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Forms of Life of Mathematical Objects

What could be more inert than mathematical objects? Nothing distinguishes them from rocks and yet, if we examine them in their historical perspective, they don't actually seem to be as lifeless as they do at first. Conceived as they are by humans, they offer a glimpse of the breath that brings them to life. Caught in the web of a language, they cannot extricate themselves from the form that the tensive forces constraining them have given them. While they do not serve a specific biological purpose, they are still, above all, possibilities of life, objects imbued with power. Though they know neither pain nor laughter, their mode or style of existence endows them with a special form of life that structures the givenness, the matter of the other entities in which they participate. Because of this, by intervening in the framework of these entities, mathematical objects condition their form of space, enjoining the entities to submit to a structure that they have not chosen.

Forms of Life and the Plane of Immanence

The expression "forms of life" seems to appear for the first time in the texts of Wittgenstein, particularly in *Philosophical Investigations* where there are no less than five examples of it. In one example, probably the most important, Wittgenstein places the form of life in the lineage of the theory of language: more exactly, in that theory's project of a general pragmatics. He gives more credit to cultural practices and linguistic variations than to architectonic, logico-structural elements: "[T]o imagine a language means to imagine a form of life."[1] Naming a thing is not just giving the name of a thing, hearing the sound of the word that designates it, but also understanding the forms of life that make this word designate what it is. We do not come to understand what the word means merely through a distinction between an expression and a content or between a signifier and a signified. Its meaning has a broader scope, involving lived experience; environmental, social and cultural practice; an intertext as much as an extratext; and a way of considering the designated thing that is shared by everyone. For Wittgenstein, all of these intraworldly relations are just so many grammatical forms, and for Bruno Latour, regimes of enunciation. Understanding the word "compassion," for example, supposes the understanding of what afflicts the other person, of the intentional forms the word contains, and of the forms of life that constitute and shape it. The interpretation or the arrangement is not limited to Hjelmslev's distinction between content and expression or Saussure's between signifier and signified: rather, it is a functional approach that abandons the sign in favor of the object. It is oriented and defined by the coalescence of a single plane of immanence that arranges the objects regardless of their origins – whether they be signs, words, sounds, entities, or forms of life – according to the forces that group them together, independently of any interpretative presupposition. A second example of the expression, also from *Philosophical Investigations*, demonstrates that forms of life are an activity that encompasses language-games. "The word 'language-*game*' is used here to emphasize the fact that the *speaking* of language is part of an activity, or of a form of life."[2] It follows that forms of life are dynamic models that, depending on who is describing these models, are distinguishable or not from lifestyles marked by their static social determinations: they can reinvent themselves at any moment. Therefore, for Jacques Fontanille, forms of life cannot – on principle and by definition – be the subjects either of any general typology or one of a sociological, anthropological or ideological nature, thereby setting them apart from any totalizing attempts at classification.[3]

Giorgio Agamben has a radically different way of conceiving forms of life. In *Means Without End*, he notes that the Greeks had two words for "life": *zoē*, which expressed the simple fact of being alive that all living beings share, and *bios*, which signified the form or way of life specific to an individual or group. The modern world has not retained this distinction, using a single expression, "life," starkly designating the shared presupposition that one can always isolate within each of the countless forms of life. "By the term *form-of-life*, on the other hand, I mean a life that can never be separated from its form, a life in which it is never possible to isolate something such as naked life."[4] For Agamben, the constitution of this bare life is the precise operation on which the political sphere is based.

> Each behavior and each form of human living is never prescribed by a specific biological vocation, nor is it assigned by whatever necessity; instead, no matter how customary, repeated, and socially compulsory, it always retains the character of a possibility; that is, it always puts at stake living itself. [...] But this immediately constitutes the form-of-life as political life. [5]

So Agamben's conception of the "form-of-life," made possible by the multitude of forms of life and influenced by Foucault's ideas on biopolitics, is founded on the impossibility of separating the individual from politics, science, arts or literature. "I call *thought* the nexus that constitutes the forms of life in an inseparable context as form-of-life."[6] Thought "must become the guiding concept and the unitary center of the coming politics."[7]

