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Was Polchinski wrong? Colombeau distributional Rindler space-time with distributional Levi-Cività connection induced vacuum dominance. Unruh effect revisited

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Abstract. The vacuum energy density of free scalar quantum field Φ in a Rindler distributional space-time with distributional Levi-Cività connection is considered. It has been widely believed that, except in very extreme situations, the influence of acceleration on quantum fields should amount to just small, sub-dominant contributions. Here we argue that this belief is wrong by showing that in a Rindler distributional background space-time with distributional Levi-Cività connection the vacuum energy of free quantum fields is forced, by the very same background distributional space-time such a Rindler distributional background space-time, to become dominant over any classical energy density component. This semiclassical gravity effect finds its roots in the singular behavior of quantum fields on a Rindler distributional space-times with distributional Levi-Cività connection. In particular we obtain that the vacuum fluctuations $\langle\Phi^2\rangle$ has a singular behavior on a Rindler horizon. Therefore sufficiently strongly accelerated observer burns up near the Rindler horizon. Thus Polchinski's account doesn't violate of the Einstein equivalence principle.

1. Introduction

In March 2012, Joseph Polchinski claimed that the following three statements cannot all be true [1]: (i) Hawking radiation is in a pure state, (ii) the information carried by the radiation is emitted from the region near the horizon, with low energy effective field theory valid beyond some microscopic distance from the horizon, (iii) the infalling observer encounters nothing unusual at the horizon. Joseph Polchinski climed that the most conservative resolution is that [1]: the infalling observer burns up at the horizon. In Polchinski's account, quantum effects would turn the event horizon into a seething maelstrom of particles. Anyone who fell into it would hit a wall of fire and be burned to a crisp in an instant. As pointed out by physics community such firewalls would violate a foundational tenet of contemporary physics known as the equivalence principle [2], it states in part that an observer falling in a gravitational field - even the powerful one inside a black hole - will see exactly the same phenomena as an accelerated observer floating in empty space [3].

However in our paper [4] the same firewalls were obtained without any reference to string theory by using QFT in curved distributional space-time. In our paper [4] the authors advocated the use of the classical De Witt-Schwinger approach [5] in order to establish QFT in general curved distributional space-time. The vacuum energy density of free scalar quantum field Φ with a distributional background space-time is considered successfully. It has been widely believed that, except in very extreme situations, the influence of gravity on quantum fields should amount to just small, sub-dominant contributions. Here we argue that this belief is false by showing that there exist well-behaved space-time evolutions where the vacuum energy density of free quantum fields is forced, by the very same background distributional space-time such as BHs, to become dominant over any classical energy density component. This semi-classical gravity effect finds its roots in the singular behavior of quantum fields on curved distributional space-times. In particular we obtain that the generalized vacuum fluctuations $(\langle\Phi_\varepsilon^2\rangle)_\varepsilon, \varepsilon \in (0,1]$ in the limit $r \to r_g = 2m$ has a singular behavior on BH horizon, i.e.

$$\lim_{\varepsilon\to 0}\langle\Phi_\varepsilon^2(r)\rangle \asymp |r-r_g|^{-2}. \tag{1.1}$$

And generalized stress tensor $\left(\langle T_\mu^\mu(r,\varepsilon)\rangle_{\mathbf{ren}}\right)_\varepsilon$ in the limit $r \to r_g = 2m$ has a singular behavior on BH horizon, i.e.

$$\lim_{\varepsilon\to 0}\langle T_\mu^\mu(r,\varepsilon)\rangle_{\mathbf{ren}} \asymp |r-r_g|^{-2}. \tag{1.2}$$

Therefore observer falling on BH burns up near the BH horizon. Recall that the classical Cartan's structural equations show in a compact way the relation between a connection and its curvature, and reveals their geometric interpretation in terms of moving frames. In order to study the mathematical properties of singularities, we need to study the geometry of manifolds endowed on the tangent bundle with a symmetric bilinear form which is allowed to become degenerate (singular). But if the fundamental tensor is allowed to be degenerate (singular), there are some obstructions in constructing the geometric objects normally associated to the fundamental tensor. Also, local orthonormal frames and co-frames no longer exist, as well as the metric connection and its curvature operator [6]. As important example of the geometry with the fundamental tensor which is allowed to be degenerate, we consider now Möller's uniformly accelerated frame given by Möller's line element [7]

$$ds^2 = -(a+gx)^2dt^2 + dx^2 + dy^2 + dz^2. \tag{1.3}$$

Of course Möller's metric (1.3) degenerate at Möller horizon $x_{hor} = -a/g$. Note that formally corresponding to the Möller's metric (1.3) classical Levi-Civit`a connection is [7]

$$\Gamma_{44}^1(x) = (a+gx), \Gamma_{14}^4(x) = \Gamma_{41}^4(x) = (a+gx)^{-1} \tag{1.4}$$

and therefore classical Levi-Civit`a connection (1.4) of course is not available at Möller horizon since at horizon formal expression (1.4) becomes infinity: $\Gamma_{14}^4(-a/g) = \Gamma_{14}^4(-a/g) = \infty$.

Remark1.1. Note that in classical Möller's paper [7], Möller mistakenly assumed that expression (1.4) gives a true Levi-Civit`a connection on whole space-time with Möller's line element (1.3). In physical literature this mistake holds until nowadays.

The main contemporary approach to avoid problems with the classical metric connections and its curvature operators, based on J. F. Colombeau generalized functions [8], namely "Distributional Geometry", during last 35 years in mathematical and physical literature was many developed [9]-[19].

In order to avoid difficultness with degeneracy of the classical metric (1.3) mentioned above, we replace now Möller's line element (1.3) by corresponding Colombeau line element [9], [10]

$$(d_\varepsilon s^2)_\varepsilon = -(\Delta_\varepsilon(x)dt^2)_\varepsilon + dx^2 + dy^2 + dz^2, \tag{1.5}$$

where $\Delta_\varepsilon(x) = [(a+gx)^2 + \varepsilon^2], \varepsilon \in (0,1]$.

Remark 1.2. Note that in contrast with a wrong formal expression (1.4) the distributional Levi-Cività connection corresponding Colombeau line element (1.5) is

$$\left(\Gamma^1_{44}(x,\varepsilon)\right)_\varepsilon = (a+gx), \left(\Gamma^4_{14}(x,\varepsilon)\right)_\varepsilon = \left(\Gamma^4_{41}(x,\varepsilon)\right)_\varepsilon = (a+\mathrm{g}x)/((a+gx)^2+\varepsilon^2)_\varepsilon. \quad (1.6)$$

Notice that the distributional Levi-Cività connection (1.6) is well defined and even regular on whole Möller's space-time.

