buckling problem), a *manipulation time scale*, Δt^*, unfolds significantly slower than the *internal relaxation time*, Δt, required for the strut to damp out and settle into equilibrium naturally. In introducing a distinct time scale, such infinitesimal compressions occasion unique semantic conceptual difficulties. Vacillations in word reference are not whimsical but can

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[15] The two forms of explanatory landscape differ insofar as an initial-boundary value problem (e.g., wave movement of a pulse inside of a metal strut) will consider the 'interior fill in' (or membrane) alongside initial conditions, the pressive arrow of time (**t**) and the boundary conditions. The explanatory landscape of a pure boundary value problem (e.g., equilibrium shape of a two-dimensional drumhead under varying edge tensions) consists of the interior membrane and boundary data.

[16] This *boundary region*, a cooperative harness through which we can obtain a viable physics for a continuous body, has a traction that we can contract into interior stress capable of interacting with the body forces and an element's inertial response.

[17] These are called 'shooting methods' because they operate much like the angle-adjustments of a bow in archery, which are readjusted iteratively, increasingly attempting to localize on the target.

be fastened to the ways in which 'cause' serves as an instrument of language management. Consider the novel usage of 'cause' that Wilson provides us, in context with the modeling circumstances we have just surveyed:

> "'Let's figure out the effects than an increased weight will cause within the strut.' Or, when we wish to alter out explanatory focus: 'No, I'm more interested in how effects from the upper weight reach the middle of the beam.'" (77).

As a specific explanatory topology is selected, 'cause' is prompted into an arena of supplementation. Altering the underlying explanatory landscape prods 'cause' into conceptual conflation with words such as 'pressure'. Through (holonomic) algebraic constraints—e.g., when we maintain that a bead stay on a wire, that a balls slides or rolls along a table top, or that a machine part remains rigid—we also glean tacit scales. On a small scale of resolution ΔL, for instance, the bead will *not* follow the curve of the wire but waver about its locus; through the coarser (observational) macroscopic ΔL^*, however, we behold the moving bead as it is perfectly situated on the wire.

Our empirical observations are under-constrained. Consider the use of a telescope to assemble an abundance of empirical observations, E, which fit within the form $<x_i, y_i, t_i>$. To amalgamate these into a linear equation ($y = mx + b$; m and b operating here as undetermined variables) provides for a problem concerning *alignment*. Our combined empirical data, E, is *over-constrained* and contains errors (ε_i). According to Gauss and Lacroix's resolution, we incorporate unknown error terms ε_i into each of our data points, such that we combine these observations, proffering $<x_i, y_i + \varepsilon_{i,} t_i>$. Including ε_i alleviates over-constraint by offering extra variables, resulting in an *under-constrained* equational system.[18] Following Lagrange's virtual "principle of least work", Wilson introduces force-like supplements (λ_i) which, in combination with the other operative forces, accord with the underlying $F = ma$ principles, akin to the manner in which the

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[18] According to equation-based object-oriented modeling languages, (e.g., Modelica, gPROMS and VHDLAMS), the existence of a single solution requires that the number of equations and variables (unknowns) are equal. If the number of equations is *greater* than unknowns, the model is said to be *over-constrained*. Conversely, if the number of unknowns is *greater* than equations, the model is said to be *under-constrained*. See: David Broman, Kaj Nyström, Peter Fritzson, "Determining Over- and Under-Constrained Systems of Equations using Structural Constraint Delta", *Proceedings of the 5th International Conference on Generative Programming and Component Engineering*, 2006, pp. 151-160.

error terms $\varepsilon_{i,}$ resolve our combination-of-observations difficulties (85). Reducing descriptive complexity in this manner, we engineer (non-visible) internal activity to cancel aspects of exterior manipulation that oppose constraints. In practical applications, Lagrangian exploits of macroscopic data simplify a complicated microscopic situation vis-à-vis the manipulation of counterfactuals—consider, for instance, applying an external force or torque to a target structure that limit or tighten lower-scale arrangements. Inferential mechanical analytics in Lagrange's manner are vital to the progress of science as a scaffold unmoored by empirical deviations and speculation; this is precisely how we erect a 'reliable' explanatory architecture that is not simply stilted by a priori (pre)determinations.

§III, "FROM THE BENDING OF BEAMS TO THE PROBLEM OF FREE WILL"

Let us return to Leibniz's metaphysical views, which stem from mathematical concerns linked to the methodological considerations of classificatory-linguistic reference. Given his pioneering work on differential equations in 1684, Leibniz is one of the first scientists to attempt the physical modeling of equations at the infinitesimal scale (i.e., differential equations), working through many of the methodological toils that Wilson takes up in *Physics Avoidance*. In particular, Leibniz work on the elastic response of loaded beams illuminates both his metaphysics and, more acutely, his conception of free will.

As most readers are likely aware, the most grating and 'unscientific' facet of Leibniz' system is the assertion that the material universe is constituted of a nested array of compressed 'monads' that are not tethered to a spatio-temporal actuality but, nonetheless, determine what we see and do, possessing desires, perceptions, and actions. While, according to Leibniz' monadology, the entire material world is controlled by monads that congregate in ordered hierarchy, this systematization *is*, nevertheless, rooted in sound modeling practice qua continuous materials. Hence, we can recover Leibniz's thought by bridging his metaphysical outpourings with concrete mechanical engineering techniques, turning to the structural requirements of continuously flexible matter.

Wilson notes that, "save for the material moduli that render wood more pliant and anisotropic than steel [….] both materials display the teleological capacity to 'remember a natural state' characteristic of all solids" (101). One of our central concerns here involves how Leibniz' monads bolster a physics where continua are required to maintain internal coherence under dilation and compression. It is, in particular, the modeling equation suitable for a loaded beam that we shall turn to, recalling how—ignoring friction—beams behave like elastic solids (that is, they possess "natural equilibrium states to which they will always strive to return"; 102), Considering the steel beam that is loaded with rocks—between its 'loaded-with-rocks' condition and its erstwhile 'straight state'—we see that there is something akin to the 'memory' or 'remembrance' that Leibniz uncovers in his work on loaded equilibrium states. A material's capacity to 'remember' an original state of molding diminishes with time, regaining a compromised and slightly curved end-state in between these two moments. Truly elastic solids display 'perfect memory' insofar as rest configurations are concerned, returning to this end-state rather quickly after being released from any binding constraints. In contrast, many materials display something akin to a 'fading memory' where, if a constrained position is maintained for a long period, such materials adept to the shape that is impressed upon them so as to minimize 'strain energy'.

Today, an amalgam of 'smart materials' have been engineered that regain various forms of earlier conditions depending upon earlier states; one such example are the temperature-respondent nickel-titanium alloy 'smart antennas' used for suborbital space vehicles such as sounding rockets. These antennas demonstrate what Leibniz presaged—that material displays a slight measure of being able to detect its neighboring media's internal conditions and respond accordingly. Leibniz conceived of two types of force, *dead* and *living* force, related to our understanding of force and kinetic energy, respectively. The Leibnizian concept of memory behavior and the teleological sense in which a material is determined by a system's displacement from its final ends is partially indebted to Edme Mariotte, a French physicist lauded for his 1673 experiments on elastic and inelastic experiments with whom Leibniz was in communication.

Let us further parse Leibniz' appeals to the desires, perception, and action of inanimate materials with special regard to such invocations of '(material) personality'. Via his work on the dynamic behavior of colliding billiard balls, Leibniz set to develop a calculus by which elastic force could be measured, recognizing that the principle of elasticity is "the true cause of all phenomena of the collision of bodies".[19] Today, when we describe two billiard balls' collision, we note that the "their incoming kinetic energy is temporally converted into internal potential energy depending upon the degree to which their contours have been distorted" and that "[a]fter these compressions reach a critical level, each ball will push back against the other by transferring its unwanted strain energy into new forms of moving energy, including spinning" (103-104). When describing such interactive events in terms laden with 'personality'—as in his use of "potency" and the Aristotelean concept "entelechy"—Leibniz correctly emphasized notions such as strain energy, and, more generally, potential energy, as being teleological, for these materials necessarily appeal to natural rest-states.[20] The prevailing models of impact in Leibniz' time were those of Huygens, Wren, and Newton, all of which assumed that billiard balls *rebound without alteration of shape*—that is, these models posited a matched asymptotics methodology that artificially collapses the scope of Δt^* (where impactive events occur) to zero duration 0.

Contra the computational compressions of such 'zero-duration-of-impact approaches', we would be shrewd to recall Leibniz' apothegm, *natura non facit saltus*, 'nature does not make jumps' (verbatim, "[i]n nature everything happens by degrees, and nothing by jumps").[21] For Leibniz, the "law of continuity" illuminates that there "is no assignable change in an instant" and that "this avoidance of leaps in the changes of bodies is due to an elastic force" that operates "in collision […] by gradual movements bodies that compress one another and then restore themselves yield to one another little by little and conserve their direction and force" (105).

Today, we can utilize quantum-mechanical modeling to examine materials at the nanoscale and parse constrained equilibrium as it relates to energetic exchange and loss, but Leibniz was relegated to tracking such complex interactions within idealized circumstances. Thus, Leibniz turned to entirely

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[19] Leibniz, *Oxford Handbook of Leibniz*, ed. Maria Rosa Antognazza, New York, Oxford University Press, 2018, p. 301.

[20] Leibniz, "Of Body and Force: Against the Cartesians" in *Leibniz: Philosophical Essays*, trans. Roger Ariew and Daniel Garber, Indianapolis, Hackett, 1989, pp. 252-254.

[21] Leibniz, "Chapter xvi: The degrees of assent" in *The New Essays on Human Understanding*, trans. Peter Remnant and Jonathan Bennett. Cambridge, UK, Cambridge University Press, 1996, p. 473.

static situations, such as the shape that a beam would assume under a load of rocks. In his own mechanical modeling work, Leibniz focused on teleological reflection in relation to the Hookean capacity of a spring to exert a restoring force in direct proportion to the degree that it has been stretched or compressed away from its configuration, constructing a plausible beam model upon this basis.[22] Physics before Leibniz (and Mariotte), such as Galileo's, approached the breaking load of a gradually weighted wooden beam as an instantaneous catastrophic event. Leibniz's consideration of internal elastic responses are linked to flexure, estimating gradual increase in internal stress that leads to material failure and, thus, relying upon sundry macroscopically-based constraints (i.e., "that beam movements transpire mainly along fibers and in plane sections"; 105).[23]

We also now know that continuous matter exhibits the property of not becoming 'simpler' at smaller size scales that leads to foundational recess. Leibniz, like most all pre-twentieth century thinkers of physics, employed appeals to ideal and infinitesimally small elements. The unavailability of PDE representations forced Leibniz to approach flexible three-dimensional bodies through the aperture of one-dimensional ODE decomposition. Conceptually, Leibniz focused on materials that seemingly remain continuous and flexible on every size scale (vide classical continuum mechanics), treating physical description at the level of spatial points as idealized. We will largely concern ourselves with a different argument concerning downward scaling when setting up a differential equation modeling of PDE type, which involves the suppression of irregular events arising upon small scale sizes.

A Hookean spring displays the simplest form of material teleology, as it immediately strives to return to its natural state in a manner that is linearly dependent upon its displacement from the condition in question. Thus, 'spring-like behavior' is a central element to Leibniz' mechanics, as it deals with continuum models. Wilson considers a complete and enclosed beam codified in

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[22] In rheology, the science of non-Newtonian formation and flow, every ideally Hookean solid—ideally elastic—contains a specific and un-breakable identity within the surface of its body (which is opposed to the ideally dissipating Newtonian liquid that perpetually extends and reproduces its surface).

[23] George Boole's work on non-numerical algebra realized Leibniz's dream of transforming reasoning into a calculus.

variational terms, carved into five equal finite length ΔL, allotted with five weights W, so as to examine how this 'possibility space', or 'function space', is variationally configured with the least overall bending. These weights act as a spring stretching within each element, exacting strain energy; the computational considerations we consider with this five-membered array of individual element configurations recall our previous reference to referential refocusing. A constrained equilibrium state represents a condition to locate an optimized state through experimental refocusing and corrective testing, where the optimally lowest tension configuration is the product of successive approximations. This search for a 'relaxed energetic state', which employs 'variational' reasoning, can be furthered by dividing ΔL into ΔL*-segmented possible elements of shorter length. Furthermore, we can relinquish ourselves of the weights and consider internal springs by assigning the bootstrapping procedure of formulating a continuous beam with an ersatz measure of "springiness determined by the true springiness within the segmented possibilities to which they lie near in our norm" (111). What this demonstrates is that real-world materials may appear to be smooth and thoroughly flexible but we can not obtain a computational handle upon their governing physics without simulating that these materials decompose into artificially curved ("kinked") and less pliable elements; "[i]n other words, flexible beams need to be assigned a measure of springiness even though they contain no springs" (111). By diffracting a continuum model into partial differential operators, infinitesimal elements can then be assembled into a final ODE that describes the one-dimensional displacement of a target object from the x-axis. This formulation of a constrained equilibrium is entirely static as our relaxation space does not contain any possibilities which change in time, such that we can superimpose such dynamic behavior to Lagrange in keeping with our elliptic signatures.

This beam formulation represents a downwards projected expression of a simplified 'personality', opening up questions regarding target materials and descriptive coherence. Our assumption of complete downward scaling is faulty for Leibniz did not assume that real-world materials behave identically at all scale sizes. As we know, if we observe wood or steel under a microscope, we see how it is comprised of:

> "a maze of cells or minute grains that individually stretch and dilate according to far more complicated rules than reveal themselves within larger hunks of the material. Small hunks of a beam no longer respond to pushes and pulls in a simple Hookean manner" (113).

Because 'nature does not make jumps', the springs that we (artificially) designate to our elements must cohere and coordinate with their adjacent elements such that they will sum to a continuous variation at the macroscopic level—i.e., infinitesimal strains sum to macroscopic displacements. Leibniz' thought shows how infinitival and extended modes of description harmonize in a non-trivial manner. *It is precisely due to this coordination of required compatibility and to underscore such harmonious cooperation that Leibniz writes about material such as wood or steel as 'machines'.* For Leibniz, calculus formulas represent downwardly projected generators of larger-scale behaviors.[24] This is a view that we generally co-opt when considering the standard equations for classical continua of everyday life (e.g., beams, strings, fluids) wherein their validity is predicated upon the downward projection of homogenous patterns observed at large-to-middling-scale lengths (114).

Wilson explicates how Leibniz fits together his two critical facets of explanation, *efficient causation* and *teleological explanation*. Leibniz's concern with *efficient causation* rests upon the fact that springs demonstrate the Hooke's law propensity to return to natural rest state. Specifically, Leibniz endorsed Descartes' "air sponge" theory of elastic rebound, noting that "even if some bodies appear denser than others, this is only because the pores of the former are filled to a greater extent with matter that belongs to the body, while, on the other hand, the other rarer bodies have the makeup of a sponge"; accordingly all "bodies of the universe" are "elastic, not though in themselves, but because of the fluids flowing

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[24] Here, we can make a curious connection with Gilles Deleuze's machine ontology. Deleuze's overall theory of machines is fundamentally flat, discontinuous, and infrastructural, as Deleuzean externality is premised upon irreducibility. In opposition to Platonism, or internalism—which results from the private depth of machines being irreducible to and unique in kind from their actualizations—our fundamental error of thinking, according to Deleuze, is to conflate the contiguity, identity and resemblance characterizing actuality as also characterizing "things in themselves." Therefore, "every entity is itself a machine, in the sense of being a causally effective agent that makes its own difference in the world" where each entity has its own unique "complex inner working." See: Arjen Kleinherenbrink, *Against Continuity: Gilles Deleuze's Speculative Realism*, Edinburgh, Edinburgh University Press, 2019, p. 7.

between them, which on the other hand consist of elastic parts, and this state of affairs proceeds in infinitum."[25] Thus, wood retains a 'memory' of how its parts can be arranged properly by relying upon circulating pressure in surrounding air that is requisite to restore its material to *teleologically desired natural equilibrium*. Material such as wood acquire a 'memory' of its natural equilibrium state via efficient causation mechanisms, which, themselves, are non-teleological and rely upon geometry and conservation principles. As the underpinnings of 'memory' become evident in lower-scale manner, Leibniz employs a bottom-up description of the processes concerning *efficient causation*, which we can contrast to final causation that is constructed through differential equation modeling (e.g., beam behavior).

