Are non-accidental regularities a cosmic coincidence? Revisiting a central threat to Humean laws

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Abstract

If the laws of nature are as the Humean believes, it is an unexplained cosmic coincidence that the actual Humean mosaic is as extremely regular as it is. This is a strong and well-known objection to the Humean account of laws. Yet, as reasonable as this objection may seem, it is nowadays sometimes dismissed. The reason: its unjustified implicit assignment of equiprobability to each possible Humean mosaic; that is, its assumption of the principle of indifference, which has been attacked on many grounds ever since it was first proposed. In place of equiprobability, recent formal models represent the doxastic state of total ignorance as suspension of judgment. In this paper I revisit the cosmic coincidence objection to Humean laws by assessing which doxastic state we should endorse. By focusing on specific features of our scenario I conclude that suspending judgment results in an unnecessarily weak doxastic state. First, I point out that recent literature in epistemology has provided independent justifications of the principle of indifference. Second, given that the argument is framed within a Humean metaphysics, it turns out that we are warranted to appeal to these justifications and assign a uniform and additive credence distribution among Humean mosaics. This leads us to conclude that, contrary to widespread opinion, we should not dismiss the cosmic coincidence objection to the Humean account of laws.

Keywords

Humean Account of Laws; Best System Account of Laws; Ignorance; Principle of Indifference; Suspension of Judgment; Coincidences.

Contents

1 Introduction

One main account of laws of nature is the Humean account. It has been developed in the last century as the so-called Best System Account (Ramsey, 1978; Lewis, 1994), and much effort has been made to improve it by solving a number of problems: how to reconcile Humean laws with objective chance, how to talk of Humean laws in the special sciences, and so on (e.g. Cohen and Callender, 2009; Loewer, 2007; Schrenk, 2014; Woodward, 2014). Its virtues and sophistication notwithstanding, since its origin this account of laws has faced a major threat. According to this account, laws are mere *descriptions*—the best descriptions of the whole history of the universe, i.e. of the Humean Mosaic—so non-accidental regularities are only apparently non-accidental; thus our ontology is freed from the mysterious notion of physical necessity. The problem is that this, however, means that the seemingly non-accidental and *extremely stable* regular behaviour *ubiquitous* in the universe is just a brute fact with no explanation, a cosmic coincidence.

The actual Humean mosaic is a brute fact, so all regularities within it are a brute fact too. But crucially, it turns out that these regularities are *rife*: our own experience and our scientific image describe a world where regular behaviour is ubiquitous. All the planets of the solar system regularly orbit around the Sun, all the stars of all the galaxies have the same life cycles, all the fermions of the universe follow the same fundamental interactions; in all, there is no empirical evidence that the laws of physics have *ever* changed their form across all space and time.

Foster (1983, 89) illustrates the cosmic coincidence objection with the example of gravitational interactions (see also Blackburn, 1990, 3; Strawson, 2014, Ch. 5 & 8 (esp. pp. 23-26); Swartz, 2018, §7):

> The past consistency of gravitational behaviour calls for some explanation. For given the

> infinite variety of ways in which bodies might have behaved non-gravitationally and, more importantly, the innumerable occasions on which some form of non-gravitational behaviour might have occurred and been detected, the consistency would be an astonishing coincidence if it were merely accidental – so astonishing as to make the accident-hypothesis quite literally incredible.

Hence, on the Humean account of laws it is a sheer cosmic coincidence that such extremely stable regularities came to obtain. Our highly patterned actual Humean mosaic seems an extremely unlikely possibility among all the possible irregular mosaics. It would be, following Strawson's (2014) analogy, like having a screen whose pixels display the random noise produced by some underlying generator and in which you find not just some fluke which surprisingly resembles a frame of a movie, but the whole of, say, Kubrick's 'Clockwork Orange' (cf. ibidem, p. 26). Thus, by 'cosmic coincidence' we mean that the occurrence of the actual Humean mosaic seems *extremely* unlikely, that is, with a probability tending to 0. Hence, it seems that the Humean posits as unexplainable something that, from any point of view, Humean or not, should be explained. In sum, an explanation of this overwhelmingly high degree of regular behaviour should be provided; otherwise the Humean account cannot be considered a serious account of laws. Figure 1 illustrates this situation.[1]

This is how the argument goes; an argument that is apparently plausible and potentially devastating for the Humean account. However, Humeans are safe for the time being, because the argument hides a false implicit premise. It is valid, as we will see in the next section, but unsound. In particular, the argument relies on the assumption that each possibility should be assigned equal probability; that each way the world could have been—each Humean mosaic—is equally likely. This premise is an instance of the Principle of Indifference, henceforth '*PoI*'. *PoI* can be formulated in many ways; for instance,

> *PoI*: Suppose that there are n mutually exclusive and collectively exhaustive pos-

[1] Recent literature has discussed an objection in the vicinity according to which Humean laws *cannot explain* their instances, which is also sometimes phrased as the complaint that there is a circularity in the Humean account, as it takes the instances to explain the laws and vice versa (see e.g. Lange, 2013; Marshall, 2015). In this paper I reassess a stronger objection: one that concerns not just the general inability of Humean laws to explain simpliciter, but rather to their inability to explain a cosmic coincidence (i.e. the *extremely* regular arrangement of the instances). This stronger objection cannot be answered merely by claiming that the Humean mosaic is a brute fact that does not need to be explained, for its extremely specific arrangement calls for an explanation—or so the objection goes, as elaborated in Section 2.

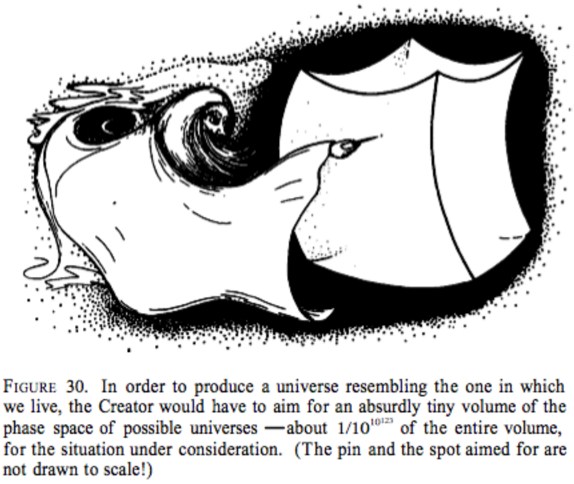

Figure 1: The cubic canvas contains all the possible universes, and each point represents a possible universe. It is a mathematical fact from combinatorics that almost all of them are disordered/chaotic universes. The set of ordered universes is represented as the very small black spot at which God is pointing. Picture from Penrose's (1989) (who used it for another argument).

