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On what counts as a translation

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Abstract: In this article, instead of taking a particular method as translation, we ask: what does one expect to do with a translation? The answer to this question will reveal, though, that none of the first order methods are capable of fully represent the required transference of ontological commitments. Lastly, we will show that this view on translation enlarge considerably the scope of translatable, and, therefore, ontologically comparable theories.

Keywords: Translation, Ontology, Ontological reduction

1 Relativity of translation

Some fundamental indeterminacies in largely used philosophical concepts were introduced by Quine in his well-known articles *Two Dogmas of Empiricism* (TD) (Quine, 2000) and *Ontological Relativity* (ORel) (Quine, 1968) and in the book *Word and Object* (WObj) (Quine, Churchland, & Føllesdal, 2013). In the first, he makes a resounding critique of the traditional distinction between synthetic and analytic sentences; from this, he concentrates on asserting the indeterminacy of translations between any two languages in the *radical translation* experiment (WObj). In the second, he shows that the absence of distinction, together with the indeterminacy of the models in Löwenheim-Skolem's theorem, implies the inscrutability of the reference relation. Thus, every reference relation is fundamentally linked to the choice of a background theory for which the existential requirements of the theory are interpreted.

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If, in TD and WObj, Quine describes the indeterminacy of the translations at the epistemological level, in ORel he relativizes the ontology of a theory to a translation in the background theory: “Specifying the universe of a theory makes sense only relative to some background theory, and only relative to some choice of a manual of translation of the one theory into the other” (Quine, 1968). Offering an ontology to a theory, therefore, amounts to reducing a theory in a background theory. The translations are, for Quine, underdetermined by any empirical experiment, but not by a kind of relativity.

Quine used the notion of model-theoretic reduction between first-order theories to show the relativity of the reference relation. But the reduction itself would not be subject to the same kind of relativity. If the translation relation has a constitutive defect, then it would be contextual - but not underdetermined. According to Quine, the translation relationship could be reconstructed from the syntactic feature he termed “proxy function”. Non-relativity, in this case, is linked to the understanding that translations are not properly objects, that such functions “need not exist as an object in the universe even of a background theory”(Quine, 1968). To take such an exit, however, is as much restricted on the scope of translations as it is invariably interpreted with respect to the mechanisms performed by the background theory. It is restricted because translations not only occur between a background theory and an object theory, but also between two unfamiliar theories to the underlying theory. It is interpreted because a first-order theory is not able to capture the claim that a given set of formulas represent-the-T-theory.

It is only possible to understand what was done by the background theory as a translation if we interpret the result outside the scope of the background theory. If PA is internalized in ZFC, this means that (1) for each formula of PA’s language, there exists a set in ZFC that represents the formula of PA, (2) for each sequence of PA’s formulas exists a set representing the sequence of PA’s formulas and finally (3) that if a sequence represents a PA’s proof, then ZFC proves that the set representing the sequence satisfies a definable property in ZFC. All these statements are not statements of ZFC, but of a metatheory that establishes the bond between the two theories. From the ZFC’s point of view, one simply proves theorems about certain sets, and the statement that these proofs speak about PA must be interpreted in another metatheory. When Gödel does the internalization of arithmetic in arithmetic itself, he does not assume to be internalizing properly the arithmetic; rather, it is only possible to understand the internalization from the theorem of the representation which is done in primitive recursive Arithmetic (ARP). Eventually, this means that the relationship of internalization only makes sense if we consider a third theory seeing the two arithmetics. Something similar occurs in the case of ZFC interpreting PA: we must take into account at least one ARP that is responsible for establishing that the representation of PA in ZFC is a representation of PA - otherwise, it is not possible to understand in this procedure more than ZFC proving interesting theorems.

More generally, we reinforce the idea that any translation relationship between theories can only be understood in a third theory. This means that the case in which we seemingly reduced a theory T in a background theory T_m is, in fact, the evaluation of the relationship between T and T_m in a third theory T_M and the reduction provided by Gödel allows us no more than to say that T_m and T_M can be syntactically the same.

