THE OFFER PARADOX

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Early this morning Margaret made me the following offer.

Margaret's offer: If you utter a sentence in the next five seconds, then whether it's true or not, you get exactly ten dollars from me. I don't care if your sentence is true, false, or the type of sentence that's neither true nor false (e.g., the sentence 'Close the door').

Given that I did utter a sentence in the required time, surely you would predict that Margaret gave me ten dollars today regardless of what my sentence was. But in fact I felt devious and my sentence was S: 'I got less than 10^{100} dollars from Margaret and the tenth sentence I uttered today wasn't true'. Was S true?

(a) Suppose S was true. Since S was true, and S is the sentence 'I got less than 10^{100} dollars from Margaret and the tenth sentence I uttered today wasn't true', it must be true that I got less than 10^{100} dollars from Margaret *and* the tenth sentence I uttered today wasn't true. So, the tenth sentence I uttered today was not true. But as a matter of fact the tenth sentence I uttered today was S! I had uttered nine sentences so far when Margaret presented me with her offer. The next sentence out of my mouth just happened to be S. Since we just concluded that the tenth sentence I uttered today was not true, and that sentence was S, S wasn't true. But this is impossible; we started out this option supposing S was true. Thus, this option (a) is ruled out.

(b) Suppose S wasn't true. Since S wasn't true, it wasn't true that: I got less than 10^{100} dollars from Margaret *and* the tenth sentence I uttered today wasn't true. So it's not true that *both* of the following hold: I got less than 10^{100} dollars from Margaret *and*, in addition, the tenth sentence I uttered today wasn't true. So either (1) I got at least 10^{100} dollars from Margaret or (2) the tenth sentence I uttered today was true. Now (2) can't be right. For if (2)--'the tenth sentence I uttered today was true'--is true then the tenth sentence I uttered today was true. But that sentence is S. Thus, if the tenth sentence I uttered today was true, since that sentence is S, S was true. But we started out supposing S wasn't true; it would be contradictory to say S was true too. So (2) doesn't hold; therefore (1) holds: I got at least 10^{100} dollars from Margaret. So even though Margaret thought she was going to give me a mere ten dollars, as I have just proved she couldn't help but give me 10^{100} dollars--making me the richest person in the world.

But this is ridiculous; (1) can't hold, for two reasons. First, surely Margaret didn't care what my sentence was; she was going to give me ten bucks, period. Second, there aren't 10^{100} dollars in existence!

So we have a contradiction: S was neither true nor not true. Thus, there is some mistake. Notice that S appears to be a perfectly good sentence; for example, if I had uttered only eight sentences when confronted by Margaret we need not have run into any problem. And Margaret's offer *seems* coherent enough. If so, then the problem is with the reasoning in (a) and (b). The *paradox* is that the line of thought leading to the contradiction seems impeccable; the *challenge* is to find the error.