Now, indeed, we must concede that time slips back in. The process air/tube wall interactions that restore the wood's webbing to its rest state configuration—where particles of air bounce off walls and interchange momentum—recall a proper account of billiard balls' collision. Tube walls and billiard balls compress temporally and distribute original kinetic energy into internal energies of deformation. The inviolable axiom of continuity requires that nature can never allot abrupt leaps such as radical shifts in direction. Thus, elastic compressions quickly push the ball away from the wall in an altered direction while compressed bodies regain desired shapes or, in Leibniz' parlance, their 'material personality'. Solely "under the assumption that these compressive events occur swiftly within a brief time interval Δt* can we preserve the 'inviolable axiom' of continuity that requires that nature never make abrupt leaps such as a radical shift in direction" (115-116). Huygens and Wren's approach to continuity approaches elastic collisions by omitting these Δt* interval events vis-à-vis the compression of hidden complexities into an instantaneous event that involves no distortion in these colliding bodies, accepting a discontinuous jump. It is through this instrument, of eliding Δt* events—or, qua Leibniz', the (higher-scale) 'final cause' account—that we can explain the elastic behavior of the wooded beam in a purist (lower-scale) *efficient causation* manner that solely regards the "pushing and pulling of

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[25] Ariew and Garber, *Leibniz: Philosophical Essay*, pp. 252-254.

contracting particles" and nothing else (117).[26]

For Leibniz, the lower-scale efficient causing model (of air particles and pore walls) is speculative and parasites upon the 'final cause' account (of differential equations for reliable modeling), which is ultimately the more fundamental strata. Therefore, it is the final cause account "which is in fact deeper and in some sense more immediate and a priori"[27] in comparison to the "primitive force or form of substance (which, indeed, fixes shapes in matter at the same time as it produces motion)."[28] For Leibniz, these alternating levels of preferred explanation are exchanged and neither aperture is wholly accurate, as each approach rests upon minor details explicable exclusively from its rival's perspective. This cyclic process repeats ad infinitum as we inspect matter upon lower ranges of microscopic detail, as "[o]ur little balls of air will themselves require pores through which an even finer air must circulate, which in turn will require pores of their own and so on […] in the manner of deMorgan's celebrated fleas-upon-fleas analogy" (118-119). Doddering through the labyrinth of the continuum, a verdant maze of never-stabilizing hierarchies and interwoven explanatory schemes, we see how a true 'final cause' narrative is always set in exchange with an 'efficient cause' description.

How does free will make its way into this model? Much like Maxwell's demon designated the physical laws that govern the inert universe as separate from the human mind's agency, repeating Descartes (and the providential direction of ambient air's elastic material reinflation towards its original configuration), Leibniz posits a Deity that mimes the same role as this benevolent Demon. Recall

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[26] Leibniz understood inclinations to be appetitive and, when treating final causes as motives, considered final causes as *essentially appetitive in nature.* In addition to involving a "mentally represented state of affairs", "final causes" are "also constituted by appetitions, the most fundamental producers of efficient causal change." This is why Leibniz' final causes are a special species of efficient cause. For Leibniz, a final cause is a motive for action and, ontologically speaking, "a mental representation of a state of affairs for which there is an appetite or desire sufficient 'to produce a complete volition'. Appetite (desire) is a constitutive component of the final cause (motive), for a mental representation alone is not a final cause, but a mental representation of some state of affairs plus an appetite for that state of affairs is a final cause, provided that such appetite moves the will". See: Laurence Carlin, "Leibniz on Final Causes", *Journal of the History of Philosophy*, vol. 44, no. 2, 2006, p. 230.

[27] Leibniz, "Discourse on Metaphysics" in *Leibniz: Philosophical Essay*, pp. 54-55.

[28] Ariew and Garber, *Leibniz: Philosophical Essay*, p. 254.

that we elucidated a beam's behavior through the teleological appeal to 'final causes'—the unstressed equilibrium conditions that wood and iron "strive to regain, with unique degrees of vim depending upon their elastic 'personalities'" (119). For Leibniz, it is God who attends to final causation's determined optimization via micro-mechanisms—if we examine wood at the scale level below its critical length ΔL^c, whereby it no longer behaves in a homogenous fashion, "we will observe air bumping into pore walls within the wood along the exact efficient causation trajectories required to knock the distorted beam back to its relaxed shape" such that there is a "machinery of the world in such a way that […] the springs in bodies are ready to act of themselves as they should at precisely the moment the soul has as suitable volition or thought"[29] and that "recoil must arise […] to conserve force" (120).[30] This is Leibniz' 'pre-established harmony' where behavior can be explained in a top-down manner according to final causation schemes which, themselves, can be understood in bottom-up fashion by virtue of efficient causation underpinnings.

Leibniz anticipates that there exists a non-trivial structural demand and this becomes registered in concrete mathematical terms with modern texts concerning continuum mechanics: unless catastrophic lower-scale events intervene (e.g., fracture), flexible bodies necessarily retain material coherence throughout all higher-size scales. By insisting that "figure is not even an entirely real quality outside of thought,"[31] therein attributing space and time as *merely ideal*, Leibniz correctly observes that every practicable description of material matter utilizing geometrical vocabulary unwittingly incorporates a fair degree of fictitious projection to unwarranted scales. As a theory of 'feigned homogeneity', this description of extension proffers a presumption that any thoroughly continuous material can be infinitely divided into any scale length ΔL. In mathematical physics, for instance, if a beam is described as curved along a definite pattern (e.g., a catenary), lower-scale complexities are artificially suppressed for the sake of convenient representation in upper-scale behavior propensities. This provides for a scheme of supervenience where, if section A of

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[29] Leibniz, "Letter to Arnauld" in *Leibniz: Philosophical Essay*, p. 84.

[30] Leibniz, *Leibniz* ed. Louis Alexandre Foucher De Careil, Paris, BiblioBazaar, 2008, p. 234.

[31] Leibniz, "Letter to Arnauld", p. 343.

this beam is apparently “more strongly curved due to the weight pressing upon section B,” then we seem to also claim that A’s “powers of influence have partially come under the sway of B’s”, describing the latter by extending the (force of the) former (124). Thus Leibniz conceives of continuity via hierarchical relationships where “something prior”—that is, the persistence of an initial “force”—finds itself repeated or diffuse, acting upon or resisted, under the guise of (behavioral) extension.[32] Despite this doctrine arguably overreaches, by utilizing it “fictitious ‘possibilities of division’ can be exploited to provide a relaxation space pathway to differential equation models like our beam formula” (122).

Of course, Wilson does not recommend we retain God’s optimization forces qua Monadism, wherein material behaviors require underpinnings within exterior reality. Instead, we can create a bridge between Leibniz’s harmonization requirements and the mathematical expression of additional compatibility equations, which insure infinitesimal relationship captures in our equations that account for stress and strain, integrating these coherently as supplementary demands on compatibility. Nonetheless, Wilson recovers a critical ambition from Leibniz’s ‘master-slave’ hierarchy, where teleological requirements of a ‘natural machine’ are determined in nested fashion, “[a]nd thus, a natural machine can never be absolutely destroyed just as it can never absolutely begin, but it only decreases or increases, enfolds or unfolds, always preserving […] some degree of primitive activity.”[33]

Consider a continuous and coherent plank of wood where each component element at a critical ΔL^C scale cooperates in “slavish fashion” with the “natural state desires of the beam as a whole” (125). Here, a small arc, a, of a wooden ring, A, is molded under tension such that it agreeably cooperates with the natural state “desires” of the whole ring A, but solely so long as a remains ‘enslaved’ to A. However, if ‘liberated’ from its ‘chains’–that is, if we cut the small arc, a, from the wooden ring, A–it displays a novel set of natural-state ‘desires’. Where Leibniz would argue that this top-down scaffolding organization within inorganic materials provides a simulacrum of the monadic master-slave relationships

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[32] Leibniz, “Letter to Basnage,” in *Philosophical Texts* ed. and trans. R.S. Woolhouse and Richard Francks, Oxford, Oxford University Press, 1998, p. 64

[33] Leibniz, “Of Body and Force” in *Leibniz: Philosophical Essays*, p. 253.

operative within living biological systems,[34] we can naturalize this description vis-à-vis behavioral coherence.

The provident manner wherein Leibniz relates God's 'arranged' construction of a 'full universe' around the desires of composite objects dominating our own scale-level[35] also informs his description of human free will. Presented with the logic of a material's possibilities of division, Leibniz provides two responses: (i) that these 'possibilities' pertain to granular monads of the 'real universe'—this determines how one possible interaction is produced from the full 'personality' of determination(s) qua intra-monadic influence and cooperation; (ii) that these 'possibilities' apply to a 'smoothed-over' continua that we attribute to the sake of the world for descriptive utility. According to (ii), the understanding of "'being divisible into segments of length ΔL' central to our beam modeling techniques merely represents a fictive aspect of our restricted upper-length scale knowledge, albeit a form of projection vital to descriptive success within mathematical physics" (127-128). As Leibniz describes, this demonstrates how the "science of continua", or the "science of possible things" contains "eternal truths" or truths that are "never violated by actual phenomena, since the difference [between real and ideal] is always less than any given amount that can be specified. And we don't have, nor should we hope for, any mark of reality in phenomena, but in the fact that they agree with one another and with eternal truths."[36] The elaborate continuum of the ideal, which contains indeterminate parts, is contrasted with the 'actual' where nothing is indefinite as, in these 'actual things', or 'real substances', every division has been made in advance.

Thus unfolds Leibniz's compatibilist doctrine of choice, a "freedom of contingency"[37] where we are set into middle-level constraints of permittance: we have access to macroscopically smoothed-over terms and, therefore, hold operative notions of contingency and necessity that reflect this middle-scale placement. Any explanation of activity X is that we execute it from desire but

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[34] We ought to note, however, that according to Leibniz every artificial/material system *does* contain a small measure of animal-like monads.

[35] Leibniz, *Theodicy*, trans. E.M. Huggard, Gloucestershire, Echo Library, 2005, pp. 157-159.

[36] Leibniz, "Letter to de Volder", *Leibniz: Philosophical Essays*, pp. 185-186.

[37] Leibniz, *Theodicy*, pp. 148-149.

that there also exists the microscopic workings of God's preplanned determination, of air particles shunting along our cranial walls, fomenting our neurons into excitation and us into action. Of course this is an exuberant fantasy but what we retain from Leibniz's conception for our contemporary modeling policies is a conception of materials as flexible matter that tacitly rest upon subtle contextual controls. This gives us an alternative from the glib appeal to "the possible world of [point-mass] physics"[38]—and, in consequence, an ODE ontology—which lacks the necessary resources to render matter stable at the atomic level.

§IV, "TWO CHEERS FOR ANTI-ATOMISM"

Now let us provide a historically oriented approach to Pierre Duhem's methodological insights on continuum mechanics and thermal observations. Here, Wilson underscores the particularly prophetic structural assumption behind Theory T philosophical thinking that advances scientific theories of a 'fundamental physics' cast by prodding the assumption that philosophy (of science/physics) should "capture the freely *autonomous behaviors* of nature within their mathematical netting" in advance (147). Wilson outlines Duhem's alternative architecture of *The Evolution of Mechanics* (1903) while focusing on scale-sensitive complexities that force 'thermal' terms such as 'temperature' and 'entropy' to obtain wide and effective referential ranges by advancing from one firmly established plateau to another via step-wise developmental arrangements.[39]

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[38] The usage of 'mass point physics' in describing the classical rendering is, indeed, unique from Newtonian physics, despite the two are often conflated. As Wilson notes, "Newton did not endorse such an attitude, which was pioneered by later writers such as Christian Wolff, R. J. Boscovich, and the French atomists. Newton himself opined that matter was probably composed of extended rigid bodies surrounded by oceans of intervening fluid. To be sure, he (mostly) modeled the solar system as a collection of point masses in ODE fashion, but this policy does not justify assigning a point-mass 'ontology' to Newton's thinking. Modern commentators frequently overlook the historical resistance to point masses (which prevailed throughout most of the classical era) because (i) they fail to compensate for the ontology-obscuring effects of the "physics always idealizes" appeals outlined in the appendix and (ii) they have been misled by contemporary instruction in "classical physics." The latter took a decidedly ODE-favoring turn in the 1920s due to the formal requirements of quantum mechanics (129).

[39] With the Quine-Duhem thesis understand science as a system of statements that are locally connected with each other: "There are statements that are far from experience; they form the center of an imagined sphere. Those statements involve logic and mathematics. Located on the outer regions of the sphere,

Through such developments, the "true significance of 'temperature' and 'entropy' become contextually localized, in a patchwork manner" (138).

This is contrasted to Theory T philosophizing, which fastens rudimentary 'natural kinds' scenographies to these words (i.e., 'temperature' and 'entropy') and, therefore, underdetermines thermodynamic thinking's contextual subtleties. Theory T thinking inherits a hex from the problem of the physical infinitesimal, wherein the complexities of arranging the dimensionally incongruent elements of continuum physics were smoothed over for the purpose of a functional manifold. This buoyed a lapse into 'essential idealizations' which maintain that a modeler much permanently *misdescribe* their targets so that the descriptive and philosophical enterprises of mathematical physics can subsist.

While Wilson does not retain all Duhemain insights (setting aside the anti-realist and phenomenalist Duhem), he focuses on the structural scientific problematic of constructing a workable (non-equilibrium continuum) thermodynamics "in which the notions of absolute temperature, heat, and entropy enter on an equal footing with the familiar qualities of standard classical physics (e.g. force, mass, potential energy, stress and strain)" (141). Circumscribing the thermal-free conceptual constraint of virtual work that informs Lagrange's method of undetermined multipliers (and Legrange's work on generalized coordinates), Duhem's proposal is to further complement this 'Old Mechanics' with a richer 'New Mechanics' that also accounts for thermal phenomena. Using the formalism of Lagrange's lower-level platform (in particular, its statics → dynamics patterns),[40] Duhem develops a thermal architecture attentive to how bidirectional forms of energy exchange occur within a dynamical setting.

The core of thermodynamic thinking was originally taken up by Rudolf Clausius, who invoked entropy to codify coherent work capacity lost and regained. Clausius and those following him addressed the issues of energetic efficiency in large-scale blocks of homogenous material (such as gas in a flask), moving from one state of constrained equilibrium to another via external

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connected to the realm of mathematics and logic, are those statements that are in connection with experience. The scientific enterprise is represented by the structure of this web of belief." See: See: Gerd Ch.Krizek, "A classification scheme for interpretations of Quantum Mechanics", 2017, p. 31.

[40] Joseph-Louis Lagrange, *Analytical Mechanics*, Dordrecht, Springer, 1997.

manipulations (e.g., increased pressures or immersion within a thermal reservoir). The claim was that entropy, temperature, and other means of thermal classification applied to these stabilized end conditions and not to the transitory states during which material *internally adjusts* to its newly altered environment. Duhem's New Mechanics program addressed energy exchange with respect to two-way entanglements situated within dynamical settings by proposing that Clausius' thermodynamic considerations be applied at the *infinitesimal* level, where coherent energy and heat exchanges take place via heat transport.

In contemporary philosophy of language, there are a number of misapprehensions concerning linguistic meaning that point o the semantical underpinnings of words such as 'heat' and 'temperature'. Following Saul Kripke and Hilary Putnam,[41] such 'thermal words' are categorized in 'natural kind' terms—herewith, such terms are prodded into referential alignment (e.g., "'[t]emperature' is held to referentially align with the mean kinetic energy per degree of molecular freedom contained within a target system"; 145). According to Wilson, "[p]roponents of natural kind stories generally underestimate the profound manner in which appeals to thermal concepts help us reduce the description of nature to tractable terms" (145). Consequently, the naturalized treatment of thermal concepts as such reduces their description of nature to tractable ersatz terms, slipping into "Fido"/Fido apriorism (the relational theory of propositional meaning that conflates the meaning of a word with the object that stands for it).[42]

Consider, for instance, the distinction performed by the architectural task of factoring a system's operative degrees of freedom when we connect thermal processes involving friction qua energy degradation: "as our pendulum swings merrily back and forth, a certain degree of frictional resistance systematically coverts some of this motion into heat" (146). Contra mean-kinetic-energy reductions (which open the doors to ersatz apriorism),[43] energy degradation—

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[41] Saul Kripke, *Naming and Necessity,* Cambridge, MA, Harvard University Press, 1980 and Hilary Putnam, "The Meaning of 'Meaning'" in *Philosophical Papers*, vol. 2, Cambridge, Cambridge University Press, 1975.

[42] Stephen Schiffer, "The 'Fido'-Fido Theory of Belief", *Philosophical Perspectives*, Vol. 1, 1987, pp. 455-580.

[43] This creates a kind of conditional necessitarianism, wherein the "[n]atural kinds doctrine claims that the slogan 'mean kinetic energy per degree of molecular freedom' reports on the hidden 'essence' of temperature", thus forging an "alignment between words and world"; by extending the "natural kind" to

crucial to decoherence events within quantum mechanics—stitches together the descriptive fabric. This does not occur within a simple two-stage process but along a hierarchical 'cascade' that is positioned via size-scales. Through these scale-centered stages, we will be able to implore the semantic complexities in thermal vocabulary.

Those opposing Duhem's position often point to a form of prospective thermal/mechanical integration which can be achieved by 'consulting' lower-scale molecular modeling, wherein applicable dynamics appeal to understanding in purely mechanical terms (i.e., the kinetic theory of heat). In *The Evolution of Mechanics*, Duhem not only shows how these reductive tropisms disregard empirical obstacles (e.g., erroneous specific heats) but also that they partake in conceptual closures, intoxicated from liberally imbibing from the well of "instinctive knowledge".[44] Duhem constructs an applicational range for amalgamated thermodynamics in layered stages; in doing so, Duhem outlines four developmental platforms as prerequisite for the successful employment of thermal vocabulary, surveying the behavioral criteria that allows one to discern coherent energy storage from incoherent means, providing a mode to understanding why, exactly, the 'natural kinds' account fails to do justice to the true utilities of thermal vocabulary:

> "(i) Establish the constrained equilibrium states of purely mechanical systems following Lagrange's virtual work policies in his *analytical Mechanics*. (ii) Following Carnot and Clausius, introduce a parallel statics for purely thermal vocabulary, again only for systems in completely constrained equilibrium. (iii) Following Lagrange once again and Willard Gibbs, lift these two forms of statics into less constrained forms of dynamics, which are then scaled down to an infinitesimal level. (iv) on the dynamical basis established under (iii), introduce the rates of quantity adjustment that allow us frictional effects to drain coherent energy out of target systems. Only at this last stage of conceptual construction can thermal and

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property identity, and thus introducing a homogenization parameter for interconnecting models that require scale-relative sensitives, "[s]implistic linguistic assumptions like the Putnam/Boyd doctrine greatly encourage the notion that 'temperature' has been referentially aligned with a simple 'essence' since its first inception" (146-147). See: Putnam, *Meaning and the Moral Sciences,* London, Routledge, 1978, p. 20.