> sibilities. If there is no evidence favouring one possibility over another, then each possibility should be assigned the same probability.

Obviously true as it may seem, *PoI* is itself problematic. This is the main reason why the cosmic coincidence objection is supposed to be unsound. In fact, instead of following *PoI*, recent formal models represent our doxastic state of total ignorance as suspension of judgment. They have then been applied to arguments in cosmology, where it is concluded that certain probabilistic inferences are unwarranted (Norton, 2010, Benétreau-Dupin, 2015). In a similar spirit, the goal of this paper is to revisit the status of the cosmic coincidence objection to Humean laws by assessing the proper representation of ignorance. To this end, I follow the methodological strategy of not deciding *a priori* which is the correct representation (the correct 'inductive logic') to use; instead, as Norton (2007, 2008, 2010, Forthcoming) has urged, the empirical (or 'material') features of the problem have to justify which inductive logic is appropriate.

By (1) focusing on the specific features of our scenario, namely the fact that it is framed within a Humean metaphysics, and (2) interpreting the probabilities referred to in the argument as subjective, I conclude that suspending judgment is an unnecessarily weak doxastic state and that we are warranted to apply *PoI*. In other words, it is epistemically rational to assign a uniform credence among Humean mosaics. Thus, conditional upon certain simplifications, I conclude that the cosmic coincidence argument against Humean laws is sound.

In more detail: in Section 2 I spell out the cosmic coincidence argument and the problems with *PoI*. Here I defend the plausibility of the other premises and set out a toy model of our scenario. Then in Section 3 I reconstruct three recent defences of the epistemic rationality of applying *PoI* in situations of ignorance. Since these are three general arguments whose applicability to our scenario is not settled, in Section 4 I assess whether they apply to our scenario. First I point out that in our argument the probabilities are to be interpreted as subjective (§4.1). These probabilities correspond to the credences of an ideal rational agent, not to any other interpretation of objective chances or subjective credences.[2] This is not necessarily the only reading of the probabilities of the argument, but it is undoubtedly coherent and the best way to vindicate the soundness of the argument. Secondly, in §4.2 I point out that since the argument is framed within a Humean framework, (i) we know that there are no primitive objective chances assigned to the ocurrence of the mosaic and (ii) we know the space of possibilities (so we are in so-called 'classical ignorance', not 'total ignorance'). These features warrant the assignment of a uniform credence among the Humean mosaics. It is also crucial, however, that the resulting credence distribution is additive, which is to say that the credence of the union of two possibilities amounts to the sum of their credences. Although intuitive, additivity should not be taken for granted. In §4.3 I argue that our scenario warrants additivity.

The conclusion is thus that, at least for our toy model, the cosmic coincidence objection to the Humean accounts of laws is sound. In spite of a potential criticism that I address in §4.4, in Section 5 I conclude that it does not seem reasonable to neglect the cosmic coincidence objection, and thus that the Humean account of laws, as an account of lawful behaviour, should be complemented by a currently missing explanation of lawful behaviour.

2 Why the cosmic coincidence argument is supposedly flawed

The cosmic coincidence argument can be spelled out as what I will refer to as 'Argument'. It begins by endorsing the Humean view that laws are just the best *descriptions* of the regularities of the Humean mosaic. In Lewis's (1973, 73) terms, the laws of nature belong to all the true

[2] 'Credence' is to be understood for now in a neutral way, leaving open whether it refers to degrees of belief or full beliefs; only later we will conclude that the credences here can be represented as precise degrees of belief.

deductive systems with the best combination of simplicity and strength. (Further details of this account are irrelevant for our purposes). This is premise 1 below. Then ARGUMENT, whose other premises will be discussed shortly, can be spelled out as follows:

	1.	Humean Laws;	[Premise]
	2.	The actual Humean mosaic displays a high degree of regularity;	[Premise]
	3.	There are overwhelmingly more irregular Humean mosaics than highly regular ones;	[Premise]
	4.	*PoI*;	[Premise]
	5.	There is no reason to regard any Humean mosaic as more likely to occur than any other;	[1]
	6.	The same probability should be assigned to the occurrence of each Humean mosaic;	[4, 5]
	7.	The probability of a highly regular mosaic occurring is overwhelmingly lower than the probability of an irregular mosaic occurring;	[3, 6]
	8.	Overwhelmingly unlikely events demand explanation;	[Premise]
∴	C.	The high degree of regularity of the actual Humean mosaic demands explanation.	[2, 7, 8]

Humeans do not provide an explanation of the high degree of regularity of the actual universe[3], contrary to what C demands. So the Humean account is flawed. Conversely, the conclusion C would be avoided by positing non-Humean governing laws, since such laws would explain actual regularities. In other words, a governing view of laws rejects 5 (which here follows from the assumed Humean viewpoint).

However, Humeans not only fail to provide an explanation; they do not even try to provide one, because they consider ARGUMENT to be unsound. While the form of ARGUMENT is valid, what has mainly been criticized is its premise 4, i.e. *PoI*.

Premise 4 looks plausible; in fact, *PoI* is often implicitly used, in philosophy, in science, and in daily life. Yet its validity has been questioned even by its originators, Laplace and Bernoulli, and indeed ever since. Recently it has been questioned again, in response to anthropic arguments in cosmology (Mosterín, 2004; Norton, 2010; Howson, 2011). While I certainly agree that misapplications of *PoI* abound, in appropriate contexts *PoI* can be correctly applied. And I will

[3]Throughout the paper we use the terms 'universe', 'world', and 'Humean mosaic' interchangeably.

conclude that, conditional upon certain simplifications that I consider innocuous, our current context warrants its application.

2.1 Other premises and an initial toy model

Before explaining the problems with *PoI*, let me defend ARGUMENT's other premises, which will not be central to our discussion, and set out a toy model with which we may assess its soundness.