We remind now that distributional black holes geometry [4], [12], [19] have an approximate distributional Rindler region near the Schwarzschild horizon [19]. For the distributional Schwarzschild metric by coordinate transformation $r = 2m\left(1+(\delta^2+\varepsilon^2)\right)$ in the limit $r \to r_g = 2m$ we obtain [19]

$$\left(ds^2_\varepsilon\right)_\varepsilon = -\left(\left(\delta^2+\varepsilon^2\right)dt^2\right)_\varepsilon + 16m^2 d\delta^2 + 4m^2 d\Omega^2_2 + O\left(\varepsilon^2\right). \quad (1.7)$$

The (t,δ) piece of the distributional metric (1.7) is distributional Rindler space-time.Thus from (1.1) and (1.2) we obtain directly

$$\lim_{\varepsilon\to0}\left\langle\Phi^2_\varepsilon(\delta)\right\rangle \sim \delta^{-4}, \lim_{\varepsilon\to0}\left\langle T^\mu_\mu(\delta,\varepsilon)\right\rangle_{\mathbf{ren}} \sim \delta^{-4}. \quad (1.8)$$

2. Distributional Rindler space-time with distributional Levi-Cività connection

Recall that fundamental tensor corresponding to the metric (1.3) were obtained in Möller's paper [7] as a vacuum solution of the classical Einstein's field equations $G^k_i = R^k_i - \frac{1}{2}\delta^k_i R = 0$, where R^k_i is the contracted Riemann-Christoffel tensor, formally calculated by canonical way by using classical Levi-Civit`a connection (1.3) and $R = R^i_i$. Thus $G^2_2(x) = G^3_3(x) = -1/2\,\Delta(x)\left[\Delta''(x) - \frac{[\Delta'(x)]^2}{2\Delta(x)}\right]$, where $\Delta(x) = (a+gx)^2$, $\Delta'(x) = \partial\Delta(x)/\partial x$ and all other components of G^k_i vanishes identically.

Remark 2.1.Therefore for any $x \neq -a/g$ we get a classical Möller's result

$$G^2_2(x) = G^3_3(x) \equiv 0.$$

Definition 2.1.We denoted by $\widetilde{\mathbb{R}}$ the ring of the real Colombeau generalized numbers [8].

Recall that there exists natural embedding $\tilde{r}: \mathbb{R} \hookrightarrow \widetilde{\mathbb{R}}$ such that for all $\in \mathbb{R}$, $\tilde{r} = (r_\varepsilon)_\varepsilon$, where $r_\varepsilon = r$ for all $\varepsilon \in (0,1]$. We say that $\tilde{r}$ is standard number and abbreviate $r \in \mathbb{R}$ for short. The ring $\widetilde{\mathbb{R}}$ can be endowed with the structure of a partially ordered ring: for $r,s \in \widetilde{\mathbb{R}}, \eta \in \mathbb{R}_+, \eta \leq 1$, we abbreviate $r \leq_{\widetilde{\mathbb{R}},\eta} s$ if and only if there are representatives $(r_\varepsilon)_\varepsilon$ and $(s_\varepsilon)_\varepsilon$ with $r_\varepsilon \leq s_\varepsilon$ for all $\varepsilon \in (0,\eta]$. For Colombeau generalized number $r \in \widetilde{\mathbb{R}}$ with representative $(r_\varepsilon)_\varepsilon$ we abbreviate $r = \mathbf{cl}[(r_\varepsilon)_\varepsilon]$ [8].

Definition 2.2.(i) Let $\delta \in \widetilde{\mathbb{R}}$. We say that δ is infinite small Colombeau generalized number and abbreviate $\delta \approx_{\widetilde{\mathbb{R}}} \tilde{0}$ if there exists representative $(\delta_\varepsilon)_\varepsilon, \varepsilon \in (0,1]$ and some $q \in \mathbb{N}$ such that $|\delta_\varepsilon| = O(\varepsilon^q)$, as $\varepsilon \to 0$. (ii) We say that $\Delta \in \widetilde{\mathbb{R}}$ is infinite large Colombeau generalized number and abbreviate $\Delta =_{\widetilde{\mathbb{R}}} \tilde{\infty}$ if $\Delta^{-1} \approx_{\widetilde{\mathbb{R}}} \tilde{0}$,[19].

Let $\left(G^k_i(\varepsilon)\right)_\varepsilon$ be the distributional Einstein tensor:

$$\left(G^k_i(\varepsilon)\right)_\varepsilon = \left(R^k_i(\varepsilon)\right)_\varepsilon - \tfrac{1}{2}\delta^k_i\left(R(\varepsilon)\right)_\varepsilon,$$

where $\left(R^k_i(\varepsilon)\right)_\varepsilon$ is the contracted distributional Riemann-Christoffel tensor calculated by using distributional Levi-Civit`a connection corresponding to the Colombeau line element (1.5) [10]-[13] and $(R(\varepsilon))_\varepsilon = \left(R^i_i(\varepsilon)\right)_\varepsilon$. Notice for any $\varepsilon \in (0,1]$ the quantity $R^k_i(\varepsilon)$ is calculated classically. Therefore for the case of the Colombeau line element (1.5) we get

$$\left(G^2_2(x;\varepsilon)\right)_\varepsilon = \left(G^3_3(x;\varepsilon)\right)_\varepsilon = -\frac{1}{2\left(\Delta_\varepsilon(x)\right)_\varepsilon}\left\{\left(\Delta''_\varepsilon(x)\right)_\varepsilon - \frac{\left[\left(\Delta'_\varepsilon(x)\right)_\varepsilon\right]^2}{2\left(\Delta_\varepsilon(x)\right)_\varepsilon}\right\} = -\left(\frac{g^2\varepsilon^2}{\Delta^2_\varepsilon(x)}\right)_\varepsilon. \quad (2.1)$$

Notice that

$$\left(C(x;\varepsilon)\right)_\varepsilon=-\left(\frac{g^2\varepsilon^2}{\Delta_\varepsilon^2(x)}\right)_\varepsilon,\varepsilon\in(0,1]$$

is Colombeau generalized function such that $\mathbf{cl}\left[\left(C(x;\varepsilon)\right)_\varepsilon\right]\in\mathbf{G}(\mathbb{R})$ and $\mathbf{cl}\left[\left(C(-a/g;\varepsilon)\right)_\varepsilon\right]=_{\widetilde{\mathbb{R}}}\tilde{\infty}$.
Remark 2.2.Note that for any standard $x\in\mathbb{R}$ such that $x\neq -a/g$ obviously $\mathbf{cl}\left[\left(C(x;\varepsilon)\right)_\varepsilon\right]\approx_{\widetilde{\mathbb{R}}}\tilde{0}$ and we obtain the desired classical result. However at horizon $\mathbf{cl}\left[\left(C(-a/g;\varepsilon)\right)_\varepsilon\right]=\mathbf{cl}[(\varepsilon^{-2})_\varepsilon]=_{\widetilde{\mathbb{R}}}\tilde{\infty}$ and therefore at horizon naive classical result completely wrong.

Thus, Colombeau generalized fundamental tensor $\left(g_{ik}(\varepsilon)\right)_\varepsilon$ corresponding to Colombeau metric (1.5) that is a non-vacuum Colombeau solution of the generalized Einstein's field equations ([4] sec. 2.3)

$$\left(R_i^k(x,\varepsilon)\right)_\varepsilon-\tfrac{1}{2}\delta_i^k\left(R(x,\varepsilon)\right)_\varepsilon=\left(T_i^k(x,\varepsilon)\right)_\varepsilon. \tag{2.2}$$

For Colombeau scalars $\left(R(x,\varepsilon)\right)_\varepsilon$, $\left(R^{\mu\nu}(x,\varepsilon)R_{\mu\nu}(x,\varepsilon)\right)_\varepsilon$, $\left(R^{\rho\sigma\mu\nu}(x,\varepsilon)R_{\rho\sigma\mu\nu}(x,\varepsilon)\right)_\varepsilon$ corresponding to Colombeau metric (1.5) we get

$$\left(R(x,\varepsilon)\right)_\varepsilon=\left(\frac{g^2\varepsilon^2}{\Delta_\varepsilon^2(x)}\right)_\varepsilon, \tag{2.3}$$

$$\left(R^{\mu\nu}(x,\varepsilon)R_{\mu\nu}(x,\varepsilon)\right)_\varepsilon=\left(R^{\rho\sigma\mu\nu}(x,\varepsilon)R_{\rho\sigma\mu\nu}(x,\varepsilon)\right)_\varepsilon=\frac{g^4\varepsilon^4}{\Delta_\varepsilon^4(x)}. \tag{2.4}$$

