

## Williamson on Fine on Prior on the Reduction of Possibilist Discourse

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Timothy Williamson's *Modal Logic as Metaphysics* [2013] (MLM) is a tour de force - comprehensive in its scope, brilliant in its argumentation, and startling in its conclusions. It merits discussion on a wide range of different fronts, but I hope I can be forgiven for focusing on chapter 7 of the book, in which Williamson criticizes my attempt to carry out Prior's project of reducing possibilist discourse to actualist discourse.

My response is in three main parts. I begin by discussing what the reductive project should be. Williamson and I disagree on this question and, although it is not important for the evaluation of my own reductive proposal, it is important for a broader understanding of the metaphysical issues at stake. I then discuss and evaluate Williamson's criticisms of my original reductive proposal. Although I believe that these criticisms can to some extent be met, they point to the need for a more satisfactory and less contentious form of reduction. Finally, I lay out the new proposed reduction; it is based on the idea of finding a general way of extending a reduction of first order discourse to higher order discourse.

The main technical innovation of the paper is the introduction of the suppositional calculus, which I hope may be of some independent interest quite apart from its connection with the present reductive project. Although I have not attempted to be rigorous, I hope it is clear from my informal remarks how a more rigorous treatment might proceed.<sup>1</sup>

### §1 What is the Problem?

Part of Williamson's aim is to reshape the debate between the actualist and the possibilist. He takes this debate to be 'hopelessly muddled' (pp. 23-4) and thinks that 'rather than trying to make sense of the confused dispute between actualism and possibilism, we can more profitably replace it with enquiry into clearer issues in roughly the same territory', where 'perhaps the deepest of those is the dispute between contingentism and necessitism' (p. 308). I must admit of a fondness for the old 'confused dispute' and something of a distaste for its proposed replacement; and so let me briefly explain why, though the question calls for a much more thorough discussion than I can give here.

As I understand it, the debate between the actualist and the possibilist rests on two distinctions. The first is the distinction between possible and actual objects; and the second is the distinction between what is and is not real. The actualist then claims that only actual objects are real, while the possibilist denies that this is so.<sup>2</sup>

This formulation of the debate is therefore as clear as the two distinctions - or notions - upon which it rests. Williamson seems to find both distinctions obscure (though obscurity in either would be

1 I should like to thank the audiences at the 2014 Prior conference in Oxford and at the 2014 meeting of the Bucharest Society of Analytic Philosophy for helpful comments. I have also benefitted from conversations with Walter Dean and from conversations with and written comments from Peter Fritz and Jeremy Goodman.

2 This is part of my general stance on ontological questions (Fine [2009]). However, the formulation in this case might be regarded as over-simple for at least two reasons. One is that the actualist might wish to make a modal claim, to say that *necessarily* only actual objects are real; another is that a philosopher might believe that only actual objects are real, not because he is an actualist, but because he is a Pythagorean, let us say, and believes that only numbers are real and that all numbers are necessarily actual. But I take it that such subtleties in the formulation of the dispute are not here at issue.

enough to damn the formulation). In §1.6 of MLM, Williamson considers various possible definitions of what it is for an object to be actual and finds them all lacking. But why is a definition required? I would have thought that the notion was perfectly clear from examples. There are various actual people - Socrates, for example, Timothy Williamson or myself - and also various merely possible people - the various children that Wittgenstein might have had, for instance. The actual people are actual objects, the actual and merely possible people possible objects. What could be clearer than that?

It might be thought that an actual person is a person while a merely possible person is not a person but something that is possibly a person and hence just as actual as a person. This is not my own way of looking at the matter nor, I think, the most natural way.<sup>3</sup> A merely possible person *is* a person, just as an actual person is a person; the difference between them lies not in whether or not they are persons but in whether or not they are actual (pace §1.3 of MLM). But no matter. Let us grant that a merely possible person is not a person but only possibly a person. Still, there is a clear intuitive distinction between being a merely possible king, let us say, and a merely possible person. Bonnie Prince Charles was not a king only possibly a king, but that does not make him a possible object; he is as actual as you or I. Likewise, a possible child of Wittgenstein will not be a person, on the view we are considering, but only possibly a person. But that is surely enough to make him a merely possible object; there is no reason, in this particular case, to suppose that the object is also actual.<sup>4</sup>

Williamson briefly discusses the idea of reality or fundamentality in §7.1 of MLM. He there takes the issue between the actualist and possibilist to be one about ‘whose quantifiers are more natural or basic’ and comments that ‘the inexpressibility of the dispute in more robust and tractable terms would be a disappointing outcome. For presumably the ordering in terms of relative naturalness or basicness is supposed to be of metaphysical interest because it is not determined just by the psychology of speakers, but rather by non-linguistic features of reality as a whole. If so, the dispute would be more perspicuous if each side could formulate its view of the disputed features in less extrinsic terms.’

I agree. But my own characterization of what it is for an object to be real *is* stated in such terms (Fine [2009]). For it is supposed that we have a conception of how things are in reality. For an object to be real is then for it to figure in reality; and so to be an actualist is to hold that how things are in reality is entirely a matter of how *actual* things are. We might also put the point by saying that we are capable in principle of fully describing reality without reference to what is non-actual, but such appeal to language or to what we do is not strictly required.

Let us turn to Williamson’s preferred formulation of the issue in terms of necessitism and contingentism (‘N/C’, for short). The necessitist holds, under an absolutely unrestricted use of the quantifier, that necessarily everything necessarily has being (is identical to something), while the contingentist denies that this is so (as explained in chapter 1 of MLM).

Whence my distaste for this formulation of the issue? One reason is that it makes me a necessitist. I am perfectly happy to concede that the unrestricted quantifiers range over all possible objects and so am happy to accept that necessarily everything necessarily has being.<sup>5</sup> But as an actualist, I also believe that

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3 I provide a defense of my own view in chapter 9 of Fine [2005], especially §9.

4 At one point (p. 23), Williamson asks ‘And why should the alternative to the view that everything does the harder thing [being actual] be a view on which everything *could* do the harder thing [be possibly actual]?’ A good question, to which the answer is that this is not the only alternative. One might think, for example, that there are sets of impossible objects which could not even be possibly actual and, indeed, such a view would naturally follow from a position which took all urelements to be possibly actual and yet took some of the urelements to be impossible.

5 I also believe that there is no intelligible notion of unrestricted quantification (Fine

there is a separate issue as to whether only actual objects are real - or some such thing, whether or not it is exactly stated in this way. Williamson's formulation of the issue is therefore completely incapable of getting at the sense in which I am an actualist. Far from clarifying the issue, it ignores it.

But it is worse than that. For I believe that *anyone* in their right mind should accept necessitism. Suppose I show you three blades and three handles and ask how many possible knives can be made from the blades and handles? Surely the correct answer is nine; there are nine possible knives that can be made from the given blades and handles. Yet, according to the contingentist, the correct answer is zero (assuming that no one of the blades is ever attached to a handle). This strikes me as absurd - perhaps not as absurd as denying that there are chairs and tables, but absurd all the same.

Of course, the contingentist, along with his fellow anti-realists, will attempt to fend off the absurdity. He will claim that although it is not strictly and literally true that nine possible knives can be made from the blades, still there is a sense in which it is correct to say that there are nine possible knives. And a number of possible reasons might be given for this: perhaps we engage in fictitious talk; perhaps we fail to use the expression 'there are' as a quantifier; or perhaps we speak metaphorically. But, to my mind, this is simply to put bad linguistics at the service of bad philosophy. For apart from the philosophical prejudice against there being possible objects, there is no good reason to take any of these hypotheses seriously. In a somewhat different connection, Williamson writes 'neither side [of the N/C dispute] is entitled to patronize the other by treating them as linguistically incompetent, unable to express their own views accurately in their own language.' (p. 311); and it seems to me that we should extend a similar degree of respect to the ordinary speakers of our language.