2.1.1 Other premises

Premise 3. This premise can be defended by appealing to the mathematical results concerning the proportions of random and non-random sequences, which state that random sequences are the overwhelming majority. The notion of the randomness of a sequence, called 'product-randomness', captures the idea of disorderliness, and has been mathematically defined in three different yet provably equivalent ways (see e.g. Earman, 1986, Ch. VIII). For the infinite case, the thesis that almost all infinite sequences will be random and disorderly and only a few will be orderly dates back to Ville (1939) (see also Dasgupta, 2011, §3; Gaifman and Snir, 1982, 534; Williams 2008, 407–11), allowing us to maintain that the set of random sequences has measure 1 (Eagle, 2012, §2.1).[4] In the case of finite sequences, it follows from cardinality considerations that most are random, as is clearly explained in (Smith, 1998, Ch. 9). In short, given the notion of product-randomness defined in terms of a sequence's degree of compressibility, we specify a criterion of what counts as random, namely a sequence that is not compressible in that it cannot be coded by an algorithm of a certain length k of bits. Then, by the very modest criterion that for a sequence to be non-random the corresponding algorithm can be 20 bits shorter than the sequence, it turns out that for sufficiently large N less than one sequence in a million will count as non-random (Smith, 1998, 152).

As I argue below when the toy model is spelled out, these results for sequences only differ in trivial respects from more sophisticated models of Humean mosaics. For as long as the

[4] Measure theory allows us to compare the sizes of uncountably infinite sets of the same cardinality, avoiding tricky orderings of these sets that make it difficult to compare them (as in Lewis, 1986, §2.5). Still, we won't follow this complex path, due to intractable problems with infinity (see § 2.2.2 and 2.2.3); rather, we will aim for results on a finite but *arbitrarily large* space, which are *approximate* but nevertheless sufficiently significant and stand on firmer ground. More on this below.

mosaic is spatiotemporally finite, its elements can be ordered and thus the full 4-D mosaic is converted into a sequence of length N.

Premise 5. On any non-Humean view, regularity is explained by the corresponding laws or some such modal notion—as held e.g. by Dretske (1977); Armstrong (1983); Tooley (1977); Cartwright (1989); Bird (2005); Mumford (2004); Mumford and Anjum (2011); Blackburn (1990); Strawson (1987, 2014). David Hume himself, according to the interpretation of Wright (1983) and Strawson (2014) inter alia, acknowledged that there might be some natural necessity, an admittedly mysterious connection, which would account for the uniformity of Nature.[5] *A fortiori*, in a world with an extremely high degree of regular behaviour, such as the actual world, non-Humean laws provide a straightforward explanation which contemporary Humeans lack.

Still, one might be tempted to think that non-Humeans should also count all the worlds without laws, so they should likewise conclude that the actual world is extremely unlikely. After all, non-Humeans have traditionally discussed possible worlds without laws in which all sorts of stuff happens; and these are metaphysically possible worlds, no less than worlds with laws!

However, this worry is misguided, as it projects the Humean point of view to the non-Humean frameworks, neglecting crucial differences. The cosmic coincidence objection arises only in the Humean framework, because their explicit metaphysical view involves positing the actual world as a primitive, brute fact, and so explicitly rules out there being any reason for this or that world being the case, leading to an arbitrary and unconstrained choice between all possible worlds (cf. §4.2). In contrast, non-Humeans do not commit at all to the whole history of the world being a brute fact. They lack this strong commitment, and they even have one or another explanation of why the states of the world are as they are—e.g. the state x_t is as it is because of any previous state x_{t-i} plus the laws. Hence, their view has never involved an

[5] So understood, Hume's scepticism did not amount to a metaphysical claim, but rather to a more modest emphasis of our epistemic limitations. (We cannot justify the rationality of inductive inference unless we justify our belief in the uniformity in Nature; yet we cannot rationally infer that Nature is uniform without making an inductive inference. But this is the *epistemic* problem of justifying the rationality of inductive inference, not the metaphysical problem of explaining the uniformity of Nature.) Also, among the contemporary Humeans who we address in this paper, that is, those who do take the further step of drawing metaphysical conclusions, it is conceded that the non-Humean does explain the regularities of the actual world: see e.g. the influential (Beebee, 2011, §2 and §6 (last paragraph)) and (Beebee, 2006, p. 527). Here it is also worth adding that Beebee dismisses the cosmic coincidence objection by appealing to *a posteriori* evidence, which begs the question, as I argue in footnote 7.

arbitrary choice between a large number of possible histories of the universe (and that is likely why, before the recent literature on Humean metaphysics, this objection never arose). [6] [7]

Premise 8. Although it seems plausible and has often been endorsed throughout the history of science and philosophy, some have disputed premise 8. See for instance Callender (2004), who argues against demanding an explanation of the arguably very special low-entropy initial condition of the universe. See also Hand (2014), who explains that very unlikely events are likely to happen given a large enough number of opportunities (as in the thought experiment of a monkey randomly hitting a typewriter and eventually producing the "Don Quixote", or as in Borges's story "The Library of Babel"). Our scenario, however, is not of this kind. Hand's thesis concerns cases with a large number of opportunities, and in our case that would amount to the actual existence of a large number of universes. In contrast, in our scenario only a *single* universe occurs. (In fact, a large number of opportunities is what proponents of a multiverse propose—a large number of universes—in order to explain the alleged fine-tuning of the actual world.)

Another worry might be raised: unlikely events, even those that are overwhelmingly unlikely, can just happen. For instance, in a large randomly generated sequence it can happen that an atypical pattern arises, without thereby demanding explanation. Similarly, in our very large universe we might find strange, atypical patterns. What we are assessing here, however, is quantitatively and qualitatively different. It is not like merely finding a very unexpected pattern; it is like finding, in an ultra-large random sequence of length N of independent integers,

[6] The metaphor of Figure 1 of God arbitrarily choosing a mosaic is inadequate from the non-Humean's point of view: the illustration was used by a non-Humean in reference *only* to the initial state of the universe! Even when restricted only to the initial state, the dialectics is *different* for the Humean and for the non-Humean. Whereas a Humean like Callender (2004) argues that there is no need to explain this (which I find unconvincing, but leave that aside), the non-Humean does not rule out that *there might be some reason* behind the very special initial state of the universe (Penrose himself proposes one). By leaving available the provision of some reason—a dialectical possibility unavailable to the Humean—the non-Humean is potentially able to avoid the scenario in which the IC has been arbitrarily actualized from a large possibility space.

[7] Finally, a temptation to avoid when reading ARGUMENT is to think that we now have *a posteriori* evidence about which mosaic is the actual world, and claim that given our current evidence it is no surprise that the world is as it is. In other words, P (the world turns out to be highly ordered | evidence) = 1, given that evidence = 'the world is highly ordered'. This trivially true statement is not what we are interested in. (This *petitio principii* is found in Beebee, 2006, p. 527.) We want to place ourselves *before* the actual world occurred, from a God's eye viewpoint, in order to assess whether the actual world is a surprising event.

only prime numbers repeated over and over again—a subsequence of length, say, N-1 (or like finding, in the screen displaying arbitrary pixels frame after frame, a whole movie). Faced with such a case, everybody should feel surprised and find it rational to seek an explanation. (Recall also the quote from Foster in Section 1, about all the gravitational interactions across all space and time.)