Remark 2.3. Note that for any standard $x\in\mathbb{R}$ such that $x\neq -a/g$ obviously
$\mathbf{cl}\left[\left(R(x,\varepsilon)\right)_\varepsilon\right]\approx_{\widetilde{\mathbb{R}}}\tilde{0}$, $\mathbf{cl}\left[\left(R^{\mu\nu}(x,\varepsilon)R_{\mu\nu}(x,\varepsilon)\right)_\varepsilon\right]\approx_{\widetilde{\mathbb{R}}}\tilde{0}$, $\mathbf{cl}\left[\left(R^{\rho\sigma\mu\nu}(x,\varepsilon)R_{\rho\sigma\mu\nu}(x,\varepsilon)\right)_\varepsilon\right]\approx_{\widetilde{\mathbb{R}}}\tilde{0}$.
However at horizon $x_{hor}=-a/g$ one obtains non-classical result:
$\mathbf{cl}\left[\left(R(x_{hor},\varepsilon)\right)_\varepsilon\right]=_{\widetilde{\mathbb{R}}}\tilde{\infty}$, $\mathrm{cl}\left[\left(R^{\mu\nu}(x,\varepsilon)R_{\mu\nu}(x,\varepsilon)\right)_\varepsilon\right]=_{\widetilde{\mathbb{R}}}\tilde{\infty}$, $\mathrm{cl}\left[\left(R^{\rho\sigma\mu\nu}(x,\varepsilon)R_{\rho\sigma\mu\nu}(x,\varepsilon)\right)_\varepsilon\right]=_{\widetilde{\mathbb{R}}}\tilde{\infty}$.
Remark 2.4. Remind that in Colombeau distributional geometry curvature scalars is well defined as point values of Colombeau generalized functions (2.3)-(2.4).Note that the curvature scalars (2.3)-(2.4) can be extended on the ring $\widetilde{\mathbb{R}}$ of the real,Colombeau generalized numbers $x=\mathbf{cl}[(x_\varepsilon)_\varepsilon]\in\widetilde{\mathbb{R}}$ by the following formulae [19]:

$$\left(R(x_\varepsilon,\varepsilon)\right)_\varepsilon=\left(\frac{g^2\varepsilon^2}{\Delta_\varepsilon^2(x_\varepsilon)}\right)_\varepsilon, \tag{2.5}$$

$$\left(R^{\mu\nu}(x_\varepsilon,\varepsilon)R_{\mu\nu}(x_\varepsilon,\varepsilon)\right)_\varepsilon=\left(R^{\rho\sigma\mu\nu}(x_\varepsilon,\varepsilon)R_{\rho\sigma\mu\nu}(x_\varepsilon,\varepsilon)\right)_\varepsilon=\left(\frac{g^4\varepsilon^4}{\Delta_\varepsilon^4(x_\varepsilon)}\right)_\varepsilon. \tag{2.6}$$

We assume now there exist fundamental generalized length $l=\eta\mathbf{cl}[(\varepsilon)_\varepsilon]\in\widetilde{\mathbb{R}}$, $\eta\in\mathbb{R}$, $\varepsilon\in(0,1]$, $\eta\ll 1$, such that

$$\mathbf{cl}[(|a+gx_\varepsilon|)_\varepsilon]\geq l. \tag{2.7}$$

By using (2.6)-(2.7) one gets the estimate

$$\left(R^{\mu\nu}(x_\varepsilon,\varepsilon)R_{\mu\nu}(x_\varepsilon,\varepsilon)\right)_\varepsilon=\left(R^{\rho\sigma\mu\nu}(x_\varepsilon,\varepsilon)R_{\rho\sigma\mu\nu}(x_\varepsilon,\varepsilon)\right)_\varepsilon=\left(\frac{g^4\varepsilon^4}{\Delta_\varepsilon^4(x_\varepsilon)}\right)_\varepsilon=\left(\frac{g^4\varepsilon^4}{[(a+gx_\varepsilon)^2+\varepsilon^2]^4}\right)_\varepsilon=$$

$$\left(\frac{g^4}{[(a+gx_\varepsilon)^2+\varepsilon^2]^2}\right)_\varepsilon\cdot\left(\frac{\varepsilon^2}{(a+gx_\varepsilon)^2+\varepsilon^2}\right)_\varepsilon\cdot\left(\frac{\varepsilon^2}{(a+gx_\varepsilon)^2+\varepsilon^2}\right)_\varepsilon\leq_{\widetilde{\mathbb{R}}}\frac{1}{(1+\eta^2)^2}\left(\frac{g^4}{[(a+gx_\varepsilon)^2+\varepsilon^2]^2}\right)_\varepsilon. \tag{2.8}$$

3. Quantum scalar field in curved distributional space-time.

Much of formalism can be explained with Colombeau generalized scalar field [13], [19]. The basic concepts and methods extend straightforwardly to distributional tensor and distributional spinor fields. To being with let's take a space-time of arbitrary dimension *D*, with a metric tensor $\left(g_{ik}(\varepsilon)\right)_\varepsilon$ of signature $(+,-,\dots,-)$ The action for the Colombeau generalized scalar field $\left(\varphi(x)\right)_\varepsilon \in \boldsymbol{G}(M)$ is

$$(S_\varepsilon)_\varepsilon = \left(\int_M d^D x \tfrac{1}{2}\sqrt{|g_\varepsilon|}\,(g_\varepsilon^{\mu\nu}\partial_\mu\varphi_\varepsilon\partial_\nu\varphi_\varepsilon) - (m^2+\xi R_\varepsilon)\varphi_\varepsilon^2\right)_\varepsilon. \tag{3.1}$$

The corresponding equation of motion is

$$([\square_{\varepsilon,x} + m^2 + \xi R_\varepsilon]\varphi_\varepsilon)_\varepsilon, \varepsilon \in (0,1]. \tag{3.2}$$

Here

$$(\square_{\varepsilon,x}\varphi_\varepsilon)_\varepsilon = \left(|g_\varepsilon|^{-1/2}\partial_\mu |g_\varepsilon|^{1/2} g_\varepsilon^{\mu\nu}\partial_\mu\varphi_\varepsilon\right)_\varepsilon. \tag{3.3}$$

With $\hbar$ explicit, the mass *m* should be replaced by $\frac{m}{\hbar}$. Separating out a time coordinate $x^0, x^\mu = \left(x^0, x^i\right), i = 1,2,3$ we can write the action as

$$(S_\varepsilon)_\varepsilon = \left(\int dx^0 L_\varepsilon\right)_\varepsilon, (L_\varepsilon)_\varepsilon = \left(\int d^{D-1}x\mathcal{L}_\varepsilon\right)_\varepsilon. \tag{3.4}$$