Williamson briefly touches on the question of triviality in his book. He writes 'Now we must consider the charge that there is no live issue for the opposite reason, because necessitism is too obviously true. The idea is that if 'something' covers merely possible coins as well as coins, then trivially this coin is necessarily something, and so on. The idea is fallacious. That 'something' is unrestricted implies that the things it ranges over include whatever merely possible coins there are, *if any* [my emphasis]' (MLM, 15). The idea is indeed fallacious. But the charge of triviality we are making is not based upon this idea but simply on the idea that it is evidently true there are nine possible knives that can be made from the three handles and the three blades. In the same way, it is evidently true, on the basis of observation, that there are chairs and table - not because 'something' covers chairs and tables whatever chairs and tables there are but because observation makes it evident to us, once 'something' is allowed to cover chairs and tables, that there are chairs and tables.

If I am right, then the issue of necessitism versus contingentism is a non-issue and any right-minded philosopher should accept necessitism without further ado. But quite apart from the question of triviality, it is far from clear to me that Williamson's approach provides any gain in clarity over the more usual approach. It certainly looks that way when we attend to the *formulation* of the issue; for the necessitist and contingentist can state their respective theses within the language of quantified modal logic, while the actualist and possibilist must appeal to the relatively obscure idioms of actuality and reality (or the like). But things look different when we attend, not to the formulation of the various theses, but to their defense.

For the necessitist needs to capture the truth in what the contingentist is saying when he says that it is possible that Socrates (and other 'contingents') do not have being. To this end, he will appeal to some feature F and say, instead, that it is possible that Socrates (and the like) are not F. The feature Williamson appeals to earlier in the book is being concrete (§1.2) and later he appeals to the related notion of being

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[2006]). But I do not press the point here, since I agree with Williamson (MLM, p. 15) that we can get at 'the point' behind the N/C dispute without appeal to the notion.

chunky in providing a possible defense of contingentism (p. 315). But what is it to be concrete? In a footnote (p. 6), Williamson remarks, ‘the term ‘concrete’ is used informally throughout this book. For present purposes, we need not decide between various ways of making it precise (being material, being in space, being in time, having causes, having effects).’

But none of these various glosses will serve the general demands made on the notion. Being material or being in space will not do since someone who thinks of Socrates as a Cartesian ego may still of him (or rather, it) as a contingent being. Being in time or having causes or effects will not do either. For consider someone who thinks that each possible world is a contingent being (having being only at that world). Then no possible world will be in space or time or have causes or effects; and, in the case of the empty possible world, it will not even be a material being or have proper parts. It therefore seems as if only the actualist’s more rarefied notion of being actual will fit the bill.<sup>6</sup> Indeed, apart from Williamson’s hostility to the notion, it is far from clear to me why the necessitist and contingentist should not simply expropriate the actualist’s notion of being actual for their own purposes. Certainly, there is nothing in their respective positions that rules it out; and, once this is done, they will be on the same conceptual footing in this regard as their actualist and possibilist counterparts.

Another apparent advantage of the N/C formulation is that it does not require appeal to the seemingly obscure idea of what is fundamental (or real). But the necessitist and possibilist must appeal to an equally obscure idea of cash-value. For the contingentist is under some obligation to find the cash value (or ‘truth’) in the statements made by the necessitist, and likewise for the necessitist (cf. MLM, 312).

But why is it not easy to discharge this obligation? Why can the contingentist, for example, not take the cash value of a necessitist’s statement A simply to be given by its ‘neutral’ consequences (those independent of their differing metaphysical commitments)? We feel that in certain cases this is not enough, that we require a neutral statement or set of neutral statements that is *equivalent* to the given statement, that otherwise something will be lost. But why and when? Williamson argues in the particular case of the statement that ‘there are infinitely many possible stars’ and of some other statements that something significant *would* be lost if the contingentist could find no neutral equivalent (MLM, 348-9); somehow the contingentist would not be able to make distinctions that they should (or perhaps would like) to make. But we are provided with no clear or principled conception of why or when an equivalent is required. For a ‘soft’ actualist like myself, by contrast, there is no such problem; for I will be willing to accept whatever the possibilist says is *true* and hence will take myself to be under an obligation to account for its truth. The contingentist may play fast and loose with the necessitist’s falsehoods, but the soft actualist cannot play fast and loose with the possibilist’s truths; and so even if the actualist’s position is more obscure in its formulation, it is much clearer what is required for its defense.

## §2 The Standard First-order Reduction

In what follows, I shall depart from Williamson’s own exposition in chapter 7 of MLM and construe the issue of reduction in terms of the debate between actualism and possibilism, since that is how I originally conceived of the issue and how I still prefer to think of it. However, it should be relatively easy to transpose what I say to fit the debate between contingentism and necessitism; and sometimes, though not always, I shall indicate what further changes this would entail.

There is a standard ‘translational’ reduction of first-order possibilist discourse to first-order actualist discourse, described in Fine [1977a]. Let us suppose that the formulas of interest to us are drawn

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<sup>6</sup> Additionally, if one thinks, as I do, that a wooden chair, say, is necessarily wooden and hence necessarily concrete then the appeal to concreteness is doomed from the start.

from a first-order language  $L$ , which contains the ‘inner’ quantifiers  $\forall$  and  $\exists$ , ranging in each possible world over the objects that are actual in that world and which also contains the ‘outer’ quantifiers  $\Pi$  and  $\Sigma$ , ranging in each possible world over all of the possible objects, i.e. over all of the objects that are actual in some possible world. (Under the N/C construal of the debate, the language  $L$  under consideration will contain just one type of quantifier which will, in effect, range over all actual objects for the contingentist and all possible objects for the necessitist; and later, when we give the suppositional reduction, we may allow the outer quantifier to range over all individuals whatever, even when they are not possible).

We shall assume that the first-order quantifiers  $\Pi$  and  $\Sigma$  or  $\forall$  and  $\exists$  range over worlds as well as over individuals of a more ordinary sort. We shall assume, more specifically, that, at each possible world, the inner domain of quantification will contain that world and that world alone, while the outer domain of quantification will contain every possible world. Thus at each world, the only world taken to be actual is that world itself. There are other ways to achieve the same effect. It could be supposed that at each world only that world *obtains*, for example, or that at each world there is only one world-proposition that is *true*. But however exactly the assumption is formulated, I shall assume that it need not be a point of contention between the actualist and the possibilist (or between the contingentist and the necessitist).

We shall also assume that the language  $L$  contains a finite stock of predicates and contains, in addition, a predicate  $W$  for being a world and a predicate  $E$  for being ‘existent’ which, in the present case, may be identified with being actual. Thus  $W$  will be true, at each possible world, of that world alone, while  $E$  will be true, at each possible world, of the individuals which are actual at that world and hence, in particular, of that world itself (under Williamson’s preferred formulation,  $E$  should be understood to mean not actuality, but chunkiness). It should be noted that the claim  $\Box\forall x\Box(Ex \equiv \exists y(x = y))$  (necessarily, every actual object is necessarily such that necessarily it exists iff there is something actual that it is) will hold for the actualist, but that the corresponding claim  $\Box\Pi x\Box(Ex \equiv \Sigma y(x = y))$  will not hold for the possibilist.