The idea reappears in the work of Yves Citton, for whom forms of life are expressions emphasizing the fact that human life is never a raw given (whether material, physical, or biological), but is constituted by a certain kind of shaping (always simultaneously social, historical and esthetic) of material conditions that could be arranged otherwise.[8] In *Lire, interpréter, actualiser* ("Reading, Interpreting, Actualizing"), Citton draws the anatomical portrait of *Homo hermeneuticus* by trying to grasp what makes it at once the product and the co-producer of our forms of social life.[9] By privileging a contemporary reading of historical texts, allowing them to be interpreted in the light of our modern world instead of reconstructing the context in which they were written, the author seeks to perform what he calls a "disruptive overcoding" (*surcodage disruptif*) in order to highlight, in the act of reading, both the analysis of contemporary forms of life and ontological reflections.

To sum up, forms of life are eminently political and are indexed on the ambiguities of being. Because of this, they interest semioticians as well as writers of fiction, architects as well as philosophers. They participate, as Agamben says, in an *ontology of style*. "What we call form-of-life corresponds to this ontology of style; it names the mode in which a singularity bears witness to itself in being and being expresses itself in the singular body."[10] Some forms of life devote themselves to a search through time; others delve into territorialized spaces. But all refer to a collective becoming, a shared immanent force that drives the object to become what it is. To illustrate our ideas, we will use two examples of mathematical objects: the Fourier transform and the monad in mathematical category theory.

Avatars of the Fourier Transform

In 1811, Joseph Fourier began his work on the propagation of heat. In 1822, five years after his election to the *Académie des sciences*, he published his 670-page magnum opus, *The Analytical Theory of Heat*. In the first two chapters, he presents the physical aspects of the question and derives the partial differential equation that regulates the change in temperature within a homogeneous mass, which is now called the *heat equation.* The solution of this equation depends upon the initial and the boundary values. The third chapter presents the use of trigonometric series. Fourier considers the canonical problem of a solid homogeneous mass contained between two planes that are vertical, parallel, and infinite, and endeavors to resolve the question of how to know what the temperature of this mass will be once the thermal equilibrium is established. This is a classic Dirichlet problem that consists in finding a harmonic function ($\Delta u = 0$) on the basis of its boundary values. By seeking a solution broken down into trigonometric series, Fourier arrives at it. He then has to demonstrate the convergence of these series and rigorously establish their decomposition, which Dirichlet will accomplish some years later. But in Fourier's text, he already sets out all the formulas that we use to calculate what we now call Fourier coefficients. He was the first to understand that this new form of trigonometrically-based analysis could be extended to many other problems besides the heat equation. He writes:

> If we apply these principles to the problem of the motion of vibrating strings, we can solve difficulties which first appeared in the researches of Daniel Bernoulli. The solution given by this geometrician assumes that any function whatever may always be developed in a series of sines or cosines of multiple arcs. Now the most complete of all the proofs of this proposition is that which consists in actually resolving a given function into such a series with determined coefficients.[11]

Since then, the Fourier transform has found a wide variety of applications. Its form participates in that ontology of style mentioned by Agamben. We see it already in Fourier's work. He takes a geometrician's problem and makes it an analytical one. The applications to mathematical physics highlight this possibility of resolving differential equations by shifting from one space to its dual. The Fourier transform converts an equation in time and space into an equation in the frequency domain that is usually easily resolved. The inverse Fourier transform allows for a return to the desired solution in time and space. Thus Euclidean space and the frequency domain, via the Fourier transform, become dual reciprocal spaces.

The extension of the Fourier transform to topological groups has shifted the problem toward other styles: algebra and topology. In mathematics, a group is a purely algebraic notion, which mathematicians have made topological as well by requiring that the group composition law and its inverse be two continuous applications. Applying a topology to a space is, for the mathematician, simply choosing what the continuous applications are on that space. Continuity is the fundamental essence of topological space, which is defined by sets of equivalent axioms, concerning open sets, closed sets or neighborhoods. In order to extend the Fourier transform to locally compact abelian (or commutative) groups – in other words commutative topological groups whose underlying space is locally compact – mathematicians have devised the notion of a group's characters and considered the dual group, formed by the set of these characters. With this notion of character, the calculations are transferred from an arbitrary group, whose objects may be fairly unusual, to the multiplicative group of non-zero complex numbers, whose calculation is well known. To be more precise, let us say that a character is a group morphism of the group G toward the complex group, i.e. it is an application that respects the group's structure. Thus the character $\chi(x)$ of an object x in G is a complex number that can be easily inserted into an integral, no matter how complex the object x is. If G is a locally compact abelian group with a Haar measure μ and with χ as a character of G, then the Fourier transform of an integrable function f of the Lebesgue space $L^1(G)$ is the integral relative to the Haar measure of the product of $f(x)$ and the complex conjugate of the character $\chi(x)$

