

## **The impacts of Logic, Paradoxes in one side and Theory of Computation in the other side**

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(The first report)

This is a presentation about the impacts of Logic and Theory of Computation. It starts by some explanations about Theory of Computation and its relations with the other subjects in science. Then we have some explanations about paradoxes and some historical points. In continuation, we present some of the most important paradoxes. Forthcoming, Five subjects around the relations between Logic and Theory of computation is introduced. Finally, we present a new approach to solve P vs NP problem via Paradoxes as it is shown in [10], [15].

Here, we explain the seminar, by describing the Power point.

1.What is the Theory of Computation?

1. Definition of Theory of Computation (Page 1,2)

The studying and research about the problems in Models of Computation (Mostly Mathematical Problems), the limitations and capabilities of these Models.

- 2. Theory of Computation Parts (Page3)
- 
- This Theory consists of
- 
- 1.Computability Theory
- 2.Complexity Theory
- 3.Theory of Automata & Languages

This Theory is based on “Church Turing” Thesis. (Page 4)

The Scientific subjects near to the Theory of Computation. (Page 5)

- 1. Logic

- 2. Mathematics (Discrete Mathematics & Continuous Mathematics)
- 3. Physics

#### 4. Theory of Algorithms, AI

A Comparison to the other Mathematical subjects.

- Comparing to the other subjects in Mathematics:
  - 1. Algebra, Analysis, Geometry
  - 2. Discrete Mathematics, Logic, Theoretical Computer Science

Here, we divide the Mathematical subjects to two groups of elder and new,

1. Geometry has a great impact on Logic in 19 century. Before that, in ancient time Logic impacts Geometry heavily (Euclid, Principles)

2. Algebra and Category Theory along with the concepts in Algebraic Geometry involves Philosophical concepts and they impact logic. Category theory could be considered as foundation of Mathematics in a natural way.

The relation between Theory of Computation and logic goes to be deeper and deeper.

Also, we have a comparison between Physics and Theory of Computation.

The deep connection between this theory and Physics is so natural. Since this is based on modeling Physical instruments & Computational Models although in theory we have some changes like infinite memory in Turing Machine.

The relation between Theory of computation and Logic are somewhat deep and bizarre. (personal point of view) (Page 7)

- 1. The relations between Theory of Computation and Physics and Logic are deep and natural.
- 2. The relations between Theory of Computation and Paradoxes are deep and somewhat bizarre.
- Page 8
- 1. The relation between Theory of Computation and Physics is deep and natural.
- Since it is based on Mathematical Modeling of some Physical instruments.
- Examples:
  - Turing Machine, Circuits, Quantum Computers
  - (Page 9)
- 2. The relations between Theory of Computation and Paradoxes are deep and somewhat bizarre.
- 3. It goes to be deeper and deeper
- We show it in below, in six sections

## The Subjects

(Page 10)

- 1. Liar Paradox and Halting, Incompleteness of Gödel
- 2. Berry Paradox and Kolmogorov Complexity and Chaitin Incompleteness Theorem
- 3. Logic and Hierarchies in Computability Theory and Complexity Theory
- 4. Curry Howard Theorem
- 5. PCP Theorem
- 6. *Unexpected Hanging Paradox and Fuzzy Time*

But first we try to explain Paradoxes more:

### What is Paradox? (Page 11)

Paradox (Cambridge Dictionary)

- A situation or statement that seems impossible or is difficult to understand because it contains two opposite facts or characteristics:
- Contra-intuitive conclusions based on an argument.
- It's a curious paradox that drinking a lot of water can often make you feel thirsty.

- Examples (Page 12)
- 
- Liar Paradox, Zeno Paradox,
- Berry Paradox, Russell Paradox
- Unexpected Hanging Paradox

Three Types of Paradoxes (Page 13)

- 1. Falsidical
- 2. Veridical
- 3. Antinomy (\*)
- 

In the progress of thought systems, paradoxes come in the middle. Usually from the first steps and attempts.

Here, as an example related to our topic, we present a part of Greek history. From the time of Epimenides and a possible spirit of shamanism to the time of writings of Euclid and Apollonius!

During taking shape of these thought systems, we face different types of paradoxes.