[44] Accordingly, such "instinctive knowledge is, after all, only a confused and unanalyzed pile of experimental givens acquired at imprecise periods of intellectual development". See: Pierre Duhem, *The Evolution of Mechanics*, trans. Michael Cole, Alphen aan den Rijn: Sijthooff and Noordhoff, 1980.

> purely mechanical cross-effects be meaningfully considered" (154).

Duhem's method to construct a unified thermomechanics capable of handling complex phenomena that resist treatment within traditional presentations of this subject is by starting with a tightly constructed systems S (i.e., textbook thermodynamics) and approaching fully autonomous trajectories $S + S^E$ via constructive 'lifts', or autonomous trajectories (e.g., the steady-state flow of a fluid around an obstacle),[45] beyond the Clausius-conceived base manifold. This is principally achieved through Duhem's conception of an enlarged New Mechanics—in particular, virtual work here maintains that a device's special states of constrained equilibrium represent the circumstances in which a machine's internal dispositions for acting against its environment can be exactly checked by an appropriate schedule of applied restraining forces (167). This shows how, within thermal contexts, potential energies comport with entropies.[46] Duhem's central demonstration in multi-stage construction is that until frictional damage and other irregular effects enter a system S, S displays uniform transfers of conserved energy between a storage facility and another facility (e.g., a shock absorber), such that there exists direction-independent reversible behavior. However, friction mars this uniform exchange pattern, robbing the mechanical system of some of its kinetic energy, such that it no longer possesses the energetic totality of its original conditions. Despite we normally presume a device's internal capacities for storing potential energy will not be damaged by friction, physical processes such as hysteresis or plastic flow damage internal energy capacity.

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[45] In a formal scientific modelling system S, if S behaves non-autonomously, we seek the larger grouping, $S + S^E$ that can evolve conjointly in fully self-contained means, "where the controls formerly exerted by S^E on S now appear as effaced approximations to the fuller $S + S^E$ evolution" (161). The result is a phase space in which the autonomous time evolutions of our $S + S^E$ system is portrayed as a single point trajectory moving through a high dimensional space; as this concerns thermal circumstances, the "'reversible trajectories' belonging to S will crawl along a close neighborhood of the S^E surface at an infinitely slow pace, whereas the heat bath and applied pressures of traditional thermodynamics will live within the static S^E sub-manifold" (162).

[46] Entropy, or the "Second Law of Thermodynamics", is initially explained wherein the total entropy of an isolated system always increases over time or remains constant within an idealized reversible process. However, this only makes sense for "systems that have been allowed to relax completely after a fresh set of external controls has been applied"; both scenarios, of "isolation" and "autonomous increase", are illegitimate when riven from the backdrop of non-equilibrium processes. See: Clifford Truesdell, *Rational Thermodynamics,* New York, Springer, 1984, p. 9.

Duhem thus adds further layers to his modeling architecture to accommodate for destructive friction-linked processes.

Duhem's analysis isolates central behavioral assumptions required to support applicational utilities of 'heat', 'temperature' and 'entropy', examining the stage-like character of these foundations. Today, scientists describe the cosmological evolution of our universe as a congregation of temperature-governed regimes. However, since science can solely produce theories of 'patchwork' constructions, the philosophical-cosmological project of 'possible worlds' is increasingly suspect. Similarly, the 'natural kinds' reference obscures applicational complexities that locate descriptive terms such as "temperature" on supportive platforms of greater architectural complexity than current philosophy of language generally admits. Since its inception, the widespread employment of 'temperature' has been far beyond its thermodynamic scope, betrothen to the perhaps mistaken application of crediting solid objects with 'temperature', treating them via a single controlled axis rather than operating in diffracted fashion. As Wilson notes, the "real dilemmas of thermodynamic usage trace to the fact that attributing a meaningful entropy to a target *S* while *S* is still engaged in internal turmoil is quite problematic, but at the same time, the classificatory utilities of the word are directed to the changes in *S* brought about these same processes of internal turmoil" (186).

In turn, we must reframe our semantic accounts such that they can more aptly deal with such concerns—as Duhem notes, "[i]n the course of a virtual change the values of [the state variables] cannot be regarded as functions of time [….] it is clear that any sequence of equilibrium states of a system, provided that it is continuous, can always be envisaged as forming a virtual change of the system."[47] The 'virtual' aspects of these modifications prod descriptive interventions, as in the 'restricted ensemble approach' where, by limiting statistical mechanics via the inclusion of configurations consistent with metastable state considered we move through relatively elevated mesoscopic scales—the true sources of target-behavior are thus found (e.g., in rubber, the higher-level ensemble required to

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[47] Pierre Duhem, *Commentary on the Principles of Thermodynamics*, ed. Paul Needham, New York, Springer, 2011, pp. 93-94.

examine the true sources of its elastic behaviors are found not at the atomic levels of the atoms that comprise the polymer chains).

When the natural ensemble of permissible variations is presented, we see that the 'temperature' extracted from the lower-scale collection may, indeed, differ considerably with the 'temperature' pertinent to its behavior at the polymer-chain level. *The shared classificatory word 'temperature' supplies different evaluations of material behavior upon different choices of length scale*, characteristic of modeling schemes which operate in multiscalar manners as earlier demonstrated. The success of a multiscalar scheme depends on its various sub-models, framed at unique choices of RVE scale, do not communicate with each other *directly* but through suitably homogenized messages. Thus, lower-scale details are converted into the coarser appearances that they present to a higher-scale size. The descriptive utilities of 'temperature', 'heat', and 'entropy' need to be considered via material-response. *The essential differences between 'pressure' and 'heating' reflect the degree of coherence retained in the material's response to particular exterior energetic effort.*

In most solids, coherence-to-incoherence degradations transpire along trickle-down hierarchies linked to natural cause-and-effect relationships that arise between RVE scales. From a ΔL^{higher} scale point of view, "coherent motion has been lost to 'heat'" while, at the Δ^{lower} level, "this same 'heat' may retain a fair amount of directional coherence" (191). Degradation occurs during the hierarchical cascade within a complex solid, necessitating that we localize temperature and entropic evaluations to codify how such losses occur via couched *context sensitivities* that explicate the coherence/incoherence relationships that act across RVE scales. This is highly relevant to linguistic adaptation: the semantics of the word 'temperature' is not equally beholden to, say, the rubber band and the iron rail as it is to the ideal gas; "[f]or such reasons, our thermal words must progressively locate their appropriate physical correlates through localized adaption" (192).

§V, "THE GREEDINESS OF SCALES"

Now we can outline working architectures of modern multiscalar modeling techniques to help us recognize the distortions and vagaries in 'Theory T thinking' or theory-as-approximation. In particular, here we focus upon the difficulties involved in describing materials that reveal large amounts of significant structure at intermediate size scales (e.g., the structural features that distinguish one igneous rock from another, or the out-of-equilibrium formations that blacksmiths fold and beat into steels). For instance, consider the diamond's long-lasting range of 'frozen order', wherein there exist strong energetic barriers within the diamond that prevent it from returning to low-pressure graphite, such that it has a long relaxation time. Similarly, most solid materials display very little inclination for maximizing their entropies. While mathematics undergoes considerable difficulties in capturing the 'natural' notion of a dominant behavior in precise terms, different scale lengths can take account of one another in dominant behavior within nature.

Central to the varieties of liberation from such computational shortsightedness are the tools of set theory and differential calculus. However, these appeals also introduce descriptive exaggerations, as demonstrated in the Greediness of Scales problem. Mathematicians currently attempt to repair these descriptive lapses via asymptotic interconnections; as we detailed, in the absence of any profound rupture or fusing, pieces of any continuous body are firmly attached throughout their distortions. This natural condition is enforced by the fact that the organized integration of behaviors across all scale sizes is scaffolded in top-down manner, as characteristic of relaxation space constructions and modern measure theory. Simultaneously, however, complete scale invariance is apparent as other process become 'secretly active' below a cutoff level ΔL^C.

The Greediness of Scales is bolstered by its core objective, to highlight a critical technique of computational architecture—*multiscalar modeling*. Wilson underscores a process of regimentation, termed *amalgamation*, that culls the previously isolated subdomains of a model into interactive communication with the supervening domains. To do so, Wilson discusses a number of obstacles that relate to the employment of differential equation models, or *puzzles of scale*, which revolve around the fractured and unequal ways in which physical information is registered at different choices of characteristic size. A central query thus emerges: how then can we consider discerning the Quinean figure of ontology, i.e., how do we formulate an explicit articulation of ontological commitments based on the value of a bound variable (commitments found in the range of values allowed for and governed by its bound variable).

To best articulate this problem, let us once again recall our choice case example of the steel beam. At the highest size scales—following Hookean first-order linear approximation—steel stretches and compresses down to approximately 10μm. At 10μm, the grain structure within steel becomes highly pertinent, as these grain structures and their components begin to stretch and compress according to a more complex set of rules than larger-scale steel. In each of these component grains is contained a number of "laminate layers which rub against one another in complicated ways [….] until we reach the tiny crystal lattices of the molecular level, whose orderly patterns are interrupted by higher-scale-irregularities called dislocations" (202-203). It is here that the differential equations that regulate behaviors nominally occurring the 'infinitesimal' level become central. The specifications relevant for the differential equations within physics are generally obtained by scaling higher-level behaviors downwards, until some simpler infinitesimal level is reached.[48] Steel, however, presents a problem to such benchmark scaling assumptions, as its behaviors stop scaling as so expected at the cutoff of ~10μm. While small sections of steel behave more or less identically at all scale lengths above this level, to capture the component grain behaviors after 10μm accurately, we are required to model it in a more laminate-based manner.

What is the Greediness of Scales? As we will recall, while RVE-submodels can be examined on account of contemporary scientific observational-measurement tools (and, in particular, advances in computer simulation that attempt to overcome the descriptive clashes we shall elaborate), it is the problem of data *amalgamation* that prevents "practitioners from profiting from this collective knowledge in a straightforward way" (203)[49]. This is because, using RVE scale-focused modeling via differential equations in bottom-down fashion (reaching towards the infinitesimal), amalgamation presents a conflict regarding the direct descriptive incompatibilities that arise concerning the same vocabulary with respects to properties that a material (such as steel) displays on small-scale

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[48] This is well codified by the apothegm that 'physics is simpler in the small'.

[49] This means that two scientists who model different select scale levels (of steel) can not simply posit their combined research results because it will result in syntactic inconsistencies where differential equation requirements overlap.

levels.[50]

An issue arises on the differential terrain: differential equations that are appropriate to two such levels of sub-modeling necessitate that the narrowly-constrained rules concerning stretching and compression *must remain applicable down to the zero-length scale*. Thus, the Greediness of Scales conflict is born due to syntactic disharmony: the differential equation model must account for all the lower-size scales available to reach the infinitesimal level, which is where differential equations articulate their stipulations. But, due to the syntactical discordance concerning the material's behavior beyond a cutoff level, inconsistent claims about the very same part of material result. This may remind the reader of Sellars' pink ice cube problem, where an ice cube's color—which appears to be pink 'through and through', for it has been made by freezing a (pink) frozen drink—is observed as ultimately homogenous because it presents itself to us as a "pink continuum" in "all the regions of which, however small, are pink."[51] The concept of 'pink' thus demands that its applications *scale continuously downwards* to the infinitesimal level, wherein this 'manifest image', or the image of as it is plainly conceived of, is rationally set in contrast to what we know through scientific measurement or, in Sellars' parlance, 'the scientific image'. For Sellars, the pink ice cube of our experience is an object which, according to science, cannot exist as we conceive of it—our 'manifest' object-image is pink 'through and through', but once we consider that the ice cube is composed of H_2o molecules, we contradict this, since those molecules are not pink.

Sellars' scope regarding the intentionality of acts of thinking involves a Kantian shift in focus from items of propositional content to intuitions. Per Kant, intuitions contain contents expressible by phrases that, themselves, are not the

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[50] With crystalline materials, for example, at low-level scales we observe segments of perfect lattice bonded together around arbitrarily oriented boundaries. At scale levels above this point, RVE behaviors around the level of conglomerations are generally isotropic (the material responds to the same rules regardless of which direction it is pulled). While higher-scale response supports modeling whereby compression and stretching behaviors are governed by Young's modulus E and the shear modulus μ, the "tiny slivers of crystal within these conglomerates will not stretch and compress in this simple manner, and RVE modellings appropriate to these tiny structures require five or six elastic modalities o capture their *anisotropic* behaviors" (203; emphasis added).

[51] Wilfrid Sellars, "Philosophy and the Scientific Image of Man" in *Science, Perception, and Reality*, New York, Humanities Press, 1963, p. 26.

content of a claim but express intentionality (Kantian intuitions are representations of *this-suches*, which have a predicative qualification built into them and locate the object within a classificatory scheme).[52] This demonstrates how intentionality presupposes understanding; thus, "the intentional directedness of an intuition at reality—its being an intuition of, say, a pink cube—is to be explained by analogical extension from the way in which, given the presence of a demonstrative phrase, say 'this pink cube,' in a form of words uttered in a certain context, anyone who understands the utterance can identify a certain object as what the utterance is about."[53] The intentionality behind thought in understanding is, thus, regarded as partially constituted by the spontaneous cognitive power that, presenting itself as the understanding, is the faculty of concepts. A crucial mediating term between the scientific and manifest image distinction and the causal/logical distinction that critics like Bas C. van Fraassen have highlighted is *normativity*.[54] Sellars considers norm governed behavior as causally but not logically reducible to regularities and dispositions. A critical difference between the scientific image and manifest image is that the latter is intrinsically normative, for it has at its core the conceptual repertoire of personhood, which Sellars considers inherently normative. The scientific image,

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[52] Willem A. deVries, "Sense-certainty and the "this-such"" in *Hegel's 'Phenomenology of Spirit': A Critical Guide* ed. Dean Moyar and Michael Quante, Cambridge, UK, Cambridge University Press, 2008, pp. 63-75.

[53] John McDowell," Sensory Consciousness in Kant and Sellars", *Philosophical Topics* vol. 32, no. 1-2, Spring and Fall 2006, p. 314.

[54] While Sellars attempts to carve a hard distinction between the vague concepts of 'the manifest image' and those determinate descriptions of 'the scientific image', critics such as Bas C. van Fraassen have responded that this doesn't hold because prior scientific theories often are vague and indeterminate in light of subsequent scientific theories, just as 'the manifest image' appears to the purported 'scientific image'. Nonetheless, while Fraassen argues for the irreducibility of this incompleteness of the manifest image to the scientific image, for Sellars the point is that the vague concepts of the manifest image related to sensible qualities are not logically/conceptually reducible, but causally reducible to the scientific image. Fraassen might respond by pointing out that ascription of causal efficacy is theory-dependent, and there's always a choice of several different theories which describe causal mechanisms adequate to our observations. The history of science does not give us adequate reason to suppose that, in the ideal limit, we will reach some theory which is ultimately adequate in its causal description. Thus, Fraassen is burdened with further detailing an account of anti-realism with which to combat Sellars' scientific realism as an account of causality sans theory-dependence lapses into the myth of the given; the Sellarsian benefits by being able to respond by giving an account of causality which is not theory-dependent, thereby supporting the terms of this discord qua realism. See: "The Manifest Image and the Scientific Image" in *Einstein Meets Magritte: The White Book - An Interdisciplinary Reflection*, eds. Diederik Aerts, Dordrecht, Kluwer, 1999, pp. 29-52.

on the other hand, is premised upon the elimination of all such normative notions from our descriptive and explanatory frameworks. Thus, while the scientific image works out the causal reducibility of human culture in practice, the manifest imagine retains its independence as the site of the logical irreducibility of the norms governing this very practice. For Sellars, sensory consciousness is the primary product of sensibility which, itself, is populated by sensations that are describable and that poses relations to the subject as modifications of statehood; they lack intentionality and must be distinguished from that which has intentionality (i.e., experiences and intuitions are composites). Although Wilson is concerned with continuum physics and the stipulations behind a scale's exacting a bottom-down monopoly—specifically as it concerns requirements of mass and stress—there is a homology here with Sellars' concern regarding the "clash between images."[55] Thus, we shall navigate the Greediness of Scales problem sympathetic to the response that homogeneity is a feature that is *represented*, not a feature of the *representing*. As demonstrated by Wilson's (and Batterman's) work on continuum idealizations, scientific descriptions are often privy to Theory T committals 'through and through', treating that which is scaled down to the 'definitely granular' with the same theoretical presuppositions as that which is 'definitively continuous'—thus, given a modeling scenario localized to the RVE level where previous scaling behavior fails, such scientific Theories are, strictly speaking, false (in a way analogous to the *pinkness* of the ice cube).

