Probabilistic Situations for Goodmanian N-universes

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ABSTRACT. I will now describe several applications of the theory of n-universes across different probabilistic scenarios. First, I will explain how n-universes can extend the probability spaces used in traditional probability theory. These extended probability spaces enable more refined modelling of complex probabilistic situations and align more intuitively with our perceptions of the physical universe. Next, I will demonstrate the use of n-universes as a methodological tool through two thought experiments described by John Leslie. Finally, I will model Goodman's paradox within the framework of n-universes, showing how they closely resemble Goodmanian worlds.

The concept of *n-universes* was introduced in Franceschi (2001, 2002) to study probabilistic situations related to several paradoxes currently under intensive investigation in the field of analytical philosophy, such as Goodman's paradox and the Doomsday Argument.[1] The purpose of this article is twofold: first, to describe how modeling within the n-universes framework extends the properties of classical probability spaces used in probability theory by providing finer modeling of certain probabilistic situations and better support for intuition; second, to demonstrate how using n-universes significantly simplifies the study of complex probabilistic situations, such as those that arise in the analysis of paradoxes.

For instance, when modeling the situation of drawing a ball from an urn, a restricted temporal space is considered, limited to the few seconds immediately before and after the draw. Events that occurred the day before or an hour before, as well as those that will happen the day after, can be completely disregarded. A very restricted time interval, reducible to one or two discrete temporal positions, suffices to characterize the corresponding situation. Similarly, our universe can be restricted to the space occupied by the urn, ignoring the space in the neighboring room and its contents. Additionally, variables such as the number of copper or molybdenum atoms in the urn, the number of photons interacting with the urn at the time of the draw, or the presence or absence of a 75 dB sound source can be omitted. In this context, it is unnecessary to account for such variables. It suffices to mention only the variables and constants relevant to the corresponding probabilistic situation, as enumerating all the constants and variables describing our entire universe would be exceedingly complicated and unnecessary. Therefore, one can legitimately limit the description to a simplified universe, mentioning only the constants and variables that play a genuine role in the corresponding probabilistic situation.

Consider the drawing of a ball from an urn containing several balls of different colors. Probability theory, to calculate the likelihood of different events related to drawing one or several balls, relies on modeling based on probability spaces. The determination of these likelihoods does not depend on modeling the physical forces governing the conditions of the draw, such as the mass and dimensions of the balls, the material they are made of, their initial spatiotemporal positions, or the characteristics of the forces applied to perform a random draw. Instead, the modeling of random phenomena using probability spaces retains only simplified elements of the physical situation, specifically the number and color of the balls and their spatiotemporal positions. This methodological approach can be generalized to other probabilistic situations involving random processes, such as drawing dice or

[1] Leslie (1996).

cards. This methodology is not an axiom of probability theory but an important tenet that could benefit from further formalization. It is also useful to explain in more detail how elements of our physical world are converted into probability spaces. In what follows, I will demonstrate how probability spaces can be extended using the theory of n-universes to better capture the structure of the part of our universe being modeled.

1. Introduction to n-universes

It is important to preliminarily describe the basic principles underlying n-universes. N-universes represent a simplified model of the physical world, studied within a probabilistic framework. By applying Occam's razor, we aim to model a physical situation using the simplest possible universe model, while still preserving the inherent structure of the corresponding physical situation. At this stage, it is necessary to highlight several important features of n-universes.

1.1. Constant-criteria and variable-criteria

The criteria of a given n-universe include both constants and variables. Although n-universes allow for modeling situations that do not correspond to our physical world, our focus here will be exclusively on n-universes that correspond to common probabilistic situations, in alignment with the fundamental characteristics of our physical universe. These corresponding n-universes must include at least one temporal constant or variable, as well as one location constant or variable. Among n-universes, we distinguish: a $\Omega T_0 L_0$ (an n-universe including a temporal constant and a location constant), a $\Omega T_0 L$ (a temporal constant and a location variable), a $\Omega T L_0$ (a temporal variable and a location constant), a ΩTL (a temporal variable and a location variable). Other n-universes may also include constants or variables related to color, direction, etc.

1.2. N-universes with a unique object or with multiple objects

Every n-universe includes one or several objects. We distinguish, for example: a $\Omega\alpha_0 T L_0$ (an n-universe including a unique object, a temporal variable, and a location constant), a $\Omega\alpha T L_0$ (multiple objects, a temporal variable, and a location constant).

1.3. multiplication with regard to a variable-criterion

It is worth highlighting the property of *multiplication* of a given object with regard to a variable-criterion in a given n-universe. Hereafter, we shall denote a variable-criterion χ with multiplication by χ^*. Any variable-criterion of a given n-universe can be multiplicated in this manner. The multiplication of an object with respect to a criterion χ is defined as the property of this object to exemplify several taxa of criterion χ. For example, consider the time criterion. The multiplication of an object with respect to time means that the object exemplifies several temporal positions. In our physical world, an object α_0 can exist at several (successive) temporal positions, and thus, it is multiplicated with regard to the time criterion. Common objects exhibit temporal persistence, which constitutes a special case of temporal multiplication. Therefore, in our universe, where time is one of the variable-criteria, it is common to observe that a given object α_0 that exists at T_1 also exists at T_2, ..., T_n. Such an object has a lifespan that spans the period from T_1 to T_n. The corresponding n-universe can thus be represented by the structure $\Omega\alpha_0 T^* L_0$ (ΩT^* for simplicity).

