First-order belief and paraconsistency

Srećko Kovač

Abstract

A first-order logic of belief with identity is proposed, primarily to give an account of possible *de re* contradictory beliefs, which sometimes occur as consequences of *de dicto* non-contradictory beliefs. A model has two separate, though interconnected domains: the domain of objects and the domain of appearances. The satisfaction of atomic formulas is defined by a particular S-accessibility relation between worlds. Identity is non-classical, and is conceived as an equivalence relation having the classical identity relation as a subset. A tableau system with labels, signs, and suffixes is defined, extending the basic language $\mathcal{L}_{\mathbf{QB}}$ by quasiformulas (to express the denotations of predicates). The proposed logical system is paraconsistent since $\phi \wedge \neg\phi$ does not "explode" with arbitrary syntactic consequences.

Keywords: appearance, belief, identity, labelled and signed tableau, object, paraconsistent, tableau suffix.

1 Introduction

Due to some specific properties and relations of objects, and due to limitations of a reasoning agent's knowledge, a multiplicity of real and possible objects can appear as fused into one "object" (appearance), and, conversely, one object can appear as split in many "objects" (appearances).[1] In such a context, *de re* contradiction in an agent's belief can arise precisely as a consequence of the agent's *de dicto* non-contradictory belief. Naturally, there are also real and possible objects which do not appear in an agent's awareness at all, and about which the agent does not believe anything at all. In this paper we present a logic of reasoning with real and possible objects, and real and possible appearances of the objects. The main distinguishing point with respect to approaches in [17, 18, 16, 15, 9, 8] (discussed in [14]) is to introduce contradictory *de re* beliefs, and to allow them to be consequences of a *de dicto* non-contradictory belief. We partially revise and further develop the semantics of [14], and propose a corresponding labelled and signed tableau system with suffixes.

There are some characteristic technical features of the logic here proposed. 1) Appearances of objects are modeled by ordered pairs $\langle d, k\rangle$, where d is an object and k an individual constant. The constant k serves as an agent's "mode" through which the object is presented and referred to.[2] 2) To allow contradictory beliefs we introduce the second accessibility relation, S, on the set of possible worlds in determining the satisfaction of an atomic formula. That results with the use of "subatomic" "quasiformulas" in the tableau system. 3) The identity relation is non-classical, and includes the classical identity relation as a subset.

2 Language and models

The language $\mathcal{L}_{\mathbf{QB}}$ is a first-order modal language for a logic of belief. Individual constants are c, c_1, c_2, ... (set $\mathcal{C}$; informally, other small Latin letters will also be used); $x, y, z, x_1, \ldots$ are individual variables (set $\mathcal{V}$); P^n, P^n_1, P^n_2, ... (other capital letters will be used informally), $=$, and E^1 are n-place predicates (set $\mathcal{P}$); there are connectives $\neg$ and $\wedge$, quantifier symbol $\forall$, abstractor λ, belief operators $B_1, \ldots, B_n$, and parentheses ($\vee, \rightarrow$, and $\exists$ are defined in the familiar way). Formulas are

[1]For the first case, see, for example, the narrative of the fusion of the two authors of the *Principia mathematica* into one apparent author [17, 18]. The second case is well known, for example, from Frege's Phosphorus–Hesperus puzzle [10].

[2]See [17, 18] for the comparison with the mode of presentation concept.

$\Phi^n t_1 \ldots t_n$, $\neg\phi, (\phi \wedge \psi), B_i\,\phi$, $\forall x\, \phi$, and $(\lambda x.\phi)(k)$ (ϕ and ψ are formulas, Φ^n is a predicate, t_i a term, k an individual constant, and $(\lambda x.\phi)$ is an abstraction term). λ-abstraction disambiguates the sense in which an individual constant should be taken. For instance, in $B_i(\lambda x.Px)(c)$, c is λ-dependent and is taken in the sense in which i understands c (*de dicto*); in B_iPc, c is taken objectively and independently of an agent i (*de re*) (see, e.g., [7]).

We try to keep the basis of the semantics classical as far as possible. To that end, the interpretation of all descriptive predicates at a world is classical, and non-classical features of the satisfaction of the formulas are achieved through the definition of the satisfaction at a world w by means of the interpretation of predicates not at w, but at worlds S-accessible to w. Only identity is interpreted at a world non-classically, precisely, as $\cong$ relation, which is conceived as an extension of the classical identity relation. Further, we introduce a special domain A of appearances (beside the classical domain D of objects), but in a way that keeps track of objects (real and possible) in their appearances (an object d is always a constituent of an appearance). Domains of an agent's accessible worlds are restricted to the objects as they appear to the agent (objects in set A). We note that frame presupposes names (set $\mathcal{C}$) of the language $\mathcal{L}_{\mathbf{QB}}$.

Definition 1 (Frame) *Frame* $\mathcal{F} = \langle W, W_A, R_1, \ldots, R_n, S, D, A, Q, \{\cong_w\}_{w \in W}\rangle$*, where*

1. *W is a non-empty set of worlds ($w \in W$),*
2. *$W_A \subseteq W$,*
3. *$R_i \subseteq W \times W_A$ (serial, transitive, and euclidean; i is a belief agent),*
4. *$S \subseteq W \times W$ (reflexive),*
5. *D is a non-empty set of objects,*
6. *$A \subseteq \{\langle d, k\rangle \mid d \in D$ and $k \in \mathcal{C}\}$ (a set of appearances),*
7. *$Q : W \longrightarrow \wp U \setminus \{\varnothing\}$, where $Q(w \in W_A) \in \wp A \setminus \{\varnothing\}$, and if wSw' then $Q(w) = Q(w')$ ('U' abbreviates '$D \cup A$'),*
8. *for each w, $\cong_w \subseteq U \times U$ such that $\cong_w$ is an equivalence relation.*

In the further text, d will be a member of D, a a member of A, and u a member of U; also

$D_w = Q(w) \cap D$

$A_w = \mathcal{Q}(w) \cap A$

$U_w = D_w \cup A_w$

Definition 2 (Model) *Model* $\mathfrak{M} = \langle \mathcal{F}, V \rangle$, *where*

1. $V(k) \in D, \qquad V(k, w) \subseteq \{d, \langle d, k \rangle \mid \langle d, k \rangle \in A\} \setminus \{\varnothing\}$,
2. $V(\Phi^n, w) \in \wp U^n$, *closed under* $\cong_w$,
3. $V(=, w) = \cong_w$,
4. $V(E^1, w) = \mathcal{Q}(w)$

As we can see, individual constants are sometimes rigid and sometimes non-rigid, and it will be determined below in which context they are used rigidly and in which non-rigidly. Non-rigid interpretation treats an individual constant as a "mode of presentation" of objects. Keeping track of the objects presented is vital for reasoning from the *de dicto* to the *de re* sense of terms.

Definition 3 (Variable assignment) Variable assignment *is a mapping* $\mathfrak{v} : \mathcal{V} \longrightarrow U$. Variant of a variable assignment $\mathfrak{v}$ *is a variable assignment* $\mathfrak{v}[u/x]$ *that differs form* $\mathfrak{v}$ *at most in assigning* u *to* x.