Possibly, people forget some of these paradoxes or their proposed solutions. But it might be soon or late these paradoxes come back to them.

In the thought systems related to Modern era specially in 19 & 20 century, the paradoxes, the old and new ones play a much important role. After introducing some Paradoxes which are more central in our topic, firstly we introduce the findings in five sections as follows

- 1. Liar Paradox and Halting, Incompleteness of Gödel
- 2. Berry Paradox and Kolmogorov Complexity and Chaitin Incompleteness Theorem
- 3. Logic and Hierarchies in Computability Theory and Complexity Theory
- 4. Curry Howard Theorem
- 5. PCP Theorem
- 6. *Unexpected Hanging Paradox and Fuzzy Time*

Forthcoming,.....

Descriptions, Truth, Arguments,

The relations between Theory of Computation and Paradoxes

- Epimenes of Crete 7th or 6th century BC (Liar Paradox)
- 'Pythagoras the Samian', c. 570 – c. 495 BC)
- Parmenides of Elea (Late sixth or early Fifth Century) & Democritus(460-370)
- Zeno of Elea (c. 490–430 BC) Zenon Paradox
- Athen
- Sophists (4<sup>th</sup> -5<sup>th</sup> BC)
- Socrates (470-399 BC), Plato(424-348 BC) , Aristotle (384-322 BC)

Systems (Page 15)

- Alexandria
- Euclid(Elements) (323–283 BC)
- Apollonius Conic Sections (240 BC – 190 BC)
- Heron of Alexandria, also called **Hero**, (flourished c. ad 62, Alexandria, Egypt), Diophantus of Alexandria AD 200 and 214

Paradoxes in Modern Era (page 16)

- 1.Zeno Paradox
- 2.Liar Paradox
- 3. Berry Paradox

- 4. Russel Paradox
- 5. Unexpected Hanging Paradox

## Again Paradoxes and Systems in 19 & 20 Century (page 17)

- Axiomatic Systems in 19 century
- Russel Paradox (Frege & ...)
- Liar Paradox & Berry Paradox (Peano Axioms, Hilbert & Formalism)
- 

1. Liar Paradox and Halting, Incompleteness of Gödel, Tarski Theorem (Page 18)
2. Liar Paradox and Halting
3. Berry Paradox

Liar Paradox and Halting, Incompleteness of Gödel, Tarski Theorem (Page 19)

- **Incompleteness of Gödel**  
**Tarski Theorem**

Berry Paradox and Kolmogorov Complexity and Chaitin Incompleteness Theorem (page 20)

- 1. Berry Paradox
- 2. Kolmogorov Complexity <https://math.ucr.edu/home/baez/surprises.html>
- <https://www.mdpi.com/1099-4300/13/3/595/htm>
- 3. Chaitin Incompleteness

## Kolmogorov Complexity (page 21)

- Definition
- $C_f(x) = \min\{|p| : f(p) = x\}$  if  $x \in \text{ran } f$ ,  $\infty$  otherwise

## Kolmogorov Complexity Theory (Page 22)

- UTM
- $M_1, M_2, \dots$
- $M_0, M_1, M_{00}, M_{01}, M_{10}, M_{11}, M_{000}, \dots$
- Complexity of string  $x$
- The least string  $y$  such that  $M_y(0)=x$ ; as an output consider  $|y|$ .

## Kolmogorov Complexity Theory (Page 23)

$K(x)$  is not computable.

Problem: The smallest Program which create a geometrical Object.

## Chaitin Incompleteness Theorem

### Chaitin Incompleteness Theorem (Page 24)

- For any UTM, there is a number  $L$  such that

for any string of bits  $x$ , we are not able to prove

$$K(x) > L.$$

### Why Chaitin Incompleteness Theorem? (Page 25)

- 1. The idea in proof
- 2. The relation with First and Second Incompleteness Theorem
- 
- Kritchman-Raz (2010)
- Chaitin Incompleteness (+ Considering Unexpected Hanging Paradox) implies
- Second Incompleteness Theorem