2.1.2 A toy model

Let us begin by considering a space of Humean mosaics in which time consists of N instants, for a fixed, arbitrarily large, but finite value of N, and where at each instant there are only two possible states for the world to be in. The histories of such Humean mosaics or worlds can be encoded in N-bit binary sequences of 0's and 1's. There are then 2^N possible Humean mosaics, which make up our possibility space. To introduce a probability measure, let the agent consider a partition Π of propositions about which Humean mosaic obtains: $\Pi = \{E_1, ...E_i..., E_{2^N}\}$ where each proposition E_i is the proposition that the Humean mosaic i obtains.

Our discussion will be unaffected by most of these simplifications; only a continuous space would complicate the tractability of the problem, and such an extension is discussed later. It is irrelevant whether time is discrete or continuous, or how rich the description of each world is. As we want to compare worlds like ours, we can choose a finitely large and fixed number N, intended to represent the current age of the actual universe, with the goal of comparing the orderliness of the actual world with all other worlds of the same "time" length N. This tractable toy model could be extended to an uncountably infinite space in which the index i would range in the bounded interval between 0 and 1, that is, $i \in [0,1]^{\mathbb{R}}$, in the partition $\Pi' = \{E_0, ...E_i..., E_1\}$. From the finitely large to the infinite case, it remains that most worlds are disordered, according to the results obtained in measure theory cited above in our comments on premise 3, for any reasonable threshold of product-randomness. However, a continuous space introduces additional well-known complications for *PoI,* as we explain in §2.2.

Then, in the partition Π (likewise in Π') we can identify two mutually exclusive and exhaustive sets: a set of worlds displaying a certain degree of randomness, which we will call Irreg, and its complement, a set of worlds displaying a certain degree of orderliness, which we will call Reg. The degree of product-randomness that marks the threshold between these two sets is determined by the value k mentioned above, which represents a world's degree of compressibility. As explained in Section 1, our actual world is supposed to belong to Reg. As previously explained in this section, REG is far smaller than IRREG.

It is not hard to imagine richer descriptions of each world, and how the two sets would be then identified. For instance, a richer description of each world could include, instead of a single binary property, a finitely large m-tuple of real-valued properties $< P_1(t), \ldots, P_m(t) >$ at each location of a spatiotemporally finite world, whose values would be specified at each (continuous or discrete) instant of time. Such descriptions have been common in metaphysics since Lewis (1986), and echo the descriptions of state-space trajectories given in physics. (For instance, consider the description of a physical system in terms of a phase space given in classical mechanics: if the system under consideration is the whole universe, it includes values for all the variables: the three coordinates of position and momentum for each of the n particles in a $6n$-dimensional phase space.) Then, as e.g. Shackel (2007, 159) elaborates in his version of *PoI* for continuum-sized sets, we can assume a measure μ on the space Π' so that that the probability of a regular world, $P(\omega_i \in \text{REG})$, is given by μ as $P(\omega_i \in \text{REG}) = \mu(\text{REG})/\mu(\Pi')$.

2.2 Why the Principle of Indifference is flawed

The problems with *PoI* that concern us are three.[8] First, there is the conceptual lack of justification in assigning a uniform distribution in a situation of ignorance. Second, the choice of a reference class can be arbitrary, and third, there is the problem of how to preserve the standard axioms of probability when dealing with an infinite possibility space.

2.2.1 Why uniformity?

The main problem with applying *PoI* is that the assignment of equiprobability—i.e. assigning a uniform probability distribution with respect to an appropriate measure over the possibility space—is unjustified. The reason: it just need not be the case that each possibility is equally likely. In our case study this problem is especially clear: we have no clue at all as to whether each possible universe is equally likely; how do we know that some universes with certain properties are not much more likely than others? It is suspicious to think that we would gain any knowledge—such as knowledge of the equiprobability of each universe—from a situation of ignorance. (This problem has been widely discussed in the foundations of statistical mechanics,

[8] I set aside other alleged problems related to updating and the impossibility of learning from experience, for they do not affect our scenario in which there is no updating.

see the clear Uffink, 2006; Frigg, 2009, or the idiosyncratic Albert, 2015, 22, fn 5.)

By presupposing that a randomization process generates the outcome, one might justify a uniform distribution: for instance, we need a deck of cards to be properly shuffled to guarantee that the probability of any one card being selected is 1/52. Accordingly, Batterman (1992) stresses that *a posteriori*, empirical factors are required to justify assigning equiprobability; for instance, he points out that chaotic dynamics serve as a randomizer (cf. Strevens 1998, 2013). Without any such empirical feature, the application of *PoI* is unjustified.

2.2.2 Relativity to a reference class

Relativity to an arbitrarily chosen reference class is the second problem threatening *PoI*. This problem concerns the lack of an objective criterion for selecting an appropriate reference class by which to classify the space of possibilities. For any given case there exist a variety of reference classes, that is, of ways to describe the possibility space. The choice of one or another seems arbitrary, but the resulting probability can vary accordingly. For example, if we really had no information at all about the physics of coin tossing, we could partition the space of possibilities into {heads, tails, edge} as well as into the usual {heads, tails}, thus yielding different probabilities (1/3 and 1/2 respectively). Which of the two partitions is better is known from our empirical evidence, but it is not discernible *a priori*. In continuous spaces this problem is even more pressing. Bertrand's (1888) paradoxes first brought this problem to light, and Van Fraassen's (1989) cube factory example clearly illustrates it. In general, there turns out to be no *a priori* and non-arbitrary way of selecting a unique set of parameters to partition the possibility space.

2.2.3 Infinite possibilities do not sum to 1

The third problem concerns dealing with an infinite possibility space. A probabilistic framework assumes that probabilities can be added (the 'additivity' axiom) and that they normalize to 1 (the 'normalization' axiom). But an infinite sum of any positive value will diverge; hence, the standard probabilistic framework seems to be forced to assign a probability of 0 to each outcome, rather than a 'very low value', whatever that may be.