Definition 4 (Designation of a term)

1. $[\![k]\!]^{\mathfrak{M},w}_{\mathfrak{v}} = V(k)$ *and* $[\![x]\!]^{\mathfrak{M},w}_{\mathfrak{v}} = \mathfrak{v}(x)$, *where* $[\![t]\!]^{\mathfrak{M},w}_{\mathfrak{v}}$ *is the designation of a term* t *in a model* $\mathfrak{M}$ *(at a world* w*) for a variable assignment* $\mathfrak{v}$, *and where* k *is an individual constant,*
2. $[u]_w = \{u' \mid u' \cong_w u\}$.

3 Satisfaction and consequence

In the definition of satisfaction below, we separately define positive, T-, and negative, F-satisfaction to enable modeling contradictory beliefs. We modalize the satisfaction of atomic formulas by S-accessibility relation and choose S-necessity for the satisfaction of atomic formulas about appearances to avoid classical inconsistencies of *de dicto* beliefs. In particular, to avoid classical inconsistencies of quantified *de dicto* beliefs of an agent i, domains of i-accessible worlds are restricted to set A (see Definition 1). For the satisfaction of atomic formulas about objects S-possibility is chosen. Such a choice of S modalities is motivated by an intuition that i will have more logical control over i's *de dicto* beliefs, than over i's *de re* beliefs.[3] Further, in a special case (2b), things are identical at w if their respective $\cong$-counterparts are each other's $\cong$-counterparts in an S-accessible world. In that way we will obtain a desired consequence that identical thing(s) do not have to share all their properties.

In the following definition, Φ^n is an n-place predicate, excluding = and E.

Definition 5 (Satisfaction)

1. (a) *If* $[\![t_1]\!]^{\mathfrak{M},w}_{\mathfrak{v}}, \ldots, [\![t_n]\!]^{\mathfrak{M},w}_{\mathfrak{v}}$ *are* $a_1 \in A, \ldots, a_n \in A$, *respectively, then*

   $\mathfrak{M}, w \overset{\mathrm{T}}{\models}_{\mathfrak{v}} \Phi t_1 \ldots t_n$ *iff* $(\forall w' wSw')\langle a_1, \ldots, a_n\rangle \in V(\Phi, w')$,

   $\mathfrak{M}, w \overset{\mathrm{F}}{\models}_{\mathfrak{v}} \Phi t_1 \ldots t_n$ *iff* $(\forall w' wSw')\langle a_1, \ldots, a_n\rangle \notin V(\Phi, w')$,

   (b) *if* $[\![t_1]\!]^{\mathfrak{M},w}_{\mathfrak{v}}, \ldots, [\![t_n]\!]^{\mathfrak{M},w}_{\mathfrak{v}}$ *are* $u_1, \ldots, u_n$, *respectively, and at least one* $u_i \in D$, *then*

   $\mathfrak{M}, w \overset{\mathrm{T}}{\models}_{\mathfrak{v}} \Phi t_1 \ldots t_n$ *iff* $(\exists w' wSw')\langle u_1, \ldots, u_n\rangle \in V(\Phi, w')$,

   $\mathfrak{M}, w \overset{\mathrm{F}}{\models}_{\mathfrak{v}} \Phi t_1 \ldots t_n$ *iff* $(\exists w' wSw')\langle u_1, \ldots, u_n\rangle \notin V(\Phi, w')$,

2. (a) *If* $[\![t_1]\!]^{\mathfrak{M},w}_{\mathfrak{v}}, [\![t_2]\!]^{\mathfrak{M},w}_{\mathfrak{v}}$ *are* $a_1 \in A, a_2 \in A$, *respectively, then*

   $\mathfrak{M}, w \overset{\mathrm{T}}{\models}_{\mathfrak{v}} t_1 = t_2$ *iff* $(\forall w' wSw')\, a_1 \cong_{w'} a_2$,

   $\mathfrak{M}, w \overset{\mathrm{F}}{\models}_{\mathfrak{v}} t_1 = t_2$ *iff* $(\forall w' wSw')\, a_1 \ncong_{w'} a_2$,

   (b) *if* $[\![t_1]\!]^{\mathfrak{M},w}_{\mathfrak{v}}, [\![t_2]\!]^{\mathfrak{M},w}_{\mathfrak{v}}$ *are* u_1, u_2, *respectively, and at least one* $u_i \in D$, *then*

   $\mathfrak{M}, w \overset{\mathrm{T}}{\models}_{\mathfrak{v}} t_1 = t_2$ *iff* $(\exists w' wSw')(\exists u'_1 \in [u_1]_w)(\exists u'_2 \in [u_2]_w)\, u'_1 \cong_{w'} u'_2$,

   $\mathfrak{M}, w \overset{\mathrm{F}}{\models}_{\mathfrak{v}} t_1 = t_2$ *iff* $(\exists w' wSw')(\exists u'_1 \in [u_1]_w)(\exists u'_2 \in [u_2]_w)\, u'_1 \ncong_{w'} u'_2$,

3. $\mathfrak{M}, w \overset{\mathrm{T}}{\models}_{\mathfrak{v}} Et$ *iff* $[\![t]\!]^{\mathfrak{M},w}_{\mathfrak{v}} \in Q_w$,
   $\mathfrak{M}, w \overset{\mathrm{F}}{\models}_{\mathfrak{v}} Et$ *iff* $[\![t]\!]^{\mathfrak{M},w}_{\mathfrak{v}} \notin Q_w$,

4. $\mathfrak{M}, w \overset{\mathrm{T}}{\models}_{\mathfrak{v}} \neg\phi$ *iff* $\mathfrak{M}, w \overset{\mathrm{F}}{\models}_{\mathfrak{v}} \phi$,
   $\mathfrak{M}, w \overset{\mathrm{F}}{\models}_{\mathfrak{v}} \neg\phi$ *iff* $\mathfrak{M}, w \overset{\mathrm{T}}{\models}_{\mathfrak{v}} \phi$,

[3] The idea of modalizing the satisfaction of formulas is familiar in paraconsistent logic. For instance, $\phi \wedge \psi$ was interpreted by S. Jaśkowski in his discussive logic [12] (see also [13]) as $\phi \wedge \Diamond\psi$. In J.-Y. Béziau [1, 2] the approach is generalized to a specific four-valued logic, where the four values $0^-, 0^+, 1^-$ and 1^+ are conceived as "necessarily false", "possibly false", "possibly true", and "necessarily true", respectively.

5. $\mathfrak{M}, w \models^{\mathrm{T}}_{\mathfrak{v}} (\phi \wedge \psi)$ *iff* $\mathfrak{M}, w \models^{\mathrm{T}}_{\mathfrak{v}} \phi$ *and* $\mathfrak{M}, w \models^{\mathrm{T}}_{\mathfrak{v}} \psi$,
   $\mathfrak{M}, w \models^{\mathrm{F}}_{\mathfrak{v}} (\phi \wedge \psi)$ *iff* $\mathfrak{M}, w \models^{\mathrm{F}}_{\mathfrak{v}} \phi$ *or* $\mathfrak{M}, w \models^{\mathrm{F}}_{\mathfrak{v}} \psi$,

6. $\mathfrak{M}, w \models^{\mathrm{T}}_{\mathfrak{v}} B_i \phi$ *iff* $(\forall w' wR_i w')\ \mathfrak{M}, w' \models^{\mathrm{T}}_{\mathfrak{v}} \phi$,
   $\mathfrak{M}, w \models^{\mathrm{F}}_{\mathfrak{v}} B_i \phi$ *iff* $(\exists w' wR_i w')\ \mathfrak{M}, w' \models^{\mathrm{F}}_{\mathfrak{v}} \phi$.