In what follows, let us use  $w, v, u$  and the like as variables for worlds.  $\forall wA(w)$ , for example, should be regarded as an abbreviation for  $\forall x(Wx \supset A(x))$ . It should be evident from the intended interpretation of  $W$  that the following two ‘axioms’ will hold:

World Existence:  $\Box\exists wT$  (necessarily, there is a world)

World Identity:  $\Box\forall w\forall v(w = v)$  (necessarily, any two worlds are the same).

Call a formula  $A$  of  $L$  *actualist* if it does not contain the quantifiers  $\Pi x$  or  $\Sigma$ . Then what the actualist would like is to construct, for each formula  $A$  of  $L$ , an actualist formula  $A^*$  equivalent to  $A$ . ‘Equivalent’ here could mean a number of things - provably equivalent within a suitable logic for the language  $L$ , semantically equivalent under the intended semantics for the language  $L$ , equivalent according to our informal understanding of the language  $L$ .

I hope it will be obvious that the formula  $A^*$  we construct below is informally equivalent to  $A$ . It could also be shown to be provably equivalent to  $A$  within a suitable formal system and semantically equivalent within a suitable semantics, though this is not something I shall attempt to establish. There are somewhat weaker standards for a successful reduction that one might lay down, not requiring equivalence but some suitable form of supervenience. But since I believe the stronger standards can be met, there will be no need for us to consider the weaker standards.

Given a formula  $A$  and world-variable  $w$ , let us use  $[A]_w$  as an abbreviation for  $\Diamond(Ew \wedge A)$  (possibly,  $w$  is actual and  $A$  is the case) and, in the particular case in which  $A$  is the atomic formula  $Rx_1x_2\dots x_n$ , let us use  $Rx_1x_2\dots x_nw$  in place of  $[Rx_1x_2\dots x_n]_w$ . Intuitively,  $[A]_w$  says that  $A$  is true at  $w$  and  $Rx_1x_2\dots x_nw$  says that  $R$  holds of  $x_1, x_2, \dots, x_n$  in  $w$ .

The basic idea behind the proposed actualist reduction of possibilist discourse is that every formula  $\Sigma xB(x)$  in  $A$  should be replaced by  $\exists w\Diamond\exists x[B(x)]_w$  (the actual world is such that possibly some object is

such that A is true at w). Thus the modalized quantifier  $\diamond\exists x$  has the effect of quantifying over all possible objects and the combination  $[B(x)]_w$  has the effect of taking the evaluation of B(x) back to the actual world. Since the idea and its variants are already familiar from the literature, I hope that there is no need to go into further detail.<sup>7</sup>

### §3 Going Higher Order

Let us now suppose that L is extended to a higher order language, with quantification over properties and relations of individuals, properties and relations of such properties and relations, and so on all the way through the type-theoretic hierarchy. As will become clear, the exact way in which the language is extended to the higher order will not much matter for our purposes. However, certain modifications to the proposed reduction would have to be made if L was allowed to contain non-logical predicates of higher order in addition to the given stock of first-order predicates.

Let us consider a particular case in which one might wish to give a higher order reduction, the case in which one has a plural possibilist quantifier  $\Sigma V$  (with V ranging over all ‘pluralities’ of possible individuals).<sup>8</sup> It will not do to follow the original first-order strategy, and replace  $\Sigma VB(V)$  with  $\square\exists w\diamond\exists V[B(V)]_w$ , for we want  $\Sigma V$ , in effect, to range over all pluralities of possible individuals, while the modalized quantifier  $\diamond\exists V$  will only range, in effect, over pluralities of *compossible* individuals, i.e. pluralities of individuals for which it is possible that all of them are actual.

To get over the difficulty, one might follow the strategy I have advocated in some previous papers (Fine 1977a, 146-8; 1977b, 161-2; 2003, 173-4) and replace the plural possibilist quantifier  $\Sigma V$  with a string of modalized quantifiers  $\diamond\exists x_1\diamond\exists x_2 \dots$ . The formulas  $\Sigma VB(V)$  will first be replaced with  $\exists w\diamond\exists x_1\diamond\exists x_2 \dots [B(V)]_w$  and each occurrence of  $V(x)$  ( $x$  is a  $V$ ) in  $B(V)$  will then be replaced with the corresponding disjunction  $x = x_1 \vee x = x_2 \vee \dots$ <sup>9</sup> I no longer wish to advocate this strategy, but it is worth considering if only because of the interest of the issues that it raises.

Williamson has two main objections against such an account: the first is to the effect that no string of modalized quantifiers  $\diamond\exists x_1\diamond\exists x_2 \dots$ , if intelligible, could be long enough to say all that needs to be said (pp. 353-6); and the second is that no infinitely long string of modalized quantifiers  $\diamond\exists x_1\diamond\exists x_2 \dots$  is even intelligible (pp. 356-60).<sup>10</sup>

Let us consider these objections in reverse order. As Williamson points out, it is not always clear what a given infinite string of quantifiers and modal operators should mean and so ‘the onus is on contingentists to explain what they intend the infinite sequence of modal operators and restricted [my actualist] quantifiers to mean.’ He then considers various proposals for fixing the intended meaning and finds them all lacking.

I think that there is a real problem here and that all but one of the proposals that Williamson considers are indeed inadequate. The one exception is the explanation of  $\diamond\exists x_1\diamond\exists x_2 \dots$  as something

7 But I should note that, strictly speaking, the reduction applies recursively to all formulas and that care should be taken to avoid clash of variables.

8 In the more general case, the outer quantifiers may also range over impossible objects - over sets of impossibles, for example.

9 This is not quite Williamson’s proposal on p. 353 of MLM, but the differences are incidental to the purpose at hand. There is also a subtlety concerning empty pluralities which I have, for convenience, ignored.

10 Fritz and Goodman [2014] point out that an especially severe form of this difficulty arises with the quantifier ‘most’.

analogous to a branching quantifier, where the branches are given by  $\diamond\exists x_1, \diamond\exists x_2, \dots$  respectively - or, better still, by  $\diamond\exists x_1\Box, \diamond\exists x_2\Box, \dots$ . He says ‘the problem [with this solution] is that a finite or infinite set of unordered sentence operators rarely has a natural meaning as an operator in its own right’ (p. 359); and so a meaning has to be given. ‘However, the only apparent way of extending the available semantic treatments of branching quantifiers to modalized examples ... is by treating the modal operators as quantifiers, just as in the possible worlds semantics, which contingentists cannot use to give the intended truth conditions’ (p. 360).

I wish to take exception to the last sentence. Indeed, there is something odd in the thought that the contingentist must somehow have recourse to the possible worlds semantics to explain the modalized quantifiers when no such recourse is required for the unmodalized quantifiers. To see why this is an unreasonable demand, consider the case of ordinary syntactic substitution. Given an expression E and some expression F that it contains, we may then substitute some other expression G for (all occurrences of) F in E, thereby obtaining the expression E[G/F]. One substitution may be performed *successively*, after another. Thus we may first substitute  $G_1$  for  $F_1$  in E and then substitute  $G_2$  for  $F_2$  in the result E[G<sub>1</sub>/F<sub>1</sub>] to obtain E[G<sub>1</sub>/F<sub>1</sub>][G<sub>2</sub>/F<sub>2</sub>]. But we may also *simultaneously* substitute  $G_1$  for  $F_1$  and  $G_2$  for  $F_2$  in E to obtain E[G<sub>1</sub>/F<sub>1</sub> | G<sub>2</sub>/F<sub>2</sub>] (as long as  $F_1$  and  $F_2$  are non-overlapping expressions in E). The result of a successive substitution will not in general be the same as the result of the corresponding simultaneous substitution. Suppose E is the formula  $p \vee q$ . Then E[q/p][p/q] will be  $p \vee p$  while E[q/p | p/q] will be  $q \vee p$ .