$$\mathcal{F}f(\chi) = \int_G f(x)\overline{\chi(x)}\, d\mu(x)$$

This bounded continuous function is an element of the Lebesgue space $L^\infty(\hat{G})$ where $\hat{G}$ is the set of characters of G, known as the dual group of G. When R^n is the real space of dimension n, the characters are the exponential functions $\chi_a(x) = e^{iax}$. When G is the torus $R/2\pi Z$, the characters are the functions $x \rightarrow e^{inx}$ for an integer n. We thus reencounter the definition of Fourier series. The generalized Fourier transform allows for an inverse transform that is the integral relative to the Haar measure V on the dual group. It has the same properties as the regular transform. The convolution product of two functions f and g, which is the mathematical representation of the notion of a linear filter, is represented analytically by the function:

$$(f * g)(x) = \int f(x-t)g(t)dt$$

The main property of the Fourier transform is that it transforms a convolution product into a simple product of the Fourier transforms

$$\mathcal{F}(f * g) = \mathcal{F}f \cdot \mathcal{F}g$$

It satisfies Parseval's theorem on the conservation of the scalar product[12]

$$\int f(x)\overline{g(x)}dx = \int \mathcal{F}f(x)\overline{\mathcal{F}g(x)}dx$$

whose corollary is the Plancherel formula[13] obtained when the functions f and g are equal:

$$\int |f(x)|^2 \, dx = \int |\mathcal{F}f(x)|^2 \, dx$$

What this generalization shows is that it transforms a problem of mathematical analysis into an algebraic problem. The new form of life of the Fourier transform is only possible because the set of characters of a locally compact abelian group is itself an abelian (i.e. a commutative) group: the dual group. This result, discovered by Lev Semenovich Pontryagin, is known as the Pontryagin duality theorem.[14] This duality allows the Fourier transform to carry the algebra under convolution $L^1(G)$ to a multiplicative algebra $L^\infty(G)$, and reciprocally by inverse transform

$$L^1(G) \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftarrows}} L^\infty(\widehat{G})$$

The generalizations of this duality did not allow the Fourier transform to change its way of life. Throughout the 20th century, it remained an object whose essence was the duality of algebraic structures. In 1938, Tadao Tannaka generalized the duality theorem to noncommutative compact groups.[15] Mark Grigorievich Krein[16] built upon Tannaka's work; William Forrest Stinespring[17] then extended duality to the case of unimodular locally compact groups (1959).[18] Pierre Eymard (1964)[19] extended the classic results of the harmonic analysis of abelian groups to the case of locally compact groups even when such groups are not unimodular. The work of Nobuhiko Tatsuuma (1967) followed that of Eymard, establishing a weak duality on three types of topological groups.[20] From around 1965 on, at the instigation of mathematical physics, the research turned toward the establishment of a duality for the Hopf and Von Neumann algebras. One of the pioneers of operator algebra, Masamichi Takesaki, sought a general theorem of duality, which was ultimately established by Leonid Vainerman and Georgiy Kac (1973)[21] and independently by Michel Enock and Jean-Marie Schwartz (1973)[22] for von Neumann and Kac algebras.