7. $\mathfrak{M}, w \models^{\mathrm{T}}_{\mathfrak{v}} \forall x \phi$ *iff* $(\forall u \in U_w)\ \mathfrak{M}, w \models^{\mathrm{T}}_{\mathfrak{v}[u/x]} \phi$,
   $\mathfrak{M}, w \models^{\mathrm{F}}_{\mathfrak{v}} \forall x \phi$ *iff* $(\exists u \in U_w)\ \mathfrak{M}, w \models^{\mathrm{F}}_{\mathfrak{v}[u/x]} \phi$.

8. $\mathfrak{M}, w \models^{\mathrm{T}}_{\mathfrak{v}} (\lambda x.\phi)(k)$ *iff* $(\forall u \in V(k, w))\ \mathfrak{M}, w \models^{\mathrm{T}}_{\mathfrak{v}[u/x]} \phi$,
   $\mathfrak{M}, w \models^{\mathrm{F}}_{\mathfrak{v}} (\lambda x.\phi)(k)$ *iff* $(\exists u \in V(k, w))\ \mathfrak{M}, w \models^{\mathrm{F}}_{\mathfrak{v}[u/x]} \phi$.

The idea of universal quantification over objects under the mode of presentation by a constant k in Definition 5, case 8, is due to R. Ye [17]. In distinction to the semantics presented here, the mode of presentation is in [17] agent dependent and allows empty set of objects.

Since disjunction and conditional are defined in the familiar way, the satisfaction cases for disjunction and conditional amount to the following:

- $\mathfrak{M}, w \models^{\mathrm{T}}_{\mathfrak{v}} (\phi \vee \psi)$ iff $\mathfrak{M}, w \models^{\mathrm{T}}_{\mathfrak{v}} \phi$ or $\mathfrak{M}, w \models^{\mathrm{T}}_{\mathfrak{v}} \psi$,
  $\mathfrak{M}, w \models^{\mathrm{F}}_{\mathfrak{v}} (\phi \vee \psi)$ iff $\mathfrak{M}, w \models^{\mathrm{F}}_{\mathfrak{v}} \phi$ and $\mathfrak{M}, w \models^{\mathrm{F}}_{\mathfrak{v}} \psi$,

- $\mathfrak{M}, w \models^{\mathrm{T}}_{\mathfrak{v}} (\phi \to \psi)$ iff $\mathfrak{M}, w \models^{\mathrm{F}}_{\mathfrak{v}} \phi$ or $\mathfrak{M}, w \models^{\mathrm{T}}_{\mathfrak{v}} \psi$,
  $\mathfrak{M}, w \models^{\mathrm{F}}_{\mathfrak{v}} (\phi \to \psi)$ iff $\mathfrak{M}, w \models^{\mathrm{T}}_{\mathfrak{v}} \phi$ and $\mathfrak{M}, w \models^{\mathrm{F}}_{\mathfrak{v}} \psi$.

Definition 6 (Satisfiability) *A set* Γ *of formulas is satisfiable iff there are* $\mathfrak{M}, w$ *and* $\mathfrak{v}$ *such that for each* $\psi \in \Gamma$, $\mathfrak{M}, w \models^{\mathrm{T}}_{\mathfrak{v}} \psi$.

Definition 7 (Consequence) $\Gamma \models \phi$ *iff, if* $\mathfrak{M}, w \models^{\mathrm{T}}_{\mathfrak{v}} \psi$ *for each* $\psi \in \Gamma$, *then* $\mathfrak{M}, w \models^{\mathrm{T}}_{\mathfrak{v}} \phi$.

Example 1 *A reasoning agent i may perhaps not know that Lewis Carroll is the same person as Charles Lutwidge Dodgson. Let a corresponding logical name for 'Lewis Carroll' be individual constant 'c', and for 'Charles L. Dodgson' individual constant 'd'. In the* de dicto *sense, the agent i distinguishes person c and person d, and hence, in the* de re *sense, i believes of the same person not to be self-identical. Further, the agent i may also think that the person which is Lewis Carroll for i is not the same person which is Lewis Carroll for an agent j.*

Let us define and picture a model $\mathfrak{M}$ *where:*

$V(c, w_1) = V(c, w_3) = \{Carroll, \langle Carroll, c\rangle\}$,

$V(d, w_1) = V(d, w_3) = \{Dodgson, \langle Dodgson, d\rangle\}$,

$V(c, w_2) = \{Carroll, \langle Carroll, c\rangle\}$,

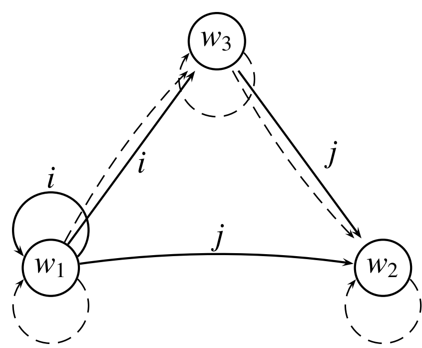
$w_1 : \langle Carroll, c\rangle \not\cong \langle Dodgson, d\rangle$, $Dodgson \not\cong \langle Carroll, c\rangle$, $Carroll \cong \langle Dodgson, d\rangle$,

$w_2 : \langle Carroll, c\rangle \not\cong \langle Dodgson, d\rangle, Dodgson \not\cong \langle Carroll, c\rangle, \langle Carroll, c\rangle \not\cong \langle Dodgson, c\rangle$,

$w_3 : \langle Carroll, c\rangle \not\cong \langle Dodgson, d\rangle, Carroll \cong \langle Carroll, c\rangle, Carroll \not\cong \langle Dodgson, d\rangle$

(as already mentioned, Carroll is classically identical with Dodgson).

In the figure bellow, full arrows represent i- and j-accessibility, while dashed arrows represent S-accessibility.



It can be shown (on the ground of Definition 5) that all the satisfaction claims 1–5 hold in the model $\mathfrak{M}$ *pictured above:*

$$\mathfrak{M}, w_1 \overset{\mathsf{T}}{\models} B_i c = c, \tag{1}$$
$$\mathfrak{M}, w_1 \overset{\mathsf{T}}{\models} B_i(\lambda x.(\lambda y.\neg x = y)(d))(c), \tag{2}$$
$$\mathfrak{M}, w_1 \overset{\mathsf{T}}{\models} B_i(\lambda x.c = x \wedge \neg c = x)(c), \tag{3}$$
$$\mathfrak{M}, w_1 \overset{\mathsf{T}}{\models} B_i \neg c = c, \tag{4}$$
$$\mathfrak{M}, w_1 \overset{\mathsf{T}}{\models} B_i(\lambda x.B_j(\lambda y.\neg x = y)(c))(c). \tag{5}$$

Note that although agent i has classically inconsistent beliefs, there is no non-classical world in $\mathfrak{M}$.