1.4. Relation one/many of multiple objects with a given criterion

At this stage, it is essential to make an important distinction. We need to differentiate between two types of situations. An object can exemplify, as previously mentioned, several taxa of a given variable-criterion. This corresponds to the case of multiplication, as described in relation to a given variable-criterion. However, another situation must be considered, which pertains only to n-universes with multiple objects. Specifically, multiple objects can instantiate the same taxon of a given criterion.

First, consider the temporal criterion. Let's imagine an n-universe with multiple objects, including a temporal variable and a constant location L_0. This can correspond to two different types of n-universes. In the first type, there is a single object at each temporal position. At any given time, only one object can exist in L_0 within the corresponding n-universe. In this case, every object in the n-universe has a *one-to-one* relationship with the temporal taxa. We denote such an n-universe as $\Omega\alpha T\alpha^* L_0$ (simplified as $\Omega\alpha T$).

Now, consider an n-universe with multiple objects, including a temporal variable and a constant location, where several objects α_1, α_2, α_3 can exist simultaneously. Here, multiple objects can occupy the same temporal position in L_0. This situation fundamentally differs from $\Omega\alpha T\alpha^* L_0$, as several objects can co-exist at a given time. In this case, the objects have a *many-to-one* relationship with the temporal taxa. We denote this n-universe as $\Omega\alpha^* T\alpha^* L_0$ such n-universe (simplified as $\Omega\alpha^* T$) .

Next, consider the location criterion. Imagine an n-universe with multiple objects, a temporal variable, and a location variable, where the objects have a *many-to-one* relationship with the temporal criterion. We need to distinguish between two types of n-universes here. In the first type, a single object can be found at a given location taxon at any given time. This scenario models, for example, the positions of pieces in a chess game. We denote such an n-universe as $\Omega\alpha^* T\alpha L$ such n-universe (simplified as $\Omega\alpha^* TL$). In this case, the objects have a *one-to-one* relationship with the location criterion.

In the second type of n-universe, several objects can occupy the same location taxon at the same time. For instance, objects α_1, α_2, α_3 can all be in L_1 at T_1. This situation is analogous to an urn (assimilated to a given location taxon) containing multiple balls at a given time. We denote this n-universe as $\Omega\alpha^* T\alpha^* L$, where the objects have a *many-to-one* relationship with the location taxa.

Finally, this differentiation also applies to the variable-criterion of color. We can distinguish between: (i) $\Omega\alpha^* T_0\alpha^* L_0\alpha C$ (simplified as $\Omega\alpha C$) where several objects co-existing at the same time in a given spatial position must all have different colors, indicating a *one-to-one* relationship with the color criterion; and (ii) $\Omega\alpha^* T_0\alpha^* L_0\alpha^* C$ (simplified as $\Omega\alpha^* C$) where several objects which can co-exist at the same time at a given space position can present the same color, because the objects are in relation *many-to-one* with the color criterion there, where several objects co-existing at the same time in a given spatial position can have the same color, indicating a *many-to-one* relationship with the color criterion.

1.5. Notation

At this stage, it is important to highlight a crucial point regarding the notation used in this discussion. We have employed both extended and simplified notations. The extended notation explicitly specifies all criteria of the considered n-universe, including both variable-criteria and constant-criteria. In contrast, the simplified notation only explicitly specifies the variable-criteria of the n-universe. The constant-criteria of time and location for the n-universe can be inferred from its variable-criteria. This inference is possible because the studied n-universes systematically include one or several objects, as well as either a variable-criterion or a constant-criterion of time and location.

Let us illustrate this with an example. Consider an n-universe that includes multiple objects, a constant-criterion of time, and a constant-criterion of location. In this case, the multiple objects necessarily exist at T_0. As a result, in this n-universe, the multiple objects are inherently related to the constant-criterion of time. Similarly, there are necessarily multiple objects at L_0, establishing a *many-to-one* relationship with the constant-criterion of location. Thus, we describe this situation as $\Omega\alpha^* T_0\alpha^* L_0$. However, given the aforementioned reasons, this n-universe can be denoted in a simplified way as $\Omega\alpha$.

he preceding observations suggest a general simplification of the notation used. Since an n-universe includes multiple objects and a constant-criterion of time, the multiple objects are necessarily related to this constant-criterion. The n-universe can thus be denoted as $\Omega\alpha^* T_0$. However, this notation can be simplified to $\Omega\alpha$. Similarly, if an n-universe includes multiple objects and a constant-criterion of location, the multiple objects are necessarily in *many-to-one* relationship with this constant-criterion of location. This n-universe can be denoted as $\Omega\alpha^* L_0$, which can be simplified to $\Omega\alpha$. Consequently, notations such as $\Omega\alpha^* L_0\alpha^* T_0$ can be simplified to $\Omega\alpha$, $\Omega\alpha^* L_0\alpha T$ to $\Omega\alpha T$, $\Omega\alpha^* L_0\alpha^* T$ to $\Omega\alpha^* T$, $\Omega\alpha^* L_0\alpha^* T^*$ to $\Omega\alpha^* T^*$, and so on.