3 Recent Justifications of the Principle of Indifference

There are a number of proposals to deal with infinities, aimed at solving the third problem. I explain them later, but they will not be strictly necessary; for as I argue, we can avoid the problems of infinity by restricting our conclusions to our finite toy model, which is also able to deal with the far more tractable finite version of the reference class problem (see §4.4). Regarding the first problem, there have been several recent independent justifications for the rationality of applying *PoI* in situations of ignorance. In this section I present the following three: Pettigrew's (2016a; 2016b) argument from accuracy, Jon Williamson's (2018; 2010) and Landes and Williamson's (2013) argument from maximum entropy and caution (understood in terms of both accuracy and pragmatic losses), and Konek's (2016) argument from maximum entropy and probabilistic knowledge. It is then crucial to argue that any of these justifications are valid in our particular scenario. I argue that this is so in Section 4.

3.1 The Argument from Accuracy

Pettigrew (2016b) proposes a non-pragmatic argument which appeals only to the cognitive value of credal states. He thus defends *PoI* on grounds of *epistemic* rationality (as opposed to pragmatic rationality). His argument aims to show that, in a situation of ignorance among possibilities, an agent who violates *PoI* in her credences risks greater *inaccuracy* than is necessary; in other words, she would be irrational not to have equal credence in each possibility. Accuracy represents how good an agent's credal state is, i.e. how closely it approximates truth. For reasons of space I cannot explain this argument in detail, but it can be summarized as follows:[9]

[9] Minimax is the rule in decision theory that demands that an agent in the absence of evidence chooses the option that minimizes its maximum disutility (see Pettigrew, 2016b, 39-40 for details and justification). Minimax applies only to what David Lewis called 'superbabies': agents at the beginning of their credal life. We are such superbabies with respect to Argument: as pointed out in footnote 7, in assessing the plausibility of the regularity of the actual Humean mosaic we have to put ourselves before the actual evidence.

1. The cognitive value of a credence function is given by any measure of accuracy that meets the conditions of being egalitarian and rendering indifference immodest;
2. MINIMAX;
3. **Theorem**: The credence function that assigns the same credence to each possibility has a maximum inaccuracy lesser than any other credence function;

∴ *PoI*

This argument rests on plausible assumptions, not disputed in our specific scenario, namely the two conditions specified in premise 1 and the decision-theoretic norm MINIMAX. Yet, there is another condition: the propositions of the possibility space must form a finite algebra. A similar "argument from accuracy" might be given in a framework of infinitesimals. This is not the place to prove this hypothesis, but it seems plausible, or at least conceivable. In any case, the argument holds for our large-but-finite toy model.

3.2 The Argument from Maximum Entropy and Caution

A second defence of *PoI* comes from an argument based on the *maximum entropy principle*. Historically, *PoI* was justified by appealing to the idea that agents should base their beliefs on *minimal information*, i.e. not on unwarranted information. Jaynes (1957, 623) concluded that "the maximum-entropy distribution may be asserted for the positive reason that it is [...] maximally noncommittal with regard to missing information".[10]

This approach has been, however, disputed (e.g. Seidenfeld, 1986). Recently, though, Williamson (2010, §3.4.4) has given a novel argument in terms of the pragmatic notion of caution, which can also be phrased in terms of the epistemic notion of accuracy (Williamson, 2018).

Williamson (2010, §3.4.4) brings into consideration the *risks* that an agent should not take. Then, favouring a *cautious* attitude before a decision between different actions—where these notions are clearly defined by the author—Williamson concludes that *the most cautious choice* is to follow *PoI*. Furthermore, Landes and Williamson (2013) generalize this argument from caution and show that *PoI*, as well as two other norms of objective Bayesianism, can be justified

[10] When an agent's credence function c is defined over finitely many outcomes E_i, its 'entropy' or 'uninformativeness' is measured by the Shannon entropy, $H(c) = -\Sigma_x c(E_i) \cdot log(c(E_i))$. When c is defined over uncountably many outcomes, its entropy is calculated as: $h(c) = -\int_{\Pi'} c(E_i) log(c(E_i)) dx$.

in terms of *minimizing worst-case expected loss.* The authors show that the belief function that minimizes worst-case expected loss is the probability function that maximises entropy.

3.3 The Argument from Maximum Entropy and Probabilistic Knowledge

A third argument also defends *PoI* by defending the Maximum entropy principle. Inspired by epistemological theories that appeal to the notion of 'probabilistic knowledge' (e.g. Moss, 2018), Konek (2016) proposes a condition that we seek to impose on our beliefs, namely that the accuracy of our beliefs is *not a matter of luck but rather of our cognitive ability.* This leads him to conclude that setting our priors according to the principle of Maximum Entropy is the best way to respect this condition, and thus constitutes a good candidate for modelling probabilistic knowledge.[11]

Appealing to the idea that the accuracy of our credences should be the result of our cognitive ability, Konek (2016, §6) argues that the accuracy of our credences should not be explained by "a prior hunch" (i.e. setting a non-uniform prior distribution) but rather by following the Maximum Entropy principle. This allows him to conclude that a uniform distribution in a situation of ignorance "does seem to deliver credences that are eligible candidates for constituting probabilistic knowledge (at least in simple inference problems)".

4 Which Inductive Logic Our Scenario Warrants

In spite of these arguments for *PoI*, we should not choose our inductive logic *a priori*; instead, as Norton (2007, 2008, 2010, Forthcoming) urges, the empirical or 'material' conditions of the problem have to justify the appropriate inductive logic. While a probabilistic framework—a precise numerical assignment of probability—is often justified, this is not always so, and using an incorrect inductive logic can lead us to incorrect predictions.

One case in which it is contended that a probabilistic framework is unjustified is the case of total ignorance. In total ignorance, *suspension of judgment* is contended to be the most appropriate doxastic state, yielding an inductive logic different from the usual probabilistic framework, the latter of which is unable to model suspension of judgment. Suspending judg-

[11] More precisely, the best way to respect the condition is to follow a slight variation of this principle, which he calls the Maximum Sensitivity principle.

ment about some proposition or set of propositions means neither believing nor disbelieving to any degree any option or options, but rather lacking any belief or degree of belief whatsoever. A number of representational frameworks have been elaborated in inductive logic to properly represent suspension of judgment. These different approaches include non-probabilistic non-numerical calculi as proposed by Norton (2008) (applied to cosmological issues in Norton 2010), and imprecise probabilities as proposed by de Cooman and Miranda (2007) (applied to cosmological issues by Benétreau-Dupin 2015). For overviews of other approaches see Halpern (2003, Ch. 2) and Dubois (2007). For recent philosophical discussions of suspension of judgment see Friedman (2013, 2015) and Tang (2015). Its origins date back to the ancient scepticism of Pyrrho (Empiricus, I c. A.C.).