4 Tableau system

We start from the basis of a paraconsistent signed tableau style like that of [3], and implement labels (for "worlds") and suffixes (for "things" satisfying a formula).[4] In the rules below in which no tableau suffix is mentioned, the suffix (if there is any) is the same for each formula. As is familiar, α rules are linear, and β rules are branching rules. In other cases, it will be annotated whether the rule in question is a linear or a branching rule.

α	α_1	α_2
$m\,\mathrm{T}\,\phi \wedge \psi$	$m\,\mathrm{T}\,\phi$	$m\,\mathrm{T}\,\psi$
$m\,\bar{\mathrm{F}}\,\phi \wedge \psi$	$m\,\bar{\mathrm{F}}\,\phi$	$m\,\bar{\mathrm{F}}\,\psi$
$m\,\mathrm{F}\,\phi \vee \psi$	$m\,\mathrm{F}\,\phi$	$m\,\mathrm{F}\,\psi$
$m\,\bar{\mathrm{T}}\,\phi \vee \psi$	$m\,\bar{\mathrm{T}}\,\phi$	$m\,\bar{\mathrm{T}}\,\psi$
$m\,\mathrm{F}\,\phi \rightarrow \psi$	$m\,\mathrm{T}\,\phi$	$m\,\mathrm{F}\,\psi$
$m\,\bar{\mathrm{T}}\,\phi \rightarrow \psi$	$m\,\bar{\mathrm{F}}\,\phi$	$m\,\bar{\mathrm{T}}\,\psi$
$m\,\mathrm{T}\,\neg\phi$	$m\,\mathrm{F}\,\phi$	
$m\,\bar{\mathrm{F}}\,\neg\phi$	$m\,\bar{\mathrm{T}}\,\phi$	
$m\,\mathrm{F}\,\neg\phi$	$m\,\mathrm{T}\,\phi$	
$m\,\bar{\mathrm{T}}\,\neg\phi$	$m\,\bar{\mathrm{F}}\,\phi$	

β	β_1	β_2
$m\,\mathrm{F}\,\phi \wedge \psi$	$m\,\mathrm{F}\,\phi$	$m\,\mathrm{F}\,\psi$
$m\,\bar{\mathrm{T}}\,\phi \wedge \psi$	$m\,\bar{\mathrm{T}}\,\phi$	$m\,\bar{\mathrm{T}}\,\psi$
$m\,\mathrm{T}\,\phi \vee \psi$	$m\,\mathrm{T}\,\phi$	$m\,\mathrm{T}\,\psi$
$m\,\bar{\mathrm{F}}\,\phi \vee \psi$	$m\,\bar{\mathrm{F}}\,\phi$	$m\,\bar{\mathrm{F}}\,\psi$
$m\,\mathrm{T}\,\phi \rightarrow \psi$	$m\,\mathrm{F}\,\phi$	$m\,\mathrm{T}\,\psi$
$m\,\bar{\mathrm{F}}\,\phi \rightarrow \psi$	$m\,\bar{\mathrm{T}}\,\phi$	$m\,\bar{\mathrm{F}}\,\psi$

B	B_0	
$m\,\mathrm{T}\,B_i\phi$	$n\,\mathrm{T}\,\phi$	any $n : mRn$
$m\,\bar{\mathrm{F}}\,B_i\phi$	$n\,\bar{\mathrm{F}}\,\phi$	any $n : mRn$
$m\,\mathrm{F}\,B_i\phi$	$n\,\mathrm{F}\,\phi$	new $n : mRn$
$m\,\bar{\mathrm{T}}\,B_i\phi$	$n\,\bar{\mathrm{T}}\,\phi$	new $n : mRn$

In the following rules, κ in suffixes is an individual constant (D-term) or a *quasiterm* $\langle o, k\rangle$ (A-term, π), where o and k are individual constants. Intuitively, o refers to an object, and k is a name of the referred object at a label (world). In a tableau, each free variable x in a formula ϕ has a corresponding suffix $[\kappa/x]$ attached to ϕ.

γ	γ_0	
$m\,\mathrm{T}\,\forall x\phi$	$m\,\mathrm{T}\,Ex \rightarrow \phi\,[\kappa/x]$	any κ
$m\,\bar{\mathrm{F}}\,\forall x\phi$	$m\,\bar{\mathrm{F}}\,Ex \rightarrow \phi\,[\kappa/x]$	any κ
$m\,\mathrm{F}\,\exists x\phi$	$m\,\mathrm{F}\,Ex \wedge \phi\,[\kappa/x]$	any κ
$m\,\bar{\mathrm{T}}\,\exists x\phi$	$m\,\bar{\mathrm{T}}\,Ex \wedge \phi\,[\kappa/x]$	any κ

δ	δ_0	
$m\,\mathrm{T}\,\exists x\phi$	$m\,\mathrm{T}\,Ex \wedge \phi\,[\kappa/x]$	new κ
$m\,\bar{\mathrm{F}}\,\exists x\phi$	$m\,\bar{\mathrm{F}}\,Ex \wedge \phi\,[\kappa/x]$	new κ
$m\,\mathrm{F}\,\forall x\phi$	$m\,\mathrm{F}\,Ex \rightarrow \phi\,[\kappa/x]$	new κ
$m\,\bar{\mathrm{T}}\,\forall x\phi$	$m\,\bar{\mathrm{T}}\,Ex \rightarrow \phi\,[\kappa/x]$	new κ

[4]Bloesch's tableau style in [3] is a many-valued tableau accomodated for paraconsistent logic. For tableaux for finite many-valued logics see, e.g., [4, 5]. See also [6].

In the following rules s is $\mathrm{T}, \mathrm{F}, \bar{\mathrm{T}}$ or $\bar{\mathrm{T}}$.

λ	λ_1	λ_2	
$m\ \mathrm{T}\ \lambda x.\phi(k)$	$m\ \mathrm{T}\ \phi\ [\langle o, k\rangle/x]$	$m\ \mathrm{T}\ \phi\ [o/x]$	linear rule; o already used for $m\ s\ \lambda x.\ldots(k)$, otherwise new o
$m\ \bar{\mathrm{F}}\ \lambda x.\phi(k)$	$m\ \bar{\mathrm{F}}\ \phi\ [\langle o, k\rangle/x]$	$m\ \bar{\mathrm{F}}\ \phi\ [o/x]$	linear rule; o already used for $m\ s\ \lambda x.\ldots(k)$, otherwise new o
$m\ \mathrm{F}\ \lambda x.\phi(k)$	$m\ \mathrm{F}\ \phi\ [\langle o, k\rangle/x]$	$m\ \mathrm{F}\ \phi\ [o/x]$	branching rule; new o
$m\ \bar{\mathrm{T}}\ \lambda x.\phi(k)$	$m\ \bar{\mathrm{T}}\ \phi\ [\langle o, k\rangle/x]$	$m\ \bar{\mathrm{T}}\ \phi\ [o/x]$	branching rule; new o

We introduce *quasiformulas* (not to be confused with "pseudo-formulas" of [17] and [18]) of the form $\llcorner\Phi\kappa_1\ldots\kappa_n\lrcorner$, $\llcorner\kappa_1 \approxeq_w \kappa_2\lrcorner$, $\llcorner not\ \Phi\kappa_1\ldots\kappa_n\lrcorner$, and $\llcorner\kappa_1 \not\approxeq_w \kappa_2\lrcorner$. Quasiformulas are used only in decomposition of atomic formulas and other quasiformulas.