The canonical momentum at a time x^0 is given by

$$(\pi_\varepsilon(\underline{x}))_\varepsilon = (\delta L_\varepsilon/\delta(\partial_0\varphi_\varepsilon(\underline{x})))_\varepsilon = \left(|h_\varepsilon|^{1/2} n^\mu \partial_\mu \varphi_\varepsilon(\underline{x})\right)_\varepsilon, \tag{3.5}$$

where $\underline{x}$ labels a point on a surface of constant x^0 the x^0 argument of $\left(\varphi(\varepsilon)\right)_\varepsilon$ is suppressed,n^μ is the unit normal to the surface, and $(|h_\varepsilon|)_\varepsilon$ is the determinant of the induced spatial metric $\left(h_{ik}(\varepsilon)\right)_\varepsilon$. In order to quantize, the Colombeau generalized scalar field $\left(\varphi(x)\right)_\varepsilon$ and its conjugate momentum $\left(\pi(\underline{x})\right)_\varepsilon$ are now promoted to hermitian operators and required to satisfy the canonical commutation relation:

$$\left(\left[\varphi_\varepsilon(\underline{x}), \pi_\varepsilon\left(\underline{y}\right)\right]\right)_\varepsilon = i\hbar\delta^{D-1}\left(\underline{x}, \underline{y}\right), \varepsilon \in (0,1]. \tag{3.6}$$

Here $\int d^{D-1}y\delta^{D-1}\left(\underline{x},\underline{y}\right) f(y) = f\left(\underline{x}\right)$ for any scalar function $f \in D(\mathbb{R}^{D-1})$, without the use of a metric volume element. We form now a conserved bracket from two complex Colombeau solutions to the scalar wave equation (3.2) by [4]:

$$(\langle\varphi_\varepsilon, \phi_\varepsilon\rangle)_\varepsilon = \left(\int_\Sigma d\Sigma_\mu j_\varepsilon^\mu\right)_\varepsilon, \tag{3.7}$$

where

$$(j_\varepsilon^\mu(\varphi_\varepsilon, \phi_\varepsilon))_\varepsilon = (i/\hbar)\left(|g_\varepsilon|^{1/2} g_\varepsilon^{\mu\nu}(\overline{\varphi}_\varepsilon\partial_\nu\phi_\varepsilon - \varphi_\varepsilon\partial_\nu\overline{\phi}_\varepsilon)\right)_\varepsilon. \tag{3.8}$$

Using equation of motion (3.2) one obtains the canonical Green functions equations. In particular for the Colombeau distributional propagator:

$$i(G_\varepsilon(x,x'))_\varepsilon = (\langle 0|T(\varphi_\varepsilon(x)\varphi_\varepsilon(x'))|0\rangle)_\varepsilon, \tag{3.9}$$

one obtains directly

$$([\square_{\varepsilon,x} + m^2 + \xi\mathbf{R}(x,\varepsilon)]G_\varepsilon(x,x'))_\varepsilon = -\left([-g_\varepsilon(x)]^{-1/2}\right)_\varepsilon \delta^n(x-x'). \tag{3.10}$$

We obtain now an adiabatic expansion of $(G_\varepsilon(x,x'))_\varepsilon$ [4].Introducing Riemann normal coordinates y^μ for the point *x*, with origin at the point x' one obtains

$$(g_{\mu\upsilon}(x,\varepsilon))_\varepsilon = \eta_{\mu\upsilon} + \tfrac{1}{3}[(\mathbf{R}_{\mu\alpha\upsilon\beta}(\varepsilon))_\varepsilon]y^\alpha y^\beta - \tfrac{1}{6}[(\mathbf{R}_{\mu\alpha\upsilon\beta;\gamma}(\varepsilon))_\varepsilon]y^\alpha y^\beta y^\gamma + \\ +\Big[\tfrac{1}{20}(\mathbf{R}_{\mu\alpha\upsilon\beta;\gamma\delta}(\varepsilon))_\varepsilon + \tfrac{2}{45}[(\mathbf{R}_{\alpha\mu\beta\lambda}(\varepsilon))_\varepsilon](\mathbf{R}^{\lambda}_{\gamma\upsilon\delta}(\varepsilon))_\varepsilon\Big]y^\alpha y^\beta y^\gamma y^\delta + \ldots \tag{3.11}$$

where $\eta_{\mu\nu}$ is the Minkowski metric tensor, and the coefficients are all evaluated at $y = 0$ Defining now:

$$(\mathcal{L}_\varepsilon(x,x'))_\varepsilon = \Big[\Big((-g_{\mu\upsilon}(x,\varepsilon))^{1/4}\Big)_\varepsilon\Big](G_\varepsilon(x,x'))_\varepsilon \tag{3.12}$$

and its Colombeau-Fourier transform $(\mathcal{L}_\varepsilon(k))_\varepsilon$ by

$$(\mathcal{L}_\varepsilon(x,x'))_\varepsilon = (2\pi)^{-n}\Big(\int d^n k e^{-ik\cdot y}\mathcal{L}_\varepsilon(k)\Big)_\varepsilon, \tag{3.13}$$

where $k \cdot y = \eta^{\alpha\beta}k_\alpha y_\beta$ one can work in a sort of localized momentum space. Expanding (3.10) in normal coordinates and converting to k-space, $(\mathcal{L}_\varepsilon(k))_\varepsilon$ can readily be solved by iteration to any adiabatic order. The result to adiabatic order four (i.e., four derivatives of the metric) is

$$(\mathcal{L}_\varepsilon(k))_\varepsilon = (k^2 - m^2)^{-1} - \Big(\tfrac{1}{6} - \xi\Big)(k^2 - m^2)^{-2}(\mathbf{R}(\varepsilon))_\varepsilon + \\ +\tfrac{i}{2}\Big(\tfrac{1}{6} - \xi\Big)\partial^\alpha(k^2 - m^2)^{-2}(\mathbf{R}_{;\alpha}(\varepsilon))_\varepsilon - \tfrac{1}{3}[(a_{\alpha\beta}(\varepsilon))_\varepsilon]\partial^\alpha\partial^\beta(k^2 - m^2)^{-2} + \\ \Big[\Big(\tfrac{1}{6} - \xi\Big)^2(\mathbf{R}^2(\varepsilon))_\varepsilon + \tfrac{2}{3}\Big(a^{\lambda}{}_{\lambda}(\varepsilon)\Big)_\varepsilon\Big](k^2 - m^2)^{-3}, \tag{3.14}$$

where $\partial_\alpha = \partial/\partial k^\alpha$,

$$(a_{\alpha\beta}(\varepsilon))_\varepsilon \asymp \\ \Big(\tfrac{1}{2} - \xi\Big)(\mathbf{R}_{;\alpha\beta}(\varepsilon))_\varepsilon + \tfrac{1}{120}(\mathbf{R}_{;\alpha\beta}(\varepsilon))_\varepsilon - \tfrac{1}{140}\Big(\mathbf{R}_{\alpha\beta;\lambda}{}^{\lambda}(\varepsilon)\Big)_\varepsilon - \tfrac{1}{30}\Big[\Big(\mathbf{R}_\alpha{}^{\lambda}(\varepsilon)\Big)_\varepsilon\Big](\mathbf{R}_{\lambda\beta}(\varepsilon))_\varepsilon + \\ +\tfrac{1}{60}\Big[\Big(\mathbf{R}^{\kappa}{}_{\alpha}{}^{\lambda}{}_{\beta}(\varepsilon)\Big)_\varepsilon\Big](\mathbf{R}_{\kappa\lambda}(\varepsilon))_\varepsilon + \tfrac{1}{60}\Big[\Big(\mathbf{R}^{\lambda\mu\kappa}{}_{\alpha}(\varepsilon)\Big)_\varepsilon\Big](\mathbf{R}_{\lambda\mu\kappa\beta}(\varepsilon))_\varepsilon, \tag{3.15}$$

and we are using the symbol $\simeq$ to indicate that this is an asymptotic expansion. One ensures that equation (3.13) represents a time-ordered product by performing the k^0 integral along the appropriate contour in figure 1. This is equivalent to replacing m^2 by$m^2 - i\epsilon$.