2. Modeling random events with n-universes

In traditional probability theory, situations often involve dice, coins, card games, or urns containing balls. It is valuable to describe how these objects can be modeled within the framework of n-universes. Additionally, it is necessary to extend the notion of a 'toss' to probability spaces within n-universes. The following models can be used:[2]

2.1. Throwing a die

How can we model a toss where the result of throwing a die is "5"? Here, we model the die as a unique object located at a space location L_0. This object can present one discrete direction from the set {1,2,3,4,5,6} at a specific time T_0. The corresponding n-universe includes a unique object, a direction variable, and a temporal constant. The unique object can present only one direction at time T_0 and is not subject to multiplication concerning the direction criterion. The n-universe is denoted as ΩO (with extended notation $\Omega\alpha_0T_0L_0O$). Traditionally, we have the sample space $\Omega = \{1,2,6\}$ and the event {5}. In this case, drawing '5' means the unique object has direction 5 among {1,2,6} at time T_0 and location L_0. Thus, the sample space is denoted by $\Omega\alpha_0T_0L_0O\{1,2,...,6\}$ and the event by $\alpha_0T_0L_0O\{5\}$.[3]

How can we model two successive throws of the same die, where the results are "5" and then "1"? Traditionally, we have the sample space $\Omega = \{1,2,...,6\}^2$ and the event {5,1}. Here, this corresponds to the die α_0 having directions 5 and 1 at times T_1 and T_2. In the corresponding n-universe, we have now a time variable, including two positions: T_1 and T_2. Moreover, the time variable is subject to multiplication because the unique object exists at different temporal positions. Therefore, the considered n-universe is ΩT^*O (with extended notation $\Omega\alpha_0T^*L_0O$). The sample space is denoted by $\Omega\alpha_0T^*\{1,2\}L_0O\{1,2,...,6\}$ and the event by $\{\alpha_0T^*\{1\}L_0O\{5\}, \alpha_0T^*\{2\}L_0O\{1\}\}$.

2.2. Throwing a coin

How can we model the outcome of a coin toss, for example, landing on Tails? We model the coin as a unique object presenting two distinct outcomes: {P, F}. The corresponding n-universe is identical to that used for modeling a dice roll, with the sole difference being that the direction criterion includes only two outcomes: {P, F}. Thus, the corresponding n-universe is ΩO (with extended notation $\Omega\alpha_0T_0L_0O$). Classically, we have: $\Omega = \{P,F\}$ and {P}. Here, the Tails outcome is associated with the unique object taking direction {P} from {P, F} at time T_0 and location L_0. The sample space is then denoted by $\Omega\alpha_0T_0L_0O\{P,F\}$ and the event by $\alpha_0T_0L_0O\{P\}$.

How can we model two successive tosses of the same coin, such that the result is "Heads" followed by "Tails"? Classically, we have the sample space $\Omega = \{P,F\}^2$ and the event {F,P}. Similar to modeling successive throws of a die, the corresponding n-universe here is ΩT^*O (with extended notation $\Omega\alpha_0T^*L_0O$). The sample space is denoted by $\Omega\alpha_0T^*\{1,2\}L_0O\{P,F\}$, and the event by $\{\alpha_0T^*\{1\}L_0O\{F\}, \alpha_0T^*\{2\}L_0O\{P\}\}$.

2.3. Throwing multiple discernible dice

How can we model the throwing of two discernible dice simultaneously, for example, one showing a "3" and the other a "5"? These discernible dice are modeled as multiple objects, each occupying a specific spatial position and capable of displaying one of six possible outcomes {1, 2, 3, 4, 5, 6} at time T_0. The multiple objects coexist at the same temporal position, indicating that they share a temporal constant. Additionally, each object can only present one outcome at time T_0, thus avoiding any multiplication with regard to the direction criterion. The fact that both dice could display the same outcome demonstrates that the objects are in relation *many-to-one* with the direction criterion. There exists also a location variable, each of the dices α_1 and α_2 being at one distinct space position.

Moreover, each die, denoted as α1 and α2, occupies a distinct spatial position, which makes the dice discernible. Here, the objects maintain a *one-to-one* relationship with the location criterion. Furthermore, the objects can only occupy one spatial position at time T_0, ensuring no multiplication with respect to the location criterion. The n-universe is then denoted as $\Omega\alpha L\alpha^*O$ (with extended

[2] It should be noted that these different models do not exhaust the possibilities for representing the corresponding objects within n-universes. Nevertheless, they resonate with our common intuitions about these objects.

[3] We can alternatively use the notation $\alpha_0T_0L_0O_5$ instead of $\alpha_0T_0L_0O\{5\}$. However, the latter notation is preferred here because it is more compatible with classical event notation.

notation $\Omega\alpha^*T_0\alpha L\alpha^*O$). Traditionally, we have: $\Omega = \{1,2,3,4,5,6\}^2$ and $\{3,5\}$. This corresponds to the dice α_1 and α_2 being found at locations L_1 and L_2, respectively, and displaying a given outcome among $\{1,2,6\}$ at time T_0. Thus, the sample space is represented by $\Omega\alpha\{1,2\}^*T_0\alpha L\{1,2\}\alpha^*O\{1,2,...,6\}$ and the event is denoted as $\{\alpha\{1\}^*T_0\alpha L\{1\}\alpha^*O\{3\}, \alpha\{2\}^*T_0\alpha L\{2\}\alpha^*O\{5\}\}$.