The Engineer J.T. Oden has described the tyranny of scales problem by remarking that all simulation methods produced until the beginning of the

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[55] For Sellars, this "relocation story" is not "simply a solution to the problem posed by mathematical physics. It is also an account of how we could come to be able to think about sense impressions in the first place. We come to be able to think about sense impressions of pink cubes by first thinking about volumes of pink that we seem to see, and then recasting the manifest pinkness as properties of perceptual states of ourselves." See: David Rosenthal, "Quality Spaces, Relocation, and Grain", in *Sellars and His Legacy*, ed. James R. O'Shea, Oxford: Oxford University Press, p. 153. Sellars' relocation picture necessitates that we conceive of sense-impressions as automatically conscious, whereby mental states' being conscious is distinct from the individual's being conscious. For Sellars, the central question of the grain problem was whether it could, in principle, be possible without a neurophysiological conceptual framework that defines states according to intrinsic character but proffers to epiphenomena. Wilson, on the other hand, is not interested in the homogeneity constraints satisfied by conscious presentational content but the syntactical overdetermination that this produces.

twenty-first century were valid solely for:

> "[L]imited ranges of spatial and temporal scales. Those conventional methods, however, cannot cope with physical phenomena operating across large ranges of scale—12 orders of magnitude in time scales, such as in the modeling of protein folding or 10 orders of magnitude in spatial scales, such as in the design of advanced materials. At those ranges, the power of the tyranny of scales renders useless virtually all conventional methods."[56]

As eluded to earlier, modeling schemes have advanced to resolve such discrepancies by allowing RVE sub-modeling layers to circumscribe their descriptive agenda to a localized and semi-autonomous 'strata', or what Robert Batterman (co-opting the term from physicist Robert Laughlin) calls a "protectorate".[57] These semi-enclosed strata/protectorates are set into communication with one-another through those 'coded messages' called *homogenizations*,[58] dividing linguistic labor and molding novel explanatory architecture.

But when Oden relays dominant behaviors that appear on/can be captured on a characteristic size scale what, precisely, is he speaking of? Natural energy cascades and capacities for transmitting coherent work illuminates how material is characterized as such. If we pound a steel beam with a hammer, macroscopic energy spreads out in both directions and, if there is no interference/energy degradation, the waves will retain their shape. However, a distinct descriptive opportunity arises if we restrain the beam on two ends (e.g., as a guitar string is tethered on two ends), as the traveling waves are continually reflected back into the interior by the endpoints. After a period of initial relaxation time, these reflected waves often coalesce into *standing wave structures*, which represent

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[56] J. T. Oden et al., "The Tyranny of Scales: The Challenge of Multiscale Modeling and Simulation" in *Simulation-Based Engineering Science*, Washington, DC, NSF Publications, vol. 5, 2006, §3.1.

[57] "The crystalline state is the simplest known example of a quantum protectorate, a stable state of matter whose generic low-energy properties are determined by a higher organizing principle and nothing else." See: Robert Laughlin and David Pines, "The Theory of Everything" *Proceedings of the National Academy of Sciences of the United States of America*, vol. 97, no. 1, 2000, pp. 28-31. As Batterman describes this more generally, the 'protectorate' is a domain of physics where behavior is independent of the microdetails found at small size scales. See: Robert Batterman, "Emergence in physics" in *Routledge Encyclopedia of Philosophy*, Abingdon, Taylor and Francis, 2010.

[58] These *homogenizations* are the very same ones that that we considered when dealing with relational propositional meaning in the 'Fido'-Fido theory of belief.

coordinated-across-the-beam patterns of movement that vibrate vertically and independent from one another. If this beam is thin, symmetrical and homogenous, than these standing waves appear as sine waves; if it is thicker, less homogenous, but still symmetrical, a less familiar kind of standing wave pattern, φ_i, appears, called an *eigenfunction* (these emergent φ_i modes create 'energy traps' by preserving "packets of energy over appreciable elements of time; 207). This last example shows a system's dominant behavior factoring into independent behavioral modes through endpoint-induced coordination, where a target media's large-scale behavior(s) are codified and captured through a smaller range of variables (φ_i) in comparison to the position coordinates (x) that track it.

Why have we so arduously detailed this last example of the ever-peculiar 'standing wave'? Once such behavior becomes prominent with our rod of steel, we can appropriately assign ΔL, the *characteristic scale length*, because this is where the (differential) movement of a localized section is fastened into coordination with other elements—i.e., this is where perfected dominant behavior patterns emerge.[59] Natural environmental boundary conditions/arrangements, such as coordinated endpoints (in our case, the 'pinned endpoints' of the steel beam-cum-string), induce any φ-variable description with "across-the-entire-system coordination" (210).

Let us now briefly consider behavioral difference according to different scale length. We say that plastic damage occurs in material, and thus a material is not perfectly elastic when RVE units distort so severely that their crystal formations are altered to an unrecoverable degree. In the case of such energetic cascade damage with our steel bar, from which we usually expect perfect elasticity, the responses at intermediate RVE levels fall out of coordination. The multiscalar computational scheme in question relies upon analyzing lower level scale behaviors in detail to assess intermediate scale damages.

Let us also consider upper scale toughness (i.e., a material's resistance to fracture) from a multi-scalar aperture. From a lower scale level point of view, the

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[59] For the behavioral regimes where (horizontally) travelling waves remain dominant, the characteristic ΔL length is smaller, "determined by the breadth of the region in which the largest part of the disturbance occurs" (208).

dislocation lines at a higher RVE scale appear to be uncoordinated and insignificant within the crystal lattice. But they move as a coherent group and, from a macroscopic point of view, such easily achieved movements shield underlying molecular bonds from shearing distortions. RVE units with plentiful dislocations retain dominant upper scale behaviors due to these dislocations, as these dislocation lines lessen the danger of fracture at the molecular lattice level. This is why steel with abundant dislocations is tougher than steel which lacks such dislocations—the latter often proves to be rather brittle, in fact. On a minute scale, dislocation movements represent structural damage—such is the becoming-macroscopic of microscopic damage vis-à-vis the history-dependent phenomena called hysteresis effect(s).

Note that most computational multiscalar models attempt to imitate natural hierarchies within computational architectures. However, this suggests that hierarchies of characteristic scales solely reflect epistemic limitations upon our representational capacities—Wilson's project toils to counteract this "by stressing the direct correspondence of dominant behaviors to objective issues of energetic transfer and degradation" (214). Thus, Wilson avoids the troubles haunting traditional top-down modelers such as Pierre Duhem, who tried to develop an all-encompassing macroscopic rule that accounted for effects of lower-scale complexity in single-level manner. The dichotomous bottom-up approach also lapses into the tyranny of scales intractability, prone to modeling error. However, "[m]ultiscalar techniques avoid this traditional top-down-versus-bottom-up dichotomy by pursuing hybrid policies that incorporate interactive modeling ingredients drawn from a wide range of intermediate RVE scales" (214).

Let us now return to the problem of mutual cooperation, of fusing individual sub-models to one another while privy to the fact that RVE-focused protectorates necessitate differential equations for formulation. This is the case despite the descriptive ranges of such differential equations must reach all the way down to the infinitesimal, meaning that there will be clashes where descriptive demands are shared/intervene upon one another. In order to obtain, descriptive pathways must travel through infinitesimal levels of description, so we can not simply superimpose a cut-off policy and confine our claims to what happens at the ΔL level and above. The solution is in *homogenization methods*, which imitate energetic hierarchies of dominant behavior relationships in nature.

The inferential pathways here are those of an eigenfunction that locates important entrapment modes φ_i via a *Fourier transform*, which is predicated upon "an underlying differential equation modeling and then decomposes our target system's superimposed complexities into the independent sub-behaviors associated with the φ_i spectrum" (216). At our cutoff ΔL level, this necessitates recourse to sophisticated decompositional techniques such as the Fast Fourier Transform, which extends elements of landmark intermediaries. This does not resolve how the infinitesimally poised differential equations operate inferentially, however, monopolizing al available scale lengths in order to draw such inferences.

Another solution proffers: simple dominating patterns emerge from the collection of divergent behavior vis-à-vis mean and variance (these collective follies are collected under the Gaussian 'bell-shaped curve'). We can bring out an asymptotic maneuver by 'blowing up', or inflating, sub-models to infinite population size, keeping the proportions of internal details intact. This asymptotic technique allots *homogenization*, which necessitates a particular kind of mathematical manipulation of imitating dominant behavior interactions. By examining a uniform stress environment we can move from the Representative Volume Element (featuring many randomly oriented grains) and homogenize these by shrinking grain sizes to 0 while keeping the energetic contributions of interior and interfacial bonds constant. This means filtering lower-scale modeling results in a manner that parallels how a characteristic scale level coordinates events arising within its lower-scale companions. A homogenization policy ably apes the physical manner in which relatively simple forms of dominating behavior, characterized by a limited set of descriptive parameters, emerge at higher scales from large, lower-scale underpinnings.[60]

What do the relationships of interconnection in RVE levels within complex materials like steel look like at lower scale RVE units? Simple randomization (untethering hierarchical RVE interconnections) does not capture the proper descriptive complexity of relationships regarding interconnected RVE levels

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[60] "Physicists dub the general phenomenon of complex behaviors that blur into higher-scale simplicity, universality" (219).

within complex material such as steel, precisely because lower-scale RVE units are often laminates or in "frozen disorder" crystal arrays. As Wilson notes, "a suitable homogenization technique must attend to both layer orientation and how interfaces are affected under distortion. Scale-based dependencies are not grounded in mere epistemology, as these homogenization policies of interstice behavior attempt to mirror natural energetic cascades which allow for a solid material to retain and eventually lose capacities for performing coherent work upon neighboring scales. Since mathematics is unable to supply us with dominant behavioral conclusions, assembling an effective computational architecture is tasked with an ontological gambit.

In short, asymptotic relationships supply the essential stitching of communication-through-homogenization. The tyranny of scales problem demonstrates that hysteresis, the microscopic migration of dislocations that eventually results in material cracks—lower-scale damage inflicted by upper-scale punishment—can not be illustrated with conventional computational modeling through single-level descriptive methods. While we cannot give an account of hysteresis by working upwards from the molecular scale in this mode, the multiscalar model evades such computational barriers by enforcing a cooperative division of descriptive labor amongst a hierarchy of RVE-centered sub-models, each of which is tasked with capturing dominant behaviors that arise within its purview. Therefore, "each local RVE sub-model directly responds only to its local environment, rather than to events that arise within distant sectors or upon alternative size scales" (222). There is a consequent step of mathematical filtering through homogenization after which we then readjust local parameters within each RVE unit until the cascade of interscalar reports is rendered self-consistent.

Returning to our beam of (pearlite) steel, let us picture it as a train track upon which a load-bearing locomotive stops on top of to load and unload cargo. After repeated sequences of loading and unloadings, the produced dislocations will collectively move close to the cementite walls, with these dislocations becoming entangled. As the capacity for protecting molecular bonds from the locomotive's shearing stresses is lost, a "corrective message must be sent to higher sub-models reporting on this damage, demanding local corrections in the key parameters of these models", forcing a stiffening and a lowering of estimated fracture strength (224). As the repeated locomotive poundings eventually result in the rail's dislocation at higher-sale behaviors, making the steel more brittle, parameters in our higher RVE sub-models must be adjusted to reflect these damage reports from below. Afterwards, we need to execute our original finite element stress calculations repeatedly so as to ascertain what effects local patches of damaged material may have on the rail altogether—this process is iteratively repeated until a self-consistent answer is reached, whereby our individual RVE sub-models reach descriptive harmony with one another (i.e., homogenize). This self-consistency amongst dominant behavior protectorates at a variety of scale sizes is a hallmark of the multiscalar technique. Each corrective message sent at other RVE scales employs the language of classical continuum physics *in compatible modes*. If we simply combine every sub-model's output as per Quinean amalgamation, syntactic inconsistency ensues. By processing every sub-model's narrow conclusions through one of the homogenization filters, thereby enlarging the local model to an infinite population and extracting the desired parameter corrections through classic 'mean and variance' asymptotics, we consolidate communication. While single-level modeling attempts fail, our computational policies imitate the relationships of physical breakdown in behavioral patterns prevalent at lower-scale size sizes ΔL* that eventually grow into substantive effects with serious repercussions for this ΔL*, a type of "trickle-down architecture" (226).

Uniquely, this strategy of multiscalar analysis begins with empirical observations rather than those derived from molecular fundamentals. Thus, outputs of multiscalar modeling supply mixed level explanations. Where nineteenth philosophy of science was divided between "rari- and multi-constant" approaches to elasticity, multiscalar techniques represent a deft compromise between purist top-down and bottom-up methodologies (226-227). Unlike Quine's amalgamationist asseverations, these RVE sub-models—which communicate with the other RVE units through homogenized messages—do not attempt to reach any univocal ontic account to describe *what is going on in the material*. In comparison to the conventional bottom-up methodologies of traditional statistical mechanics, our multiscalar sequencing vis-à-vis a linked equilibrium communication provides a rich descriptive mapping. A common criticism of multiscalar modeling is that it is difficult to determine the operative parameters at hand regarding its 'first principles'.[61] However, this is an incorrect categorization as the complex computational architecture that we are dealing with is an assembly of localized forms of constrained equilibrium sub-models, "linked together through homogenized relationships that have been selected to imitate the cross-scalar dependencies found within the target material" (232). Thus, the structural contents of these modeling practices does not conform to some initial value problem and its concomitant bottom-up expectations, as higher-level empirical observations are incorporated within our explanatory architectures.

The implications of the Greediness of Scales dilemma is widespread. As an engine of effective inference, the applicative range of interior modeling formulas as such must be extended beyond the reaches of descriptive applicability, down to the infinitesimal. In order to contend with descriptive distortions, multiscalar methods impose corrective filters qua homogenization, with these compensatory tactics prodding patches of RVE modeling to cooperate with one another fluidly and profitably (234). This integrated framework has a deep-seated implication: "we can no longer regard each individual sentence within the encompassing scheme as directly 'capturing the entire truth' about nature's relevant activities" (234). The descriptive merits of any multiscalar modeling necessitate that distributed implementation, rather than interior equations alone, be recognized in order to comprehend explanatory strategy. The sub-models within a multiscalar modeling scheme provides us with a treatment of manifolds. A fruitful analogy is the atlas-of-charts mode recalled in the parable of a group of blind men touching and reporting an elephant's various body parts, with these local bits of information comprising 'understanding' solely when they are woven together in pragmatic fashion (i.e., the homogenization of data registration). Thus, we generally obtain our differential equation models by artificially extending higher dimension scaling rules down to the infinitesimal scale-level, despite being entirely aware that these behavioral assumptions fail at the lower-

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[61] Gene Mazenko, *Fluctuations, Orders and Defects,* New York, Wiley, 2003, pp. 57-58.

size scales—for instance, the isotropic scaling behaviors of "well-made steel fail when we reach the level of its component grain, yet we relentlessly plow past these limitations in setting up the standard modeling equations for our subject" (273).

§VI, "BELIEVERS IN THE LAND OF GLORY"

Here we critique contemporary metaphysics' tacit reliance upon the course categories of Theory T thinking, which has "misdirected philosophical attention away from the puzzles of applied mathematical technique that originally concerned Leibniz" (xv). The term 'cause' serves, at least partially, as a central instrument of linguistic management insofar as we utilize it to arrange *component strategies* within an extended reasoning process. Such applications endow 'cause' with robust physical content (e.g., the 'causal processes' involved in wave motion) but we can consider, with equal prudence, a scenario where the majority of customary physical referents are relinquished through mechanical constraints, leaving behind a pure exemplar of procedural significance.[62]

Analytic metaphysics has, following David Lewis, been prodded into considerations regarding the *early a priori*, wherein there is a quasi-Kantian expectation that metaphysical categories exist which serve as fundamental prerequisites of descriptive thought. The first of these is 'genetic on consideration', and emphasizes how, via linguistic training, we learn to reason about 'parts' and 'wholes', 'causes' and 'effects'. Relying upon such categories, we scaffold inferential skills. Thus, in parsing the concept of mereology—the branch of classificatory doctrine that is apparently demanded by our conceptual rendering of 'parts' and 'wholes'—philosophers such as L.A. Paul pose a necessitarian position concerning 'cause' and 'effect' wherein:

> "the metaphysics tells us what it is to be a sum or physical object composed of these structured arrangements of parts, and thus tells us how the physical object is metaphysically constructed (composed) from its parts. In contrast, chemistry tells us what some of the parts and the arrangements of the parts are for different kinds of molecules, and it also tells us how to causally manipulate the world in order to

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[62] Wilson delineates a sewing machine with complex parts where the circular motion on the right-hand crank is converted into back-and-forth motion at the highest extremity suitable for sewing machine stitching; in this example, there are two mechanical pathways that lead to a triangular piece, making it difficult to visually extrapolate whether this piece will turn clockwise ore counter-clockwise.

bring such arrangements into existence".[63]

Other analytic metaphysicians cite future scientific development rather than the prerequisites of knowledge formation wherein metaphysics is considered to be purely "speculative, and rarely if ever results in certainty" contrary to "continuity with science".[64] Wilson's position is closer to Paul's, whereby the vocabulary we use to explicitly distinguish cases and reorient behavior necessitates that we have a vocabulary of language management. On the other hand, Wilson identifies Sider's position with Theory T thinking, as it suppresses the importance of the contextual complexities in the linguistic tools of management and architectural subtleties therein. Accordingly, the terminologies with which we can articulate novel reasoning architectures to ourselves and others who may benefit from learning these routines are necessarily entangled in functions concerning multiscalar architecture qua the fulcrum of change (as is the case when concerning questions like"'[w]hen the small parts of granite recrystallize, what changes do these cause on a macroscopic level?'"; 243). Accordingly, we require linguistic tools in order to manage architectural subtleties of the languages we speak. Here, Wilson's claim concerning the pragmatic factors driving language in progressive fashion is in keeping with Quine's derision on the "analytic and synthetic"[65] although novel in its structural conception.