This issue becomes even more evident if we consider the environment of independence proofs in set theory. When, by abbreviation and convenience, we call the Gödel's constructibles L a model for ZFC, we do no more than actually state that there is an *interpretation* (Shoenfield, 1967, p. 57 - 61) between the two theories. In this case, it is not possible to do a complete internalization in the same sense applied in the cases already mentioned. If, in the earlier cases, internalization would provide us with a truth predicate for the internalized theory, the same is not valid for these set theories - if consistent, none of them is capable of internalizing one truth predicate into the other. Notably, the background theory (ARP) for the interpretation is not able to offer an ontology even relative for set theories, and yet it is able to speak about the "translation" between them.

The question still remains open: (QEx) who asserts that the syntactic resources stated in the third theory are a translation relation? Indeed, this seems a new question of representation, in a sense similar to that of the question in the case of reduction, although more serious. As Boghossian (1996) recalls, Quine himself in TD examines two distinct types of analyticity: the first in which the substitution of synonymous terms for forming logical truths is based on the synonymy between a term T and a term T' introduced by definition; and the second in which the basis is a synonymous relation intuited by a competent speaker. In the first case, the relation of synonymy is trivial and of little influence in Quine's critic, whereas the second one is the focus of harsh criticisms. Indeed, QEx is a question whose answer suffers from the same problems as the empiricist dogmas. If we want to analyze translations so that they do not suffer from such criticisms, we must deny them the status of a question capable of expressing meaning and - (1)

assume that the two theories analyzed in the translation were defined in the context of the background theory T_B and (2) assume that "being a translation" is a theoretically defined element in T_B. To assume (1) is excessively convenient, though hardly problematic; to assume (2) is artificial. Let's look at these two hypotheses in more detail.

The analysis of the assumption (1) starts with the question of what it means to take a given theory as a metatheory or background theory. That question is not frequently raised because it is obvious or by negligence. When, for example, we study the relationship of satisfaction in ZFC, through model theory, we do much more than using ZFC as the background theory. Before even evaluating the satisfaction of a T theory, one must internalize T's syntax into ZFC; and the fact that this internalization represents T is a statement from another background theory. The notion that an internalized theory "is a theory" must be a ZFC predicate whose (2') representability is guaranteed in a similar fashion to the assumption (2). Even if, again, we take the assumption (2') as unproblematic, we would have to admit that it is not possible to understand a theory that was not internalized in the first place. This means that if an agent A_1 has T_M as background theory and a second A_2 agent presents a theory T_x unknown by A_1, then A_1 cannot offer any understanding about T_x, since he or she could not present a representation predicate "this internalization represents T_x"[2].

Let us assume problem (1) as solved and proceed with the question pointed out in supposition (2). As we have seen, the translation must be defined as a predicate in a background theory that establishes by theorem the link between two theories. This necessity imposes precisely what Quine wanted to avoid concerning translations, i.e. that they were objects. In this case, it is relatively simple to show that what counts as a translation is relative to the choice of a meta-metatheory. Consider, for instance, the case of interpretation between PA and ZF without the axiom of infinity and with the addition of the negation of the axiom of infinity (ZF^{-Inf}). Explicitly, this result can be described as a proof in ARP that if $Th_T(\alpha)$ is the predicate internalized in the ARP that states that α is theorem of theory T and being

[2]This problem of representation is not easily solved without extrapolating the first-order environment. I endorse, in this sense, the thesis defended by Freire (2017) that the identity of a mathematical theory is not a formal system, but a normativity instituted by the practice. In the case of a purely formal theory such as the T_x, I add that its identity is instituted in practice with formal systems. This movement toward normativity allows the communication between agents to be the exchange of information at the normative level and the representation of T_x for A_1 is given by the same relation that A_1 already has with the theories of A_1's known scope.