2.4. Throwing several indiscernible dice

How can we model the throwing of two indistinguishable dice, such as simultaneously tossing one "3" and one "5"? Both indistinguishable dice are modeled as multiple objects located at the spatial position L_0, each capable of presenting, at time T_0, one of the six possible spatial directions {1, 2, 3, 4, 5, 6} at a given location. These multiple objects coexist at the same temporal position, so that the objects are in relation *many-to-one* with the temporal constant. The multiple objects can only present one single direction at time T_0, and thus are not subject to multiplication with respect to the criterion of direction. The fact that both dice may exhibit the same direction indicates that the objects are in relation *many-to-one* with the direction criterion. Both dice, denoted as α_1 and α_2 are at the same location L_0, rendering them indiscernible. Additionally, the multiple objects are in relation *many-to-one* with the constant-criterion of location. Finally, the objects can only occupy one single spatial position at time T_0, and are not therefore subject to multiplication with respect to the location criterion. The corresponding n-universe is then $\Omega\alpha^*O$ (with the extended notation $\Omega\alpha^*T_0\alpha^*L_0\alpha^*O$). Classically, we have: $\Omega = (i, j)$ with $1 \leq i \leq j \leq 6$ and $\{3,5\}$. This corresponds to the fact that dice α_1 and α_2 are both in L_0 and present a given direction among $\{1,2,...,6\}$ at T_0. The sample space is then denoted by $\Omega\alpha\{1,2\}^*T_0\alpha^*L_0\alpha^*O\{1,...,6\}$ and the event by $\{\alpha\{1\}^*T_0\alpha^*L_0\alpha^*O\{3\}, \alpha\{2\}^*T_0\alpha^*L_0\alpha^*O\{5\}\}$.

2.5. Drawing a card

How can we model the drawing of a card, for example, card #13, from a set of 52 cards? In this model, cards are represented as distinct objects, each associated with a unique color from the set {1, 2, ..., 52}. The numbers of the cards are equated with color taxa, numbered from 1 to 52. Each object can only exhibit one color at any given time, ensuring that there is no multiplication concerning the color criterion. Consequently, a specific card can display only one color at any given moment. Thus, the objects are uniquely related to the color criterion. In addition, a given card can only present one single color at the same time. Thus, the objects are in relation *one-to-one* with the color criterion. Furthermore, multiple objects can occupy the same spatial location simultaneously (e.g., on a table). The objects are then in relation *many-to-one* with the location criterion. Lastly, the objects can coexist at the same temporal point. Thus, they are in relation *many-to-one* with the time criterion. The corresponding n-universe can be denoted as $\Omega\alpha C$ (with the extended notation $\Omega\alpha^*T_0\alpha^*L_0\alpha C$). To model the drawing of a card, we traditionally consider the sample space $\Omega = \{1,2,...,52\}$ and the event $\{13\}$. In this framework, drawing card #13 corresponds to the object whose color is #13 being at T_0 and location L_0. Thus, the sample space is denoted by $\Omega\alpha\{1,2,...,52\}^*T_0\alpha^*L_0\alpha C\{1,2,...,52\}$ and the event is denoted by $\alpha\{1\}^*T_0\alpha^*L_0\alpha C\{13\}$.

The same modeling approach applies to drawing two cards simultaneously or drawing two cards in succession.

2.6 Drawing of a ball from an urn containing red and blue balls

How can we model the drawing of, for example, a red bowl, from an urn containing 10 balls among which 3 red balls and 7 blue balls? Here, the balls are modeled as distinct objects, each presenting a colour from the set {R, B}. Consequently, a color variable exists in the corresponding n-universe. Moreover, multiple objects can share the same color. These objects are therefore in a relation *many-to-one* with the variable-criterion of color. Additionally, the objects are also related by the constant criteria of time and location. Hence, the corresponding n-universe is represented as $\Omega\alpha^*T_0^*\alpha^*L_0\alpha^*C$ (with the simplified notation $\Omega\alpha^*C$). Classically, the sample space is given by $\Omega = \{R,R,R,B,B,B,B,B,B,B\}$ and the event of drawing a red ball is represented by $\{R\}$. Thus, the sample space can be denoted by $\Omega\alpha\{1,2,...,10\}^*T_0^*\alpha^*L_0\alpha^*C\{R,B\}$, and the event is denoted by $\{\alpha\{1\}^*T_0^*\alpha^*L_0\alpha^*C\{R\}\}$.

The process of drawing two balls simultaneously or the successive drawing of two balls is modelled in the same manner.

3. Dimorphisms and isomorphisms

The comparison of the structures of the extended (to n-universes) sample spaces corresponding to two given probabilistic situations allows us to determine whether these situations are isomorphic from a probabilistic viewpoint. Examining the structures of the sample spaces helps identify the isomorphisms or, conversely, the dimorphisms. Let us provide some examples.

Consider a first type of application where we examine whether two probabilistic situations are comparable in nature. To do this, we model the two distinct probabilistic situations within the n-universes. The first situation is modeled in a $\Omega\alpha^*T_0\alpha^*L_0\alpha^*C$ (simplified notation $\Omega\alpha^*C$), and the second one in a $\Omega\alpha^*T_0\alpha^*L_0\alpha C$ (simplified notation $\Omega\alpha C$). We then observe a dimorphism between the n-universes that model the two probabilistic situations. In the first situation, multiple objects are in *many-to-one* relationship with the color criterion, indicating that several objects can share the same color at a given moment and location. In contrast, in the second situation, the multiple objects are in relation *one-to-one* with the color criterion, what corresponds to the fact that each object has a different color at a given time and location. This dimorphism at the level of the multiplication of the variable-criterion of color in the two n-universes indicates that the two probabilistic situations are not comparable in nature.

Now consider a second type of application. The throwing of two discernible dice is modeled, as we did see it, in a $\Omega\alpha\{1,2\}T_0{}^*\alpha L\{1,2\}\alpha^*O\{1,...,6\}$. Next, let us consider a headlight that can display one of six colors numbered from 1 to 6 at a given time. If we consider two such headlights, the corresponding situation can be modeled in a $\Omega\alpha\{1,2\}T_0{}^*\alpha L\{1,2\}\alpha^*C\{1,..., 6\}$. In this case, the variable-criterion of color replaces the criterion of orientation. At this stage, it becomes evident that the structure of this n-universe (simplified notation $\Omega\alpha L\alpha^*C$) is isomorphic to the n-universe where the throwing of two discernible dice was modeled (simplified notation $\Omega\alpha L\alpha^*O$). This allows us to conclude that the two probabilistic situations are of a comparable nature.