The word 'cause' adjusts to its semantic bearings as we move from one explanatory architecture to the next, contingent upon descriptive architectural adjustments. Recall that differential equational models that capture genuine causal processes generally possess a formal feature, or a hyperbolic signature, which is within those equational sets that capture non-evolutionary physical circumstances ("such as equilibrium condition[s], generally of elliptical signature"; 245). Thus, physics has a need for considerations regarding 'cause' when modeling equations that seek to accurately capture evolutionary developments as they unfold in causal processes. In order to do this, we

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[63] L.A. Paul, "The Handmaiden's Tale," *Philosophical Studies*, vol. 160, 2012, p. 3.

[64] Theodore Sider, "Introduction" in T. Sider. J. Hawthorne, and D. Zimmerman, eds. *Contemporary Debates in Metaphysics,* Hoboken, Wiley-Blackwell, 2007, p. 18.

[65] W.V.O. Quine, "Two Dogmas of Empiricism" in *From a Logical Points of View,* Cambridge, MA, Harvard University Press, 1980.

characterize causal processes according to 'finite difference', characterizing infinitesimal relationships within differential equations in terms of finitary spatiotemporal 'steps'.

As it concerns these 'early a priori' childhood acquisitions, we begin to speak of nature's causal processes in terms of finite difference. Rendering such finite differences into language means turning these into the Humean propositions of 'cause now, effect later'. Wilson characterizes those co-opting the Humean framework of inference as "calculus-avoiding philosophical descendants, who pressure […] that the 'laws of nature' invariably take the form 'for all x and all t, if F(x) holds at t, then G(x) will hold at t + Δt'" (246). True causal processes are, however, captured in terms of differential equation relationships that we can only approximate *if we do not possess the appropriate calculus*. However, this does this mean that we agree with Ernst Mach and Bertrand Russell's critiques on 'cause', as both philosophers fail to contend with Jacques Hadamard's distinction between elliptical and hyperbolic signatures.

When, for instance, the energy input into a violin string remains trapped within travelling wave-front packets, 'cause' is shaped along time's progressive forwards-arrow, allowing us to co-opt a straightforward evolutionary modeling. But with *real strings* energetic confinement does not smooth out into standing wave patterns. Rather, "smeared out pulses continue to travel back and forth across the string for appreciable periods" where "the applied work across the entire string" is redistributed in a manner that, after a short relaxation time, sees input energetic resettlement such that we can characterize the string's movement as a "superposition of standing wave patterns that retain their individual energies for significant periods of time" (248). This altered representation returns us to *Fourier analysis*, where there is a shift of basis vectors within a common descriptive arena—moving from a position representation to energy representation, with a change in descriptive basis representing coeval adjustments in reasoning architecture. Fourier factoring demonstrates how the *real string*, in the absence of energy dissipation, cycles through a number of simple processes independently of one another.

In order to consider the varied vibratory modes of the string at 'turnaround points', or those points when stored energies are expressed completely in a potential manner, we need to relocate string calculations through potential energy vis-à-vis an eigenfunction problem. This culls a different explanatory landscape wherein we no longer are concerned with 'time' at all but, merely, configurations of maximal potential energy (which is not contingent upon time but, instead, energetic storage capacity). In order to consider this, we need to adjust our computational strategy as a control problem rather than as an initial value problem, applying standard separation of variable techniques. This kind of adjustment is accompanied by a shift in how the word 'cause' shifts, as it is no longer attached to any evident 'causal process' in the evolutionary modeling manner.

Specifically, when we are speaking about the least potential energy calculated according to curvature, invoking a description of a string in a Fourier-like way, our causal attention shifts to features of the central control variable involved, where questions of cause are contingent upon changes in variables such as angle changes. These are called *manipulationist counterfactuals*, where the outcome of various potential manipulations is centered upon target variables that proceed processually.[66] A change in strategic focus, or instruction, accompanies an adjustment in the appropriate questions associated. This admixture is characteristic of guiding terms such as 'cause' and philosophers such as James Woodward have examined the manipulationist concerns involved.

The Fourier paradigm is particularly seductive because of its "strong physics avoidance virtues—the invariant nature of our string's modes allows us to push most issues of temporal development off the table and to concentrate instead upon the time-removed question of what the system's eigenfunction modes will look like when frozen into their positions of pure potential energy" (252). However, causal process, as it applies to continuous wave progression as we originally examined (the wave motions which carry a violent string *forward in time from one state to another*) inherently deal with temporal developmental processes. This is a product of our 'early a priori' training concerning the developmental etiology of 'cause'—we progressively deal with target variables such as counterfactual construction, causation, and manipulations, which are refined

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[66] Woodward discusses how a 'cause' operates in accordance to manipulationist conditionals that arise from a wider set of operative circumstances. See: *Making Things Happen*, Oxford University Press, 2003.

over time through mixing linguistic instruction with factual report qua reasoning judgment (e.g., we can solve the problem of maximal potential energy shapes that 'cause' a refined preserve energy via shooting method trials, locating distinct eigenfunctions according to the number of times they cross the x-axis.). As in Woodward's oft-overlooked analyses on manipulationist conditionals, effective techniques concerning counterfactuals depend upon the inferential methods regarding specialized search spaces.[67]

'Cause' performs a different function with respect to linguistic instruction than with factual reporting. Temporally deracinated, the word 'cause' no longer attaches itself in the temporal manner of a wave traveling along a violin string. Instead, its focus becomes at least partially architectural, concerning how alterations arising within a modeling format α (Δl^{ower}) can be matched to alterations that appear within modeling format β (Δ^{higher}). Such is also the case with the use of 'cause' in multiscalar modelings (e.g., adjustment in the stresses around the mineral grain within granite can 'cause' that portion to shear elastically or recrystallize). Such multi-layered reasoning architectures mimic reasoning policies found in nature, with collections of linked sub-models centered around various scale lengths and communicating with one another via homogenization techniques, rather than through the straightforward amalgamation of data. Enforced changes on scale level Δ^{lower} affect behaviors on scale level Δ^{higher} according to adjustments in stress that affect elasticity adjustments. As it concerns distinguishing granite from pumice on the macroscopic scale, lower-scale adjustments manifest themselves by shearing elastically (behaving like standard granite) or transmuting into gneiss (recrystallization); pumice, which lacks significant lower scale grain, has an architecture that requires counterfactuals of a similar type but also contains many

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[67] This is opposed to Nelson Goodman and Stalnaker's Theory T thinking, where significantly distinct forms of explanatory architecture are collapsed into the format of as an initial value problem sans reliability-enhancing homogenization (the helpful kind of 'physics avoidance') and salient modeling equations are endowed an evolutionary character of hyperbolic signature. See: Nelson Goodman, *Fact, Fiction and Forecast*, Cambridge, Harvard University Press, 1983. Robert Stalnaker, "A Theory of Conditionals" in Nicholas Rescher, ed., *Studies in Logical Theory*, Oxford, Blackwell, 1968.

trapped gas bubbles that can erupt when their obsidian walls melt under higher temperatures. As in the example of thermodynamic effort, which positions a crucial distinction between coherent and incoherent effort, the notion of controlled manipulation is central to conceptual endeavors such as how a spring's original coherence responds to macroscopically manipulated push or pull with increased internal pressure—etiological question arise, such as "how much of a specified manipulation will *cause* an increase in pressure and how much will *cause* a rise in temperature?" (258; emphasis added). Through the collection of cross-scalar counterfactuals of the same general type—that is, a compilation of reliable Woodward-style counterfactuals adequate to the computational architecture we should employ—we are able to address questions concerning the principles that dictate 'causal' change by grounding our laws in counterfactual claims.

The type of autonomous causal processes of the sort typified by wave motion do not invoke control variable considerations. Thus, we are not claiming that 'cause' necessitates an induced manipulated change at lower or higher size scale. Rather, the specializations that we are concerned with are related to reasoning stratagems that are applicationally and circumstantially 'mixed and matched' according to early a prior training modules. The rigid requirements pertaining to how 'causes' relate to 'effects' depends on local architectures and distinctive bonds between word-and-world, just as "the strength of the thread does not reside in the fact that some one fiber runs through its whole length, but in the overlapping of many fibers".[68]

According to Paul and fellow analytic metaphysicians, whose position opposes the rigidified semantics within contemporary philosophy of language, during our early a priori training we attach a central 'meaning' to 'cause', with firm extensions in *all* 'possible worlds'. The essential pattern of word/world(s) attachments takes such shape, and thus we, in agreeance with Quine's naturalist portraiture of linguistic development, reject the necessitarian assumptions of standard semantic essentialisms. However, this does not mean that we also ought not to be critical of how Quine is privy to Theory T thinking at times as well, grounding counterfactual claims in scientific laws. Thus, Quine is partial to the

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[68] Ludwig Wittgenstein, *Philosophical Investigations*, Oxford, Blackwell, 2001, §67.

Gaussian patchwork where early a priori verities can collapse under inferentially patterned adoptions:

> "Operating as a term of mixed descriptive and management import, the word 'cause' tags along with the architectural decisions we make in adapting established strands of parent reasoning into strategically modified sons and daughters. Borrowing terminology from the mathematicians, we can say that the newer employments represent natural continuations or *prolongations*…" (260).

Wilson gives us good reason not to reduce every explanatory setting to evolutionary modeling circumstances, as those analytic metaphysicians who overlook the fact that counterfactual claims "make perfectly good sense within explanatory circumstances that are equilibrium-center or which eschew direct consideration of temporal consideration through other means" are writ to do (265). Insofar as Paul's emphasis on early a priori considerations is concerned, Wilson's objection to her defense of analytic metaphysical doctrine is rooted in the fact that her means of improving descriptive practice rests upon tearing our inferential doctrines away from the simpler demands upon which they were originally formed (267-268). Rather than appeal to a Theory T "fundamental theory" of futurist appeal, as philosophers such as Ted Sider do, *Wilson denies distinctions that rely upon 'perfectly natural properties', 'internal versus relational properties' and 'counterfactuals sustained by explicitly articulated laws'*. This is precisely why Wilson takes such arduous time to make the case that the history of classical mechanics is characterized by a dependable resistance to suggesting plausible 'laws' regarding the basic cohesion of solid matter, preferring to co-opt the more metaphysically reliable "dodge of relying upon constraints and allied evasive crutches" (268).

What do differential equations within our science teach us about classificatory concepts? First and foremost, and in agreement with the inferentialist Hegelianism of Robert Brandom, statements of scientific law should be understood as making explicit something that is implicit already in ordinary empirical descriptions of how things are. Such equations are not directly anticipated within the subject's pre-assigned syntax (thus the necessary task of 'making explicit')—the law is present in appearance, but it is not the entire presence of appearance. Under unique circumstances, the scientific law of nature has an ever different actuality; the laws of nature determine how things actually interact *only* when supplemented by actual boundary conditions, or applications. In fixing which antecedents are factual, lawful necessity is expressed under actual conditions, which single out some of those hypotheticals as worthy of detaching conclusions from.[69] Considering the Fourier-style characteristics of an uneven string, the decompositional traits capture basic behaviors of the string in a direct manner despite differential equation vocabularies' 'recursive orbit' rarely captures terminological specificities; these are, instead, born from the "fixed point limits of holistic approximations, whose existences must be established by set theoretic means" (270). Despite, as made clear by the Greediness of Scales problem, we do not regard differential equations, themselves, as positing directly descriptive accounts of physical behavior(s) below a cut-off level where scaling assumptions fail (instead regarding formulas as convenient bottlenecks of descriptive overextension as we search for tractable conclusion), Fourier-like models obtained from such infinitesimal seeds demonstrate that target systems' reliable characteristics, as these are captured upon a macroscopic, dominant behavior basis. Our string's modal properties do not obtain status as 'important traits' from laws or differential equations in and of themselves, but, instead, from the means through which such interior considerations provide bridges to boundary conditions, interfaces, basic modeling assumptions, *and, more importantly, unformalized appeals to dissipation, relaxation times, and steady-state conditions.* That is, Fourier string modes obtain descriptive centrality by way of the vibrating string's endpoints—continually redirecting traveling wave energy back towards the interior—which couples with relaxation time dispersion. Such boundary condition behaviors designate unique physical factors, which are registered within the interior string equation. Without taking account of such operative cooperative partnership, the string loses its capacity for storing energy in standing wave containers.

Leibniz' metaphysics is similarly concerned with descriptive overreach in differential equations. For Leibniz, differential equations do not directly reflect physical reality naively but require being parsed "in a manner that accurately recognizes the expressive limitations of the tools of applied mathematics. The

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[69] Robert Brandom, *A Spirit of Trust*, Harvard, Harvard University Press, 2019, pp. 189-190.

metaphysical entities he [Leibniz] reaches via these reflections are his strange monads" (272). While we do not endorse such monads, Leibniz' interpretative problem is still with us, as we cannot extract a plausible ontology from physical doctrine in the straightforward syntactic manner that Quine[70] and others posit (in their Theory T modalities). Appealing to the terms of Sider's 'internal' and 'relational' property distinction elides the critical importance that homogenization and allied techniques play in supplying mathematical surrogates for the environmental and interscalar relationships that determine how natural behaviors on varied length scales unify. What are the considerations to be privy to when concerning cooperation-of-linguistic-labor?

Syntactic labels that we assign to significant physical behaviors "often derive from the mathematical terrain in which the advantages of strategies like factoring are clearly registered […] rather than first appearing within the grammatical orbits directly spawned by the original differential equation modelings" (276). In Galileo's description of triangles and rectangles,[71] boundary region ascriptions do not accord with differential calculus tools used to describe their interiors—the corners concentrate stresses, undercutting the validity of the interior equation. Nonetheless, the idealized notion of perfect triangles also facilitates many helpful reasoning practices that we are barred to otherwise. To solve this, scientists since the 1950s have invoked functional analysis corrections to continually confront such discrepancies–Theory T thinking is oblivious to such cooperative repair due to its static conception of 'ontology'.

As Wilson notes, "Paul's emphasis on early a priori learning properly directs our attention to the linguistic question of how we competently manage a wide variety of differently strategized explanatory schemes" (278). Our disagreements with Paul are at the level of Paul's rigidified semantic assumptions, which presume that words like 'cause' retain a constant and metaphysically analyzable 'meaning' throughout all of its helpful ministrations—following Wilson and other critics of permanent necessity (which includes Quine), we reject this premise. Sider does not repeat Paul's mistake; instead, Sider makes a range of Theory T assumptions

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[70] Quine, *From a Logical Point of View*, Cambridge, MA, Harvard University Press, 1980.

[71] Galileo, "Selections from 'The Assayer'" in Maurice A. Finocchiaro, ed. *The Essential Galileo*, Indianapolis, Hackett, 2008.

concerning how predicates that are posited within science are stratified hierarchically with respect to 'importance'. According to Wilson, such a static position "locks science with a conceptual straitjacket that fails to account for the subtle adjustments at the core of its improving practices" (278). We can remedy Sider's conception of metaphysics as pre-scientific enterprise and a necessitarian doctrine that rests upon syntactic convictions and a hypothetical 'final physics' by affirming Woodward's manipulation conditionals. In unison with Wilson's robustly adaptive semantic pragmatism, we pronounce that referential ties to the natural world ultimately are rooted in language's practical entanglements with it, in action-enjoining contextual manners which frequently employ complex modes of data registration.

§VII, "IS THERE LIFE IN POSSIBLE WORLDS?"

We ought to approach causation in 'possible worlds' by examining the dappled nature of practical considerations that are contingent upon rationalizing appeals to counterfactual possibilities within effective science. In doing so, this essay creates a bricolage with Wittgenstein's later thought. As we noted earlier, Lagrangian descriptive tactics codify constrain-based higher scale knowledge in the form of virtual manipulationist conditionals (e.g., "[if] child i is moved through a vertical distance δ in a virtual manner, this manipulation will cause X amount of opposing work to arise as a reaction on behalf of the rest of the system"; 265). Lagrangian techniques collect such specific forms of assertion into 'possibility spaces'/'function spaces' which guide reasoning through improving approximation techniques until counterfactuals are framed by equilibrium (based on the inductive enlargement of experimental results).

In *Naming and Necessity*, Saul Kripke describes "possible worlds" as little "more than the mini-worlds of school probability blown large".[72] If we take two six-faced dice, there are thirty-six possible states, with only one actual outcome; all the others remain relegated to the realm of possibility. Scott Soames opens these contours onto the globalized view of possibility, wherein "[a] possible world is a possible world-state—a way that everything could have been. It is, in effect, a

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[72] Kripke, *Naming and Necessity*, Cambridge, MA, Harvard University Press, 1972, pp. 16-18.

maximal property that the universe *could have had*."[73]

In Wittgenstein's *Tractatus*, there is a particular focus upon the precept that logical possibilities comprise a well-defined collection over which a competent speaker possesses an absolute and wholly a priori command. However, in *Philosophical Investigations*, Wittgenstein appears to believe that the variegated appeal to the 'possible' and 'impossible' are localized and revisable insofar as proper characteristics are concerned, meaning that the difference between the 'possible' and 'impossible' is located in the contours of our present forms of life—that is, "the phrase 'logical possibility' represents something of a misnomer because the underlying motives for erecting a local possibility/function space will typically vary from quarter to quarter and display little of the shared commonality that the modifier 'logical invites"' (289). Thus, we have no reason to presume that circumscribed spaces can be sensibly combined into those aggregate Kripke-like completions. Accordingly, philosophical expectations otherwise rest upon a substantive misconstrual of the pragmatic utilities that localized talk of possibilities commonly facilitate.