Further, assigning the same probability to each possibility is not all that is disputable, but also the step of *adding* the probabilities. And we do intend to add the individual probabilities of the members of the two sets, Reg and Irreg, to conclude that an ideal agent's credence in the latter would be overwhelmingly greater than in the former. The axiom of additivity, one of the Kolmogorov axioms of probability, states that, for any mutually disjoint sets of possibilities A and B, $P(A \cup B) = P(A) + P(B)$. This axiom would lead us conclusions that the advocate of suspension of judgment would find suspicious. Intuitive as it may be, additivity would lead us to conclude that the probability that either world_1 or world_2 occurs is *exactly two times*, no more no less, than the probability that world_1 occurs. But what in the physics would license this? In a situation of total ignorance, nothing in the physics licenses these precise confidences. Accordingly, it has been contended that in situations of ignorance, additivity is unwarranted. In its stead, an axiom of *non-additivity* has been proposed which states that, for any sets of possibilities A and B, $P(A) = P(B) = P(A \cup B) = I$, where '$I$' (which stands for 'ignorance' or 'indifference') is a non-numerical doxastic state. (Variations of *PoI* which incorporate non-additivity have been proposed in Norton, 2008; Eva, 2018.)

Then, at first sight, it might seem that we should suspend judgment in Argument. However, upon closer inspection, we will see that there are two distinctive features of our scenario which will tip the balance towards endorsing a credence stronger than the mere suspension of judgment, namely: (1) we will interpret the probabilities in Argument as *subjective probabilities*, i.e. the credences of an ideal rational agent, not as objective probabilities (§4.1); and (2) since we are assuming a Humean metaphysics (§4.2), we know (2a) that there are no objective chances concerning the occurrence of the Humean mosaic and (2b) which is the real space of possibilities. These features are sufficient to justify assigning a *uniform* and *additive* credence among the possibilities.

4.1 Only Subjective Probability, Not Objective

A first step is to interpret the probabilities in Argument as referring not to the objective probability of a Humean mosaic obtaining, but rather to the *credence* that an ideal rational agent should have that a certain Humean mosaic obtains. In general, there might or might not be objective chances. Still, in scenarios such as ours in which we do not know their value, there is no requirement that our credences match the chances. In other words, the so-called *Principal Principle* does not apply. It is this principle that connects epistemic credences with metaphysical chances, stating that the former should match the latter. For example, if we know that the chance of the result 'Heads' for a coin flip that's about to occur is 0.5, then our credence should be the same, 0.5.[12] However, this principle applies only if we know the chances, or at least have some degrees of belief concerning them.

We might say that ignorance puts normative constraints on what our credences should be, which is not to extract knowledge about the world. As White (2009) says, we find it reasonable to think that one *needs a reason* to give more credence to one outcome than to another. If we lack any such reason (as we do in our state of ignorance), we should assign the outcomes the same credence. Any other assignment of beliefs would be unjustified. What is more, *we do have reasons* to assign a uniform credence: the three independent reasons presented in Section 3 that appeal to accuracy, caution, and probabilistic knowledge.

At this point, it might seem that we can already justify assigning a uniform credence. However, this is still insufficient. For it is still *more cautious* to suspend judgment than to assign a uniform credence. This is especially clear in our present context of philosophical inquiry. It would be acceptable to endorse a uniform credence in epistemic contexts in which an agent is forced to choose, or willing to take a risk (as when betting in a game). But philosophical and scientific inquiry are characterized by the norm of prioritizing the avoidance of falsehoods over the search for truths—what is known as 'strict evidentialism' (in contrast with William James's (1979) non-evidentialist "will to believe"). In fact, if we make predictions using a uniform as-

[12] The Principal Principle states that, if p is a certain proposition about the outcome of some chancy event and E is our background evidence at t, which must be admissible evidence, then: $Cr(p|Ch(p) = x \wedge E) = x$. (Evidence E is admissible relative to p if it contains no information relevant to whether p will be true, except perhaps information bearing on the chance of p.) See Lewis (1980, 86), Hoefer (2019, Ch. 3).

signment of credences, we risk arriving to conclusions very far from the truth (for more on this see Filomeno (20xxa)). As explained below, this is bolstered by the fact that we sometimes assign credences to apparent possibilities which are not really possible, thus aggravating the inaccuracy of our predictions.

In other words, in the defences of *PoI* discussed in Section 3, the uniform distribution is implicitly compared *only* with other probability distributions—thus resulting as the most accurate, most cautious and most in tune with probabilistic knowledge. Nevertheless, suspending judgment is a still more cautious doxastic state than the uniform distribution. Since in our context our priority is avoiding falsehoods, a uniform distribution is riskier than suspending judgment. Hence, the elements put forward so far—a subjective interpretation of the probabilities and the various defences of the rationality of *PoI*—still do not suffice to justify a uniform distribution.

4.2 The Lack of Chances in a Humean Metaphysics

So far we have been open to scenarios in which there are objective chances, whose value we ignore. Since, however, our scenario is framed within a Humean metaphysics, we know (1) that there are no objective chances assigned to the occurrence of each Humean mosaic and (2) exactly which is the real space of possibilities. (That there is no objective chance concerning the occurrence of the actual world can of course be conceived without endorsing a Humean metaphysics; although we should then somehow otherwise motivate its plausibility.) This means that the metaphor of God choosing the actual world at random is not what we are modelling here; Figure 1 does not accurately depict our scenario.[13]

Regarding feature (2), knowing which is the so-called 'real' space of possibilities, that is,

[13] The point is that there is not a chance distribution related to the *obtaining* of this or that mosaic. Lewis (1994) tried to make sense of the whole mosaic having an objective chance, but in reductionist, Humean terms. He defined the fit of a system of laws to be equal to the chance that the system gives to the full mosaic that it supervenes on (cf. Loewer (2004); Albert (2012, Ch.1); Hoefer (2007, 2019)). Yet, this is of course a different sense from the objective chance related to bringing about the mosaic. In any case, this attempt has been criticized in several ways, e.g. by Hoefer, 2007, and more recently in Hoefer (2019, Ch. 4) where, for instance, it is shown that Humean chances must be restricted to small-scale phenomena within the mosaic in order for his proof of the Principal Principle to go through. Thus, we can talk of Humean chances within the mosaic—indeed we should, insofar as we are assuming the Humean point of view!—while accepting that no chance is assigned to the occurrence of the mosaic. It is, remember, just a brute fact.

knowing which worlds have non-zero probability of being the case, turns out to be crucial for the choice of an epistemic representation of our ignorance. If we know the real possibility space, we are said to be in 'classical ignorance' (Hansson, 1994). If we do not know exactly which is the real space of possibilities, we are said to be in 'total ignorance', which means that some of the alleged possibilities under consideration might have probability 0, or that we might not be taking into account all the possibilities.