We can usually give no clear meaning to an infinite successive substitution. If E is an expression containing the infinitely many distinct sentence letters  $p_1, p_2, p_3, \dots$ , then what meaning is to be given to E[p<sub>1</sub>/p<sub>2</sub>][p<sub>2</sub>/p<sub>3</sub>] ...? But infinite simultaneous substitution *will*, in general, make sense. If E is an expression containing the infinitely many non-overlapping expressions  $F_1, F_2, F_3, \dots$ , then E[G<sub>1</sub>/F<sub>1</sub> | G<sub>2</sub>/F<sub>2</sub> | G<sub>3</sub>/F<sub>3</sub> | ...] is the well-defined result of simultaneously replacing each of  $F_1, F_2, F_3, \dots$  with  $G_1, G_2, G_3, \dots$  respectively.

Let us now ask: how are we capable of understanding an infinite simultaneous substitution E[G<sub>1</sub>/F<sub>1</sub> | G<sub>2</sub>/F<sub>2</sub> | G<sub>3</sub>/F<sub>3</sub> | ...]? A natural answer suggests itself. Each of the terms  $G_1/F_1, G_2/F_2, G_3/F_3, \dots$  corresponds to a syntactic operation on expressions, the operation that is performed when  $F_i$  is replaced in the given expression E with  $G_i$ . Moreover, these operations are in a clear sense independent of one another. If we were to assign a subordinate to execute each of the operations, then each of the subordinates could proceed with his task without any interference from the others. And it is because of the independence in this sense of the different syntactic operations corresponding to the terms  $G_1/F_1, G_2/F_2, G_3/F_3, \dots$  that we are able to make sense of performing all of these operations simultaneously, i.e. to make sense of the infinite simultaneous substitution E[G<sub>1</sub>/F<sub>1</sub> | G<sub>2</sub>/F<sub>2</sub> | G<sub>3</sub>/F<sub>3</sub> | ...].

Similarly for any other operations or ‘actions’. Suppose that there are infinitely many light switches in heaven and God commands that each of the switches be flipped from Off to On (let there be light!). Then since the actions of flipping a switch are independent of one another, there is no difficulty in making sense of all of them being performed simultaneously. Generally, given any independent actions  $\alpha_1, \alpha_2, \alpha_3, \dots$ , we can make sense of the action  $\alpha = (\alpha_1 | \alpha_2 | \alpha_3 | \dots)$  which is the simultaneous execution of each of them.

What goes for syntactic operations, or for actions in general, also goes for semantic operations. The semantic operations  $\alpha_1, \alpha_2, \alpha_3, \dots$ , corresponding respectively to the modalized quantifiers  $\diamond\exists x_1\Box, \diamond\exists x_2\Box, \diamond\exists x_3\Box, \dots$  are independent of one another in the relevant sense (as long, of course, as the variables  $x_1, x_2, x_3, \dots$  are all distinct from one another). This is a fact that can be recognized by the actualist (or contingentist) as much as by the possibilist (or necessitist) and no more requires appeal to some ‘higher

order' apparatus than in the analogous case of substitution. But then the actualist is in a perfectly good position to make sense of  $(\diamond\exists x_1\Box, \diamond\exists x_2\Box, \diamond\exists x_3\Box, \dots)$  as the simultaneous application of the semantic operations signified by the individual prefixes  $\diamond\exists x_1\Box, \diamond\exists x_2\Box, \diamond\exists x_3\Box, \dots$ . I have no definition of independence that might apply across the board to all these different cases, but that is no reason to doubt the intelligibility of the notion or the legitimacy of its application to 'parallel processing' in any given case.<sup>11</sup>

For this reason, I am not inclined to take Williamson's second objection seriously. The first is more serious, but it raises difficult issues in the theory of classes and in the metaphysics of modality about which I am not sure what to say. What we would like is: (i) to have as many variables as possible objects; and (ii) to allow all variables to appear in a single formula, perhaps many times over. Since any object can be considered a variable, (i) is tantamount to the claim (i)' that there should be as many (actual) objects as there are possible objects; and since any formula can be regarded as a class, (ii) is tantamount to the claim (ii)' that there should be a class of all actual objects (and also suitable classes of classes of actual objects). There are versions of the theory of classes that allow for (ii)' and perhaps may serve our purposes.<sup>12</sup> But (i)' is more dubious and it is not even clear how the actualist might state it, since there will presumably be no actual one-one correspondence between the actual individuals and all possible individuals. Perhaps it is sufficient for the possibilist to accept (i)', or something analogous, as true. He will then be in a position to recognize the correctness of the actualist's infinitary reduction even if the actualist herself cannot state the assumptions upon which its correctness depends.

But this is all very speculative. Add to it the desirability of providing a finitary reduction (MLM, 353), and we have reason enough to look for an alternative approach.

#### §4 The Suppositional Approach

Let me explain in broad outline the idea behind the alternative approach before proceeding to greater detail.

There is something which we might call the 'modal pluriverse'. This is constituted by the totality of possible worlds, the totality of possible individuals, and the pattern of instantiation of certain properties and relations in each of the possible worlds. Thus a simple 'toy' pluriverse might be constituted by three possible worlds  $w$ ,  $v$  and  $u$  and two possible individuals  $a$  and  $b$ , with  $w$  containing no actual individuals,  $v$  containing two actual individuals,  $a$  and  $b$ , both with the property  $F$ , and  $u$  containing one actual individual  $a$ , without the property  $F$ .

Our interest will not be in all properties or relations whatever, or even in the fundamental properties or relations, but merely in those that can be expressed by means of the atomic predicates of the possibilist sentences whose reduction is in question. Thus we will view the pluriverse through the 'lens' of the sentences by which it might be described. We shall also suppose that the pluriverse is 'centered', in that one of the worlds will be picked out as the actual world. A pluriverse will therefore correspond, at least under a set-theoretic representation, to one of the models of Kripke [1963] for the quantified system of modal logic S5.

The possibilist is relatively well-equipped to describe the pluriverse, since he can freely quantify over all possible worlds and all possible individuals. The actualist is not so well equipped; and indeed it is

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11 The study of parallel processing or 'process algebra' has become a significant branch of computer science in its own right (see Boeten [2005] for a brief survey). I might also note that the case of substitution shows that Williamson's putative tests for independence (idempotence and commutativity, p. 358) are clearly off the mark.

12 The theory of classes advocated in Fine 2005b is one such example.



not even clear that he can form a complete and intelligible conception of what the pluriverse is. Our first step is therefore to describe the possibilist's pluriverse from an actualist point of view, i.e. by only making appeal to resources available to the actualist.<sup>13</sup> Now if every possible individual were actual, the expressive edge that the possibilist enjoys over the actualist would disappear. So what we will have the actualist do is describe a hypothetical pluriverse, one which is just like the actual pluriverse but for the fact that every possible individual is actual. Within such a pluriverse, the distinction between what is possible and what is actual will disappear and so it must be reinstated as a further distinction made within the realm of what is actual.

We might imagine that each possible world within the possibilist's pluriverse contains a barrier between what is actual and what is possible - with all the actual objects of the world lying to the right of the barrier and all the merely possible objects of the world lying to its left. The actualist first slides the barrier all the way to the left so that all possible objects lie to its right and he then introduces a dummy barrier to mark where the actual barrier once lay (under the N/C construal of the debate, the new dummy barrier will mark the distinction between what is and is not 'chunky').