In the late 20th and early 21st century, the Fourier transform opened itself to yet another form of life through developments in category theory, which Samuel Eilenberg and Saunders Mac Lane had created in the 1950s. We have just seen that the Fourier transform over abelian groups enabled the emergence of the Pontryagin duality, which is generalized in the non-commutative case as a Tannaka-Krein duality. We will now see that what lies behind this Tannaka duality is precisely the category of representations of a group. The representation of a group is a way of describing a set of abstract and unwieldy algebraic objects like the elements of a group by a set of matrices and to transform the operations on these algebraic objects or group elements into simple operations like adding or multiplying matrices. When the representation is one-dimensional, matrices are reduced to numbers. For example, instead of working with sets of algebraic operators like rotations, it is more convenient to work with matrices that represent them. In this case, a simple product of matrices represents the composition of two rotations, the rotation at a given angle following another rotation. When the determinant of matrices (or their volumes) is equal to unity, we say that the representation is unitary. Representations are either reducible or irreducible. An irreducible representation is one that has no subrepresentation (other than itself and $\{0\}$). It therefore presents a character of uniqueness that reducible representations do not share.

When the group G is commutative, its dual is the group of characters, in other words the group of unitary one-dimensional representations. Calculations become possible because the dual of G is a group. But when the group is no longer abelian, the group of characters no longer exists. Its equivalent is the set of equivalence classes of irreducible unitary representations. The analogue of the product of characters is the tensor product of representations. Furthermore, since the irreducible representations of a given group generally do not form a group, we are limited to considering the monoidal (or tensor) category of all the irreducible representations of finite dimensions equipped with the tensor product of representations. Tannaka then provides a way of constructing a compact group on the basis of the category of its representations, while Krein gives the necessary and sufficient conditions for a category to be a dual object of a compact group G. The Tannaka-Krein duality theory is therefore the study of the interrelations between a group and the category of its representations. This allows us to grasp the transition from an algebraic to a categorical way of life.

To be somewhat more exact, we should say that the Tannaka duality was a product of the development of mathematical physics, low-dimensional topology (knot and link theory) and quantum groups. As André Joyal and Ross Street[23] emphasize, Shahn Majid[24] demonstrated that one could use the Tannaka duality to construct the quasi-Hopf algebras introduced by Drinfel'd in relation to the solution of the Knizhnik-Zamolodchikov equation. Many questions in mathematical physics are linked to the Tannaka duality, such as the theory of the composition of angular momenta, Racah-Wigner algebras, knot invariants and Yang-Baxter operators. All of these questions, which we cannot fully explore here, are centered on quantum groups and are deeply connected to the theory of monoidal categories.

In a 2011 article, Brian Day[25] gave a categorical construction of the Fourier transform. A category is a broader mathematical notion than the set. Unlike the set, the category has its own collection of operators and acts, involving objects, arrows and morphisms linking these objects together, which satisfy elementary properties such as the transitivity of morphisms. Between two categories, functors are defined that carry both the objects of one category to the objects of the other and the arrows of one to the arrows of the other. They are called functors, not functions, as they apply to two kinds of entities at once: objects and morphisms. When a category possesses a tensor product that satisfies certain axioms of compatibility, we say that the category is monoidal; if it only satisfies some of them, we say that the category is promonoidal. In order to define a Fourier transform in the categorical sense, Day starts by defining a promonoidal category for which he establishes two convolution products of two functors: an upper convolution and a lower one. Day then gives the definition of a multiplicative kernel K that is used to present the Fourier transform of f as the left Kan extension

$$\overline{K}(f)(x) = \int^{a} K(a,x) \otimes f(a)$$

and its dual transform as the right Kan extension

$$\overline{K}^{\vee}(f)(x) = \int_{a} [K(a,x), f(a)]$$

He then demonstrates that the Fourier transform K preserves the upper convolution

$$\overline{K}(f \mathbin{\bar{*}} g) = \overline{K}(f) \mathbin{\bar{*}} \overline{K}(g)$$

and that its dual preserves the lower convolution

$$\overline{K}^{\vee}(h \mathbin{\underline{*}} k) = \overline{K}^{\vee}(h) \mathbin{\underline{*}} \overline{K}^{\vee}(k)$$

Next, for the product defined as the coend of the tensor product of f and g

$$\langle f, g \rangle = \int^{a} f(a)^{*} \otimes g(a)$$

Day establishes the Parseval relation

$$\langle f, g\rangle \cong \langle \overline{K}(f), \overline{K}(g)\rangle$$

To sum up, we have just seen that the Fourier transform has, in different realms, experienced ways of life inspired by key moments in the development of mathematics. First used in mathematical analysis where it was linked to harmonic theory, in its first generalization it became an essentially algebraic object. The Pontryagin duality was then the Fourier transform's driving force. Its generalization to the case of non-commutative groups brought about the Tannaka-Krein duality, which allowed for the emergence of strong categorical connections. The definition via coends produced by Kan extensions breathed new life into the Fourier transform. That new life reminds us that one of the subsections in Mac Lane's book was titled "All Concepts Are Kan Extensions."[26]