I the interpretation of PA into ZF^{-Inf}, then

$$ARP \vdash \forall \alpha \in \mathcal{L}_{PA}(Th_{PA}(\alpha) \rightarrow Th_{ZF^{-Inf}}(\alpha^I)) \tag{1}$$

We take an undecidable formula δ of PA. If an ARP model (in this case, relativized to the meta-metatheory) satisfies $Th_{PA}(\delta)$, then the number representing the proof must be a non-standard number. Therefore, it is possible to build an ARP model such that $Th_{PA}(\delta)$ is true, while δ is false - say this model is $\mathcal{M}$. Notably, for $\mathcal{M}$, the formula α^I is true in ZF^{-Inf} and, at the same time, it is a translation of the formula α of PA. Still, the model sees this same formula as false. Then we come across the strange situation in which a false formula is translated into a true formula. In this case, the interpretation I does not count as a translation relative to the choice of $\mathcal{M}$.

Even so, one could insist on contextualization, stating that the translation of a theory T_1 into another theory T_2 makes sense only when the ontology is fixed for each of the theories. That is, we would think the translation between two theories in the context in which all their sentences were decided by the stipulation of the models $\mathcal{M}_1$ and $\mathcal{M}_2$. Although we take this as an error in establishing priority, we will consider it for the moment. In the article *Satisfaction is not absolute*, Hamkins and Yang (2014) show that two models for ZFC can agree on what the standard model for PA is, and still disagree on which formulas the standard model satisfies. This means that the decision on meta-metatheory can determine the truth value of certain formulas of the model of T_1 or T_2. If, therefore, we translate a formula with this property into another theory, we would not know if we should map it to a true or false sentence in the other theory until we fix the meta-metatheory. Thus, even if the context of the background theory is fixed as a condition for the translation, it is still underdetermined in relation to the meta-metatheory.

But, as stated, the problem is rather one of priority. Offering an ontology to a theory is more complex than offering a translation between two theories - evidence for this is the fact that one can speak much of the translation between two theories without having to touch the subject of an ontology. The ontology for a theory is the answer to the question “what does the theory commit to?”, While translation is the answer to the question “how can one theory offer an understanding for the commitment of another theory?”. We can have a more understandable and consensual answer to the question “what is the translation of unicorn in Japanese?” before we have an answer to the question “are there unicorns?” - and to say that the answer to the first depends on the response to the second seems unreasonable.

Therefore, translations are relations between two theories established in a third theory. And a theoretical environment must be responsible for asserting that the mapping in question preserves what one wants to preserve. Because of that, as we have seen, the relativity of translation takes effect. Accepting this thesis, however, is not possible in Quine's program, since a significant part of this relies on the concept of ontological reduction.

2 Translation Idealized

One way to restore the treatment of the philosophy of mathematics and physics after the attack on meaning is to attribute legitimacy to the ontological reductions between theories. It is for this reason that Quine emphasizes the treatment of ontological reductions. However, we have shown that translations are also subject to the same kind of relativity as the notion of meaning. I understand that Quine himself would impose a limit on his relativism if he came to that conclusion. Despite this, I am still sympathetic to his inquiries - though not to his conclusions. Therefore, we review Quine's implications for meaning, analyticity, and, by extension, for translations.

In *On what there is*(Quine, 1948), Quine establishes the concept of ontological commitment of first-order theories. We would be committed to the existence of entities capable of assuming the role of the bound variables in the axioms, making them true. Further on, in ORel, Quine shows that we are unable to present a determinate ontology to the criterion of ontological commitment in an absolute sense, and so we are forced to speak of ontology in a relative sense. Notably, Quine first asks about what "existence must accomplish" and then shows that we can only relatively accomplish such a requirement. However, for him, the same does not occur with the concept of translation: translations are simply defined as a tool that preserves truth and boolean structure in a very particular way. It is therefore necessary to rekindle the question of what counts as a translation by introducing the question: "What should a translation do?" Or "What does someone who performs a translation expect to do?"

To properly formulate this question, we correlate with the question about existence. If, in the case of existence, we want to know what the statement of the theory requires it to exist, in the case of translation we want to replace the language in such a way that the requirement of existence is preserved. So we start from the definition: With a translation of a theory T_1 into another theory T_2 it is expected that **what T_1 is committed to exist** be transferred to **what T_2 is committed to exist**.