This is the first stage in the proposed reduction. We are still not done since, even if we have been successful in the first stage, we will only have provided a *first-order* description of the pluriverse, one describing the first-order behavior of the possible individuals in different possible worlds but not one also describing the higher order behavior of the possible individuals or the behavior of properties or relations or other such entities in the different possible worlds.

At the next stage of the reduction, we appeal to the general idea that the higher order behavior of the pluriverse can be fully determined on the basis of its first-order behavior. Of course, inner quantification over actual properties and relations will in general behave differently from outer quantification over properties and relations, just as quantification over actual individuals in general behaves differently from quantification over possible individuals. But, in the present case, the hypothetical pluriverse is one in which each possible individual is actual and hence one in which each property or relation of the possibilist is actual; and so there will be no discrepancy between inner quantification over these entities in the hypothetical pluriverse and outer quantification over these entities in the actual pluriverse. The actualist is thereby able to simulate possibilist discourse within the actual pluriverse by means of the corresponding actualist discourse within the hypothetical pluriverse.

There is a certain resemblance between my approach and that of Sider [2002]<sup>14</sup>, since we both take a possibilist sentence to be true if it is true according to the possibilist's pluriverse. But, as will become apparent, there are two significant differences. First, and most significantly, the way in which we specify the pluriverse is different: I specify it via suppositions while he specifies it directly by means of an infinitary sentence or a modal model. This means that my specification is finitary, not subject to worries over the cardinality of the set of possible objects or set of worlds and complete (no possibilist sentence is left undecided), while his specification is infinitary, subject to worries over cardinality and possibly incomplete (so that some possibilist sentences may be left undecided). Second, we have a different understanding of how the pluriverse determines the truth of a possibilist sentence: he makes use of the ordinary notion of first order logical consequence (extended to an infinitary language), while I make essential use of a broader notion of logical consequence which allows higher order truths to be determined

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13 This is a place where appropriate adjustments will need to be made under the N/C construal of the debate.

14 A similar approach to Sider's is considered in Fine [1977a], 147 and developed in §3 of Fine [2002]. My own approach also avoids the problems raised by Leuenberger [2006] and extended in the formal appendix to Fritz & Goodman [2014].

on the basis of first-order truths. I am thereby able to capture the higher order features of possibilist discourse, which is not something that can be properly accommodated within his own approach.

### §5 The Suppositional Calculus

Let us turn to the details of the alternative approach. We begin by developing the suppositional calculus within which the actualist will provide his description of the pluriverse.

The suppositional calculus (SC) is a calculus in which we are able to reason explicitly about the structure of suppositions and what follows from a supposition. It is of the greatest importance for understanding what is distinctive about the calculus to appreciate that a suppositional term, such as ‘suppose that A’, is taken to signify an act - the intellectual act of supposing that A. Given that a suppositional term signifies an act, we may then use the general means available to us for specifying complex acts in terms of simpler acts to specify complex suppositional acts. So, for example, just as I can say ‘do  $\alpha$  and then do  $\beta$ ’, so I can say ‘first suppose A and then suppose B’. Moreover, just as I can use indexed modalities to indicate what will follow from a given act  $\alpha$  - where  $\Box_{\alpha}A$  is taken to mean that the truth of A will result from the performance of  $\alpha$ , so we can use indexed modalities to indicate what will follow from a given suppositional act  $\sigma$  - where  $\Box_{\sigma}A$  is now taken to mean that the truth of A will result from the supposition of  $\sigma$ .<sup>15</sup>

Of course, systems of natural deduction - especially when laid out in the style of Fitch - constitute a kind of suppositional calculus. We can see from a derivation within such a system that a given formula will follow from the supposition of other formulas. But such a system does not enable us to explicitly talk or reason about what follows from a supposition. It does not enable us to say, let alone show, for example, that some formula does *not* follow from the supposition of other formulas; and, as we shall see, it provides very limited means for saying what the suppositions might be.

As the previous paragraphs have perhaps made clear, the expression ‘supposition’ is multiply ambiguous. It may refer to a suppositional term, such as ‘suppose A’, or to the instruction or act expressed by such a term, or the sentence or proposition (A or  $\langle A \rangle$ ) that is supposed. In case there is need of disambiguation, we can refer to the ‘suppositional clause’, the ‘suppositional act’ and the ‘supposed sentence or proposition’.

In what follows, we shall simply describe the language of the suppositional calculus and indicate in an informal manner how it is to be interpreted. In a fuller treatment, we should also provide a semantics and proof theory for the calculus. This is an interesting project in its own right; and I am hopeful, once it carried out, that we be able to provide a more formal vindication of the proposed reduction, under which the possibilist formula to be reduced will be both provably and semantically equivalent to its actualist counterpart.

Although my sole aim in this paper has been to put the calculus at the service of the actualist, it seems plausible that the calculus will have a number of other applications. Within logic, the calculus provides us with a means of generalizing the notion of a sequent, where the suppositional term  $\sigma$  provides us with a more general notion of an antecedent and where a sequent  $\sigma \vdash A$  (or  $\sigma \vdash A_1, A_2, \dots, A_n$ ) will now be expressed by the modal formula  $\Box_{\sigma}A$  (or  $\Box_{\sigma}[A_1, A_2, \dots, A_n]$ ); and within the semantics for natural language, the suppositional terms  $\sigma$  provide us with a means of keeping track of the structure and flow of suppositions within a given piece of discourse.

The calculus contains two main kinds of clause: suppositional and indicative. For ease of expression, we call the former ‘S-terms’ or ‘terms’ and the latter ‘I-formulas’ or ‘formulas’. We suppose

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<sup>15</sup> A symbolism of this sort is familiar from dynamic programming logic (Harel [1984]), another well-developed branch of computer science.

that the initial formulas are ‘sourced’ from the given language  $L$  (so, strictly speaking, the notions of formula and term will be relative to  $L$ ).

Terms and formulas are defined by the following clauses (informal interpretation to follow):

- (i) if  $A$  is an formula then  $/A/$  is a term;
- (ii)  $\backslash\backslash$  is a term;
- (iii) if  $\sigma$  and  $\tau$  are terms, then so is  $(\sigma | \tau)$ ;
- (iv) if  $\sigma$  and  $\tau$  are terms, then so is  $(\sigma; \tau)$ ;
- (v) if  $A$  a formula and  $\sigma$  a term then  $A \rightarrow \sigma$  is a term;
- (vi) if  $\sigma$  is a term then so is  $\forall x\sigma$ ;
- (vii) if  $\sigma$  is a term then so is  $\Box\sigma$ ;
- (viii) if  $A$  is a formula of  $L$  then  $A$  is a formula
- (ix) formulas are closed under the usual operations for forming formulas (negation, conjunction, actualist quantification etc.)
- (x) if  $\sigma$  is a term and  $A$  a formula, then  $\Box_{\sigma} A$  is a formula.

Let us make some comments on the various types of construction:

The terms under (i) and (ii) constitute the two basic types of supposition, all other suppositions being formed from them by means of certain basic operations.  $/A/$  (read ‘suppose that  $A$ ’) allows us to suppose the truth of an indicative formula, while the back step constant  $\backslash\backslash$  (read ‘revert to the previous supposition’) allows us to go back to what (if anything) was previously supposed. Thus within a context in which one supposition is immediately embedded within another,  $\backslash\backslash$  has the effect of taking us back to the immediately preceding supposition (this corresponds to the operation of moving one step leftward within a Fitch-style system of natural deduction); and when the context is one in which there is no immediately preceding supposition,  $\backslash\backslash$  has the effect of going to the ‘empty’ suppositional state, in which nothing has been supposed.