The Monad and the Fold

Mathematical category theory is currently the focus of many developments both in low-dimensional topology and in quantum field theory. As we have just seen, a category – in the mathematical sense – is a collection of objects and arrows that satisfy elementary axiomatic properties. In a category, all objects have become featureless. They belong to one and the same level and are ontologically equal, without distinction or quality. Category theory is therefore a flat ontology where all things are equal.[27] But in the world we live in, things are placed in a hierarchy, structured, made of differences and intensities of all kinds. Indeed, we must take into consideration the action of the ontological fields by which things are structured, mutate and become differentiated objects, objectively differentiable (through the interplay of a category's arrows) both by the mathematician and the philosopher. The disembodied thing becomes a full, entire object, with all its references, relations and distances, which is *something* qua *what it is*. It is both what constitutes it as a particular reality caught in the forms of its existence, and what produces it like the categorical objects from which it is derived, which are caught in the web of the world. Thing and object are distinct, complex entities that are not limited to the inclusion of things within things and subsequently to their transformation into objects. But there are no hierarchies of objects and things in categorical ontology, just as there is no zoology of first- and second-level entities. Thing and object are sufficient for the ontological interpretation of categories. The definition of universality, the Yoneda lemma and Diaconescu's theorem are the most immediate examples of this toposic hermeneutics.[28]

In category theory, the notion of the monad was introduced in the 1960s. In 1958, Roger Godement,[29] for reasons associated with homological algebra, constructed the first monad (a *comonad*, to be more precise) as the embedding of a sheaf in a flasque sheaf.[30] Three years later, Peter Huber[31] demonstrated that each pair of adjoint functors results in a monad. Heinrich Kleisli,[32] as well as Samuel Eilenberg and John Coleman Moore,[33] independently demonstrated the reciprocal theorem (every monad is the result of an adjunction), Eilenberg and Moore referring to monads as "triples." The first to use the term "monad" was Saunders Mac Lane in 1971. This monad relies upon two mathematical concepts in category theory, *adjunction* and *functoriality*, both of which recall Leibniz's philosophy, as well as Gilles Deleuze's rereading of it. Functoriality, or the functorial character, is the existence of a functor making it possible to envelop and represent a multitude in a unit and to define the monad. Adjunction is this twofold character represented by the two leaves of the Deleuzian fold, symbolized mathematically by the two natural transformations that participate in the definition of the monad and that justify the principle of individuation. As Gilles Châtelet emphasized repeatedly, the monad is a living mathematical object.[34]

As mentioned earlier, the existence of functors between two categories is used to carry properties from one category to another, like the property of isomorphism, but also to transform categories, like the forgetful functor that abandons the structure of the initial category, or the abelianization functor that makes the laws of the final category commutative. The non-existence of a given functor is another important result: the lack, for example, of a functor from the category of symplectic varieties, which form the mathematical framework of classical mechanics, to the category of Hilbert spaces, which constitute the framework of quantum mechanics, poses the problem of quantification or the transition from classical to quantum mechanics.

In mathematical category theory, the monad is a functor equipped with two natural transformations, the unit and the multiplication, whose axioms of identity and associativity mimic the behavior of an algebraic germ. This triad (on which the former name of "triple" was based) defines the monad of the category theorists. For Leibniz, the monad is a simple substance that folds the world into a unit endowed with perception and appetite. It is an astoundingly categorical conception: considering the monad as an object (a unit) bearing morphisms (of perception and appetite). Going even further, the universe itself is seen as a category whose objects are the monads and the morphisms are the resulting phenomena.