In order to observe how these misconceptions operate, we can examine counterfactual conditions that comprise contemporary metaphysics. Consider, for instance, sentences of the form]if A is altered in manner B, condition C would result'—this encourages an inflation the likes of David Lewis' grounding relation, that the possible world w_1 embraces a provision of variant 'nearby' conditional composition, which means that it obeys a fundamental set of physical laws that prevail within our world w_0.[74] Wilson, however, shifts our attention towards appropriate modeling, once again: for instance, if we are planning to erect a building that we do not want to collapse, we will attend our architectural reasonings and model our proposed edifice along a molecular physics basis, favoring the collections of descriptive parameters and computational policies that

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[73] Scott Soames, *Philosophical Analysis in the Twentieth Century,* Vol II, Princeton, Princeton University Press, 2003, 355. This inflationist position allows constructions that descent into infinitesimal oblivion.

[74] David Lewis, *Counterfactuals,* New York, Wiley-Blackwell, 2001. Such patterns as Lewis' demonstrate uncritical faith in coarse structural distinctions; Theory T thinking as such was canonically formulated by the logical empiricists of the 1950s.

best insure against injurious collapse. In doing so, we seek established constraints in the realm of macroscopic facthood without worrying needlessly about lower-scale complications. Such traits are generally abstract in their conceptual contours and, therefore, we have to specify these specialized collections of localized possibilities within search spaces, thus insulating counterfactuals. Therefore, "[w]ell-selected and localized possibility spaces commonly serve the important purpose of insulating macroscopic descriptive claims about substantive reliance upon the complexities of microscopic fact" (290).

How, then, do we contend with the fact that the central utilities of 'possibility spaces' are anchored by the means in which they allow us to evade reliance upon lower-scale details? By co-opting Lagrangian methods, we can: i) eschew speculative assumptions regarding unnecessary constructional details, and ii) direct our reasonings to "important energetic traits hidden within a target system through the guidance of carefully assembled search spaces" (291). Rather than law-based speculation, by relying upon empirical experimentation we can, therein, derive counterfactual data. In doing so, we bar ourselves from the kinds of 'free enlargement' in Kripke and Soames. Rather, we follow the late Wittgenstein in agreeing that the presumption of off-handed appeals to absolutist possibly merits critical scrutiny.

In order to further illuminate Wittgenstein's thought we ought to concern ourselves with those "local packets of guiding 'possibilities' that appear peculiar in their contours", such that these counterfactual claims "ask what happens under strange antecedent circumstances or possibilities that can't be sensibly enlarged into a science fiction story of any kind, no matter how otherworldly" (293).[75] Rather than a priori possibility, drawing upon such counterfactuals shines a light on the data registration requirements of specific inferential engineering.

As in the processes of optical design arrangement when creating an algorithm with which to design the basin of a Peaucellier mechanism, occasioning for motion conversion, finding the orientation of any optimal arrangement lies in

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[75] Notably, Quentin Meillassoux terms frames such counterfactuals under the domain space of "extro-science fiction", wherein a target system does not follow from modified possible worlds that still rest upon the "reliable physics" of w_0. See: *Science Fiction and Extro-Science Fiction*, Minneapolis, University of Minnesota Press, 2015.

restricting our attention to the possibilities of representation. Searching for an optimal sizing vis-à-vis proper setting for physical modeling(s), we select a space of restricted possibilities that are pertinent to locating the proper solution to a particularized problem. By drawing on a 'guiding norm' through refinement processes (e.g., a sequence of refinement processes that reduces the measured error on an output curve-to-straight-line motion), we can examine the movement related to "such-and-such possibilities of movement in a machine" without reducing it to "the ideally rigid machine that can only move in such-and-such way."[76] This does not mean drawing solely on the physical conditions for moving but *play*, as in the "play between socket and pin, the pin not fitting too tight in the socket […] The possibility of a movement is, rather, supposed to be like a shadow of the movement itself," by which we do not mean "some picture of the movement", but, instead, veer towards optimality such that we designate "the possibility [of this movement]" as "the possibility of *just this movement*."[77]

In *Tractatus*, Wittgenstein links hypothetical capacities with an a priori understanding of all the variations in absolute possibility that a given term will accept. The early Wittgenstein, as demonstrated by this stage of his thought, believed that unfettered knowledge of possible variation constitutes an essential aspect of the 'local grammar' of a term whereby this 'grammar' can be enacted but not coherently hermeneutically unraveled by being 'spoken of'. Wittgenstein later abandoned such absolutist conceptions of possible variation, although he retained the notion of a 'logical grammar' when considering the applications of specialized acquisition vis-à-vis the 'firm control' of a localized possibility space. As it concerns our pragmatic position, this means that the utilities of a specific specialized spare are linked to *the empirical and algorithmic opportunities* patterned from reasoning. There is no "a priori expectation that the distinct possibility spaces we frame within different applicational circumstances will fit together nicely at all, let alone cohere into grand possible worlds of a Kripke-like strip" (300). Rather, we can expect incongruent conceptions of possibility when switching our focus from the affordances of a localized design patch's optimal design.

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[76] Wittgenstein, *Investigations*, §194.

[77] Ibid., emphasis added.

What does Wittgenstein mean by 'logical grammar'? As his thought progresses, Wittgenstein notes that "the movement of the machine-as-symbol is predetermined in a different sense from that in which the movement of any actual machine is predetermined."[78] Logical grammar can be explicated in terms of 'spaces of possibility' associated with usage, but now such spaces of variation do not arise from a priori attachment to absolutist possible worlds, but practical advantages pertinent to human proceedings. This is a 'connection in grammar' because it is not causal or experiential but, "stricter and harder, so rigid even, that the one thing somehow already is the other, is always a connection of grammar".[79] Our intuitive connections (of 'machine essence') fade when we discover alternative pathways of machine improvement over time; localized control remains critically determinant of cognizing over shorter periods of time. Despite this it is not *entirely* incompatible with inferentialism, Wittgenstein's 'logical grammar' (a remainder from his *Tractatus*) is closer to the revisable a priori than those narrower task-focused concerns we share with Wilson.

Instead, let us co-opt Lagrange's virtual work methods, which execute a task similar to the employment of homogenization techniques when computationally mixing data obtained from different choices of scale-size. With Lagrange, we direct these methods towards data amalgamation, but in a "somewhat simpler setting where search within a restricted space of tweaked possibilities effectively blends data types that otherwise do not fit together well […] we begin within configuration spaces allied to the mobility spaces [already] canvassed but in which we introduce applied forces able to push the parts of a system around" (302). In doing so, rather than abiding by Wittgenstein's notion of an intuited 'logical grammar', we create a combination that exacts a deeper symbiosis between a specialized space of counterfactual claims and practical utilities that they underwrite. Higher-scale knowledge that is registered here is contracted via *constraints*, or geometric restrictions placed on the possible mobility of the variegated elements/parts of any target system. The virtual work measures of Lagrangian technique manipulate counterfactual possibilities—while keeping our time interval Δt small enough to reiterate this marching method computation

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[78] Wittgenstein, *Investigations*, §193-194.

[79] Wittgenstein, *Remarks on the Foundations of Mathematics*, Oxford: Blackwell's, 1964, §128.

over and over, plotting evolving behavior over long spans of time—that guide us to the characteristic we seek to focus upon, supplying us with norm-guided searches that measure how far a given approximation departs from target behavior until we reach certitude stability standards. Such manipulations compute virtual work value by considering and modifying various possibility counterfactuals via effective functional compromises, exploiting reliable manipulation data gleaned from an experiment. These virtual possibilities determinately resist those ready enlargements into richer possible worlds as discussed earlier. *Rather than appearing as redactions from a wider collection of a priori possibility, the origins of virtual work's categories are directly related to modeling utilities.*

Thus, virtual work illustrate the investigative recommendations that the later Wittgenstein recommends. Where, in the *Tractatus*, he maintained the possibility of an absolutist universe of logical possibility "from which any localized specimen has been plucked, and further claimed that we aren't ready to employ language at all until a complete mastery of this grand space of permissible variation has been acquired," the later Wittgenstein commits to non-degenerate possibilities as being directly produced via utility; thus, principles such as 'virtual' reflect "strategic contours" and "look peculiar only as they are encountered within a vast ocean of amalgamated possibilities fed by many inlet streams" (311). As Lagrange's method demonstrates, we assign truth-values to otherwise peculiar 'virtual work' counterfactual conditionals by way of the exploitation of higher-scale constraints. Contra bottom-up endeavors, exploiting localized possibility founded upon upper-scale knowledge of material behavior, we can identify important physical structure, such as available work potential. Thus, we co-opt Wittgenstein's notion of long-term mutability regarding those epistemic "river-beds of thought" which "may shift"[80] according to alternate strategic methodologies and technological salience. This warns us against conceit in any historically restricted intellect, which is always relative to a computational compass. The standard 'possible world' analyses of modal logic have, indeed, assisted our unraveling common patterns of linguistic behavior with respect to how we communicate necessity and possibility. But this also means that such talk has latched onto *usage* and *training*

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[80] Wittgenstein, *On Certainty*, trans. Denis Paul and G.E.M. Anscombe, Oxford, Blackwell, 1969, §96-97.

(i.e., the 'early a priori'). Just as "computer scientists take their algorithms for trial runs over a representative sample of well-understood situations to see if any unanticipated glitches threaten their proposed reasoning procedures" (313), we similarly utilize inferential rules for structuring reasoning and, using these reasoning rules, cultivate counter-factual spaces as corrective checks for such deductive procedures. Despite our inferential repertory undergoes referential refocuses, a "strong psychology of classificatory confidence accompanies these adaptive extensions, in the sense that we are predisposed to assume that we already possess a suitable label for everything we encounter, no matter how unfamiliar" (314)—for instance, we confidently label a spindly blot 'spider-like'.

Despite vocabularies such as 'possibility' and 'necessity' *should* be generic in their contours, as when we first learn the term 'force' in a loosely focused manner, the chief grammatical structure of our language shapes how such early a priori reasoning co-opts inferential patterns that reflect grammatical distinction. Specifically, if reasoning capacities include the 'searching through' of relevant spaces for salient possibility, then this should mean that we integrate basic grammatical reasoning "in roughly the syntax-linked-to-possible-world-semantics-pattern" (315). However, the computational demands of manipulating syntax when dealing with the external natural world often run against the current of such originalist expectations, with our early a priori presumptions altered and overwritten (as in Neurath's boat). Wilson provides a prudent example:

> "we swiftly acquire an early understanding of the word 'rainbow' by assimilating its usage to the geometrical reasoning patterns we associate with words like 'arch'. But these inferential expectations must be corrected by the time a speaker becomes an adult, for the underlying physical phenomena demand a different set of reasoning policies [….] Making these corrective adjustments [….] we need to suppress the many ersatz possibilities that loom large [….]" (315).[81]

That is, we require a linguistic vehicle for weaning from 'rainbow possibilities' concerning textual images (those storybook rainbows) and moving towards rainbows based in meteorology and optics. Tools of linguistic management provide the verbal instruments through which we can reach towards 'real possibility', redirecting patterns of usage as circumstances require. Via the

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[81] Wilson, *Wandering Significance*, pp. 22-24.

bottleneck of such corrective agencies, our conceptions of 'what is possible' thus enlarge and contract over time. However, these have not just been latent within our linguistic skills since their inception—in fact, this is precisely the conceptual misconception that bolsters Wittgenstein's static view of classificatory semantics and applicational development in *Tractatus*. Despite we rarely notice these inferential retoolings, for they seem to be camouflaged within our general manner of conceptualizing the world in scenario-centered terms, truth-values of a restricted class of counterfactual possibility are exploited in a manner similar to Lagrange's, stemming from direct induction upon manipulative experimentation. The essential utility to Lagrangian modeling methods is that they exploit our upper-scale knowledge of media (e.g., rigid wires and beads, prompting effective 'physics avoidance' techniques). Such exploits permit us to facilitate 'cut offs' in regards to modelling complex (interior) processes that arise inside media (e.g., the waves arising inside a wire as they are buffeted by scuttling beads). To claim that truth-values of such counterfactuals are necessarily dependent upon the laws that they are designed to circumvent, grounding them in laws as analytic metaphysicians often do, overshadows how reliability-enhancing utilities develop.

The informational thesis of language poses that the truth-conditions of descriptive sentences ultimately reflect the manner in which they segregate absolutist possibilities into two groups: 1) the world they accept; 2) the worlds they reflect. Accordingly, we anticipate that underlying semantics for counterfactual claims are anchored in allied discriminations amongst possible worlds. If this is true, "any English-speaker competent in counterfactual constructions must recognize how informational basis operates" (316). However, as the examination of Lagrangian technique suggests, we can readily invert this notion of truth-conditioning and informational dependency. The information that we exploit within any technique is contingent upon macroscopic experiment rather than sub-atomic 'carryings-on', with corresponding truth-values running in parallel. Thus, if we characterize wires and rods as 'rigid', we apply applicational standards along the appropriate macroscopic scales of length and time. If we shift attention to other characteristic scales, we may describe such similar physical circumstances in alternative ways. Contextually based evaluations as such illuminate linguistic efficiencies—thus, the 'truth conditions' of virtual work conditionals must be regarded as informationally rooted within the means through which we decide upon truth (i.e., direct inductions from experiment that are consequently corrected for virtual work effects). If, as in Wittgenstein's *Tractatus*, we hinge informational content upon mental abilities, we limit how epistemological dependences develop, segregating the 'absolute a priori' into these aforementioned two groups of possibility.[82]

§VIII, "SEMANTIC MIMICRY"

Let us now concentrate upon the supportive rationalities and problems of describing continuous material in a coherent manner, wherein the 'function spaces' or 'possibility spaces' previously describes are sewn into a cooperative logic that reinforces strategic functionalities that provide us with localized utilities. In the history of science, we see how methodological reappraisals appear where simple inferential transition from sentence A to sentence B turn upon a hidden machinery. Such are the circumstances of *semantic mimicry*, "situations where a language's employers fancy that sentences A and B signify physical underpinnings of a certain type, when the real-world support of their modeling successes lies elsewhere" (327). Thus, surface syntactic resemblances can be deceiving, and it is by culling extension elements that we can improve our inferential-derivational landscape, in the same manner that adding complex points towards infinity to the regular Euclidean plane produces improved deductive efficiency.

The treatment of physical modelling that is suppressed under the shadow of celestial mechanics floods back into the stage with finite dimensions, boundary and interfacial conditions, constraints, Fourier decompositions, and so on. Such a treatment requires a shift towards partial differential equations (PDEs) and away from evolutionary ODE-driven evolutionary processes. Theory T thinking

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[82] We ought to admit that Wittgenstein's notion of "hardened propositions" in *On Certainty* suggest that his conception of logical grammar may indeed be more akin to the notion of a revisable a priori as conceived of by Hans Reichenbach and, in a different form, Michael Friedman in their discussion on linguistic determination qua "space-time metric[s]" and their employment, rather than those narrower task-focused concerns Wilson is concerned with. See: Hans Reichenbach, *The Theory of Relativity and A Priori Knowledge,* Berkeley, University of California Press, 1965. Michael Friedman, *Dynamics of Reason*, Paolo Alto, Center for the Study of Language and Information, 2001.

has recast the reasoning patterns pertinent to an ODE equational basis into the format of elementary logic, accomplishing such trickery by converting the "smoothly flowing stages of an ODE-driven evolutionary process into long chains of modus ponens instantiations based upon alleged 'general laws' such as '[a]t all time t, if conditions A hold at t, then condition B will hold at time t + Δt'" (340-341). Replacements of this stripe present an inferential advance within mathematical physics that are supplied by rules of elementary logic, rather than the skills involved in seeking out *helpful inferential adjuncts*, as in Green's functions.

Given an elastic material, we may observe that circular membrane—such as a banjo head—do not present us with corner singularities. However, we can not decompose or tesselate an arbitrary shape into circles without significant number of gaps. By creating such a decompositional passage, square, triangles, and other geometrical figures insinuate themselves into our mechanical prominence. Despite the impulse to dismiss such oddities as the result of 'excessive idealization', this is a haughty position, however, and, following Wilson, we will see how tessellation exacts descriptive demands, traversing between representational tools that mathematics makes available to us and the uncooperative natural facts behind such "mismatched representational" constraints and modeling parameters (336).

Generally, we select boundary conditions appropriate to a problem depending upon the natural possibilities that are available. This is a kind of idealization with respect to underlying physics, of the same ilk as those approximations introduced with the representation of geometry, with loads and material behavior. Thus "when one thinks of permissible error in an approximate solution, it is understood to be relative to exact solutions of the governing equations that inherently contain various approximations."[83] We conceived of our original formation of how the interior of a membrane behaves and how its boundary region behaves as completely independent descriptive tasks—this is incorrect, however. Insofar as it applies to boundary membrane disturbance, the expressive analytic data that we obtain through algebraic information within Green's function formula can be contrasted with the less informative numerical

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[83] J. N. Reddy, *Theory and Analysis of Elastic Plates*, Boca Raton, CRC Press, 1999, p. 73.

data obtained from conventional computer simulation. Employing Green's functional formula, and thus co-opting a functionalist and integrated perspective, our algebraic $g_\alpha(x,y)$ expression "serves as a key out-of-territory way station along a very profitable inferential pathway" (331). In short, 'out-of-territory' reasoning, formerly regarded as 'useful idealizations', re-adjust the appraisal of descriptive practices and emerge as integral parts of a smoothly functioning portion of descriptive machinery.