Similarly, the adiabatic expansions of other Green functions can be obtained by using the other contours in figure 1. Substituting equation (3.14) into equation (3.13) gives [4] equation (3.16)

$$(\mathcal{L}_\varepsilon(x,x'))_\varepsilon = (2\pi)^{-n} \times \Big(\int d^n k e^{-iky}(k^2 - m^2)^{-1}\Big[a_0(x,x';\varepsilon) + a_1(x,x';\varepsilon)\Big(-\tfrac{\partial}{\partial m^2}\Big) + \\ a_2(x,x';\varepsilon)\Big(\tfrac{\partial}{\partial m^2}\Big)^2\Big]\Big)_\varepsilon, \tag{3.16}$$

where $(a_0(x,x';\varepsilon))_\varepsilon = 1$ and, to adiabatic order 4, equation (3.17)

$$(a_1(x,x';\varepsilon))_\varepsilon = \Big(\tfrac{1}{6} - \xi\Big)(\mathbf{R}(\varepsilon))_\varepsilon - \tfrac{i}{2}\Big(\tfrac{1}{6} - \xi\Big)[(\mathbf{R}_{;\alpha}(\varepsilon))_\varepsilon]y^\alpha - \tfrac{1}{3}[(a_{\alpha\beta}(\varepsilon))_\varepsilon]y^\alpha y^\beta \\ (a_2(x,x';\varepsilon))_\varepsilon = \tfrac{1}{2}\Big(\tfrac{1}{6} - \xi\Big)(\mathbf{R}^2(\varepsilon))_\varepsilon + \tfrac{1}{3}\Big(a^{\lambda}{}_{\lambda}(\varepsilon)\Big)_\varepsilon \tag{3.17}$$

with all geometric quantities on the right-hand side of equation (3.17) evaluated at x' in equation (3.16), then the $d^n k$ integration may be interchanged with the ds integration, and performed explicitly to yield (dropping the $i\epsilon$.)

$$(\mathcal{L}_\varepsilon(x,x'))_\varepsilon = -i(4\pi)^{-n/2}\Big(\int_0^\infty ids(is)^{-n/2}\exp\Big[-im^2 s + \tfrac{\sigma(x,x')}{2is}\Big]\mathcal{F}_\varepsilon(x,x';is)\Big)_\varepsilon, \tag{3.18}$$

where $\sigma(x,x') = 1/2y_\alpha y^\alpha$ The function $\sigma(x,x')$ which is one-half of the square of the proper distance between x and x' while the function $\left(\mathcal{F}_\varepsilon(x,x';is)\right)_\varepsilon$ has the following asymptotic adiabatic expansion:

$$(\mathcal{F}_\varepsilon(x,x';is))_\varepsilon \asymp (a_0(x,x';\varepsilon))_\varepsilon + is(a_1(x,x';\varepsilon))_\varepsilon + \\ +(is)^2(a_2^{\pm}(x,x';\varepsilon))_\varepsilon + \ldots \tag{3.19}$$

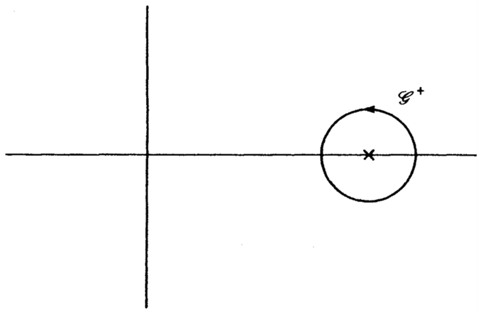

Figure 1. The contour in the complex k^0 plane C to be used in the evaluation of the integral giving L^+. The cross indicates the pole at $k^0 = (|\mathbf{k}|^2 + m^2)^{1/2}$.

Using equation (3.12), equation (3.18) gives a representation

$$(G_\varepsilon(x,x'))_\varepsilon = -i(4\pi)^{-n/2}\left(\left[(\Delta^{1/2}(x,x';\varepsilon))_\varepsilon\right]\int_0^\infty ids(is)^{-n/2}\exp\left[-im^2s + \frac{\sigma(x,x')}{2is}\right]\mathcal{F}_\varepsilon(x,x';is)\right)_\varepsilon, \tag{3.20}$$

where $(\Delta(x,x';\varepsilon))_\varepsilon$ is the distributional Van Fleck determinant

$$(\Delta(x,x';\varepsilon))_\varepsilon = -\det[\partial_\mu\partial_\nu\sigma(x,x')]\left([g_\varepsilon(x)g(x',\varepsilon)]^{-1/2}\right)_\varepsilon. \tag{3.21}$$

In the normal coordinates about x' that we are currently using, $(\Delta(x,x';\varepsilon))_\varepsilon$ reduces to $\left([-g_\varepsilon(x)]^{1/2}\right)_\varepsilon$. The full asymptotic expansion of $\left(\mathcal{F}_\varepsilon(x,x';is)\right)_\varepsilon$ to all adiabatic orders is:

$$(\mathcal{F}_\varepsilon(x,x';is))_\varepsilon \asymp \sum_{j=0}^{\infty}(is)^j(a_2(x,x';\varepsilon))_\varepsilon \tag{3.22}$$

with $(a_0(x,x';\varepsilon))_\varepsilon = 1$ the other $\left(a_j(x,x';\varepsilon)\right)_\varepsilon$ being given by canonical recursion relations which enable their adiabatic expansions to be obtained. If (3.22) is substituted into (3.20) the integral can be performed to give the adiabatic expansion of the Feynman propagator in coordinate space:

$$(G_\varepsilon(x,x'))_\varepsilon \asymp -(4\pi i)^{-n/2}\left(\Delta_{\pm}^{1/2}(x,x';\varepsilon)\sum_{j=0}^{\infty} a_j(x,x';\varepsilon)\left(-\frac{\partial}{\partial m^2}\right)^j \times \right. \\ \left. \times\left[\left(-\frac{2m^2}{\sigma}\right)^{\frac{n-2}{4}} H^{(2)}_{(n-2)/2}\left((2m^2\sigma)^{\frac{1}{2}}\right)\right]\right)_\varepsilon \tag{3.23}$$

which, strictly, a small imaginary part $i\epsilon$. should be subtracted from σ.

4. Effective action for the quantum matter fields in curved distributional space-time.

As in classical case one can obtain Colombeau generalized quantity $(W_\varepsilon)_\varepsilon$, called the effective action for the quantum matter fields in curved distributional space-time, which, when functionally differentiated, yields

$$\left(2(-g(\varepsilon))^{-\frac{1}{2}}\frac{\delta W_\varepsilon}{\delta g^{\mu\nu}(\varepsilon)}\right)_\varepsilon = (\langle \mathbf{T}_{\mu\nu}(\varepsilon)\rangle)_\varepsilon. \tag{4.1}$$

Note that the generating functional

$$(Z_\varepsilon[\mathbf{J}_\varepsilon])_\varepsilon = \left(\int D[\varphi_\varepsilon]\exp\left\{iS_{\mathbf{m}}(\varepsilon) + i\int \mathbf{J}_\varepsilon(x)\varphi_\varepsilon(x)d^n x\right\}\right)_\varepsilon \tag{4.2}$$

was interpreted physically as the vacuum persistence amplitude $(\langle \boldsymbol{out}_\varepsilon, 0|0, \boldsymbol{in}_\varepsilon\rangle)_\varepsilon$ The presence of the external distributional current density $(\mathbf{J}_\varepsilon)_\varepsilon$ can cause the initial vacuum state $(|0, \boldsymbol{in}_\varepsilon\rangle)_\varepsilon$ to be unstable, i.e., it can bring about the production of particles.