Φ-*atom* (only with A-terms)	Φ-$atom_0$	
$m\ \mathrm{T}\ \Phi x_1\ldots x_n\ [\pi_1,\ldots,\pi_n/x_1,\ldots,x_n]$	$n\ \llcorner\Phi\pi_1\ldots\pi_n\lrcorner$	any $n : mSn$
$m\ \bar{\mathrm{F}}\ \Phi x_1\ldots x_n\ [\pi_1,\ldots,\pi_n/x_1,\ldots,x_n]$	$n\ \llcorner\Phi\pi_1\ldots\pi_n\lrcorner$	new $n : mSn$
$m\ \mathrm{F}\ \Phi x_1\ldots x_n\ [\pi_1,\ldots,\pi_n/x_1,\ldots,x_n]$	$n\ \llcorner not\ \Phi\pi_1\ldots\pi_n\lrcorner$	any $n : mSn$
$m\ \bar{\mathrm{T}}\ \Phi x_1\ldots x_n\ [\pi_1,\ldots,\pi_n/x_1,\ldots,x_n]$	$n\ \llcorner not\ \Phi\pi_1\ldots\pi_n\lrcorner$	new $n : mSn$

Φ-*atom* (with a D-term)	Φ-$atom_0$	
$m\ \mathrm{T}\ \Phi t_1\ldots t_n\ [\kappa_i/t_i]$	$n\ \llcorner\Phi\kappa_1\ldots\kappa_i/t_i\ldots\kappa_n\lrcorner$	new $n : mSn$
$m\ \bar{\mathrm{F}}\ \Phi t_1\ldots t_n\ [\kappa_i/t_i]$	$n\ \llcorner\Phi\kappa_1\ldots\kappa_i/t_i\ldots\kappa_n\lrcorner$	any $n : mSn$
$m\ \mathrm{F}\ \Phi t_1\ldots t_n\ [\kappa_i/t_i]$	$n\ \llcorner not\ \Phi\kappa_1\ldots\kappa_i/t_i\ldots\kappa_n\lrcorner$	new $n : mSn$
$m\ \bar{\mathrm{T}}\ \Phi t_1\ldots t_n\ [\kappa_i/t_i]$	$n\ \llcorner not\ \Phi\kappa_1\ldots\kappa_i/t_i\ldots\kappa_n\lrcorner$	any $n : mSn$

In the rule above, $\kappa_j = t_j$ if t_j does not occur in a suffix.

$=$-*atom* (only with A-terms)	$=$-$atom_0$	
$m\ \mathrm{T}\ x_1 = x_2\ [\pi_1,\pi_2/x_1,x_2]$	$n\ \llcorner\pi_1 \approxeq \pi_2\lrcorner$	any $n : mSn$
$m\ \bar{\mathrm{F}}\ x_1 = x_2\ [\pi_1,\pi_2/x_1,x_2]$	$n\ \llcorner\pi_1 \approxeq \pi_2\lrcorner$	new $n : mSn$
$m\ \mathrm{F}\ x_1 = x_2\ [\pi_1,\pi_2/x_1,x_2]$	$n\ \llcorner\pi_1 \not\approxeq \pi_2\lrcorner$	any $n : mSn$
$m\ \bar{\mathrm{T}}\ x_1 = x_2\ [\pi_1,\pi_2/x_1,x_2]$	$n\ \llcorner\pi_1 \not\approxeq \pi_2\lrcorner$	new $n : mSn$

$=$-*atom* (with a D-term)	$=$-$atom_0$	$=$-$atom_1$ *linearly*	$=$-$atom_2$	
$m\ \mathrm{T}\ t_1 = t_2\ [\kappa_i/t_i]$	$m\ \llcorner\kappa'_1 \approxeq \kappa_1\lrcorner$	$m\ \llcorner\kappa'_2 \approxeq \kappa_2\lrcorner$	$n\ \llcorner\kappa'_1 \approxeq \kappa'_2\lrcorner$	new $n : mSn$, new κ'_j
$m\ \bar{\mathrm{F}}\ t_1 = t_2\ [\kappa_i/t_i]$	$m\ \llcorner\kappa'_1 \approxeq \kappa_1\lrcorner$	$m\ \llcorner\kappa'_2 \approxeq \kappa_2\lrcorner$	$n\ \llcorner\kappa'_1 \approxeq \kappa'_2\lrcorner$	any $n : mSn$
$m\ \mathrm{F}\ t_1 = t_2\ [\kappa_i/t_i]$	$m\ \llcorner\kappa'_1 \approxeq \kappa_1\lrcorner$	$m\ \llcorner\kappa'_2 \approxeq \kappa_2\lrcorner$	$n\ \llcorner\kappa'_1 \not\approxeq \kappa'_2\lrcorner$	new $n : mSn$, new κ'_j
$m\ \bar{\mathrm{T}}\ t_1 = t_2\ [\kappa_i/t_i]$	$m\ \llcorner\kappa'_1 \approxeq \kappa_1\lrcorner$	$m\ \llcorner\kappa'_2 \approxeq \kappa_2\lrcorner$	$n\ \llcorner\kappa'_1 \not\approxeq \kappa'_2\lrcorner$	any $n : mSn$

In the rule above, $\kappa_j = t_j$ if t_j does not occur in a suffix. For new κ' we chose an individual constant.

id	ϕ	id_0	
$m \llcorner\kappa_1 \approxeq \kappa\lrcorner, m \llcorner\kappa_2 \approxeq \kappa', \lrcorner, n \llcorner\kappa_1 \approxeq \kappa_2\lrcorner$	$m\ s\ \phi(\kappa)$	$m\ s\ \phi(\kappa'//\kappa)$	mSn

where ϕ is a literal or a quasiformula.

E	E_0
$m\ \mathrm{T}\ Et\ [\kappa/t]$	$m \llcorner E\kappa/t\lrcorner$
$m\ \bar{\mathrm{F}}\ Et\ [\kappa/t]$	$m \llcorner E\kappa/t\lrcorner$
$m\ \mathrm{F}\ Et\ [\kappa/t]$	$m \llcorner not\ E\kappa/t\lrcorner$
$m\ \bar{\mathrm{T}}\ Et\ [\kappa/t]$	$m \llcorner not\ E\kappa/t\lrcorner$

In the rule above, $\kappa = t$ if t does not occur in a suffix.

We *close* a path (by putting $\times$ under the path) if it contains, under the same label (world), some quasiformula and it's negation, or a quasiformula $\pi \not\approxeq \pi$. A tableau is closed iff it has each path closed, otherwise a tableau is open.