Then, given that we are assuming a Humean metaphysics, which explicitly contends that any rearrangement of the entities and properties of a world itself counts as a possible world, we should rule out no such possible world: any could have been the case, as is explicitly stated in Lewis's principle of recombination. In other words, no Humean mosaic has a zero chance of being the case. (There are not even chances assigned to the worlds.) Any such possible world is a so-called 'real possibility': both for the finite and for the uncountably infinite toy model, we know which are the real possibilities; and we are thus in classical ignorance. This excludes one potential source of error in predictions based on a uniform distribution, namely that in a space of total ignorance, uniformity might be assigned to the wrong space of possibilities—be it too wide or too narrow or both—leading to incorrect predictions. (As we will see below, classical ignorance also helps to justify the required property of additivity and to avoid the reference class problem in a finite space.)

The other risk involved in assigning a uniform credence is, as mentioned above, that an unknown biased objective chance distribution might be the case, thereby undermining any prediction based on a uniform credence. This risk is here avoided, since in a Humean framework there are no irreducible objective chances whatsoever, and in particular there is no objective chance concerning the occurrence of the actual Humean mosaic. Rather, the occurrence of the mosaic is a *contingent primitive fact.* The mosaic could have turned out differently, but not as the result of a chancy process.

The point here is that it is not possible for a biased chance distribution to exist. As explained above, a uniform distribution cannot be justified *a priori* because there could have been biased chances—in a standard case such as a coin toss, the coin could well be unfair. To justify uniformity in such cases, we have to appeal to empirical, *a posteriori* reasons; for instance the underlying chaotic dynamics, which randomizes the outcomes, which then guarantees uniformity. Of course in our scenario we cannot use this *a posteriori* justification because we have no underlying dynamics. However, we do not face any objection concerning the possibility of

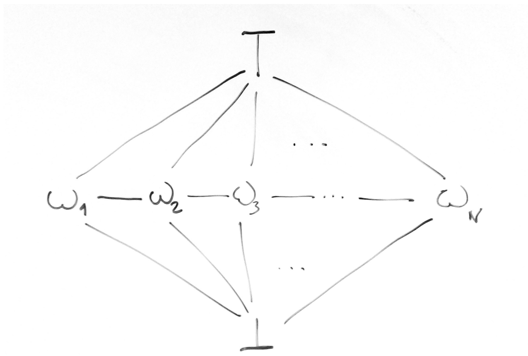

Figure 2: A representation of our order of confidence, where ⊤ means maximum confidence, or certainty, and ⊥ means minimum confidence. In between we have the *same* confidence, as indicated by the horizontal lines, in each proposition/world ω_i. Here it is not granted that our confidence in the union of some possibilities ω_i is larger than our confidence in each individual possibility ω_i, so the situation depicted is compatible with non-additivity.

a biased chance distribution, because we *know* that there is no chance distribution. Hence, our particular scenario avoids the risks that made suspending judgment a more cautious doxastic attitude than any credence distribution. Thus, following any of the defences of *PoI* discussed in the previous section, we are warranted in assigning the same credence to each possibility.

4.3 Additivity

A uniform credence among atomic possibilities is, however, still insufficient for our purpose of arguing that we should be *much more* confident in *much larger* sets of possibilities. The axiom of additivity of possibilities is needed. As Norton advocates, and as has been stressed at least since Fine (1973), additivity, like the other axioms of probability, should not be taken for granted. Rather the particular scenario under study must warrant its use.

Non-additivity, as explained at the beginning of this section, is admittedly counterintuitive. Yet even some of those who advocate Bayesianism as "*the* true inductive calculus" recognize that additivity is hard to *universally* justify—see for instance the explicit diagnosis offered by Titelbaum (in progress, Ch. 2.4 and Ch. 10.3.3). Here is not the place to defend the legitimacy of non-additivity, but let me at least note the existence of different inductive logics without additivity, used in real applications such as artificial intelligence, as discussed by Halpern (2003, Ch. 2) and Dubois (2007).

Before assessing whether additivity is warranted, let me recapitulate our discussion so far. We have justified at least a comparative belief structure with a finite set of possibilities each *as probable as* any other, where their union is at the extremum representing certainty, and the lack of any of them is at the other extremum representing full disbelief; as depicted in Figure 2.[14]

As we are only interested in a situation of classical ignorance, a vindication of additivity is only required in the situation in which the size of the possibility space is known. Pettigrew (2016b, §6.1) claims that this is so, namely that *PoI* implies the probabilism of superbabies (i.e. agents lacking any empirical evidence; see footnote 9 in p. 14). He does not justify this, so let us verify it in what follows.

One general argument for probabilism, including additivity, is a Dutch book argument. Dutch book arguments state that if an agent does not set her own beliefs according to the axioms of probability, then she is vulnerable to having a book made against her.[15] Assuming that the agent's betting quotients violate the axioms, a bookie can guarantee herself a profit. While these arguments are given in terms of pragmatic rationality, there is a similar argument given in terms of epistemic rationality that uses the previously introduced notion of accuracy, due to Joyce (1998). There are some objections to these general arguments, but in any case we can now cite specific results for the simple comparative structure that we are dealing with here.

As noted by Suppes (1994, 1), the fact that all the atomic events are equiprobable suffices in the finite case for a comparative structure to agree with a numerical probability measure. In particular, it has been proved that a comparative belief structure such as ours admits and is compatible with a finitely additive numerical quantitative probability: if certain conditions are met by the comparative order, Theorem 1 of Fine (1973, IIB.2) entails the existence of a quantitative probability, and Theorem 4 (*ibid.*) states the existence of a function that can be finitely additive if further conditions are met. Several candidate conditions (necessary, sufficient, or necessary and sufficient, which hold for finite, infinite or both spaces) have been proposed in the literature for compatibility with finite additivity; we can mention here Scott's (1964) sufficient conditions, since these are justified by Joyce (1998, 601-2) for our context of a

[14] Optionally we could also add—although it is not necessary for our justification of additivity—that *strict* subsets are more probable than their constitutive elements. This has been argued in (Filomeno, 20xxa). See Figure 3.

[15] The book consists of a set of bets, each of which the agent views as fair, but which together guarantee that she will lose money come what may.