Let us now consider a concrete example. John Leslie (1996, 20) describes the *Emerald case* in the following terms:

> Imagine an experiment planned as follows. At some point in time, three humans would each be given an emerald. Several centuries afterwards, when a completely different set of humans was alive, five thousands humans would again each be given an emerald in the experiment. You have no knowledge, however, of whether your century is the earlier century in which just three people were to be in this situation, or the later century in which five thousand were to be in it. Do you say to yourself that if yours were the earlier century then the five thousand people wouldn't be alive yet, and that therefore you'd have no chance of being among them? On this basis, do you conclude that you might just as well bet that you lived in the earlier century?

Leslie draws a parallel between a real situation involving some emeralds and a probabilistic model concerning balls in an urn. Let's model the real, concrete situation described by Leslie in terms of n-universes. Firstly, the situation is characterized by the presence of multiple objects: the emeralds. Thus, we find ourselves in an n-universe with multiple objects. Additionally, the emeralds are located in one place: the Earth. Hence, the corresponding n-universe has a constant location (L_0). Leslie also distinguishes two discrete temporal positions in the experiment: one corresponding to a given time and the other several centuries later. Therefore, the corresponding n-universe includes a time variable with two positions: T_1 and T_2. Moreover, the emeralds existing in T_1 do not exist in T_2, and vice versa. Consequently, the n-universe corresponding to the *emerald case* is an n-universe which is not with temporal multiplication. Furthermore, multiple emeralds can exist at the same temporal position T_i: three emeralds exist in T_1 and five thousand in T_2. Hence, the objects have a *many-to-one* relation with the time variable. Lastly, several emeralds can coexist in L_0, indicating a many-to-one relation with the location constant. Considering the above, it appears that the *Emerald case* occurs in an $\Omega\alpha^*T$ (extended notation $\Omega\alpha^*T\alpha^*L_0$), an n-universe with multiple objects, a constant location, and a time variable to which the objects have a *many-to-one* relation.

Now, let's compare this with the situation of the Little Puddle/London experiment, also described by Leslie (1996, 191):

> Compare the case of geographical position. You develop amnesia in a windowless room. Where should you think yourself more likely to be: in Little Puddle with a tiny situation, or in London? Suppose you remember that Little Puddle's population is fifty while London's is ten million, and suppose you have nothing but those figures to guide you. (…) Then you should prefer to think yourself in London. For what if you instead saw no reason for favouring the belief that you were in the larger of the two places? Forced to bet on the one or on the other, suppose you betted you were in Little Puddle. If everybody in the two places developed amnesia and betted as you had done, there would be ten million losers and only fifty winners. So, it would seem, betting on London is far more rational. The right estimate of your chances of being there rather than in Little Puddle, on the evidence on your possession, could well be reckoned as ten million to fifty.

The latter experiment is based on a real, concrete situation, which can be related to an implicit probabilistic model. First, the situation is characterized by the presence of multiple inhabitants: 50 in Little Puddle and 10 million in London. Consequently, the corresponding n-universe is one with multiple objects. Second, the experiment occurs at a single point in time, thus the corresponding n-universe has a single time constant (T_0). Additionally, two spatial positions, Little Puddle and London, are distinguished, allowing us to model the situation with an n-universe comprising these two spatial positions: L_1 and L_2. Each inhabitant is either in Little Puddle or London, but no one can be in both places simultaneously, indicating that the corresponding n-universe does not involve local multiplication. Moreover, several inhabitants can be at a given spatial position L_i simultaneously: there are 50 inhabitants in Little Puddle (L_1) and 10 million in London (L_2). Therefore, the objects are in a *many-to-one* relation with the spatial variable. Similarly, multiple inhabitants can be in either Little Puddle or London at time T_0, establishing a *many-to-one* relation between the objects and the time constant. Considering these factors, the Little Puddle/London experiment occurs in a $\Omega\alpha^*L$ (extended notation $\Omega\alpha^*T_0\alpha^*L$), an n-universe with multiple objects that includes a time constant and a spatial variable, with the objects in a *many-to-one* relationship with both.

In contrast, the emerald case occurs in a $\Omega\alpha^*T$, whereas the Little Puddle/London experiment occurs in a $\Omega\alpha^*L$. This comparison highlights the isomorphic structure of the two n-universes modeling the experiments. Consequently, the probabilistic model applicable to one is also valid for the other. Furthermore, both $\Omega\alpha^*T$ and $\Omega\alpha^*L$ are isomorphic with $\Omega\alpha^*C$, facilitating the determination of the corresponding probabilistic model. Thus, the situations in both the emerald case and the Little Puddle/London experiment can be modeled by drawing a ball from an urn containing red and blue balls. In the emerald case, the urn contains 3 red balls and 5000 green balls. In the Little Puddle/London experiment, the urn includes 50 red balls and 107 green balls.

4. Goodman's paradox

The n-universes serve as a valuable methodological tool for elucidating complex situations, such as those encountered in the study of paradoxes. In the following, I will illustrate the contribution of n-universes in this context through the analysis of Goodman's paradox.[4]

Goodman's paradox was described in *Fact, Fiction and Forecast* (1954, pp. 74-75). Goodman explains his paradox as follows: All emeralds observed so far have been green. Intuitively, we predict that the next observed emerald will also be green. This prediction is based on the generalization that all emeralds are green. However, if we consider the property 'grue,' defined as 'observed before today and green, or observed after today and not-green,',[5] we notice that this property is also satisfied by all previously observed emeralds. The resulting prediction, based on the generalization that all emeralds are *grue*, is that the next observed emerald will be not-green. This contradicts the earlier conclusion, which aligns with our intuition. The paradox arises because applying enumerative induction to the same instances, with the predicates *green* and *grue*, leads to contradictory predictions. This contradiction lies at the heart of the paradox. Therefore, one of the inductive inferences must be fallacious. Intuitively, the conclusion that the next observed emerald will be not-green seems erroneous.