Following canonical calculation one obtains [4]

$$(Z_\varepsilon[0])_\varepsilon \propto \left([\det(-G_\varepsilon^{\pm}(x,x'))]^{\frac{1}{2}}\right)_\varepsilon, \tag{4.3}$$

where the proportionality constant is metric-independent and can be ignored. Thus we obtain

$$(W_\varepsilon)_\varepsilon = -i(\ln Z_\varepsilon[0])_\varepsilon = -\frac{i}{2}\left(\mathbf{tr}\left[\ln\left(-\hat{G}_\varepsilon\right)\right]\right)_\varepsilon. \tag{4.4}$$

In equation (4.4) $\left(\hat{G}_\varepsilon^{\pm}\right)_\varepsilon$ is to be interpreted as an Colombeau generalized operator which acts on an linear space $\mathfrak{I}$ of generalized vectors $(|x,\varepsilon\rangle)_\varepsilon$,$\varepsilon \in (0,1]$ normalized by

$$(\langle x,\varepsilon|x',\varepsilon\rangle)_\varepsilon = \delta(x-x')\left([-g_\varepsilon(x)]^{1/2}\right)_\varepsilon \tag{4.5}$$

in such a way that

$$(G_\varepsilon(x,x'))_\varepsilon = \left(\langle x,\varepsilon|\hat{G}_\varepsilon|x',\varepsilon\rangle\right)_\varepsilon. \tag{4.6}$$

Remark 4.1. Note that the trace $(|\mathbf{tr}[\![\cdot]\!])_\varepsilon$ of the Colombeau generalized operator $(\mathcal{R})_\varepsilon$ which acts on a linear space $\mathfrak{I}$ is defined by

$$(\mathbf{tr}[\Re_\varepsilon])_\varepsilon = \left(\int d^n x[-g^{\pm}(x,\varepsilon)]^{\frac{1}{2}}\Re_{xx;\varepsilon}\right)_\varepsilon = \left(\int d^n x[-g_\varepsilon(x)]^{\frac{1}{2}}\langle x|\Re_{xx;\varepsilon}|x'\rangle\right)_\varepsilon. \tag{4.7}$$

Writing now the Colombeau generalized operator $\left(\hat{G}_\varepsilon^{\pm}\right)_\varepsilon$ as

$$\left(\hat{G}_\varepsilon\right)_\varepsilon = -(\mathcal{F}_\varepsilon^{-1})_\varepsilon = -i\left(\int_0^\infty ds\exp[-s\mathcal{F}_\varepsilon]\right)_\varepsilon, \tag{4.8}$$

by equation (3.20) we obtain

$$(\langle x|\exp[-s\mathcal{F}_\varepsilon]|x'\rangle) = i(4\pi)^{-n/2}\left[(\Delta^{1/2}(x,x';\varepsilon))_\varepsilon\right]\exp\left[-im^2 s + \frac{\sigma(x,x')}{2is}\right]\mathcal{F}_\varepsilon(x,x';is)(is)^{-n/2}. \tag{4.9}$$

Proceeding in standard manner we get equation (4.10) [4]:

$$(W_\varepsilon)_\varepsilon = \frac{i}{2}\left[\left(\int d^n x[-g^{\pm}(x,\varepsilon)]^{\frac{1}{2}}\right)_\varepsilon\right]\times$$
$$\times\left(\lim_{x\to x'}\int_{m^2}^\infty G_\varepsilon(x,x';m^2)dm^2\right)_\varepsilon. \tag{4.10}$$

Interchanging now the order of integration and taking the limit $x \to x'$ one obtains

$$(W_\varepsilon)_\varepsilon = \frac{i}{2}\left(\int_{m^2}^\infty dm^2\int d^n x[-g_\varepsilon(x)]^{\frac{1}{2}}G_\varepsilon(x,x;m^2)\right)_\varepsilon. \tag{4.11}$$

Colombeau generalized quantity $(W_\varepsilon)_\varepsilon$, is called as the generalized one-loop effective action.
In the case of fermion effective actions, there would be a remaining trace over spinorial indices.
From equation (4.11) we may define an effective Lagrangian density $\left(L_{\varepsilon;\mathbf{eff}}^{\pm}(x)\right)_\varepsilon$ by equation

$$(W_\varepsilon)_\varepsilon = \left(\int d^n x[-g_\varepsilon(x)]^{\frac{1}{2}}L_{\varepsilon;\mathbf{eff}}(x)\right)_\varepsilon \tag{4.12}$$

whence one get

$$(L_\varepsilon(x))_\varepsilon = \left([-g_\varepsilon(x)]^{\frac{1}{2}}\mathcal{L}_{\varepsilon;\mathbf{eff}}(x)\right)_\varepsilon = \frac{i}{2}\left(\lim_{x\to x'}\int_{m^2}^\infty dm^2 G_\varepsilon(x,x';m^2)\right)_\varepsilon. \tag{4.13}$$

5. Stress-tensor renormalization.

Note that $(L(x))_\varepsilon$ diverges at the lower end of the s integral because the $\sigma/2s$ damping factor in the exponent vanishes in the limit $x \to x'$. (Convergence at the upper end is guaranteed by the $-i\epsilon$ that is implicitly added to m^2 in the De Witt-Schwinger representation of $\left(L_\varepsilon^\pm(x)\right)_\varepsilon$ In four dimensions, the potentially divergent terms in the DeWitt-Schwinger expansion of $\left(L_\varepsilon^\pm(x)\right)_\varepsilon$ are

$$(L_{\varepsilon;\mathrm{div}}(x))_\varepsilon = -(32\pi^2)^{-1}\left(\lim_{x\to x'}\left[(\Delta^{1/2}(x,x';\varepsilon))_\varepsilon\right]\int_0^\infty \frac{ds}{s^3}\exp\left[-im^2 s + \frac{\sigma(x,x')}{2is}\right]\times \left[a_0(x,x';\varepsilon) + isa_1(x,x';\varepsilon) + (is)^2 a_2(x,x';\varepsilon)\right]\right)_\varepsilon, \tag{5.1}$$

where the coefficients a_0, a_1 and a_2 are given by equation (3.17). The remaining terms in this asymptotic expansion, involving a_3 and higher, are finite in the limit $x \to x'$.