With this definition, we want to emphasize that the relation between the concept of translation and the mappings we carry out in the first order theories is the relation of *formalization*. Much of what is observed in the literature of the subject is the assumption that translations carry out the transport of existential requirements and the assertion of translation between two theories guarantees the reducibility of one ontology into the other. We consider this an error - an exaggerated transparency as to what it may or may not count as a translation. We now turn to a more accurate analysis on what this question can offer us.

We will take a step back, trying to understand what would possibly be an *ideal translation*. We do not want to establish a methodology that obtains ideal relations of translation, but to affirm some necessary (not sufficient) properties so that a method can imply a translation in its maximum sense. Nor do we want to say that this is a translation to be sought, on the contrary, we want to use this abstract experiment to reinforce a subtle problem, namely, that satisfying what is desired with an idealized translation is not at all trivial.

Initially and (in my view) without prejudice, we consider a rather simplified conceptual scheme, in which the defined names and descriptions have only reference relations to objects in the world. We will avoid the problem of the *radical translation* pointed out in TD and WOb, assuming the existence of an ideal mediator (IM). Quine showed that it is necessary for two speakers to have equivalent linguistic/conceptual structures so that they can establish any effective communication about a translation between the theories used by each of them. This IM is able to understand both languages that one wants to translate. In this case, she will be responsible for ensuring that speakers actually refer to the same objects when they, in fact, perform a correct defined description that replaces the description used by another speaker in their own language. Assuming the IM as our mediator, we avoid the epistemological problem and focus on what is ideally a translation[3].

An important aspect that makes this type of translation possible is: we are assuming as fixed the object of the references. At least in principle, there is a non-linguistic way of accessing object, either by sight, hearing,

[3] In view of these considerations, it seems possible to establish a translation between two speakers of two different languages. When one of them, A, refers to an object by a defined description a_1, the other speaker, B, could, by trial and error, finally hit the equivalent defined description b_1 in its own language. Ideally, A and B could begin to seek more complex levels of language until an effective translation is established.

touch or any indirect way of capturing that same information. If a speaker describes “the stone”, we can see, hear or touch that referent. In the case of a description of the type “the mayor of the city”, identifying this referent can occur through obtuse and complex ways; if one of the languages does not have a single predicate that is equivalent to “being a mayor”, one could still use a predicate like “head of X”. The reference of “head of X” still depends on the concept of “citizens of Y” and this concept must be translated in a progressive nesting of attempts and errors until an effective translation can be reached.

Indeed, we have dealt only with the formulas that have a reference to ensure understanding. However, this does not include all cases of translation, we can, as we well do, translate sentences that have no referents, as the sentence “the king of France is bald”. Although we take it simply as false, we cannot say that this sentence has no translation in another language. We can say that the translation preserves the sense of Frege or the positivist’s method of verification; independently of this, the sentence, although without reference, has a "potential reference" and this must be preserved. If France is to have a king again and this king is bald, the sentence in both languages must cease to have false truth-value or “ be meaningless ” and become true. This does not mean a denial of Quine’s holism of meaning - in fact, it is perfectly acceptable to admit that, rather than simply accepting that the sentence becomes true, another theoretical assumption is reviewed. It is enough that both speakers are sensitive to the truth-value change of the proposition in question. And if it is the case that this change entails the understanding (through IM) that the speakers are not using the same referent, then this will adjust in the next iterations with IM.

It is not the case that the simple mapping of the formulas with reference must preserve the truth-value, this only guarantees that the translation "works" for the particular experiential universe that the speakers live in the moment. In order for there to be a translation in the strongest sense, it is necessary (and reasonable) that languages preserve translation even if reality changes or if there is any new discovery of the sciences. Therefore, the ideal translation must be able to fix the reference relations, the arrows that link the names to the objects.

We have considered only theories that make direct reference to objects of the world. There is much controversy whether such theories would even be possible. We assume as possible only as an abstract experiment. This will not influence the discussion, since ultimately we want to talk about theories that do not make direct reference as theories of the first order.