[4] This analysis of Goodman's paradox, presented in a simplified form with several adaptations, corresponds to the version originally described by Franceschi (2001). The variation of the paradox considered here is derived from Goodman's original (1954) formulation, but it is applied to a single emerald.

[5] Given two predicates, P and Q, the predicate 'grue' is defined as follows: (P and Q) or (~P and ~Q).

Now, let us model Goodman's experiment using n-universes. It is necessary to accurately describe the *universe of reference* in which the paradox occurs. Goodman mentions the properties green and not-green as applicable to emeralds, making color one of the variable criteria of the n-universe in which the paradox occurs. Furthermore, Goodman distinguishes between emeralds observed *before time T* and those observed *after T*. Thus, the corresponding n-universe also includes a variable criterion of time. As a result, we can describe the minimal universe in which Goodman situates his paradox as a colored and temporal n-universe, i.e., a ΩCT.

Goodman also mentions several instances of emeralds. It might seem natural to model the paradox in an n-universe with multiple objects, both colored and temporal. However, it is not necessary to use an n-universe with multiple objects. To avoid a combinatorial explosion of cases, it is preferable to model the paradox in the simplest type of n-universe: one with a unique object. In this version of the paradox, we consider a single emerald whose color may vary over time. The emerald currently observed was green every time it was observed before. Therefore, by induction, we conclude it will be green the next time it is observed. However, the same inductive reasoning leads to the conclusion that it will be *grue*, and thus not-green. This variation consistently leads to the emergence of the paradox. This version takes place in an n-universe with a unique object and variables of color and time, i.e., a ΩCT. Given that the original statement of the paradox is ambiguous in this respect, and that the minimal context is a ΩCT, we distinguish between two situations: one situated in a ΩCT, and another in a ΩCTβ (where β denotes a third variable-criterion).

Let us place ourselves first in the context of a coloured and temporal n-universe, i;e. a ΩCT. In such universe, to be *green*, is to be *green* at time T. In this context, it appears completely legitimate to project the shared property of colour (*green*) of the instances through time. The corresponding projection can be denoted by C°T. The emerald was green every time where I observed it before, and the inductive projection leads me to conclude that it will be also green next time when I will observe it. This can be formalized as follows (V denoting *green*):

(I1)	$VT_1 \cdot VT_2 \cdot VT_3 \cdot ... \cdot VT_{99}$	instances
(H2)	$VT_1 \cdot VT_2 \cdot VT_3 \cdot ... \cdot VT_{99} \cdot VT_{100}$	generalization
(P3)	$\therefore VT_{100}$	from (H2)

The previous reasoning appears entirely correct and aligns with our inductive practice. However, can we conclusively determine from this that the *green* predicate is projectible without restriction in the ΩCT? It seems not. The preceding inductive enumeration applies to an n-universe where the temporal variable corresponds to our present time, such as a period of 100 years surrounding our current epoch, specifically the interval [-100, +100] years. But what if the temporal variable extended much further, including, for example, a period of 10 billion years around our current time, specifically the interval [-10^{10}, +10^{10}] years? In that case, the emerald would be observed in 10 billion years. At that time, our sun would have burned out and progressively become a white dwarf. The temperature on our planet would have increased significantly, reaching 8000°C: the observation would reveal then that the emerald—as with most minerals—had undergone significant transformations and was no longer green. Why is the projection of *green* correct in the ΩCT where the temporal variable is defined by *restriction* relative to our present time, but incorrect if the temporal variable assimilates itself by *extension* to the interval of 10 billion years before or after our present time? In the first case, the projection is correct because the different instances of emeralds are representative of the reference class to which the projection applies. An excellent way of obtaining representative instances of a given reference class is by choosing the latter randomly. On the other hand, the projection is incorrect in the second case, as the different instances are not representative of the considered reference class. Indeed, the 99 observations of emeralds come from our modern time, while the 100th concerns an extremely distant time. Thus, the generalization (H2) results from 99 instances that are not representative of the ΩCT[-10^{10}, +10^{10}] and does not legitimately support induction. Consequently, *green* is projectible in the ΩCT[-10^2, +10^2] but not projectible in the ΩCT[-10^{10}, +10^{10}]. At this stage, it is evident that *green* is not projectible in the absolute sense but is projectible or not projectible *relative* to a specific n-universe.

In light of the preceding discussion, we are now in a position to highlight the fallacies in the generalization that ‘all swans are white.’ In 1690, this hypothesis arose from the observation of numerous instances of swans in Europe, America, Asia, and Africa. The n-universe in which this projection took place included multiple objects, with variables for color and location. To simplify, we can assume that all instances were observed at a constant time T_0. The corresponding inductive projection C°L led to the conclusion that the next observed swan would be white. However, this prediction proved to be false with the discovery of black swans in Australia by the Dutch explorer Willem de Vlamingh in 1697. In the n-universe where this projection was made, the location criterion implicitly covered the entire planet. However, the generalization that 'all swans are white' was based on observations of swans from only one part of the reference n-universe. Consequently, the sample was biased and not representative of the reference class, thus invalidating the generalization and the corresponding inductive conclusion.