Let us determine now the precise form of the geometrical $\left(L_{\varepsilon;\mathrm{div}}(x)\right)_\varepsilon$ terms, to compare them with the distributional generalization of the gravitational Lagrangian that appears in [4]. This is a delicate matter because (3.1) is, of course, infinite. What we require is to display the divergent terms in the form $\infty \times$[geometrical object]. This can be done in a variety of ways. For example, in n dimensions, the asymptotic (adiabatic) expansion of $\left(L_{\varepsilon;\mathbf{eff}}(x)\right)_\varepsilon$ is

$$(L_{\varepsilon;\mathbf{eff}}(x))_\varepsilon \asymp 2^{-1}(4\pi)^{-n/2}\left(\lim_{x\to x'}\left[(\Delta^{1/2}(x,x';\varepsilon))_\varepsilon\right]\sum_{j=0}^\infty a_j(x,x';\varepsilon)\times \times\int_0^\infty ids(is)^{j-1-n/2}\exp\left[-im^2 s + \frac{\sigma(x,x')}{2is}\right]\right)_\varepsilon \tag{5.2}$$

of which the first $\frac{n}{2}+1$ terms are divergent as $\sigma \to 0$. If n is treated as a variable which can be analytically continued throughout the complex plane, then we may take the limit $x \to x'$:

$$\begin{gathered}(L_{\varepsilon;\mathbf{eff}}(x))_\varepsilon \asymp 2^{-1}(4\pi)^{-n/2}\left(\sum_{j=0}^\infty a_j(x;\varepsilon)\int_0^\infty ids(is)^{j-1-n/2}\exp[-im^2 s]\right)_\varepsilon = \\ 2^{-1}(4\pi)^{-n/2}\sum_{j=0}^\infty a_j(x;\varepsilon)(m^2)^{n/2-j}\Gamma\left(j-\frac{n}{2}\right), a_j(x;\varepsilon) = a_j(x,x;\varepsilon).\end{gathered} \tag{5.3}$$

It follows from equation (5.3) we shall wish to retain the units of $\left(L_{\varepsilon;\mathbf{eff}}(x)\right)_\varepsilon$ as $(\text{length})^{-4}$, even when $n \neq 4$ It is therefore necessary to introduce an arbitrary mass scale μ and to rewrite equation (5.3) as

$$(L_{\varepsilon;\mathbf{eff}}(x))_\varepsilon \asymp 2^{-1}(4\pi)^{-n/2}\left(\frac{m}{\mu}\right)^{n-4}\left(\sum_{j=0}^\infty a_j(x;\varepsilon)(m^2)^{4-2j}\Gamma\left(j-\frac{n}{2}\right)\right)_\varepsilon. \tag{5.4}$$

If $n \to 4$ the first three terms of equation (5.4) diverge because of poles in the Γ - functions:

$$\Gamma\left(-\frac{n}{4}\right) = \frac{4}{n(n-2)}\left(\frac{2}{4-n}-\gamma\right) + O(n-4), \Gamma\left(1-\frac{n}{2}\right) = \frac{4}{(2-n)}\left(\frac{2}{4-n}-\gamma\right) + O(n-4),$$
$$\Gamma\left(2-\frac{n}{2}\right) = \frac{2}{4-n} - \gamma + O(n-4). \tag{5.5}$$

Denoting these first three terms by $\left(L_{\varepsilon;\mathrm{div}}(x)\right)_\varepsilon$ we have

$$\left(L_{\varepsilon;\mathrm{div}}(x)\right)_\varepsilon = -(4\pi)^{-n/2}\left\{\frac{1}{n-4}+\frac{1}{2}\left[\gamma+\ln\left(\frac{m^2}{\mu^2}\right)\right]\right\}\times \left(\left[\frac{4m^4 a_0(x;\varepsilon)}{n(n-2)} - \frac{2m^2 a_1(x;\varepsilon)}{n-2} + a_2(x;\varepsilon)\right]\right)_\varepsilon. \tag{5.6}$$

The functions $a_0(x;\varepsilon), a_1(x;\varepsilon)$ and , $a_2(x;\varepsilon)$ are given by taking the coincidence limits of (3.17)

$$\begin{gathered}(a_0(x;\varepsilon))_\varepsilon = 1, (a_1(x;\varepsilon))_\varepsilon = \left(\tfrac{1}{6}-\xi\right)(\mathbf{R}(\varepsilon))_\varepsilon,\\ (a_2(x;\varepsilon))_\varepsilon = \tfrac{1}{180}(\mathbf{R}_{\alpha\beta\gamma\delta}(x,\varepsilon)\mathbf{R}^{\alpha\beta\gamma\delta}(x,\varepsilon))_\varepsilon - \tfrac{1}{180}(\mathbf{R}^{\alpha\beta}(x,\varepsilon)\mathbf{R}_{\alpha\beta}(x,\varepsilon))_\varepsilon - \\ -\tfrac{1}{6}\left(\tfrac{1}{5}-\xi\right)(\Box_{\varepsilon,x}\mathbf{R}(x,\varepsilon))_\varepsilon + \frac{1}{2}\left(\tfrac{1}{6}-\xi\right)(\mathbf{R}^2(x,\varepsilon))_\varepsilon.\end{gathered} \quad (5.7)$$

Finally we get [4]

$$(L_{\varepsilon;\mathbf{ren}}(x))_\varepsilon \asymp -\frac{1}{64\pi^2}\left(\int_0^\infty ids\ln(is)\frac{\partial^3}{\partial(is)^3}\left[\mathcal{F}_\varepsilon(x,x;is)e^{-ism^2}\right]\right)_\varepsilon. \quad (5.8)$$

All the higher order $j > 2$ terms in the DeWitt-Schwinger expansion of the effective Lagrangian (5.4) are infrared divergent at $n = 4$ as $m \to 0$ we can still use this expansion to yield the ultraviolet divergent terms arising from $j = 0,1,$ and 2 in the four-dimensional case. We may put $m = 0$ immediately in the $j = 0$ and 1 terms in the expansion, because they are of positive power for $n\sim 4$. These terms therefore vanish. The only non-vanishing potentially ultraviolet divergent term is therefore $j = 2$:

$$\left(\Xi(x,\varepsilon)\right)_\varepsilon = 2^{-1}(4\pi)^{-n/2}\left(\tfrac{m}{\mu}\right)^{n-4}\Gamma\left(2-\tfrac{n}{2}\right)\left(a_2(x,\varepsilon)\right)_\varepsilon, \quad (5.9)$$

which must be handled carefully. Substituting for , $a_2(x;\varepsilon)$ with $\xi = \xi(n)$ from (5.7), and rearranging terms, we may write the divergent term in the effective action arising from (5.10) as follows

$$\begin{gathered}(W_{\varepsilon,\mathbf{div}})_\varepsilon = 2^{-1}(4\pi)^{-n/2}(\tfrac{m}{\mu})^{n-4}\Gamma\left(2-\tfrac{n}{2}\right)\left(\int d^n x[-g(x,\varepsilon)]^{\frac{1}{2}}a_2(x,\varepsilon)\right)_\varepsilon = \\ 2^{-1}(4\pi)^{-n/2}(\tfrac{m}{\mu})^{n-4}\Gamma\left(2-\tfrac{n}{2}\right)\left(\int d^n x[-g(x,\varepsilon)]^{\frac{1}{2}}\left[\widetilde{\alpha}\mathsf{F}_\varepsilon(x)+\widetilde{\beta}G_\varepsilon(x)\right]\right)_\varepsilon + O(n-4),\end{gathered} \quad (5.10)$$

where

$$\begin{gathered}(\mathsf{F}_\varepsilon(x))_\varepsilon = (R^{\alpha\beta\gamma\delta}(x,\varepsilon)R_{\alpha\beta\gamma\delta}(x,\varepsilon))_\varepsilon - 2(R^{\alpha\beta}(x,\varepsilon)R_{\alpha\beta}(x,\varepsilon))_\varepsilon + \tfrac{1}{3}(R^2(x,\varepsilon))_\varepsilon,\\ (G_\varepsilon(x))_\varepsilon = (R^{\alpha\beta\gamma\delta}(x,\varepsilon)R_{\alpha\beta\gamma\delta}(x,\varepsilon))_\varepsilon, \widetilde{\alpha} = \tfrac{1}{120}, \widetilde{\beta} = -\tfrac{1}{360}.\end{gathered} \quad (5.11)$$