Definition 8 (Derivability, $\vdash$) *$\Gamma \vdash \phi$ iff a tableau for the labelled signed set $m\ \mathrm{T}\ \Gamma \cup \{m\ \bar{\mathrm{T}}\ \phi\}$ is closed.*

Definition 9 (Consistency) *A set Γ is consistent iff there is an open tableau for the labelled signed set $m\ \mathrm{T}\ \Gamma$.*

Example 2 *Consistent de dicto beliefs can have a* de re *self-contradictory consequence. In the following example, let 'v' stand for 'Venus', 'p' for 'Phosphorus' and 'h' for 'Hesperus'.*

$$\{B_i(\lambda x.(\lambda y.\neg x = y)(p))(h), B_i(\lambda x.x = v)(p), B_i(\lambda x.x = v)(h)\} \vdash B_i\neg v = v$$

1	0	$\mathrm{T}\ B_i(\lambda x.(\lambda y.\neg x = y)(p))(h)$	
2	0	$\mathrm{T}\ B_i(\lambda x.x = v)(p)$	
3	0	$\mathrm{T}\ B_i(\lambda x.x = v)(h)$	
4	0	$\bar{\mathrm{T}}\ B_i\neg v = v$	*neg. cons.*
5	1	$\bar{\mathrm{T}}\ \neg v = v$	*4,* $\bar{\mathrm{T}}\ B_i$, $0R1$
6	1	$\mathrm{T}\ (\lambda x.(\lambda y.\neg x = y)(p))(h)$	*1,* $\mathrm{T}\ B_i$
7	1	$\mathrm{T}\ (\lambda x.x = v)(p)$	*2,* $\mathrm{T}\ B_i$

8	1	$\mathrm{T}\ (\lambda x.x = v)(h)$	*3,* $\mathrm{T}\ B_i$
9	1	$\mathrm{T}\ (\lambda y.\neg x = y)(p)\ [\langle c, h\rangle/x]$	*6,* $\mathrm{T}\ \lambda$
10	1	$\mathrm{T}\ (\lambda y.\neg x = y)(p)\ [c/x]$	*6,* $\mathrm{T}\ \lambda$
11	1	$\mathrm{T}\ \neg x = y\ [\langle c, h\rangle, \langle c_1, p\rangle/x, y]$	*9,* $\mathrm{T}\ \lambda$
12	1	$\mathrm{T}\ \neg x = y\ [\langle c, h\rangle, c_1/x, y]$	*9,* $\mathrm{T}\ \lambda$
13	1	$\mathrm{T}\ \neg x = y\ [c, \langle c_1, p\rangle/x, y]$	*10,* $\mathrm{T}\ \lambda$
14	1	$\mathrm{T}\ \neg x = y\ [c, c_1/x, y]$	*10,* $\mathrm{T}\ \lambda$
15	1	$\mathrm{T}\ x = v\ [\langle c_1, p\rangle/x]$	*7,* $\mathrm{T}\ \lambda$
16	1	$\mathrm{T}\ x = v\ [c_1/x]$	*7,* $\mathrm{T}\ \lambda$
17	1	$\mathrm{T}\ x = v\ [\langle c, h\rangle/y]$	*8,* $\mathrm{T}\ \lambda$
18	1	$\mathrm{T}\ x = v\ [c/y]$	*8,* $\mathrm{T}\ \lambda$
19	1	$\llcorner v_2 \approxeq \langle c_1, p\rangle \lrcorner$	*15,* T *=-atom*
20	1	$\llcorner v_1 \approxeq v \lrcorner$	*15,* T *=-atom*
21	2	$\llcorner v_2 \approxeq v_1 \lrcorner$	*15,* T *=-atom,* $1S2$
22	1	$\llcorner v_4 \approxeq \langle c, h\rangle \lrcorner$	*17,* T *=-atom*
23	1	$\llcorner v_3 \approxeq v \lrcorner$	*17,* T *=-atom*
24	3	$\llcorner v_4 \approxeq v_3 \lrcorner$	*17,* T *=-atom,* $1S3$
25	1	$\mathrm{T}\ \neg v = y\ [\langle c_1, p\rangle/y]$	*11, 22–24 id*
26	1	$\mathrm{T}\ \neg v = v$	*25, 19–21 id*
27	1	$\bar{\mathrm{F}}\ v = v$	*5,* $\bar{\mathrm{T}}\neg$
28	1	$\mathrm{F}\ v = v$	*26,* $\mathrm{T}\neg$
29	1	$\llcorner v_5 \approxeq v \lrcorner$	*28, =-atom*
30	1	$\llcorner v_6 \approxeq v \lrcorner$	*28, =-atom*
31	2	$\llcorner v_5 \not\approxeq v_6 \lrcorner$	*28, =-atom*
32	2	$\llcorner v_5 \approxeq v_6 \lrcorner$	*27, =-atom*
		$\times$	

Example 3 *Classically inconsistent beliefs do not explode.*

$$B_i(P_1c \wedge \neg P_1c) \nvdash B_iP_2c$$

Tableau proof is left as an exercise.

Proposition 1

$\{\phi \wedge \neg\phi\} \vdash \psi$ *(only* λ*-dependent terms occur)*
$\{\neg(\phi \vee \neg\phi)\} \vdash \psi$ *(only* λ*-dependent terms occur)*
$\{\phi \wedge \neg\phi\} \nvdash \psi$
$\{\neg(\phi \vee \neg\phi)\} \nvdash \psi$

Proof Each case can be proved in the defined tableau system.

$\phi \wedge \neg\phi$ does not "explode" with arbitrary syntactic consequences.[5] Hence, the proposed logical system is paraconsistent.

In the following proposition, '$\phi \dashv\vdash \psi$' is short for '$\{\phi\} \vdash \psi$ and $\{\psi\} \vdash \phi$'.

Proposition 2

$$
\begin{aligned}
&\{(\lambda x.\phi)(k) \wedge (\lambda x.\psi)(k)\} \dashv\vdash (\lambda x.\phi \wedge \psi)(k)\\
&\{(\lambda x.\phi)(k) \vee (\lambda x.\psi)(k)\} \dashv\vdash (\lambda x.\phi \vee \psi)(k)\\
&\{(\lambda x.\phi)(k)\} \nvdash \phi(k/x)\\
&\{\phi(k/x)\} \nvdash (\lambda x.\phi)(k)\\
&\{\phi(k) \wedge (\lambda x.x = k)(k)\} \vdash (\lambda x.\phi(x))(k)\\
&\{(\lambda x.\phi(x) \wedge x = k)(k)\} \nvdash \phi(k/x))\\
&\{(\lambda x.(\lambda y.\phi(x) \wedge \neg\phi(y))(k_2))(k_1), (\lambda x.(\lambda y.(k_1 = x \wedge k_1 = y)(k_2))(k_1)\} \vdash\\
&\qquad (\phi(k_1/x) \wedge \neg\phi(k_1/x))\\
&\{B_i(\lambda x.(\lambda y.\phi(x) \wedge \neg\phi(y))(k_2))(k_1), k_1 = k_2\} \nvdash B_i\psi\\
&\{\forall x\,\phi \wedge (\lambda x.Ex)(k)\} \vdash (\lambda x.\phi)(k)\\
&\{(\lambda x.\phi \wedge Ex)(k)\} \nvdash \exists x\phi\\
&\vdash k = k\\
&\{\neg k = k\} \nvdash \psi\\
&\{(\lambda x.\neg x = x)(k)\} \vdash \psi
\end{aligned}
$$

Proof Each case can be proved in the defined tableau system.