Now, let us consider the projection of *grue*. The use of the *grue* property, which along with *bleen* constitutes a taxon of tcolor*, reveals that the criteria system used originates from the ΩZ. The n-universe in which the projection of grue occurs is then a ΩZ, an n-universe to which the ΩCT reduces. The presence of two color taxa (*green*, *not-green*) and two time taxa (*before T*, *after T*) in the ΩCT determines four different states: *green before T*, *not-green before T*, *green after T*, *not-green after T*. In contrast, the ΩZ only determines two states: *grue* and *bleen*. The reduction of the ΩCT to the ΩZ is is achieved by transforming the color and time taxa into taxa of tcolor*. The classical definition of *grue* (*green before T* or *not-green after T*) allows for this transformation. In this context, the paradox remains. It manifests in the following form: the emerald was *grue* every time I observed it before, and I conclude inductively that the emerald will also be *grue*, and thus *not-green*, the next time I observe it. The corresponding projection Z°T can then be formalized (G denoting *grue*):

(I4*)	$GT_1 \cdot GT_2 \cdot GT_3 \cdot \ldots \cdot GT_{99}$	instances
(H5*)	$GT_1 \cdot GT_2 \cdot GT_3 \cdot \ldots \cdot GT_{99} \cdot GT_{100}$	generalization
(H5'*)	$VT_1 \cdot VT_2 \cdot VT_3 \cdot \ldots \cdot VT_{99} \cdot \sim VT_{100}$	from (H5*), definition
(P6*)	$\therefore GT_{100}$	prediction
(P6'*)	$\therefore \sim VT_{100}$	from (P6*), definition

What is it then that leads to deceive our intuition in this specific variation of the paradox? It appears here that the projection of grue comes under a form which is likely to create an illusion. Indeed, the projection Z°T which results from it is that of the tcolor* through time. The general idea which underlies inductive reasoning is that the instances are *grue before T* and therefore also *grue after T*. But it should be noticed here that the corresponding n-universe is a ΩZ. And in a ΩZ, the only variable-criterion is tcolor*. In such n-universe, an object is *grue* or *bleen* in the *absolute*. By contrast, an object is *green* or *not-green* in the ΩCT *relative* to a given temporal position. But in the ΩZ where the projection of *grue* takes place, an additional variable-criterion is missing so that the projection of *grue* could be legitimately made. Due to the fact that an object is *grue* or *bleen* in the absolute in a ΩZ, when it is *grue before T*, it is also necessarily *grue after T*. And from the information according to which an object is *grue before T*, it is therefore possible to conclude, by deduction, that it is also *grue after T*. As we can see it, the variation of the paradox corresponding to the projection Z°T presents a structure which gives it the appearance of an enumerative generalization but that constitutes indeed a genuine deductive reasoning. The reasoning that ensues from it constitutes then a disguised form of induction, a *pseudo-induction*.

Let us now consider the case of a colored, temporal n-universe that includes an additional variable-criterion β, denoted as ΩCTβ. An n-universe with variable criteria of color, time, and location,[6] denoted as ΩCTL, will be suitable for this discussion. In a ΩCTL, to be *green* means to be *green* at time T and location L. Furthermore, the ΩCTL reduces to a ΩZL, an n-universe where the variable criteria are tcolor* and location. The taxa of tcolor* are *grue* and *bleen*. To be *grue* in the ΩZL means to be *grue* at location L.

It is important to note that the projections C°TL and Z°TL do not require separate analyses. These projections have the same structure as the previously studied projections C°T and Z°T, except for the additional criterion of location. The conditions under which the paradox dissolves when comparing the

[6] Criteria other than colour and time, such as mass, temperature, and orientation, are also appropriate.

projections C°T and Z°T apply identically to the variation of the paradox that arises when considering the projections C°TL and Z°TL.

Additionally, it is pertinent to compare the projections CT°L and Z°L, which occur in the ΩCTL and ΩZL, respectively. Let us begin with the projection CT°L. Here, the shared criteria of color and time are projected through the differentiated criterion of location. The taxa of time are defined as *before T* and *after T*. In this context, the projection of *green* is as follows: the emerald was *green before T* in every place it was observed, leading to the conclusion that it will also be *green before T* in the next place it is observed. The corresponding projection C°TL can then be formalized as follows:

(I7)	$VTL_1 \cdot VTL_2 \cdot VTL_3 \cdot \ldots \cdot VTL_{99}$	instances
(H8)	$VTL_1 \cdot VTL_2 \cdot VTL_3 \cdot \ldots \cdot VTL_{99} \cdot VTL_{100}$	generalization
(P9)	$\therefore VTL_{100}$	prediction

At this stage, it appears entirely legitimate to project the properties *green* and *before T*, shared by previous instances, through a differentiated criterion of location. We can then predict that the next emerald observed at location L will exhibit the same properties.