(iii) and (iv) provide two ways of forming composite suppositions.  $(\sigma | \tau)$  (read ‘ $\sigma$  and  $\tau$ ’) allows one *jointly* to suppose  $\sigma$  and  $\tau$ . Thus  $(/A/ | /B/)$ , in particular, tells one jointly to suppose  $A$  and  $B$ .  $(\sigma; \tau)$  (read ‘ $\sigma$  and then  $\tau$ ’) allows one *successively* to suppose  $\sigma$  and  $\tau$ . Thus  $(/A/; /B/)$ , in particular, tells one successively to suppose  $A$  and *then* to suppose  $B$ , thereby embedding the supposition of  $B$  within the supposition of  $A$ . Our understanding is that when one forms an embedded supposition, the previous suppositions remain in force. If one were to suppose, alternatively, that the previous suppositions do not remain in force, then the present conception of an embedded supposition should be defined as  $(\sigma | (\sigma; \tau))$ . The composite suppositions  $(/A/ | /B/)$  and  $(/A/; /B/)$  are, of course, to be distinguished from the corresponding conjunctive supposition  $/A \Box B/$ . For in the former cases, the two conjunct propositions are each supposed, either jointly or successively, while, in the latter, case the single conjunctive proposition is supposed.

(v) allows us to form a conditional supposition  $A \rightarrow \sigma$  (read ‘if  $A$  then  $\sigma$ ’). Thus one might only want  $Ga$  to be supposed if  $Fa$  is the case; and this would then be expressed as  $Fa \rightarrow /Ga/$ . A conditional supposition, such as  $Fa \rightarrow /Ga/$ , should, of course, be distinguished from the supposition of the corresponding conditional  $/Fa \rightarrow Ga/$ . A conditional supposition only comes into effect if its antecedent is in fact the case, while the supposition of the corresponding conditional will come into effect regardless of the circumstances. We may say, in general, that a supposition is *weak* or *conditioned* if what it takes to be given is dependent on the actual circumstances and that otherwise it is *strict* or *unconditioned*. Thus the construction  $A \rightarrow \sigma$  (along with the construction  $\forall x\sigma$ , to be considered below) provides us with the means of forming conditioned suppositions.

(vi) allows us to form a generalized supposition  $\forall x\sigma$  (read ‘for all  $x$ ,  $\sigma$ ’).  $\forall x\sigma$  requires the joint

supposition of  $\sigma$  for each value of the variable  $x$ . The generalized supposition  $\forall x/A(x)/$  is, of course, to be distinguished from the corresponding general supposition  $/\forall x(A)/$ . The latter requires the supposition of a general proposition while the former requires the supposition of all of its instances. The generalized construction may be usefully combined with the conditional construction. For example,  $\forall x(\text{Man}(x) \rightarrow / \text{Woman}(x)/)$  will require us to suppose, of each man, that he is a woman.

There is an important respect in which generalized suppositions differ from other generalized injunctions. Suppose I state ‘climb every mountain!’, something which might be formalized as  $\forall x(\text{Mountain}(x) \rightarrow \text{Climb}(x))$ . Then in order to execute this injunction, I must, for each mountain  $x$ , perform the individual act of climbing  $x$ . But suppose now that I state ‘for each mountain, suppose that it is climbed’, something which might be formalized as  $\forall x(\text{Mountain}(x) \rightarrow / \text{Climbed}(x)/)$ . Then in order to ‘execute’ or make this supposition, it is not necessary that, for each mountain  $x$ , I perform the individual act of supposing that  $x$  is climbed. All I need to do is to state ‘for each mountain, suppose that it is climbed’. There is a performative aspect to suppositional statements, which means that nothing beyond the statement of the supposition itself is required to ensure that the supposition is made.

(vii) is perhaps the most contentious of our constructions and allows us to turn a conditioned supposition  $\sigma$  into an unconditioned supposition  $\Box\sigma$  (read ‘necessarily,  $\sigma$ ’). The necessity here should be taken to be metaphysically necessity; and  $\Box\sigma$  then strengthens the supposition  $\sigma$  by requiring it to take effect not merely in the actual circumstances but in any counterfactual circumstances that might obtain. Suppose, for example, that  $\sigma$  is the conditional supposition  $A \rightarrow /B/$ , requiring the supposition of  $B$  when  $A$  is the case.  $\Box\sigma$  will then require the supposition of  $B$  when  $A$  is possibly the case since, in any actual or counterfactual circumstance in which  $A$  is the case,  $\sigma$  would require the supposition of  $B$ .

Note that we do not understand  $\Box(A \rightarrow /B/)$  to mean that  $B$  would be supposed if  $A$  were the case. Indeed, we can see on general grounds that this cannot be so, since we always want  $\sigma$  to tell us to form a supposition. But  $\Box(A \rightarrow /B/)$ , on the proposed reading, is an indicative sentence and only tells us what *has* been supposed under certain counterfactual circumstances, not what *is to be* supposed. Rather,  $\Box(A \rightarrow \sigma)$  should be taken to mean ‘necessarily, if ever  $A$  then form (i.e. actually form) the supposition  $\sigma$ ’.

The sole use we shall make of  $\Box$  is in constructions of the form  $\Box\forall x\Box(A(x) \rightarrow /B(x)/)$ . This construction tells us, in effect, to form the supposition  $B(x)$  for any possible object  $x$  for which it is possible that  $A(x)$ . Thus  $\Box\forall x\Box$  here serves as a quantifier over possible objects and so  $\Box\forall x\Box(A(x) \rightarrow /B(x)/)$ , were we allowed to use the possibilist quantifier  $\Box$ , might also be expressed in the form  $\Box\forall x(\Box A(x) \rightarrow /B(x)/)$ .

Even if the above use of  $\Box$  is for some reason rejected, it is still plausible that the actualist should be able to make sense of generalized suppositions of the form  $\Box\forall x(\Box A(x) \rightarrow /B(x)/)$ . For he can already make sense of the indicative claim  $\Box\forall x(\Box A(x) \rightarrow B(x))$  (for every possible object  $x$  for which it is possible that  $A(x)$  it is the case that  $B(x)$ ). But if he can make sense of this claim, then what is to prevent him from making sense of the corresponding suppositional sentence  $\Box\forall x(\Box A(x) \rightarrow /B(x)/)$  (for every possible object  $x$  for which it is possible that  $A(x)$  suppose that  $B(x)$ ), given that the only difference between them lies in the substitution of a supposition for a judgement?

Consider a related case. The actualist can certainly make sense of the claim that every possible act of killing is forbidden ( $\Box\forall x(\Box K(x) \rightarrow Fx)$ ), since he can express it in the form  $\Box\forall w\Box\forall x(\Box K(x) \rightarrow Fxw)$ .<sup>16</sup> But then surely he can also make sense of the injunction that every possible act of killing is to be forbidden ( $\Box\forall x(\Box K(x) \rightarrow F!(x))$ ) - something someone might bear in mind were they to envisage killing

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<sup>16</sup> This claim makes most sense if we take it to be essential to an act of killing that it be an act of killing.

someone! Perhaps this injunction can be stated in corresponding fashion as  $\Box\forall x(\Diamond K(x) \rightarrow F!(x))$ , where the injunction  $F!(x)$  automatically brings one back to the actual world. But even if it cannot, there would appear to be no essential difficulty in moving from the intelligibility of the indicative to the intelligibility of the corresponding injunction.