The ideal translation between two theories of **direct reference** T_1 and T_2 should be a mapping between the formulas of T_2 in T_1 such that the existential requirement of a formula of T_2 is the same as the existential requirement of the correspondent in T_1. In the case of theories that do not make direct reference, the picture changes. The terms of the theory do not point directly to the world, but only require that those objects captured by the quantifiers satisfy a certain set of properties. In these cases, we no longer require that a translated sentence point to the same object as the original sentence - it is only necessary that the collection of objects pointed out by both theories be isomorphic.

A first order theory does not fix the references, but just how each object captured by the quantifiers relates to each other [4]. That is why, when we speak of first order theories, we affirm reference relations to collections of objects "under isomorphism". In order to have a soundness criterion for the translation, we need to have a criterion to say that the theories refer to the "same" object as we had with the IM. Given a possible reference model $\mathcal{R}_1$ for the theory T_1 and $\mathcal{R}_2$ for the theory T_2, then the reference r_1 of a defined description $d^{trans(T_1)}$ is the same as the reference r_2 of the original description d in T_2 if, and only if, r_1 relates to all other objects translated into $\mathcal{R}_1$ in the same way as r_2 relates to all other objects of $\mathcal{R}_2$.

Yet, why, in the case of theories that do not fix the reference, should we maintain that translation preserves meaning? Given a defined description $d(x)$, we can not know if the description has a reference until the theory proves that $\exists x d(x)$. However, even if we do not know whether this is the case or not, the translation must be able to establish this "transfer of existential requirement". Indeed, some first-order theories are decidable, and for this reason there is a procedure that determines whether or not the predicate refers. For cases of translation between decidable theories, it is arguably valid to assume that the translation need not focus on fixing the reference relations, but only that all descriptions are mapped isomorphically. However, this is not valid in any case, if a theory is incomplete, there are descriptions $d(x)$ we cannot know whether they have reference or not - in this case, we must preserve the existential requirement in the same way that we must preserve the translation for the "king of France" even though he does not exist at the moment. Nevertheless, we now stop talking about "fixing the reference relation" and we proceed to say that the translation must "fix

[4] The article In defense of a dogma (Grice & Strawson, 1956) emphasizes this aspect criticizing Quine's TD.

the reference relation in an isomorphic context".

In order to be able to assert an "equality of meaning" criterion independent of the models fixed for the theories, it would be necessary for the descriptions $d^{Trans(T_1)}$ and d to have the same reference for any two models $\mathcal{R}_1$ and $\mathcal{R}_2$ of T_1 and T_2. This criterion, however, cannot be applyed unrestricted as we have argued in the last section. Thus it does not seem, at least in principle, possible to establish an ideal translation between any two theories.

As we have seen, the fundamental characteristic of a translation is to transfer existential requirements. That is, it should occur at the level of the reference arrows, rather than occur at the level of the formulas. At the level of the formulas we hope to formalize an apparatus capable of fixing the desired transfer to the level of the reference arrows. If all relations of reference to objects of one theory can be converted into relations of reference to objects of another theory, we could say that the second manages to capture every ontological import of the other theory and thus manages to preserve the existential requirements of the first theory. We call this transfer of existential requirements: *ideal translation*. However, dealing directly with the idea of translation is to return to the same problem of fixing the intended model for a theory, since the relation of reference presupposes the two points of the arrow relation fixed: formulas and objects of the model.

2.1 Conditions for Idealization

What is traditionally called translation is a mapping Map between formulas of a theory T_1 into formulas of another theory T_2 which (i) preserves some properties of the translated theory and, mainly, (ii) guarantees the result of relative consistency between the theories (if T_2 is consistent, then T_1 is consistent). We consider, in line with tradition, that a proof of relative consistency is strongly linked to the concept of translation. However, if we want to talk about the idea of *ideal translation*, we must minimize our expectations by stating that a mapping implies a (partial or total) aspect of the *ideal translation*.