What about the projection of *grue* in the ΩCTL? The use of *grue* indicates that we are considering ourselves within a ΩZL, an n-universe to which the ΩCTL reduces, and whose variable criteria are tcolor* and location. Being *grue* is relative to the variable criterion of location. In the ΩZL, to be *grue* is to *be* grue at location L. The projection thus relates to a taxon of tcolor* (*grue* or *bleen*) shared by instances, through a differentiated criterion of location. Consider the classical definition of *grue* (*green before T* or *non-grue after T*). Hence, the emerald was *grue* at every location where it was observed before, and I predict it will also be *grue* at the next location where it will be observed. If we take T to be 1010 years, the projection Z°L in the ΩZL then appears as a completely valid form of induction (with V~T denoting *green after T*):

(I10*)	$GL_1 \cdot GL_2 \cdot GL_3 \cdot \ldots \cdot GL_{99}$	instances
(H11*)	$GL_1 \cdot GL_2 \cdot GL_3 \cdot \ldots \cdot GL_{99} \cdot GL_{100}$	generalization
(H11'*)	$VT{\sim}V{\sim}TL_1 \cdot VT{\sim}V{\sim}TL_2 \cdot VT{\sim}V{\sim}TL_3 \cdot \ldots \cdot VT{\sim}V{\sim}TL_{99} \cdot VT{\sim}V{\sim}TL_{100}$	from (H11*), definition
(P12*)	$\therefore GL_{100}$	prediction
(P12'*)	$\therefore VT{\sim}V{\sim}TL_{100}$	from (P12*), definition

As pointed out by Frank Jackson (1975, 115), this type of projection applies legitimately to all objects whose colors change over time, such as tomatoes or cherries. Furthermore, if we consider a very long period, extending up to 10 billion years as in the example of emeralds, this property virtually applies to all concrete objects. Finally, it can be noted that the contradiction between the two concurrent predictions (P9) and (P12'*) has now disappeared since the emerald turns out to be green before T in L_{100} (VTL_{100}) in both cases.

As we see in the present analysis, a predicate turns out to be projectible or not projectible *relative* to a given universe of reference. Similar to *green*, *grue* is not projectible in an absolute sense but becomes projectible in some n-universes and not projectible in others. This contrasts with several classical solutions offered to solve Goodman's paradox, which suggest that a predicate is projectible or not projectible in an *absolute* sense. Such solutions lead to the definition of criteria distinguishing projectible predicates from unprojectible ones, based on temporal/non-temporal, local/non-local, qualitative/non-qualitative, etc. Goodman himself aligns the distinction between projectible/unprojectible with the distinction between entrenched/unentrenched. However, Goodman's further reflections in *Ways of Worldmaking*,[7] emphasize the non-absolute nature of the projectibility of *green* or *grue*: 'Grue cannot be a relevant kind for induction in the same world as green, for that would preclude some of the decisions, right or wrong, that constitute inductive inference'. As a result, *grue* can be projectible in a Goodmanian world and not projectible in another. *Green* and *grue*, for Goodman, belong to different worlds with different category structures. Thus, the present solution is based on a form of relativism that is essentially Goodmanian in nature.

[7] Cf. Goodman (1978, 11).

5. Conclusion

From the preceding discussion, particularly in light of Goodman’s paradox, it becomes evident that the n-universes possess a fundamentally Goodmanian essence. From this perspective, the essence of n-universes is pluralistic, allowing for multiple descriptions of the same reality using different systems of criteria. A characteristic example, as previously observed, is the reduction of the criteria of color and time in a ΩCTL to a single criterion of tcolor* in a ΩZL. Thus, n-universes can be seen as an implementation of Goodman’s program defined in *Ways of Worldmaking*. Goodman proposes constructing worlds through composition, emphasis, ordering, or deletion of elements. In this context, n-universes enable the representation of our concrete world through various systems of criteria, each corresponding to a relevant viewpoint or a way of perceiving the same reality. Favoring one system of criteria over another leads to a truncated view of reality, and the exclusive choice of a particular n-universe without objective justification results in a biased perspective.

However, the genuine nature of the n-universes is inherently ambivalent. The similarity of the n-universes to Goodmanian worlds does not preclude a purely ontological approach. Alternatively, n-universes can be considered solely from an ontological viewpoint, serving as methodological tools to model specific concrete situations directly. In this sense, n-universes represent various universes with different properties based on combinations of unique or multiple objects, their relationships, and criteria such as time, location, and color. Goodmanian n-universes also enable the construction of diverse universes with different structures, which sometimes align with real-world properties and at other times exhibit exotic properties. For instance, the simplest of these, ΩL^* is an n-universe containing a single ubiquitous object that exists simultaneously in multiple locations.[8]

At this stage, it is worth noting several advantages of using n-universes for modeling probabilistic situations. One significant advantage is the improved intuitive understanding of a given probabilistic situation by *emphasizing* essential elements and *removing* superfluous ones. For example, by distinguishing whether the situation involves a constant or time-variable, a constant or space-variable, a single object, or multiple objects, n-universe modeling provides better intuitive support. Furthermore, distinguishing whether objects are duplicated or in *one-to-many* relationships with various criteria allows for precise classification of different probabilistic situations encountered.

Additionally, using notation for probability spaces extended to n-universes eliminates the ambiguity sometimes associated with classical notation. For instance, $\{1,2,...,6\}^2$ can ambiguously denote both the sample space of simultaneously throwing two discernible dice in T_0 and that of two successive throws of the same die in T_1 and T_2. With n-universe notation, this ambiguity is resolved: the sample space for simultaneously throwing two discernible dice in T_0 is $\Omega\alpha\{1,2\}^*T_0\alpha L\{1,2\}\alpha^*O\{1,2,...,6\}$, whereas for two successive throws in T_1 and T_2, it is $\Omega\alpha_0T^*\{1,2\}L_0O\{1,2,...,6\}$.

Finally, a key advantage of modeling probabilistic situations in n-universes is the ease of comparing multiple probabilistic models, highlighting isomorphisms and corresponding dimorphisms. However, the primary advantage of using n-universes as a methodological tool, as demonstrated through Goodman’s paradox, lies in clarifying complex situations encountered in the study of paradoxes.[9]

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[8] N-universes with non-standard properties warrant further detailed investigation, which is beyond the scope of this study.

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