4.1 Soundness and completeness

Let us sketch a soundness and a completeness proofs with some preliminaries. We call all formulas occurring in tableaux *tableau formulas*. The set of tableau formulas includes, beside $\mathcal{L}_{\mathbf{QB}}$ formulas, also labelled signed formulas with suffixes and labelled quasiformulas. Accordingly, we extend a model $\mathfrak{M}$ to a tableau model $\mathfrak{M}^T$ with a world corresponding to each label of a tableau formula, and define $V^T(\langle o, k\rangle) = \langle V^T(o), k\rangle$. The satisfaction by a tableau model $\mathfrak{M}^T$ and $\mathfrak{v}$ is merely a reformulation of a satisfaction by $\mathfrak{M}, w$ and $\mathfrak{v}$, where

$$\mathfrak{M}^T \models_{\mathfrak{v}} l \ \mathrm{T}\ \phi[\kappa/x] \text{ iff } \mathfrak{M}, w_l \overset{\mathrm{T}}{\models}_{\mathfrak{v}[V^T(\kappa)/x]} \phi,$$

[5] Note, for example, that $\bar{\mathrm{T}}\ \neg(Pc \wedge \neg Pc)$ has a closed tableau, while $\mathrm{T}\ Pc \wedge \neg Pc$ has an open tableau.

$\mathfrak{M}^T \models_{\mathfrak{v}} l\ \bar{\text{T}}\ \phi[\kappa/x]$ iff $\mathfrak{M}, w_l \not\models^{\text{T}}_{\mathfrak{v}[V^T(\kappa)/x]} \phi$,

similarly for F and $\bar{\text{F}}$,

and where the satisfaction of labelled quasiformulas is defined in the following way:

$\mathfrak{M}^T \models_{\mathfrak{v}} l \llcorner \Phi\kappa_1 \dots \kappa_n \lrcorner$ iff $\langle V^T(\kappa_1), \dots, V^T(\kappa_n)\rangle \in V^T(\Phi, w_l)$,

$\mathfrak{M}^T \models_{\mathfrak{v}} l \llcorner\ \textit{not}\ \Phi\kappa_1 \dots \kappa_n \lrcorner$ iff $\langle V^T(\kappa_1), \dots, V^T(\kappa_n)\rangle \notin V^T(\Phi, w_l)$,

$\mathfrak{M}^T \models_{\mathfrak{v}} l \llcorner \kappa_1 \cong \kappa_n \lrcorner$ iff $V^T(\kappa_1) \cong_{w_l} V^T(\kappa_2)$,

$\mathfrak{M}^T \models_{\mathfrak{v}} l \llcorner \kappa_1 \ncong \kappa_n \lrcorner$ iff $V^T(\kappa_1) \ncong_{w_l} V^T(\kappa_2)$.

Definition 10 (Distributed satisfiability of a set Γ of tableau formulas) *A set Γ of tableau formulas is distributively satisfiable iff there is a tableau model $\mathfrak{M}^T$ and a variable assignment $\mathfrak{v}$ that satisfy each member of Γ.*

We call a tableau T (distributively) satisfiable iff it has a distributively satisfiable path.

4.1.1 Soundness

Let us outline main steps of the soundness proof.

(i) It should be shown, by mathematical induction, that if a tableau T is distributively satisfiable, then, after the application of any tableau rule, the resulting tableau T' remains distributively satisfiable. For example, suppose that $m\ \text{T}\ \lambda x.\phi(k) \in \text{p}$, where p is a distributively satisfiable path of a tableau T. If $\mathfrak{M}^T \models_{\mathfrak{v}} \text{p}$, then also $\mathfrak{M}^T \models_{\mathfrak{v}} \text{p} \cup \{m\ \text{T}\ \phi\ [\langle o, k\rangle/x], m\ \text{T}\ \phi\ [o/x]\}$ (with o new to the path or already used for λ-dependent k in accordance with the rules). This follows from the fact that, in terms of $\mathfrak{M}$ satisfiability, if $\mathfrak{M}, w_m \models^{\text{T}}_{\mathfrak{v}} \lambda x.\phi(k)$, then $\mathfrak{M}, w_m \models^{\text{T}}_{\mathfrak{v}[\langle d,k\rangle/x]} \phi$ and $\mathfrak{M}, w_m \models^{\text{T}}_{\mathfrak{v}[d/x]} \phi$, where $d \in V(k, w_m)$ and $[\![o]\!]^{\mathfrak{M},w}_{\mathfrak{v}} = d$.

(ii) After that, it should be proved that if a set Δ of tableau formulas has a closed tableau, then Δ is not distributively satisfiable.[6] The proof is indirect. Suppose that Δ, having a closed tableau, is distributively satisfiable. If Δ is distributively satisfiable, the tableau for Δ should eventually also be distributively satisfiable (see (i)). That is impossible, since the conditions under which a tableau for

[6]For comparison, see Lemma 2 in [3].

Δ eventually closes make the tableau distributively unsatisfiable. Thus Δ, having a closed tableau, cannot be distributively satisfiable.

(iii) In a special case, suppose that ϕ and each $\psi \in \Gamma$ are $\mathcal{L}_{\mathbf{QB}}$ formulas, and that $l \; \mathsf{T} \; \Gamma \cup \{l \; \bar{\mathsf{T}} \; \phi\}$ has a closed tableau. Therefore (by (ii)) $l \; \mathsf{T} \; \Gamma \cup \{l \; \bar{\mathsf{T}} \; \phi\}$ is not distributively satisfiable. Hence, if $\mathfrak{M}^T \models_{\mathfrak{v}} l \; \mathsf{T} \; \Gamma$ then $\mathfrak{M}^T \models_{\mathfrak{v}} l \; \mathsf{T} \; \phi$, and thus, if $\mathfrak{M}, w \overset{\mathsf{T}}{\models}_{\mathfrak{v}} \Gamma$ then $\mathfrak{M}, w \overset{\mathsf{T}}{\models}_{\mathfrak{v}} \phi$. Therefore, if $l \; \mathsf{T} \; \Gamma \cup \{l \; \bar{\mathsf{T}} \; \phi\}$ has a closed tableau, then $\Gamma \models \phi$, that is, the soundness theorem holds.

4.1.2 Completeness

We give a sketch of the completeness proof.

(i) A labelled and signed Hintikka set H with suffixes should be defined according to the tableau rules. More specifically, if for an atomic sentence ϕ, $m \; \mathsf{T} \; \phi \in H$, then an appropriate labelled quasiformula $l \llcorner \phi' \lrcorner$ (see the tableau rules for the appropriate quasiformulas) should also be a member of H. Regarding quasiformulas, it is not the case that for a quasiformula $\llcorner \phi \lrcorner$, $l \llcorner \phi \lrcorner \in H$ and $l \llcorner \neg\phi \lrcorner \in H$, or $\pi \not\cong \pi \in H$. Also, to give another example, if a Hintikka set H contains a signed formula $\mathsf{T} \; B_i\phi$ with a label m, then H contains the signed formula $\mathsf{T} \, \phi$ for all labels previously introduced in the tableau from the label m (according to B rules), or for a new label n if previously no label is introduced from m.

(ii) It should be shown that every open path is a subset of a corresponding Hintikka set. This follows from the fact (clear from (i)) that in building a Hintikka set, we add each formula that can be added in accordance with the tableau rules and, at the same time, we never fulfil the tableau closure conditions.