With Theory T diagnostic practices, we see the dilution of crucial differences concerning explanatory architecture. All explanatory differences are reformulated under the banner that 'for all times t, if conditions A hold at t, then condition B will hold at time $t + \Delta t$', which elides the unfolding dynamic processes qua evolutionary equations in equilibrium-based constrained state calculation(s). As Hadamard stressed, we can not comprehend the explanatory purposes of a set of differential equations without appreciating how, exactly, they fit together with their anointed side conditions (e.g., initial and boundary region values, interfacial jumps, and so on). To merely treat these considerations as undifferentiated auxiliary conditions, conflating physical descriptions with "initial conditions", as Ernest Nagel does, prevents any hope of understanding how harmonization operates.[84]

A related question concerns how do sentences that appear within our inferential practices encode pertinent information about the physical world? Those formulas that we have considered as merely 'approximation statements' encode central physical considerations—"[a] modern diagnosis holds that we mistakenly viewed the 'semantical contents' of the sentences appearing in our reasonings as individually interpreted claims, rather than as a collection of statements that must work together within a balanced descriptive network" (331).

We must enlist descriptive harmony repairs between the mismatches that arise between applied mathematics and physical descriptions. According to engineers such as J. N. Reddy, when descriptive disharmonies and other such oddities arise within our descriptive policies, we ought to examine the mathematical

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[84] Ernest Nagel, *The Structure of Science* (New York: Harcourt, Brace, and World, 1961), 32. Carl Hempel ascribes to as similar position with her treatment of "antecedent conditions" as does Karl Popper (with "Initial conditions) and Israel Scheffler (who employs all three phrases synonymously).

characteristics of all contributing parties, which includes our interior equation laws, for evidence of descriptive overstatement and approximations—"[t]he approximations are introduced through several sources, including representation of the geometry and boundary conditions, loads and material behavior."[85] Such suitable correctives involve the shuttling of informational content between former characterizations of 'boundary', 'interior', 'interface', and so on, such that their respective contents are adjusted until they reach descriptive accord. Mathematically, we can reach improved descriptive harmonies insofar as *interior harmonies* are concerned by examining "influence function."[86]

Cooperative symbiosis obtained between interior and boundary descriptions, where Green's function $V_\alpha(x)$ acts as a natural inferential bridge, shows how semantic meanings are revised so as to remove inequities to provide harmonious cooperation. In pure mathematics, functions that are straightforwardly composed from elementary operations rarely capture targeted physical behaviors accurately over wide expanses of space and time. Thus, they "fall out of accurate correlation and must be correctively repaired" (346).

In the 1890s, for instance, Hilbert provided us with a new means of resolving harmonization problems in disparate branches of mathematics (e.g., Riemann's appeals to Dirichlet's principle within complex function theory). Hilbert's direct method within the calculus of variations was inspired by Richard Dedekind's policies for filling in irrational 'holes' within the real number system through allied constructions. Under Hilbert's method, we bestow energy norms that can be uniquely extended to a wider measure onto an enclosed, smooth and non-optimal space, thus extending the non-optimal evaluative reach of descriptive labor onto a solution space α. By instrumentalizing such a space of approximate possibility, which is kept relatively small and appropriately focused, we 'fill in' topological surface holes via extension techniques within the space of functions. As such, we proffer possibilities such as when "two regions of surface contain the

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[85] Reddy, Theory and Analysis of Elastic Plates, p. 73.

[86] Recall that, previously, we concerned ourselves with the problems of harmonizing square boundaries with uncooperative interior equations via Green's functions of 'boundary element'. In this new setting, we are concerned with "base characteristic manifold" shifts in regards to the static circumstances of, for example, "a stationary string loaded with weights [....] Here the pertinent interior formula is $kd^2y/dx^2 = f(x)$, where f captures the local gravitational force density weighing the string down at position x" (343).

same strain energy content if they affect all measuring instruments in the same way, an assertion that we can apply to modeling gizmos like our sharp-cornered Green's functions despite the fact that they lack the derivatives requires to possess a 'strain energy' in the old-fashioned, unenlightened manner" (350).[87]

As the context of our virtual work considerations is concerned, by combining trial functions and their potential manipulations into a common space, we formulate a product of modeling functions that are unified along endpoint conditions. These conditions remain statically fixed and prompt a harmoniously enlarged setting, which produces new computational opportunities. We can search within a subspace along the Rayleigh-Ritz principle, locating an approximation substance along a sum of piecemeal linear (hat) functions (S_{FE} broken line approximations). Additionally, we can restrict our inner product computations to our approximation subspace S_{FE} along lowest energy demands within the specific S_{FE} space employed. This "subdivision into simpler pieces" and

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[87] Laurent Schwartz is perhaps the best example to consider such this semantic rebalancing. Consider, for instance, the following description: "Schwartz sets up a special space of test functions D' comprised of completely smooth functions θ that he will employ to smear out $V_\alpha(x)$ enough that the mollified results will possess regular calculus derivatives. He does this because he wants to view our target function $V_\alpha(x)$ through the lens of how it appears to the measuring instruments modeled by the θ's within D'. So he treats $V_\alpha(x)$, not as a regular function, but as a functional that acts on all of the test functions θ and produces a mesh of smooth curves that surrounds the two sides of the erstwhile function $V_\alpha(x)$ as closely as we might like. Intuitively, the 'derivative' that we would like our functionalized $V_\alpha(x)$ to possess should be the broken-line function ____----, indicating that $V_\alpha(x)$ possesses a constant negative slope up to the point α and a constant upward slope thereafter. How does Schwartz persuade the functionalized version of ____---- to serve as the first derivative of $V_\alpha(x)$? Answer: by asking all of the regular calculus derivatives of the θ- smoothed companions of $V_\alpha(x)$ to surround the two pieces of ____---- in a similar mesh. If we repeat this demand a second time, we obtain a functional second $V_\alpha(x)$ that isn't similar to a normal function at all, but rather represents a Dirac δ 'function' blip" (350-351). This re-orientation by way of privileging mollifiers is also highly related to Albert Lautman's diagrammatic 'phase space' of rigorous structural appropriation, where energetic possibilities govern collective behavior, portending Deleuze's fully immanent "virtual multiplicities". As perhaps made most explicitly clear by Deleuze's disjunctive synthesis, the second synthesis, this 'phase space' invokes the possibilities of a system that cannot be reduced to its 'vector field'. That is, these 'virtual multiplicities' are akin to concrete universals rather than the Aristotelian scenography of 'essences' (i.e., abstract archetypes). Thus, Deleuze writes that "[a]bstract machines do not exist only on the plane of consistency, upon which they develop diagrams; they are already present enveloped or "encasted" in the strata in general, or even erected on particular strata upon which they simultaneously organize a form of expression and a form of content." See: Gilles Deleuze and Felix Guattari, *A Thousand Plateaus*, trans. Brian Massumi, Minneapolis: University of Minnesota Press, 1987, p. 165.

"the equations of equilibrium and compatibility between the pieces"[88] means recasting our original physical problems into an enlarged and harmonized setting (355). Theory T thinking's general approximation technique can lead to mistaking localized virtual work manipulation upon the region of a trial function with a direct interaction between neighboring plates, confusing manipulations with trial functions.

Consider inspecting a small part of a three-dimensional piece of steel, where we find two unique kinds of force at work: i) 'so-called body forces' like gravitation, which act across distances to pull directly upon local points p within our steel, and ii) traction forces, which "push and pull on the smallish surfaces that surround p" (357). The traction forces sustain stress waves that ripple through our steel media object when we hit it with a hammer. A reconciliation problem arises, as these two varieties of force have both attached to geometrically incompatible regions—the 'body forces' to *points* and the 'traction forces' to *surfaces*; both types of force represent a density which requires a volume or surface integral to supply a finite force. Even if we examine so small a region of our steel media object, such that essentially only one body force acts inside it, we nonetheless have to confront the fact that its surrounding surface still carries an infinite bristle of traction vectors, which need to be processed in a way such that they can interact sensibly with the solitary body force at p. In order to do so, we need to invoke the notion of a 'stress tensor' located at p by fabricating a p-centered force vector that interacts with body forces "after we slice through p with a selected plane" wherein "different slices produce different forces", shrinking little volumes of our metal around these p points as in the shrinking-processes pioneered by Cauchy in the 1820s (357-358). Consequently, we coordinate and reconcile body forces with tractions by focusing on *the functional analysis recalibration of the finite element*, structuring these reconciliations qua central continuum mechanics structures obtained through a search within a subspace. Thus, "a linear triangular element becomes a pair of hinged rods; a quadratic element becomes a set of springs, and so forth" (359).

In short, despite the descriptive tools that seem to capture natural

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[88] William Gilbert Strang and George J. Fix, *An Analysis of the Finite Element Method,* Wellesley, Wellesley-Cambridge Press, 1973, p. 2.

circumstances aptly may not inferentially harmonize properly with those descriptions that seem appropriate to their target-media, there are mathematical means to enlarge conceptual constructions in set theory that, following Riemann, Dedekind, Schwartz, and Sobelev, we can occasion. This necessitates functional analysis, probing interfacial and intermedial queries concerning how should the information encoded within a linear differential operation be properly viewed? It is the infinitesimal element that prods our functional analysis' adaptive developments and, in parallel, the adaptive developments of language. From algorithmic assistance supplied by seemingly 'out-of-territory sentences' which serve as a harbinger of unanticipated networks that implement novel forms of descriptive strategy to the inferential explanations that are registered contextually in the guise to tacit heuristic reliance, "sophisticated forms of informational content remain encoded exclusively within the manner in which the usage contextually unfolds and may not be available to immediate inspection in the guise of stand-alone declarative sentences" (361).

Wilson's scientific realism is of a particularly contextual stripe, for here he is a rationalist who believes that our ongoing science of gradual accumulation bolsters a large and ever-increasing set of reliable 'Truths'. Nonetheless, Wilson does not believe that the 'truth-rules' for these 'Truths' can be captured by the simple isomorphisms favored by ontic structural realists such as Steven French, James Ladyman, and Don Ross, who enlist a naturalized metaphysics. According to Wilson, ontic structural realism, as well as homologous positions such as John Worrall's naturalized structural realism, "suffer from insufficiently flexible opinions with respect to the semantics of language" (361).[89] This is, of course, to

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[89] See: John Worrall, "Structural Realism: The Best of Both Worlds" in David Papineau, ed., *The Philosophy of Science*, Oxford, Oxford University Press, 1996. James Ladyman and Don Ross, *Every Thing Must Go*, Oxford, Oxford University Press, 2007. Steven French, *The Structure of the World: Metaphysics and Representation*, Oxford, Oxford University Press, 2014. Notably, one of the most direct analogs to Deleuze's machine ontology is analytic philosophy is Ladyman and Ross' Ontic Structural Realism (OSR)—also known as information theoretic structural realism (ITSR)—whereby the understanding of being is conceived of as relational and without substance. As a philosophy of entanglement, OSR/ITSR rejects the idea that reality is ultimately composed of self-subsisting entities, individuals, or trans-temporal objects with intrinsic properties and "primitive thisness", haecceity, etc. As Johanna Seibt notes, "[a]ccording to OSR the world has an objective modal structure that is ontologically fundamental, in the sense of not supervening on the

say nothing of his approach to mathematics where he is a would-be empiricist who would "rather not allow any portion of language to escape so flagrantly from the tribunal of experience" (363). For Wilson, the adequate responsibility of an exterior corrective is what prevents philosophy's fall into apriorism.

§IX, "A SECOND PILGRIM'S PROGRESS"

Lastly, let us consider the critique of the sundry contemporary formulations of 'naturalism' from the perspective of applied mathematics, particularly insofar as mistaken presumptions about the role that set theory plays within naturalism are concerned. For Wilson, philosophy of mathematics' direction towards the imperatives of 'naturalist philosophy'—as distinguished by texts such as Quine's *From a Logical Point of View* and Benacerraf and Putnam's *Philosophy of Mathematics*[90]—has been unfortunate, to say the least. While Penelope Maddy attempts an admirable corrective,[91] she seeks to detach her mathematical endeavors from concerns over linguistic reference. Wilson, on the other hand, views mathematics as playing a critical role in helping us understand how, exactly, subtle referential strategies operate. If we develop a conception of 'naturalistic obligation' that does not reflect the linguistic policies that take advantage of mathematical thought, then any such conception of 'naturalism' will be improper. What do we know about our computational position with nature? The answer relies upon scale and temporal dependency. The differing world(s)[92] between organisms are determined by varied ranges of dominant objects and properties, "in the sense of representing the characteristic ways in which an organism attends to the ambient data available to it" (367). Wilson uses the example of a fly-catching frog, concocting its visual system to examine how a landscape is filtered away, in cone-like fashion, of topographical noise/irregularities.

According to Descartes' constriction problem, we do not possess sufficient

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intrinsic properties of a set of individuals." See: "Quantum Mechanics" in Johanna Seibt et al., ed., *Handbook of Mereology*, Analytica, Philosophia, Munich 2017, p. 467.

[90] Quine, *From a Logical Point of View*, Cambridge, MA, Harvard University Press, 1980. Hilary Putnam and Paul Benacerraf, *Philosophy of Mathematics: Selected Readings*, 2nd ed., Cambridge, Cambridge University Press, 1984. According to Quine, mathematics serves science; Wilson rejects this claim.

[91] Penelope Maddy, *Second Philosophy*, New York, Oxford University Press, 2009.

[92] Knut Schmidt-Nielsen, *Scaling: Why is Animal Size so Important?*, Cambridge, UK, Cambridge University Press, 1984, p. 9.

mathematical capacities for tracking the adjusted geometry of a continuous flow (e.g., water flowing through a pipe constriction). We ought not forget that this problem is also related to Descartes' position that matter is, at its core, granular—every component particle possesses a fixed size and shape at every moment. For Descartes, empty space could also never appear between such particles; they had to always be in contact. Thus, "at every particular instant every spatial array of fluid decomposes into a tight mosaic of contacting particles" (369). However, today we have to account for how a moving fluid passes into a narrowed portion of the pipe and the relevant rearrangements. For Descartes, mechanisms split integral particles into smaller pieces that are rejoined into different geometries later on. Nonetheless, consider the constriction process of fracture and fusing, which occurs instantaneously and leaves no vacuum gaps. Descartes claimed we are simply incapable of following "such an infinitary process through all of its component stages" and that "[w]e must bluntly acknowledge that natural processes frequently pass through stages of 'indefiniteness' that our finite minds cannot track with the limited reasoning tools available to us" (369).

Descartes' position is an early articulation of *mathematical opportunism*, wherein nature offers us restricted occasions wherein we can follow developing processes with reasoning tools available to us with mathematics. This need not proffer via anti-realist positions in respect to the external world—like Descartes, we can be robustly realist, while recognizing out limited capacities for tracking generic physical processes accurately over long spans of time by utilizing mathematical tools alone. This is precisely why Jacques Rohault, Descartes' discipline, noted that "there are in nature things which are vastly more fine and subtle; we shall clearly see that what exceeds our imagination is not therefore impossible."[93]Despite our inability to inferentially track intermediate stages that link incoming flow to outgoing flow across the tube, we can asymptotically match descriptive materials to capture the smooth flow, as in the Huygens-Wren treatment of billiard ball collision.

Does not Descartes merely lack the differential equation tools to examine the flow of a fluid while it moves continuously through the narrowing opening

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[93] Jacques Rohault, *Rohault's System of Natural Philosophy* Vol 1, trans. Samuel Clarke, Farmington, Gale Ecco, 1723, p. 37.

without forming vacuum-like gaps? This illuminates part of the answer, which concerns mathematical optimism, but does not deal with how the data concerning those events which unfold at the infinitesimal level can be converted back at the finite-size scale without appealing to numerical approximations. By drawing upon Euler's rule reasoning and its coeval mildly transcendental realms, where differential equations posit unfamiliar curves without direct human experience, we can amend Descartes and offer how mathematical thought offers an "enlarged landscape in which the viability of a reasoning procedure of a computational stripe can be judiciously assessed" (375).

If we re-express our governing equation in elliptic coordinates on a target system, a modification of Euler's method to suit altered coordinates may supply reliable results at a much larger step size. Thus, we can improve our causal computation with strategies such as: i) taking a Cartesian coordinate computation and creating an elliptic coordinate system, as well as ii) examining perturbation computation (e.g., taking account of the side-to-side weaving perturbations in a sinusoidal flight path of a goose flying around its nest). Returning to the fly-catching frog and how it is embedded within a structural landscape within which the frog identifies strategic visual opportunity, we can take heed of the insect's curves around the frog. Modeling considerations here depend upon a particular scale length (between organism, geographical environment, and target system) which we can characterize as 'dominant behavior' terms relative to scale selection, wherein fine-grained interactions are less reliably controlled. As a mapping-to-cusp singularity of data registration, the frog's fly-catching routine is mapped within a strategic environment qua adaptive behaviors:

> "[i]f the insect victim's reversing curves are very tight and their trajectories remain approximately coplanar at these reversing points, the escape paths can be nicely projected onto two-dimensional curves with cusp singularities" (379).

Thus, we recast the problem of understanding adaptive behavior via the diagnosis of strategic environment in which these behaviors emerge by considering the relationship of profitable-strategy-to-opportunity as, essentially, a mathematical question. This does not mean, like Quine, that we posit mathematics as a projected science but, instead, side with Riemann, continually consulting general mathematical *experience* as a fount of useful strategic borrowings and often unexpected ploys which stem from a unique structural domain, fine-tuning the descriptive holds of our descriptive encoding (e.g., "we first learn the advantages of factoring by working with prime numbers and later transfer these techniques, after suitable tinkering, to garbage can lids"; 383).