In his classes on Leibniz and the fold, Deleuze[35] explains that the world is folded, that the fold has a particular inflection or curvature and that this curvature, as in the case of the foci of an ellipse, determines one or several points of view. From this point of view, we can measure the curvature and become aware of the fold's inflection. The curvature of things, Deleuze says, demands a point of view. And the point of view is consequently the condition for the emergence or the manifestation of truth in things. But why, asks Deleuze, are things folded? Because what is folded is necessarily enveloped in something that occupies the point of view. And what envelops the points of view is precisely the monad, which for Deleuze is the individual: an individual, however, who encompasses the infinite on their own, like the assemblages of monads or the bodies of which Leibniz speaks. Saying the world is folded means that it can be individualized. We must therefore comprehend the fold as an abstract notion, a functor, and not as the geometric pleats of physical space. It follows that via a monadic interpretation, the principle of individuation intertwines with the functoriality of the fold.

In category theory, the fold is created by the natural transformations of the monad, and individuation is the condition for a category's algebraicity. Saying a category is individualized means that it can be likened to an algebra or to the category of its representations, that it can be the subject of a calculation. Beck's theorem[36] is precisely what stipulates the conditions of monadicity. In order for there to be monadicity of a given category B toward a category C, one must be able to describe category B on the basis of a monad of category C. Given a functor of B on C, category B is called *monadic* if B can be considered the category of algebras of a monad of C. The existence of this monad enables algebraic calculation on B as if everything took place in category C or that C is calculable. The monad then becomes the condition of algebraicity (and therefore of calculability) of a category. As Lawvere has partly demonstrated, algebraic theories are the monads of mathematicians. Even in the most recent cases of Hopf monads that are used to comprehend the differences between braided and non-braided universes, mathematics returns to Leibniz's idea that the monad contains a representation of the world and that in the universe defined as the ontological closure of things, the individuation of a category ultimately corresponds to its monadicity. The monad becomes the indicator of the algebraic representations of a category and its possibilities of calculation.

NOTES

[1] Ludwig Wittgenstein, *Philosophical Investigations*, trans. G.E.M. Anscombe, P.M.S. Hacker, and Joachim Schulte (Hoboken: John Wiley & Sons, Ltd., 2009), 11e.

[2] Wittgenstein, *Philosophical Investigations*, 15e.

[3] Jacques Fontanille, *Formes de vie* (Liège BE: Presses universitaires de Liège, 2015), 5.

[4] Giorgio Agamben, *Means Without End: Notes On Politics*, trans. Vincenzo Binetti and Cesare Casarino (Minneapolis: Univ. of Minnesota Press, 2008), 2-3.

[5] Agamben, *Means Without End*, 3.

[6] Agamben, *Means Without End*, 8.

[7] Agamben, *Means Without End*, 11.

[8] Yves Citton, *Lire, interpréter, actualiser* (Paris: Éditions Amsterdam, 2007), 341.

[9] Citton, *Lire, interpréter, actualiser*, 26.

[10] Agamben, *The Use of Bodies*, trans. Adam Kotsko (Stanford: Stanford University Press, 2015), 233.

[11] Joseph Fourier, *Théorie analytique de la chaleur* [1822] (Paris: Jacques Gabay, 1988), 249 [*Analytical Theory of Heat* [1878], trans. Alexander Freeman (Cambridge UK: Cambridge University Press, 2009), 198].

[12] Marc-Antoine Parseval des Chênes, "Mémoire sur les séries et sur l'intégration complète d'une équation aux différences partielles linéaire

du second ordre à coefficients constants" [1799], in *Mémoires présentés à l'Institut des sciences, lettres et arts, par divers savans, tome premier* (Paris: Baudouin, 1805), 638-648.

[13] Michel Plancherel, "Contribution à l'étude de la représentation d'une fonction arbitraire par les intégrales définies," in *Rendiconti del circolo matematico di Palermo* 30 (1910): 289-335.

[14] L.S. Pontryagin, "The Theory of Topological Commutative Groups," in *Annals of Mathematics* 35.2 (1934): 361-388.

[15] Tadao Tannaka, "Über den Dualitätssatz der nichtkommutativen topologischen Gruppen," in *Tôhoku Mathematical Journal* 45 (1939): 1-12.

[16] Mark Grigorievich Krein, "A Principle of Duality for Bicompact Groups and Quadratic Block Algebras," in *Doklady Akad. Nauk. SSSR* 69.6 (1949): 725-728.