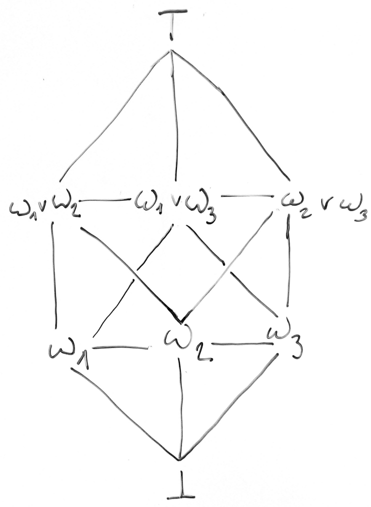

Figure 3: In this representation we only add a weak additivity, namely the additivity between strict subsets. This representation is still compatible with non-additivity, unless additivity is justified.

credence distribution.[16] According to Joyce, Scott's condition amounts to the requirement that one would impose if one wanted credences to be 'gradationally accurate', which means that they are prevented from being less accurate than they need to be. And indeed we want our credences to be gradationally accurate; for otherwise we would be epistemically irrational.

4.4 The Remaining problems

There remain the two problems with *PoI* discussed in §2.2.2 and §2.2.3. The problem that infinities do not normalize to 1 of course only affects infinite models. Benci et al. (2016) propose a formal model for an uncountably infinite lottery, which uses so-called non-Archimedean infinitesimal probabilities.[17] Another usual solution for uncountable spaces is a finitistic approach that partitions the space into a coarse-grained finite space of equivalence classes.

The reference class problem, in contrast, affects both finite and infinite models. Given our state of ignorance, we do not know which reference class to choose: for example, {heads, tails} or an expansion such as {heads, tails, edge}. Similarly, we do not know whether to

[16] Cf. the other conditions proposed in (Kraft et al., 1959; Luce, 1967; Savage, 1972; Suppes and Zanotti, 1976); for discussion see Fine (1973, IIC and IIID), Krantz et al. (2006, Ch.5), and Suppes (1994).

[17] The use of infinitesimals has been disputed by Pruss (2012, 2013, 2014) and Williamson (2007). Benci et al. (2016) and Weintraub (2008) attempt to reply to such objections.

choose {heads, tails} or a refinement such as {heads oriented within [0°,180°), heads oriented within [180°, 360°), tails}. The problem in the finite model thus arises due to expansions or refinements of the possibility space.

In our Humean scenario, however, refinements and expansions cannot occur. Lewis's principle of recombination uniquely determines the full space of possibilities, so we are in classical ignorance and the space of possibilities is fixed. Thus, no refinement or expansion of the space is an acceptable representation of our scenario.

The reference class problem is hardly tractable in the case of an infinite model. There are infinitely many partitions that follow some symmetry and thus all seem equally acceptable; hence each possible world belongs to multiple partitions. The aforementioned method of coarse-graining the infinite model to a finite one would also be affected by this relativity. This problem seems to be inescapable: classical probabilities in infinite domains must be relativized to a reference class.

Thus, we should opt to stick to the finite model, as long as it does not leave out any relevant feature that would alter the conclusion. I believe that this is so: the finite toy model laid out in Section §2.1.2 and any enriched version of it would hardly leave out any relevant feature, since what is needed just is that the size of Reg is overwhelmingly larger than the size of Irreg, which is the case for any extension of the finite arbitrarily large possibility space. Furthermore, one might think that the problems that arise *only* in infinite spaces are merely technical rather than real problems, at least in relation to our argument, which concerns (i) a finite universe (the actual Humean mosaic) and (ii) what we should believe about such universe. It has often been noted that the role of the infinite is just to facilitate our reasoning about the finitely large universe (read here as 'Humean mosaic') in which we dwell (e.g., Hilbert, 1925). In which case, the chief constraint on the behaviour of the infinite is that it conserves the behaviour of the finite. In fact, precisely with respect to formal epistemology Williamson (2010, 153) says: "it was important that in defining objective Bayesianism on an infinite predicate language it should behave very much like objective Bayesianism on a large but finite predicate language."

The Humean may insist that a *standard* approach to modelling a (spatiotemporally finite) world recurs to an infinity of real-valued properties—something like the infinite partition Π' laid out in §2.1.2. Continuous models are not only entertained by metaphysicians, but are common and fruitful in physics. However, are we *required* to recur to such models? It does not seem so, and in the case of the physicists that use such models it is in fact especially clear that they do not have to ontologically commit to the continuum (for instance, nobody blames researchers in Loop Quantum Gravity, according to which reality is fundamentally discrete, for using standard calculus). For all these reasons, it does not seem reasonable to just neglect the cosmic coincidence objection, which has been shown to hold for any enrichment of the toy model including an extension of the domain to *any arbitrarily large* finite size, by appealing to the intractable problems in infinite domains.

5 Conclusion

This journey, which began with wondering about the ubiquity of regularity in nature and has ended with assessing an intuitive but disputable axiom of probability theory, has left several questions open; but it suggests at least a restricted conclusion. If the discussion of the previous section is correct, our particular scenario, with respect to which we know its possibility space and its lack of chances, warrants the assignment of an additive and uniform probability measure, interpreted as an ideal subjective credence distribution among the Humean mosaics. This leads us to conclude that, at least for the toy model with an arbitrarily large but finite space of possible Humean mosaics, Argument is sound.

More specifically, in a class of models describing possible Humean mosaics of length N, where N is a sufficiently large number emulating the finitely large time length of the actual Humean mosaic and each state at a time is described by a binary property (extendable to a richer description with more properties and more possible values per property), it is a cosmic coincidence that a highly regular Humean mosaic such as the actual one obtains. That is, *an ideal rational agent's credence in a highly regular Humean mosaic obtaining is overwhelmingly low*. This is because, while we have argued that the justifications of *PoI* cited in Section 3 are not always sufficient—since suspension of judgment can be a more cautious doxastic attitude—we have also argued in Section 4 that the specific features of our scenario warrant us to depart from an unnecessarily cautious suspension of judgment.

Hence, assuming that this toy model does not leave out any relevant feature of the actual Humean mosaic that could alter the conclusion of Argument, we are led to conclude that a Humean account of laws should provide an explanation, currently missing, for the overwhelmingly unexpected degree of regularity of the actual world. Positing the mosaic as a fundamental *unexplained* fact is unsatisfactory, in absolute terms; and even more so in comparative terms, that is, in comparison with non-Humean proposals. Of course, while I consider this objection to be serious, it does not mean that it is insurmountable. A path for future research is thus opened for the Humean: provide an explanation of the ubiquity of regular behaviour without appealing to governing laws or any other primitive physical necessity. This is, in fact, the project explored in (Filomeno, 2014, 2019, 20xxb,x) and references therein.[18]

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