### The Hardest Logic Puzzle Ever (Break, Page 26)

- George Boolos, 'The Hardest Logic Puzzle Ever'. *The Harvard Review of Philosophy*, Volume 6 (1996), pp.62-65 <https://doi.org/10.5840/harvardreview1996615>.
- Raymond Smullyan
- Hilary Putnam

### 3. Logic and Hierarchies in Computability Theory and Complexity Theory (Page 27)

- [Computability Theory - Recursive Enumerable Sets | Ray \(oneraynyday.github.io\)](#)

### 4. CURRY HOWARD CORRESPONDENCE (Page 28)

- BESIDES MANY SUBJECTS, Type Theory....
- <https://prism.ucalgary.ca/bitstream/handle/1880/112761/Zach%20-%202019%20-%20The%20Significance%20of%20the%20Curry-Howard%20Isomorphism.pdf?sequence=1&isAllowed=y>
- <https://www.pédrot.fr/slides/inria-junior-02-15.pdf>

### 5. PCP Theorem (Page 29)

- the PCP theorem by Arora, Lund, Motwani, Sudan, and Szegedy in 1998, 2001 Gödel Prize
- In 2005 Irit Dinur discovered a significantly simpler proof of the PCP theorem. She received the 2019 Gödel Prize for this. <sup>[5]</sup>
- 
- The equivalent Logical Form of PCP (Page 30)
- ....

The ongoing research:

#### 6. Unexpected Hanging Paradox and Fuzzy Time (Page 31)

- 1. Unexpected Hanging Paradox....
- 2. History of Unexpected Hanging Paradox
- 3. New version, a contradiction not a paradox
- 4. Solutions( Epistemological, Logical) (Quine)
- 4.1 It is not a Paradox (Quine)
- 4.2 Changing Logic
- 4.3

History (Page 32)

- 1. Lennart Ekblom (1943-1944) & Daniel John O'Connor (1948)
- (Pragmatic Paradoxes, DJO'Connor, Mind, volume lvii, Issue 227, 1948)
- 2. Quine (1953)
- Quine, W.V.O. "On a So Called Paradox." Mind 62, 65-67, 1953
- 

History (Page 33)

- 3. Gardner
- "It was first introduced to the public in Martin Gardner's March 1963 Mathematical Games column in *Scientific American* magazine." wikipedia
- **The Unexpected Hanging and Other Mathematical Diversions First PB Edition, 1991**
- 4. Kripke. Philosophical Troubles: Collected Papers, Volume 1, On Two Paradoxes of Knowledge, Saul Kripke, Oxford University Press, 2002,

History (Page 34)

5. Timothy Chow

(American Monthly)

The Surprise Examination or Unexpected Hanging Paradox, 1998

6. Dean Clark, How expected is the Unexpected? <https://www.jstor.org/stable/2690559>

Solutions (Epistemological, Logical) (Page 35)

- In general, What do we mean by the solution of a Paradox?
- In comparison to Mathematical Solving Problem.
- 

Solutions (Page 36)

- Solving a paradox is not solving a Mathematical Problem.
- Usually in Mathematical problem, the logic, theory and Model is clear.

Solutions (Page 37)

- A solution to the surprise exam paradox in constructive mathematics • Authors
- Mohammad Ardeshir, Rasoul Ramezani
-

Shira, Krichman (Page 38)

- Incompleteness (+ Considering Unexpected Hanging Paradox) implies
- Second Incompleteness Theorem
- 

Modified version of the Paradox and the result (Page 39)

- Is Classical Mathematics Appropriate for Theory of Computation?
- 

Modified version of the Paradox and the result (Page 40)

- 1.  $\forall \varphi \exists i \in \mathbb{N} \varphi(i)$
- 2.  $\sim \varphi(n) \rightarrow [A: [\sim \varphi(n)]]$  (In the evening of each day, A understands he is not destroyed and he declares it).
- 3.  $\varphi(k) \rightarrow \bigwedge \sim \varphi(i) \forall i=1, i \neq k$  (If he is destroyed in a day, we conclude that in the other days he wasn't destroyed).
- 4.  $(\bigwedge A: [\sim \varphi(i)] \forall i=1, \dots, k-1) \wedge (\bigwedge A: [\sim \varphi(i)] \forall i=k+1, \dots, 7) \rightarrow A: k [\varphi(k)]$
- 5.  $A: k [\varphi(k+1)] \rightarrow \sim \varphi(k+1)$  (If A utters in the  $i$ th day that he will be executed in the  $i+1$ th day, he will not be executed in  $i+1$ th day,  $i=0,1,\dots,6,7$ )
- 