[17] William Forrest Stinespring, "Integration Theorems for Gages and Duality for Unimodular Groups," in *Trans. Amer. Math. Soc.* 90 (1959): 15-56.

[18] A locally compact group is unimodular if its right Haar measure coincides with its left Haar measure. Every abelian (i.e. commutative) group is unimodular, as is every compact group and every discrete group.

[19] Pierre Eymard, "L'algèbre de Fourier d'un groupe localement compact," in *Bulletin de la Société Mathématique de France* 92 (1964): 181-236.

[20] Nobuhiko Tatsuuma, "A Duality Theorem for Locally Compact Groups," in *J. Math Kyoto Univ.* 6 (1967): 187-293.

[21] Leonid Vainerman, Georgiy Kac, "Nonunimodular Ring Groups and Hopf-von Neumann Algebras," in *Soviet Math. Dokl.* 14 (1974): 1144-1148.

[22] Michel Enock, Jean-Marie Schwartz, "Une dualité dans les algèbres de von Neumann," in *Bull. Soc. Math. France, Supp. Mémoire* 44 (1975): 1-144.

[23] André Joyal, Ross Street, "An Introduction to Tannaka Duality and Quantum Groups," in *Lecture Notes in Mathematics* 1488 (1991): 411-492.

[24] Shahn Majid, "Tannaka-Krein Theorems for Quasi-Hopf Algebras and Other Results," in *Contemp. Math.* 134 (1992): 219-232.

[25] Brian Day, "Monoidal Functor Categories and Graphic Fourier Transforms," in *Theory and Applications of Categories* 25.5 (2011): 118-141.

[26] Saunders Mac Lane, *Categories for the Working Mathematician* (New York: Springer, 1971), 248.

[27] On these questions, see Tristan Garcia's book, *Forme et objet: Un traité des choses* (Paris: PUF, 2011) [*Form and Object: A Treatise on Things*, trans. Mark Allan Ohm and Jon Cogburn (Edinburgh: Edinburgh University Press, 2014)].

[28] On these issues and their relationships to Alain Badiou's metaphysics, see Franck Jedrzejewski, *Ontologie des catégories* (Paris: L'Harmattan,

2011). A *topos* is a non-pathological category where all the particular properties of categories are in general satisfied.

[29] Roger Godement, *Théorie des faisceaux* (Paris: Hermann, 1958).

[30] Developed in algebraic topology by Jean Leray, the notion of the *sheaf* is a generalization of the notion of the set of sections of a vector bundle. The sheaf is *flasque* if for any open subset, the application is surjective.

[31] Peter J. Huber, "Homotopy Theory in General Categories," in *Mathematische Annalen* 144.5 (1961): 361-385.

[32] Heinrich Kleisli, "Every Standard Construction Is Induced by a Pair of Adjoint Functors," in *Proc. Amer. Math. Soc.* 16 (1965): 544-546.

[33] Samuel Eilenberg, John Coleman Moore, "Adjoint Functors and Triples," in *Illinois J. Math.* 9 (1965): 381-398.

[34] Compare with Leibniz's monad, *Monadology* § 63: "The body belonging to a monad, which is its entelechy or soul, constitutes together with the entelechy what may be called a living thing, and with the soul what is called an animal. Now this body of a living thing or animal is always organic; for since every monad is in its way a mirror of the universe, and the universe is regulated in a perfect order, it must be the case that there is also an order in whatever represents it, that is, in the perceptions of the soul, and consequently in the body, in accordance with which the universe is represented in it." *Leibniz’s Monadology: A New Translation and Guide*, ed. and trans. Lloyd Strickland (Edinburgh: Edinburgh University Press, 2014), 27.

[35] Gilles Deleuze, *Le Pli* (Paris: Minuit, 1986) [*The Fold*, trans. Tom Conley (Minneapolis: University of Minnesota Press, 1993)]. See also the session of his seminar on 16 December 1986 in Saint-Denis: "Le pli, récapitulation" ["Leibniz and the Baroque, Lecture 04, 16 December 1986," trans. Charles Stivale, deleuze.cla.purdue.edu, n.p., November 2019, Web, 15 July 2020].

[36] Jon Beck, *Triples, Algebra and Cohomology,* PhD thesis, Columbia University, New York, 1967.