Of course, the actualist may be in no position to make some of the individual suppositions  $/B(x)/$ . But this is equally true of the possibilist when he forms the supposition  $\Box x\sigma(x)$  or even of the actualist when he forms the supposition  $\forall x\sigma(x)$ . Nor would it appear to be required that each of the individual suppositions should somehow be collected together in a single entity or that it should be possible to regard each of them as the supposition of a proposition that actually exists. For if such a constraint is not required for the intelligibility of the indicative sentence  $\Box x(\Diamond A(x) \rightarrow B(x))$ , then why should it be required for the intelligibility of corresponding suppositional sentence  $\Box x(\Diamond A(x) \rightarrow /B(x)/)$ ?

In what follows, I shall take the liberty of using  $\Box x(\Diamond A(x) \rightarrow /B(x)/)$  as an abbreviation for  $\Box\forall x\Box(A(x) \rightarrow /B(x)/)$ , but with the understanding that there may be some alternative way of making the intended meaning of  $\Box x(\Diamond A(x) \rightarrow /B(x)/)$  acceptable to the actualist.

(x) provides us with our sole means, beyond usual apparatus of connectives and quantifiers, for forming new formulas. Given a term  $\sigma$  and a formula  $A$ , it allows us to form the ‘entailment’  $\Box_{\sigma}A$ , which may be read ‘under the supposition  $\sigma$ , it logically follows that  $A$ ’ and which we may write in the form ‘ $\sigma \Box A$ ’ to avoid confusion with  $\Box A$ . Here  $\sigma$  is the *antecedent* or *suppositional base* and  $A$  the *consequent* of the entailment. But note that the antecedent is an S-term, not an indicative sentence, and signifies a complex suppositional act for which the propositions to be supposed may be several rather than single, embedded one within the other, and dependent upon the circumstances. We may abbreviate the formula  $\Box\Box A$  to  $\Box^{-1}A$ . Thus  $\Box^{-1}A$  tells us that  $A$  follows from what was previously supposed.

The intended notion of logical consequence is logical consequence in a broad sense which goes beyond classical first order logical consequence in a number of ways. First, it encompasses higher order logic and logic with generalized quantifiers - such as ‘for finitely many’ or ‘uncountably many’ - as long as these quantifiers are of a purely logical character, and it also encompasses any other higher order constructions - such as plural quantification or quantification over classes - whose use by the possibilist may be in question. Second, it accommodates modal reasoning. We will take  $A$ , for example, to be a logical consequence of  $\Box A$ . There may, of course, be different opinions as to how higher order modal logic should be formulated. But it is important, if we are not to prejudge the issue between the actualist and possibilist or between the contingentist and necessitist, that the logic should be acceptable to both and that it should, in particular, remain non-committal on the validity of the Barcan Formula (and its converse). Third, we should allow the consequence relation to hold between what, in effect, are singular positions. Quine has inveighed against quantifying into modal contexts, but his concerns may be met in the present case if we take the identity and distinctness of the individuals in the different singular proposition to be an aspect of their logical form (as proposed in Fine [2005], 108-112); and this is how we shall understand the notion in what follows.

## §6 The Suppositional Base

We now want to specify some suppositions that will serve to describe the possibilist’s pluriverse from an actualist point of view. We will do this by means of a suppositional term of the form  $\sigma; \tau$ , with one supposition within the other and with each of the component suppositions  $\sigma$  and  $\tau$  formulated in terms acceptable to the actualist.

So let us specify each of the terms  $\sigma$  and  $\tau$  in turn. The unembedded supposition  $\sigma$  will be the joint supposition  $(\sigma_1 | \sigma_2 | \sigma_3 | \sigma_4 | \sigma_5) (= (((\sigma_1 | \sigma_2) | \sigma_3) | \sigma_4) | \sigma_5))$  of the following suppositions, for  $R$  any  $n$ -

place predicate of  $L^{17}$ :

- $\sigma_1: \Pi x/\Box\exists y(x = y)/$   
 $\sigma_2: \Pi x_1\Pi x_2\dots\Pi x_n\Pi w(Rx_1x_2\dots x_nw \rightarrow /Rx_1x_2\dots x_nw/)$   
 $\sigma_3: \Pi x_1\Pi x_2\dots\Pi x_n\Pi w(\neg Rx_1x_2\dots x_nw \rightarrow / \neg Rx_1x_2\dots x_nw/)$   
 $\sigma_4: / \Box\exists wEw \wedge \Box\forall w\forall v(Ew \wedge Ev \supset w = v)/$   
 $\sigma_5: \forall w/\Pi x_1\Pi x_2\dots\Pi x_n(Rx_1x_2\dots x_nw \equiv Rx_1x_2\dots x_n)/$

The embedded supposition  $\tau$  is simply:

$$\tau: / \Pi x(\vdash^{-1} \exists y(x = y))/$$

Let me make some comments on the interpretation and role of these various suppositions:

$\sigma_1$  is the sole supposition of  $\sigma$  to be counterfactual, i.e. to be incompatible with what is the case. It tells us to suppose, of each possible object, that necessarily there is some actual thing that it is. Of course, the actualist believes no such thing, but he may still intelligibly hypothesize that this is how things are.

$\sigma_2$  tells us to suppose, whenever a given  $n$ -place predicate  $R$  holds of some possible individuals in a possible world, that this is the case; and  $\sigma_3$  tells us to suppose, whenever a given predicate does not hold of some possible individuals in a possible world, that this is not the case. Thus the behavior of the given properties and relations will be the same in the hypothetical pluriverse as it is in the actual pluriverse.

It is important to bear in mind that the predicate  $E$  for existence has been taken to be one of the given atomic predicates. Thus, as a special case of  $\sigma_2$  and  $\sigma_3$ , we have:

- $\sigma_2': \Pi x\Pi w(Exw \rightarrow /Exw/)$   
 $\sigma_3': \Pi x\Pi w(\neg Exw \rightarrow / \neg Exw/)$

These clauses tell us that the behavior of the existence predicate is the same in the hypothetical pluriverse as it is in actual pluriverse. Since we have also, in effect, supposed  $\Box\forall x\Box\exists y(x = y)$  (that necessarily each object is necessarily something), we will no longer have the usual connection  $\Box\forall x\Box\exists y(x = y) \equiv \Box\forall x\Box(Ex \equiv \exists y(x = y))$  between existence and actuality. Existence in the hypothetical pluriverse will be the shadow, so to speak, of actuality in the actual pluriverse.

It should also be noted that there is no need to assume Williamson's 'being constraint' of chapter 4 of MLM (also called the 'falsehood principle' in Fine [1981]), according to which an atomic predicate can only be true in each world of the actual objects of that world. For even if a dyadic predicate  $R$ , say, is true of two non-actual objects  $x$  and  $y$  in a given world  $w$ ,  $\sigma_2$  will still trigger the supposition that this is so.

$\sigma_4$  tells us to suppose that possible worlds behave as possible worlds. We can no longer suppose, of course, that World Existence and Identity hold since worlds, like everything else, will necessarily be actual. But we make the corresponding suppositions that necessarily some world exists and necessarily any two existent worlds are the same.

$\sigma_5$  tells us to suppose that the actual world behaves as the actual world, with truth of an atomic predication in the actual world being tantamount to truth simpliciter.

Finally, the embedded supposition  $\tau$  requires us to suppose that each possible object is identical to one whose being follows from the preceding supposition. Since it is only the possible objects  $a, b, \dots$  from the original actualist pluriverse whose being will follow from the preceding supposition, the present embedded supposition requires, in effect, that each possible object should be identical to one of  $a, b, \dots$ . It is in this way that we can guarantee that the original ontology of possible objects is exhaustive of the possibilist ontology without using an infinitary sentence.

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17 If only the identity, and not also the distinctness, of individuals is an aspect of logical form, then we should add the supposition  $\Pi x\Pi y(\neg x = y \rightarrow / \neg x = y/)$  (suppose, of any two distinct possible objects, that they are distinct).