This way of dealing with a concept puts us back on the floor and allows us to speak significantly of concepts such as translation, analyticity and meaning. To recover these concepts as idealizations is, at the same time, to recover the normative use of the dichotomy that they impose and to preserve the idea that we cannot perfectly fulfill their requirements. It is to recognize, then, that I can admit that there are no analytic sentences and at the same time say that philosophical and scientific activity are fundamentally different, because of the normative dichotomy imposed by the analytization or not of concepts. Indeed, there would be no fundamental difference between any sentence of a philosopher and of a scientist – all being subject to empirical revision – but there would be a fundamental difference in attitude to each sentence. On the concept of "bachelor", one prefers to argue about the sentence "bachelors are generally younger than married men" and the other prefers to discuss whether "bachelors are unmarried " – and we all know who is one and who is the other.

It is not, though, allowed to any concept $\mathcal{C}$ the possibility of idealization. It is necessary (1) that there may be good reasons to say that certain X's are more $\mathcal{C}$ than other Y's; and (2) that the context in which the comparison is done is subject to less relativity than the context of the statement of $\mathcal{C}$. In fact, a large portion of philosophical concepts can be justified precisely in this way, and the idea of inscrutability of meaning and analyticity is simply the result of the nonobservance that ultimately any philosophical concept would be subject to a greater or less degree of inscrutability. If there are no good reasons for a α or β sentence to be ideally analytical, there are good reasons to say that $\alpha \vee \beta$ is more analytic than α and also that β; if there is no good reason to say that the extensional meaning of a sentence is measured by Tarski's semantics, there are good reasons to say that other options are bad; if, finally, there are good reasons to say that interpretations fail to capture everything a translation should accomplish, there is good reason to say that interpretations do this better than the simple mapping of true sentences into true sentences[5].

It is from this argument that we recover the translation: although there is no mapping between formulas of a language in formulas of another language that completely transfers the existential requirements of a theory in the first language to the existential requirements of a theory in the other language, there are still reasons to understand certain methods as capturing aspects of translation neglected by other methods. This, on the other hand, makes us liberalize what counts as "translation". Any mapping that in some sense transfers ontological commitments between two theories can count as a translation. The point here is the observation that certain reductions do not count as a complete transport from one ontology to another, and a more accurate analysis on how this imperfection contaminates the analy-

[5]Benacerraf (1965) stresses a view, related to this argument, that I find inspiring. For him, although numbers are not sets, it does not mean that taking them as such is worthless: this strategy "cast some sobering light on what it is to be an individual number".

sis of ontologies is necessary. If, however, we suppose that interpretations are the only legitimate method of translation, then the case in which neither of the two theories interprets the other would be intractable; in this case, the liberalization of what counts as a translation can offer ways to compare ontologies where it was not possible.

3 The sense of translation in a proof of relative consistency

What should we preserve in the mapping between the formulas of two theories so that it is possible to affirm that there is a transfer of existential requirements? One possible answer to this question is simply to state that all the true formulas of a theory must be mapped into true formulas of another theory. This results from the observation that the opposite seems absurd: if "$2+2=4$" is taken to be a false sentence of any theory, then no translation occured. However, the very notion 'true formulas' is subject to indetermination. Do we take as "true" all the formulas that are true in some model? We take as "true" the formulas that are true in an intended model? We take as "true" only formulas that are theorems of a theory? It is at least reasonable to admit that "$2+2=4$" is translated into a formula that one know is not false. It is also acceptable to admit that "$2+2=4$" is translated into a true formula in the "intended model", as it is to be translated into a theorem.

We suppose, provisionally, that all theorems of a T_1 must be mapped in theorems of a T_2. As well noted by the tradition in the subject, a mapping that imposes only this restriction is too flexible: suffice, for example, that all the theorems of T_1 in some theorem of T_2 have been mapped. However, by imposing the condition that if α is mapped to β, then $\neg\alpha$ is mapped to $\neg\beta$ - we are able to perform some ontological analysis. The case where T_1 is inconsistent no longer supports such a mapping in a consistent theory T_2. Notably, this means that an inconsistent theory is ontologically irreducible to a consistent theory. In fact, an inconsistent theory commits itself to the existence of any object, whereas a consistent theory is only committed to the existence of a particular universe of objects.

Other types of requirements can be imposed for mappings so that we can affirm closer links between ontologies. We can impose the preservation of the boolean structure, as we can impose that existential quantifications are maintained existential quantifications. Each of these constraints has a role in effecting the transfer of ontological commitment, and in general we will say that the sum of all these constraints results in the mapping method.