Fuzzy Time (Page 41)

- Brouwer and Husserl
- Physics (Time as an operator)
- Solution of unexpected Hanging Paradox
- 
- Fuzzy Time (Page 42)
- The Problem Has Physical aspects.
- Time is a Physical concept.
- Have we any clue in Physics?
- Quantum Mechanics and different interpretations of it (Page 43)
- **Copenhagen interpretation**
- **Many worlds**
- **Bohmian Theory**
- 

Signs (Page 44)

*Probabilistic time, C. Wetterich, Foundation of Physics, 2012, Springer*

One step back (Page 45)

- Explanation of PhD Thesis Problem
- Logical approach
- 

Why this problem has not been solved yet? (Page 46)

- The proposed solutions are usually based on Theorems and definitions in the field setting of Combinatorics, Probability, Models of Computations....
- Models of Computations: Circuits, Turing Machines,...

- Three major ways which are failed:
- 1.Paralizacion, relativization
- 2.Algebrization
- 3. Natural Proof
- 

Possibly, we should find a way in much fundamental parts (Page 47)

- Logic
- Shub, Smale,....
- B.Poizat
- Huroshovski suggestion
- The Thesis

The Continuation of the work (Page 48)

- Rapid elimination of Quantifiers
- Checking the other ways
- 1. Combinatorics
- 2. Orders
- 3. Employing Paradoxes +
- 4.Physical Models and Bioinformatics....

Employing Paradoxes (Page 49)

- Similar to Gödel ways
- The letter of Gödel to Von Neumann
- 

P vs NP (Page 50)

- P, BPP, NP

BQP, QAM

Reductions

4 conclusions (Page 51)

- **TC + CON(TC\*) + P ≠ NP**
- 2. **TC\* + P\* ≠ NP\* , P\* = BPP\***
- 

4 conclusions (Page 52)

- **¬CON(TC)**
- **4. CON(QM) → CON(TC\*)**
- (Page 53)

By CON (T) for a theory T we mean this theory is consistent and the existence of at least a model which theory holds in that.

Computing Fuzzy Time Function (Page 54)

- Schrodinger Equation, Dirac Equation
- The Integral Equation
- How to solve it?

Equations (Page 55)

$$1. X(t, x, y, z) = \int_{z=-\infty}^{z=+\infty} \int_{y=-\infty}^{y=+\infty} \int_{x=-\infty}^{x=+\infty} \int_{t=-\infty}^{t=+\infty} X(x - x', y - y', z - z', t - t',) f(t - t',) dt dx dy dz$$

$$= \int_{t=-\infty}^{t=+\infty} \left( \int_{x=-\infty}^{x=+\infty} \int_{y=-\infty}^{y=+\infty} \int_{z=-\infty}^{z=+\infty} X(x - x', y - y', z - z', t - t',) f(t - t',) dx dy dz \right) dt$$

$$2. X = \psi$$

$$3. i\hbar \frac{\sigma}{\sigma(t)} \psi(\vec{r}, t) = \left[ \frac{-\hbar^2}{2m} \nabla^2 + V(\vec{r}, t) \right] \psi(\vec{r}, t)$$

Computing Fuzzy Time Function (Page 56)

1. *Computing Fuzzy Time Function, F.Didehvar, Philpaper, SSRN, 2022*
2. *Does accepting Fuzzy Time-Particle interpretation of Quantum Mechanics, refutes the other interpretations? (Is Fuzziness of time checkable experimentally?), F.Didehvar, 6/1/2022, SSRN*

P vs NP, Article & Conclusions (Page 57)

- $P \neq NP$ , by Accepting to Make a Shift in the Theory (Time as a Fuzzy Concept)
- P vs NP, Article & Conclusions (Page 58)
- Zeno Paradox, Unexpected Hanging Paradox (Modeling of Reality & Physical Reality, A Historical-Philosophical view), Philpapers

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