If the predicate  $E$  is allowed to apply to higher order entities, then we might also include such suppositions as:

$$\tau': / \prod \prod w (EP_w \rightarrow \vdash^{-1} EP_w) /.$$

This guarantees that  $E$  will not have an application to such higher order entities beyond its original application in the actual pluriverse. It should be noted that it is only in the suppositions  $\tau$  or  $\tau'$  that we make use of the back space operator  $\backslash$ ; and so even if we have reservations about the general use of the operator, it is only its use in the present context that is required to legitimate the present reduction.

We are now in a position to state the reduction. Let  $A$  be a possibilist statement from the higher order version of the language  $L$ , possibly containing the outer quantifiers and the existence predicate at each type, but none of the inner quantifiers. Let  $A'$  be the result of replacing all of the outer quantifiers by the corresponding inner quantifiers.  $\prod x \square \sum y (x = y)$ , for example, becomes  $\forall x \square \exists y (x = y)$ . Of course,  $A'$  will not be an acceptable actualist substitute for  $A$  since  $A'$  may often be false when  $A$  is true, as in the previous example. However, the formula  $A'$  will be useful as an intermediate step in giving the required reduction  $A^*$  of  $A$ . For given a possibilist formula  $A$  from the higher order version of the language  $L$ , the actualist reduction  $A^*$  may be taken to be the formula  $\sigma; \tau \vdash A'$ , to the effect that  $A'$  follows from the supposition  $\sigma; \tau$ .

We may convince ourselves of the adequacy of the reduction as follows. The statement  $A$  will be true iff  $A$  logically follows from a complete first-order description of the possibilist's first-order pluriverse (in which it is said what objects there are, what worlds there are, which world is actual and how the objects behave in each of the worlds).  $A$  will follow from such a description iff  $A'$  follows from a complete first-order description of the actualist analogue of such a pluriverse (in which necessarily each actual object is necessarily actual). But  $\sigma; \tau$  constitutes a complete first-order description of the actualist analogue of the possibilist pluriverse; and so  $A'$  will follow from this complete first-order description of the actualist analogue iff  $A'$  follows from the supposition  $(\sigma; \tau)$ .

There are two key aspects of the proposed 'demonstration'. First, it is supposed that a complete first-order description of the pluriverse will logically entail whatever holds at the higher level; the first-order is taken, in this sense, to be *determinative* of the higher order. Second, we postulate devices of supposition that enable us to provide what is, in effect, a complete first order description of the possibilist pluriverse. Of course, we have only demonstrated equivalence of truth-value. But the demonstration is broadly logical in character and so, under any reasonable conception of content,  $A$  will have the same content as  $A^*$ .

It is also very plausible, as I have tried to argue, that the reductive statement  $A^*$  will indeed be acceptable to the actualist - nothing in the ideology of  $A'$  or in the suppositional apparatus of  $\sigma; \tau$  will require appeal to mere possibilia; and so we have what would appear to be an adequate and acceptable account of higher order possibilist discourse in actualist terms.

### §7 Comparisons, Consequences, Objections

The present reduction has a number of advantages over my previous reduction. The main advantage is that it is completely finitary and, in particular, requires no appeal to infinitely long conjunctions or disjunctions or to infinitely long strings or branches of quantifiers interlaced with modal operators. But it also has a certain flexibility in its favor. For the general idea of determining higher order truths on the basis of first-order truths will have application to other forms of higher order truth (such as quantification over pluralities or sets) and to areas other than quantified modal logic.

I had always thought that there should not be a separate problem about effecting a reduction of higher order possibilist discourse, that the intelligibility of higher order discourse should somehow follow

from the intelligibility of first-order discourse. But I could come up with no satisfactory way to articulate the thought. The present account shows how this might be done. For it shows how, once we are able to state how things are at the first-order level, we can appeal to a general notion of logical consequence to say how things are at the higher level.

However, the proposed reduction is not completely painless. The ideology of the actualist's reduction will need to go beyond the ideology of the possibilist's discourse in a number of key respects. For it will need to make reference to worlds or the like, even when the possibilist makes no such reference. Of course, in this respect, the higher-order reduction is not so different from the first-order reduction (though for the purposes of the first-order reduction, we can use Vlach's up and down arrows in place of quantifiers over worlds). More significantly - and this *is* a major difference from the first-order case - our actualist will need to employ a concept of logical consequence that is hyper-intensional; consequence will not be preserved under the 'substitution' of different necessarily equivalent propositions. And this might be regarded as a reason in itself for rejecting the reduction.

But quite apart from a general prejudice against hyper-intensional notions (which is perhaps no better founded than the earlier prejudice against intensional notions), it is not clear that there is anything especially problematic about our notion of logical consequence. There is, of course, the need to fix the relevant sense of consequence; and it may also be necessary to fix the intended sense of the higher-order quantifiers. But once this is done, then it is far from clear that we do not have a notion that is in reasonably good order.

In a recent paper, Fritz and Goodman [2014] raise the specter of 'revenge'. Suppose that the actualist uses some further modality, such as our broad notion of logical necessity, in attempting to reduce possibilist discourse. Then does not the problem arise anew for objects which are possible with respect to the new modality? In our own case, even if we can explain away metaphysical possibilities with the help of a logical modality, how do we explain away the logical possibilities?

There *is* a possible danger of this sort. But it does not necessarily arise, since it cannot simply be taken for granted that the Barcan Formula (or its converse) will be true for any given modality under an unrestricted reading of the quantifier, even if this is so for other modalities. The present case strikes me as a case in point. Let us suppose, for the sake of argument, that there are just five objects *a*, *b*, *c*, *d* and *e*; and let *a*, *b*, *c*, *d* and *e* be names for these objects. Then under a reading of  $\Box$  as 'it is logically necessary that' and using  $\Pi x$  for the unrestricted quantifier,  $\Pi x \Box (x = a \vee x = b \vee \dots \vee x = e)$  should hold, since every object *x* will be identical to *a* or *b* or ... or *e*. But surely  $\Box \Pi x (x = a \vee x = b \vee \dots \vee x = e)$  will not hold; it will not be a *logical* necessity (something guaranteed by logical form alone) that every object *x* will be identical to *a* or *b* or ... or *e*. This means that there can be no intelligible notion of possible object corresponding to the new modality, since otherwise quantification over such objects would conform to the Barcan formula; and hence the possibility of revenge does not even arise.<sup>18</sup>

Fritz and Goodman have also objected to a counterfactual reduction, in which the possibilist statement *A* is, in effect, replaced with the statement that *A'* would be true if necessarily everything were necessarily to exist. I myself have general misgivings about counterfactual analyses (of necessity, knowledge, cause and the like). For the use of any counterfactual requires us to take something as 'fixed'; and there is always the danger that the only way to get the counterfactual analysis to come out as correct is to presuppose the very notion to be analyzed in understanding what it is that is to be kept fixed (similar

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18 As Jeremy Goodman has pointed out to me, the contingentist also does not think that the Barcan Formula holds for metaphysical necessity and so this may be a respect in which there is a significant difference between the A/P and N/C formulations of the debate.



misgivings also arise over fictionalist analyses).

There is a way in which my own reduction is counterfactual in character, since I have the actualist counterfactually supposing that necessarily every actual object is necessarily actual. However, I do not simply rely on this counterfactual supposition in drawing out its consequences. I also specify in complete detail everything else that must obtain, so that it is simply a matter of logic what the consequences should be. Thus general misgivings over the import or circular character of counterfactual forms of analysis will have no bearing in the present case.

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