(iii) By a construction of an appropriate canonical tableau model, it should be proved that each labelled and signed Hintikka set with suffixes is distributively satisfiable. We now briefly sketch that step of the completeness proof. To simplify the metalanguage notation, we will write $\phi(\kappa)$ instead of $\phi(x)\;[\kappa/x]$.

Definition 11 (Equivalence class) *Equivalence class* $[k]$ *of an individual constant k with respect to a tableau H is the set* $\{k' \mid m \llcorner k \cong k' \lrcorner \in H$ *for some* $m\}$.

Definition 12 (Canonical frame) *Canonical frame* $\mathcal{F}^H$ *for a Hintikka set H is an n-tuple* $\{W, W_A, R_i, \ldots, R_n, S, D, A, Q, \{\cong_w\}_{w \in W}\}$, *where*

1. *W is a non-empty set of labels of H,*

2. $W_A \subseteq W$,

3. $R_i \subseteq W \times W_A$ *(serial, transitive, and euclidean),*

4. $S \subseteq W \times W$ *(serial, reflexive),*

5. *D is a set of equivalence classes of individual constants in tableau $\mathcal{L}_{\mathbf{QB}}$ formulas of H and in suffixes of H if there are any such constants, otherwise $D = \{[c]\}$,*

6. $A = \{\langle [o], k\rangle \mid \langle o, k\rangle$ *occurs in a quasiformula or a suffix of* $H\}$,

7. $Q(m) = \{[k] \mid m \llcorner Ek \lrcorner \in T\} \cup \{\langle [o], k\rangle \mid m \llcorner E\langle o, k\rangle \lrcorner \in H\}$,

8. *for each m, $\approxeq_m = \{\langle u_1, u_2\rangle \mid m \llcorner \kappa_1 \approxeq \kappa_2 \lrcorner \in H\}$, where*

$$u_i = \begin{cases} \langle [o], k\rangle & \text{if } \kappa_i = \langle o, k\rangle \\ [k] & \text{if } \kappa_i = k. \end{cases} \tag{6}$$

Definition 13 (Canonical model) *Canonical model $\mathfrak{M}^H$ for a Hintikka set H is a pair $\langle \mathcal{F}, V\rangle$, where*

1. $V(k) = [k]$, $\quad V(k, m) \subseteq \{[o], \langle [o], k\rangle \mid \langle [o], k\rangle \in A\}$, $\quad V(\langle o, k\rangle) = \langle [o], k\rangle$,

2. $\langle u_1, \ldots, u_n\rangle \in V(\Phi^n, m)$ *iff* $m \llcorner \Phi\kappa_1 \ldots \kappa_n \lrcorner \in H$,

3. $V(=, m) = \approxeq_m$,

4. $V(E, m) = \{u \mid m \llcorner E\kappa \lrcorner \in H\}$,

under the condition (6) above.

Now it should be proved that each labelled and signed Hintikka set H with suffixes is distributively satisfied by the canonical model $\mathfrak{M}^H$ (under a given variable assignment $\mathfrak{v}$, if any). Let us take quasiformulas as an example. Suppose that $m \llcorner \Phi\kappa_1 \ldots \kappa_n \lrcorner \in H$. Thus, $\langle u_1, \ldots, u_n\rangle \in V(\Phi^n, m)$ under condition (6) (see Definition 13), and therefore $\mathfrak{M}^H \models_{\mathfrak{v}} m \llcorner \Phi\kappa_1 \ldots \kappa_n \lrcorner$.

(iv) Finally it follows from (iii) that, if a set Δ is not distributively satisfiable, then Δ is not a subset of any Hintikka set. Accordingly, if Δ is not distributively satisfiable, then Δ has a closed tableau, since each open path of a tableau is a subset of a Hintikka set (see (ii)). As a special case, if a set $l \top \Gamma \cup \{l \bar{\top} \phi\}$ is distributively unsatisfiable (and hence $\Gamma \models \phi$), then it has a closed tableau (that is, $\Gamma \vdash \phi$), which establishes the completeness theorem.

4.1.3 Acknowledgment

I am grateful to the referee for finding an tableau related error.

References

[1] Béziau, J.-Y. Paraconsistent logic from a modal viewpoint. *Journal of Applied Logic 3* (2005), 7–14.

[2] Béziau, J.-Y. A new four-valued approach to modal logic. `http://www.lia.ufc.br/~locia/artigos/modal4.pdf`, 200X.

[3] Bloesch, A. A tableau style proof system for two paraconsistent logics. *Notre Dame Journal of Formal Logic 34* (1993), 295–301.

[4] Carnielli, W. A. Systematization of finite many-valued logics through the method of tableaux. *Journal of Symbolic Logic 52* (1987), 473–493.

[5] Carnielli, W. A. On sequents and tableaux for many-valued logics. *The Journal of Non-Classical Logic 8* (1991), 59–78.

[6] Fitting, M. *Proof Methods for Modal and Intuitionistic Logics*. D. Reidel, Dordrecht, Boston, Lancaster, 1983.

[7] Fitting, M. *First-Order Modal Logic*. Kluwer, Dordrecht, Boston, London, 1999.

[8] Fitting, M. First-order intensional logic. *Annals of Pure and Applied Logic 127* (2004), 171–193.

[9] Fitting, M. FOIL axiomatized. *Studia Logica 84* (2006), 1–22.

[10] Frege, G. Über Sinn und Bedeutung. In *Funktion, Begriff, Bedeutung*, G. Patzig, Ed., 6. ed. Vandenhoeck und Ruprecht, Göttingen, 1986, pp. 40–65.

[11] Jaśkowski, S. Propositional calculus for contradictory deductive systems. *Studia Logica 24* (1969), 143–157. (In Polish 1948).

[12] Jaśkowski, S. On the discussive conjunction in the propositional calculus for inconsistent deductive systems. *Logic and Logical Philosophy 7* (1999), 57–59. (In Polish 1949).

[13] Jaśkowski, S. A propositional calculus for inconsistent deductive systems. *Logic and Logical Philosophy 7* (1999), 35–56. (A modified version of [11]).

[14] Kovač, S. Contradictions, objects, and belief. In *Perspectives on Universal Logic*, J.-Y. Béziau and A. Costa-Leite, Eds. Polimetrica, Monza - Milano, 2007, pp. 417–434.

[15] Kracht, M., and Kutz, O. The semantics of modal predicate logic II. Modal individuals revisited. In *Intensionality*, R. Kahle, Ed. A K Peters, Wellesley, Ma., 2005.

[16] Kutz, O. New semantics for modal predicate logics. In *Foundations of Formal Sciences II* (Dordrecht, Boston, London, 2003), B. Löwe et al., Eds., Kluwer.

[17] Ye, R. *Belief, Names and Modes of Presentation: A First-Order Logic Formalization*. PhD thesis, City University of New York, 1999.

[18] Ye, R., and Fitting, M. Belief, names, and modes of presentation. In *Advances in Modal Logic*, e. a. F. Wolter, Ed., vol. 3. World Scientific, New Jersey, etc., 2002, pp. 389–408.

Institute of Philosophy
Ul. grada Vukovara 54
10000 Zagreb, Croatia