For Hilary Putnam and Richard Boyd, over time predicative expressions in science evolve to registering information concerning the world in simple predicate/natural kind pairings (e.g., "that 'is 12° C' will eventually link tightly to some property involving mean molecular kinetic energy"; 383).[94] For Wilson, this is not the case and empirically false as there is no reason why linguistic improvements inevitably should follow in so simplistic a mode. Such a rigidified semantical position assigns standalone informational content(s) to individual sentences. This is problematic because of the operative terms necessary to fasten harmonious cooperation between the modeling ingredients (e.g., "interior equations versus the boundary conditions with which they must coordinate"; 384). As Wilson notes, Putnam and Boyd would have more accurately noted that, as a science develops, its reasoning patterns and measurement techniques increasingly register correct information dealing with the physical world without making further simplistic presumptions concerning the means by which such information becomes encoded within linguistic practices.

Contra those aforementioned would-be 'naturalists' cast under Quinean fantasies of 'all-at-once' Theory T postulation, Wilson prompts an ongoing process of semantic fine-tuning. Indeed, one of the primary reasons that we humans adapt more readily to varying circumstances compared to frogs is because we can "readily borrow and retune reasoning stratagems developed for task A to become novel routines for achieving task B, despite the fact that the relevant subject matters may scarcely resemble one another" (384). Such conceptual adaptations necessitate us to think in terms of applied mathematics, examining strategies in formal conditions and consistently engaging in semantic realignments as we enhance our referential grip to the world around us.

How can we better develop the paradigm for understanding inferential strategies by mapping an original problem within a richer setting? One example

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[94] Hilary Putnam, *Meaning and the Moral Sciences*, London: Routledge & Kegan Paul, 1978, p. 20.

is with the unraveling of series behavior, which Cauchy and his followers achieved by imbedding a real line upon the plane of complex numbers as, for instance, in reallocating a line upon an appropriate Riemann surface. Working through a probabilistic enclosure, where one seeks to find a required answer via an efficient set of refining questions (in which initial questions divide the available search space into spatial partitions of the same size), mathematicians say that such strategies operate through the *contractive* (or *coercive*) *entrapment* of the desired answer—"we progressively pose inquiries that ultimately squeeze in upon a 'final point'" (390). The search-shortening capacities of probabilistic algorithms are composed of a cycling filter upon the enclosure.[95] An example of this is with transferred mapping—for instance, considered in its own light, "the singular point at the center of the 2-D bow tie curve on the bottom plane […] exhibits puzzling behaviors with respect to its points of intersection with other geometrical figures, but the proper rules become immediately clear if we blow up the point into the setting of a 3-D curve" (392). This is precisely how 19th century Italian geometers untangled complex singularities. We can transcend ingrained limitations by mapping opaque circumstances into strategic settings, wherein we recognize tangent singularities, as in the example of an algebraic curve. That is, we recognize the tangent singularities of an algebraic curve more readily when

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[95] Here we can create another bricolage between Wilson and Deleuze by focusing on the corrective answer via operative probabilistic strategies (Wilson) and relations being external to their terms (Deleuze). For Deleuze, a connection between two multiplicities immediately generates something that exceeds them. These syntheses neither concern perceptions of objects nor human experience. In the context of Deleuze's machine ontology, to be what Deleuze calls a "Body without Organs," a "Figure," or a "problem" means to contextualize relations via neither a phenomenological nor an epistemological thesis. Syntheses are passive contemplations, pulling other (passive) entities into an (active) experience and, ontologically, all relations are thus expressed as a *contraction*. Therefore "every organism, in its receptive and perceptual elements, but also in its viscera, is a sum of contractions, of retentions and expectations." See: Gilles Deleuze, *Difference and Repetition*, trans. Paul Patton, New York, Columbia University Press, 1994, p. 73. For Deleuze, to be a multiplicity is to assemble other entities via contractions, proffering difference while also contracting each entity in turn (through repetition). As actuality transpires in the form of indirect contact (e.g., through sense or the event) and is relational, manifesting in the unity of machines, Deleuzean 'difference' is comprised of the virtual being fully absent from the actual—this is precisely how Deleuze theorizes of the radical distinction between 'corporeal things' and 'incorporeal events'. This is related to how Wilson's description of the *contractive* concerns itself with progressive questions asked gradually to squeeze an already narrowed search space into yet smaller components, creating a difference through (self-improving) repetition.

they are recast into a point-singularity setting, despite both forms of irregularity are entirely coequal, mathematically speaking.

To comprehend the special functions that naturally arise from the equations of mathematical physics, their behaviors must be examined over a wider territory than the real line. This means recasting them via the complex plane, which is what led Cauchy to several of his breakthroughs in complex analysis when "investigating the convergence of series solutions to Kepler's equation, which describe where a planet is in its orbit at any given time".[96] By mapping a function's singularities upon the complex plane, we can understand the otherwise shrouded computational failures relevant to computing standard expansion techniques. Thus, such inferentially transferred setting considerations offer great value. While Descartes could only subscribe to a limited thesis concerning descriptive opportunism, today we know that we can accurately capture natural occurrences within mathematical thinking of a differential ilk. As the 'mildly transcendental' true solution is produced vis-à-vis *contractive approximations*, we can consider this logic beyond the limits of finite-step termination that can be concretely verified. For instance, consider an infinite number of improvements as required before Euler's method approximations, which squeeze in on fixed-point targets and where set theoretic reasoning provides careful explication in order to find paths of resolution. Indeed, instead of providing ephemeral ontological reductions, "by replacing our intuitive pictures of curves and approximations by hard data on how coercive nets of real-valued n-tuples relate to one another through residual terms" we can utilize set theoretic thinking to achieve strategic ratification (398).

Set theory's extended hierarchies elaborate contractive processes and allied strategies, illuminating otherwise mystified modes of inferential technique by codifying mildly transcendental relationships between concrete calculation and target objects that have become obscured in the philosophical tradition ("due to Russel's and Quine's dubious methodological fables of Ockham's razor parsimony and ontological reduction"; 398). However, as Dedekind's work in set theory demonstrates—and as Sturm and Liouville's work, which can be characterized as 'ur-set theoretic' illuminates, as it relies upon assumptions about contractive

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[96] Tristan Needham, *Visual Complex Analysis,* Oxford, Oxford University Press, 1887, p. 64.

'fixed points' that demand set theoretic tools for clarification—it is important for naturalists to recognize that set theory provides the natural vocabulary for articulating relations "that we lowly calculators bear to nature's more abundant collection of processes" (398). [97]

In logic, we do not introduce new predicates, Px, through non-creative definition (viz., Px ≡…x… wherein P does not appear in the matrix) but via phraseology that contains *definitive descriptions*: (tx) (x is a Sturm-Liouville factor). Such introductions qualify as legitimate definitional extensions *if and only if* the implied existence claim can be ratified beforehand. In the absence of a combinatorial operations based proof, set theoretic construction plays a critical role in amplifying concerns of 'physical quantity' to workable proportions concerning target systems. Rather than rigidified semantics, via Sturm and Liouville, we see how special traits can be located within a 'mildly transcendental' manner, "as the system invariants that, allow us, inter alia, to carry out a long series of shooting method computations that provide an increasingly accurate fix on the target system's true behaviors", factoring upon complex behaviors to enclose upon substantial traits (e.g., topology; 404).

The classic remedy, the *vector equation over a manifold*, is an intuitive corrective, one where we periodically replace those charts of a descriptive problem—for instance, when we calculate a bird's flight—with alternative maps centered upon new locales to modify apparent coordinate magnitudes by suitable

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[97] Similarly, on the continental side, Alain Badiou appeals to set theory to constitute the register in which one thinks consistently of inconsistency, presenting the form of inconsistency that underlies all consistent presentation. Drawing the historical context for his dialectical alternative, in *Being and Event*, Badiou systematically develops a meta-ontological narrative that identifies ontology with the theory of inconsistent multiplicity, and the latter in turn with Zermelo-Fraenkel axiomatized set theory. In relation to the ontological order of the pure multiple and the objective order of presentation, becoming or subjectivation emerges as unwarranted or "illegal" interruptions of the stability and stasis of the ontological order, implying a process which Badiou names an "event." This initiates a creative process of construction, or "truth-procedure." These "truth-events" are manifested in ordinary situations (i.e., "worlds") across four domains of thinking and practice, functioning as the "conditions" which philosophy aims to think together, relative to its historical moment: science, art, politics, and love. Within these domains, "truth-functions" form as an exception to knowledge or representation, in the sense of disrupting the objective distribution of "bodies and languages" through which one discerns coherent parts within a situation or "world". For Badiou, the "truth-event" signals the emergence of a "strong singularity," making the "inexistent" of a world appear with maximal intensity. See: Alain Badiou, *Being and Event*, trans. Oliver Feltham, New York, Continuum, 2006.

adjustment factors. Insofar as we consider a bird flying across the globe, with its dominion of low numerical calculators, we can supplement the manifold, itself, with set-theoretic constructions based on computational *opportunities* (i.e., Euler's rule calculations that can parallel the bird's flight to a reasonable degree of accuracy, which we will improve). Thus, we ascertain correctible relationships between a target reality and concrete computational capacity. This remedy, pioneered by Dedekind, Weyl, and company, utilizes equivalence class techniques, abstracting an invariant target from more concrete appearances and using set theoretic techniques of framing to make more precise repairs, invoking transcendental departures from raw computational capacity.

Unlike Leibniz, Euler accepted the 'mildly transcendental' nature of the differential equation, itself, as an adequate mathematical description. We follow transcendental consideration when considering how the word 'manifold' has, throughout this progression, co-opted a rich descriptive model for understanding a physical target, codifying the inferential procedures that follow in its reasoning. Rather than merely reducing set theory's key objectives as 'reductive', (i.e., reducing ontological commitments) or 'mereological' (i.e., capturing a notion of 'part' and 'whole'), we can use it to articulate an enlarged portrait of our conceptual abilities.

Can applied mathematics resolve Descartes' concerns? We have considered his remarks regarding applied mathematics' contention with indefinite behaviors in constricting pipe vis-à-vis "patched-together" asymptotics. So, we can alleviate Descartes' pipe-constriction problem by employing differential equations.[98] Nonetheless, Descartes also articulated a second set of inadequacies regarding those curved paths that readily appear in nature, which are beyond the reach of geometrical description. Such curves can not be described as mechanical or imaginary "on the grounds that their contours can only be represented as images within the faculty of the imagination and not through rules cognizable by our

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[98] In addition, actual circumstances also necessitate that we contend with "counterflows" that involve sheets of fluid sliding past one another in Helmholtz's fashion, forcing mathematicians to address non-trivial questions of jump conditions. See: Anatoly I. Ruban and Jitesh S. B. Gajjar, *Fluid Dynamics: Part I: Classical Fluid Dynamics,* Oxford, Oxford University Press, p, 204. Vladimir Shtern, *Counterflows: Paradoxical Fluid Mechanics Phenomena,* Cambridge, Cambridge University Press, 2012.

purely intellectual powers" (409). Accordingly:

> "[g]eometry should not include lines that are like strings, in that they are sometimes straight and sometimes curved, since the rations between straight and curved lines are not known, and I believe cannot be discovered by human minds, and therefore no conclusion based upon such ratios can be accepted as rigorous and exact."[99]

For instance, take the random configuration that a string assumes when casually placed upon a table. While the ancient Greek geometers accepted these curves as merely geometrical, according to Descartes, all points in such curves which we may call 'geometric' necessarily "bear a definite relation to all points of a straight line and this relation must be expressed by means of a single equation."[100] Descartes' concern is with the methodological possibility of mathematics in grouping those successive states of curve-behavior together through inherently mathematical relationships uniquely distinct form the processes nature, itself, employs. According to mathematics' strong rules of analytic continuation, pieces are bound together in a single-generated function untrue to nature's fluid particles. One basic computational solution seems to construct partial solutions by extracting proposals from a modeling equation and matching them with an interior, introducing a replacement rule and a connection formula. This also introduces non-trivial questions of data harmonization (i.e., the problem of connecting data across a tear). This involves matching conditions across an interface where, to work with standard differential equation models successfully, we seek suitable descriptive locales where we can opportunistically position the interfaces, tears, boundary conditions, and so forth in order to patch them together through interior modelings in cooperative harmony. Thus, we forge an entanglement "with side condition requirements that encode other forms of physical process in *a more compressed manner*" (413; emphasis added).

There is, indeed, great benefit to modeling media with such alien ingredients, using crude repairs to profitably redirect our inferential forays. This shows a reason to resist the static conception of mathematics' obligations within science

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[99] René Descartes, *The Geometry of René Descartes*, trans. D. E. Smith and M. L. Latham, New York, Dover, 1954, p. 91

[100] Ibid., p. 48.

that is promoted by Quine. According to Quine's presumption, it is profitable to conceive of science as operating via all-at-once postulation, where at any given moment we accept an all-embracing Theory T that supplies us with a fixed vocabulary that we employ when operating within T's empirical ambit. According to Quine, as T's empirical prowess turns towards a different direction, we can seek a replacement T' and proceed as before, weaving together a patchwork via necessities of strategic adaptation, innovation, and inferential *monitoring*. Thus Quine's aperçu, that "the only mathematics that physics requires for its own purposes is whatever expressive tools are required to convey its fundamental postulates", reducing physics' needs to a low level with in set theory's analytic hierarchy (416). Epistemologically, however, with the Quinean Neurath's boat we are confronted with a moment in natural history where we face great conceptual difficulty in contending with syntactic disharmony. We simply can not gauge the degree to which future developments in concept formation will allow us to improve our descriptive grip on nature in a manner that can completely transcend the necessities to move from one descriptive opportunity to another. One day, we may move past this patchwork of descriptive fabric, stitched together in accordance to dominant behavior. Perhaps the enlarged assistance of set theory's conceptual tools will eventually allow for every process in nature to be fitted to an "appropriate set of *** without abrupt corrections," with suitable side conditions adjoined, "where the *** capture some wider sense of mathematical process than our 'differential equations'" (415). However, it is equally possible that we may have to eventually resign ourselves to a subdued descriptive opportunism, acknowledging that the successful application of mathematics to the world and, more specifically, to media, requires abrupt stitching on order to keep our reasoning running. Such strategic 'avoidance(s)' do not promote an anti-realistic philosophical doctrine; rather, they simply allow us to record the fact that certain forms of unfolding natural processes can not be adequately and perfectly tracked by effectively programmable numerical methods.

This does not mean that we have to abandon Quine's pragmatic empiricism with respect to linguistic meaning, however. Instead, it is the semantic rigidity of Putnam and Benacerraf that we oppose where, with respect of physical terminology (and not mathematics) we possess a firm and constant conception of the referential facts that must obtain within the external world for the sentential applications of that vocabulary to qualify as truth or false (417). This thesis is stilted by the a priori confidence of Frege, where nature and algorithmic struggles neatly contend. According to Benacerraf, any "physical proposition p places restrictions on what the world can be like" and "our knowledge of the world, combined with our understanding of the restrictions placed by p, given by the [referential] truth-conditions of p," tells us that a given individual can or can not "come into possession of evidence sufficient to come to know that p."[101] According to this position, a Tarskian theory of truth, we have a firm referential grip of truth-conditions and how they structurally relate to the inferential policies we apply to propositions: "logical relations are subject to uniform treatment: they are invariant with subject matter" and, "[i]ndeed, they help define the concept of 'subject matter'", for the very "same rules of inference may be used and their use accounted for by the same theory which provides us with our ordinary account of inference."[102]

According to this doctrine, our physical claims experience no difficulties in aligning themselves with exterior truth-values despite mathematics cannot tie its own references causal bonds. That is, physical vocabulary earns its inferential and referential credence in a direct manner, an appeal to truth-conditionings that is entirely based on component-decompositional policies of reference-linkage. However, reality proves stochastic, where such Tarski-style soundness proofs of the external world—which support such a simple predicate-to-extension picture—collapse under scrutiny. Indeed, these inferential warrants are always provisional, "hostage to the consideration that they may rest upon a faulty picture of how physical information is actually encoded within a descriptive language" (419). Again, *none of this is to suggest that logic is empirical.* Rather, those semantic pictures—i.e., theses about word/world relationships—that we rely upon when deciding if our vocabularies conform to familiar logical categories or not can, indeed, prove erroneous. So, the problem is not with logic, but with semantic diagnosis. While logic creates the inferential purview of a scientific discipline, its real-world application limits its correlational portraiture; no matter how clearly

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[101] "Mathematical Truth" in Putnam and Benacerraf, *Philosophy of Mathematics*, p. 413.

[102] Ibid., p. 411.

laws codify scientific premises, real-world application produces isolated patches, or protectorates, that are held down solely through homogenization and other forms of counterfactual asymptotic stitching. The inferential imperatives of implementing a useful strategic opportunity outweigh the utilities of obeying logical rules. In turn, a true naturalist position accepts that any substantial mathematics is required to ascertain how we reason with respect to everyday physical vocabularies, rather than accepting the picture of word/world correlation. Our naturalism is adaptive, diversifying in response to shifting currents and scales, profiting from new data and settings, providing us with a continuous picture of mathematical thinking. Such a process of coeval logical and metaphysical explication thereby captures the 'movement' or necessary relations of material incompatibility ('determinate negation') and consequence ('mediation') that holds between concepts and states of affairs alike, which is always implicit in the way consciousness inferentially relays its object.