Finally we get [4], [19]:

$$\begin{gathered}\left(\langle T^\mu_\mu(x,\varepsilon)\rangle_{\mathbf{ren}}\right)_\varepsilon = -(1/2880\pi^2)\left[\widetilde{\alpha}\left(\mathsf{F}_\varepsilon(x) - \tfrac{2}{3}\Box_{\varepsilon,x}R(x,\varepsilon)\right)_\varepsilon + \widetilde{\beta}(G^\pm_\varepsilon(x))_\varepsilon\right] = \\ -(1/2880\pi^2)\times \\ [(R_{\alpha\beta\gamma\delta}(x,\varepsilon)R^{\alpha\beta\gamma\delta}(x,\varepsilon))_\varepsilon - (R_{\alpha\beta}(x,\varepsilon)R^{\alpha\beta}(x,\varepsilon))_\varepsilon - (\Box_{\varepsilon,x}R(x,\varepsilon))_\varepsilon].\end{gathered} \quad (5.12)$$

Thus, for the case of the distributional Möller space-time using equation (2.5)-(2.6) and equation (5.12) in the limit $x \to -a/g$ we obtain [19]:

$$\left(\langle T^\mu_\mu(x,\varepsilon)\rangle_{ren}\right)_\varepsilon = -(2880\pi^2)^{-1}O(g^4)([(a+gx)^2+\varepsilon^2]^{-2})_\varepsilon \quad (5.13)$$

thus, QFT in distributional curved space-time predict that the in falling observer burns up near the Möller (Rindler) horizon. In order to avoid singularity at horizon $x = -a/g$ in equation (5.13) one can applies the Loop Quantum Gravity approach [19].

Multiplicative renormalization.

We willing to choose in Eq.(5.9) $n = 4 - \varepsilon^4, \varepsilon \in (0,1]$ and therefore from Eq.(5.5) and Eq.(5.9) we get

$$\left(L_{\varepsilon;\mathrm{div}}(x)\right)_{\varepsilon} = -(4\pi)^{-2}\left\{(\varepsilon^{-4})_{\varepsilon} + 0.5\left[\gamma + \ln\left(\frac{m^2}{\mu^2}\right)\right]\right\} \times$$

$$\left(\left[\frac{m^4 a_0(x;\varepsilon)}{2-\varepsilon^4} - \frac{2m^2 a_1(x;\varepsilon)}{2-\varepsilon^4} + a_2(x;\varepsilon)\right]\right)_{\varepsilon}. \quad (5.14)$$

The only non-vanishing potentially ultraviolet divergent term is therefore $j = 2$:

$$\left(\Xi(x,\varepsilon)\right)_{\varepsilon} = 2^{-1}(4\pi)^{-2}\left(\left(\frac{m}{\mu}\right)^{\varepsilon^4}\right)_{\varepsilon} \times \left[2(\varepsilon^{-4})_{\varepsilon} - \gamma + \left(O(\varepsilon^4)\right)\right]. \quad (5.15)$$

Finally from (2.5),(2.6), (5.12)-(5.15) we get

$$\left(\langle T_{\mu}^{\mu}(x,\varepsilon)\rangle_{ren}\right)_{\varepsilon} = -(2880\pi^2)^{-1} O(\mathrm{g}^4)([(a+gx)^2+\varepsilon^2]^{-4})_{\varepsilon}, \quad (5.16)$$

or in the following equivalent form

$$\left(\langle T_{\mu}^{\mu}(x,\varepsilon)\rangle_{ren}\right)_{\varepsilon} = -(2880\pi^2)^{-1} O(\mathrm{g}^4)([(a+gx)^2+(\varepsilon^2)_{\varepsilon}]^{-4}), \quad (5.17)$$

where $x \in \mathbb{R}$.

6. Unruh effect revisited.

The Unruh effect is the prediction that an accelerating observer will observe blackbody radiation where an inertial observer would observe none. The Unruh effect was first described by Stephen Fulling in 1973, Paul Davies in 1975 and W. G. Unruh in 1976 [20]. The Unruh temperature, derived by William Unruh in 1976, is the effective temperature experienced by a uniformly accelerating detector in a vacuum field. It is given by [20]:

$$T = \frac{\hbar \mathrm{g}}{2\pi c k_B}, \quad (6.1)$$

where g is the local acceleration, k_B is the Boltzmann constant, $\hbar$ is the reduced Planck constant, and c is the speed of light. Thus, for example, a proper acceleration of $2.47\times10^{20}\, m\cdot s^{-2}$corresponds approximately to a temperature 1 K. Notice that for a proper acceleration of $2.47\times10^{20}\, m\cdot s^{-2}$ the event horizon very close to observer by distance $|x_{hor}| = c^2/a \cong 1/\,2748.2m^{-1} \cong 3.6387\times 10^{-4}m$.It is currently not clear whether the Unruh effect has actually been observed, since the claimed observations are disputed. There is also some doubt about whether the Unruh effect implies the existence of Unruh radiation. Although Unruh's prediction that an accelerating detector would see a thermal bath is not controversial, the interpretation of the transitions in the detector in the non-accelerating frame is. It is widely, although not universally, believed that each transition in the detector is accompanied by the emission of a particle, and that this particle will propagate to infinity and be seen as Unruh radiation.

The existence of Unruh radiation is not universally accepted. Some claim that it has already been observed [21], while others claim that it is not emitted at all [22].While the skeptics accept that an accelerating object thermalizes at the Unruh temperature, they do not believe that this leads to the emission of photons, arguing that the emission and absorption rates of the accelerating particle are balanced. Remind that the temperature measured by the observer located near the BH horizon,

$$\mathrm{T}_{\mathrm{loc}} \cong \left(1 - \frac{r_{\mathrm{g}}}{r}\right)^{-1/2}, \quad (6.2)$$

grows infinitely near the BH horizon [23]. From Eq.(1.2) and Eq.(6.2) one obtains

$$\sigma \mathrm{T}_{\mathrm{loc}}^4 \cong \left|r - r_{\mathrm{g}}\right|^{-2} \quad (6.3)$$

and therefore near the BH horizon Stefan-Boltzmann law holds. By the Einstein equivalence principle Stefan-Boltzmann law holds near the Möller horizon. Therefore by the Eq.(5.13) the temperature measured by the observer located near the Möller horizon is

$$\mathrm{T_{loc}} \cong \mathrm{g}(1 + \mathrm{g}x)^{-1}. \tag{6.4}$$

For example, observer with a proper acceleration of $2.47\times10^{20}\ m \cdot s^{-2}$ burns up near the Möller horizon.

Conclusion

On a Riemannian or a semi-Riemannian manifold, the metric determines classical invariants like the classical Levi-Civita connection and the classical Riemann curvature. If the metric becomes degenerate (as in singular semi-Riemannian geometry), these constructions no longer work, because they are based on the inverse of the metric, and on related operations like the contraction between covariant indices. In order to avoid difficultness mentioned above, the apparatus the Colombeau distributional geometry by using Colombeau generalized functions is proposed. Appropriate generalization of classical GR based on Colombeau generalized functions is proposed. We pointed out that Polchinski's account [1] doesn't violate the Einstein equivalence principle [2].

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