In particular, a mapping method frequently used in the study of ontological reductions: relative consistency proofs (RCP). By the first order completeness theorem, we know that the consistency of a theory implies an ontology. For this reason, the proof that the consistency of a theory T_1 implies the consistency of a theory T_2 is a good reason to assume that the ontology of T_2 is reducible to the ontology of T_1. However, as we saw in the previous sections, this is not enough to affirm the reduction. In this case, we will call the *the sense of translation* of a RCP what counts as transference of ontological commitment in this RCP.

3.1 The general scheme for translation

Interpretations impose excessive restrictions on what counts as a translation. This method requires that each α formula of a T_1 theory be interpreted as a unique formula α^I in T_2's language. The procedure for determining α^I is

1. Regular: predicates, constants, and functions of T_1 are interpreted by predicates, constants, and functions definable in T_2.
2. Uniform: predicates, constants, and functions are always interpreted in the same way, regardless of where or how they occur in the T_1 formulas.
3. Universally regular: interpreted quantifiers are quantifiers limited by a single predicate in T_2.

In the treatment of natural languages, we are accustomed to make translations in which the context weighs substantially on the process that generates the translated sentence. Indeed, the case where the exact words of the dictionary substituted in one sentence form a sentence in the other language with the desired meaning is of a special type and is usually associated with rather simple constructions of both languages. However, in formal languages the requirement for uniformity seems more natural, though not necessary: it is not at all strange to suppose a translation of the relation of membership which means something when we speak of "sets of one kind" and another when we speak of "sets of another kind".

A Similar argument can be used to deny the necessity of universal regularity. It is possible to imagine that the context of the quantifiers changes according to the sentence that one wishes to translate. The predicate that defines the universe of interpretation could be variable according to the formulas being analyzed. Of course, it is necessary that this universe alternates in an ordered way, maintaining the necessary cohesion so that the RCP is possibly obtained. However, nothing in principle prevents the universe of interpretation from varying with context.

Therefore, to create a flexible notion of interpretation, we start with the analysis of two antagonistic forces: make the conditions (2) and (3) more flexible, while maintaining a version of the theorem of interpretation in such a way that it still implies relative consistency.

The interpretation theorem states that if all the axioms of a theory T_1 are interpreted in theorems of a theory T_2, then all the theorems of T_1 are interpreted in the theorems of T_2. This means that the interpretation preserves the logical structure of the arguments in T_1. Similarly, we expect a flexible version to satisfy: If T_2 **sees as true** each axiom of T_1 **brought by the mapping into a universe comprehensible to** T_2, then the same holds for any theorem of T_1.

We convert this condition into symbolic language using the following notation: $\alpha^{Tr(T_2)}$ denotes "α brought into the comprehensible universe of T_2"; and $\vdash^s$ denotes “seeing as true” in some compatible fashion to the definition of $Tr(T_2)$.

It follows, therefore, in a symbolic language the general scheme of interpretation: given two theories T_1 and T_2 and being that, for every axiom α_i of T_1, $T_2 \vdash^s \alpha_i^{Tr(T_2)}$, then $T_1 \vdash \alpha \Rightarrow T_2 \vdash^s \alpha^{Tr(T_2)}$.

Finally, in order to obtain the RCP, it is enough to impose on the $\vdash^s$ and $Tr(T_2)$ the following condition: if $T_2 \vdash^s \alpha^{Tr(T_2)} \wedge \neg\alpha^{Tr(T_2)}$, then there is a formula β in T_2 such that $T_2 \vdash \beta \wedge \neg\beta$.

A method that satisfies these conditions presents a great claim for transferring existential commitments. However, it is still necessary to answer whether or not there is a method that satisfies those requirements other than the method of interpretation itself. In fact, it is possible in this scheme to capture the RCP in which the assumption of the existence of a model for a theory implies the existence of a model for another theory[6]. However, this demonstration goes beyond the scope of this paper and will be presented in an upcoming article.

[6]Many of the relative consistency proofs by model-theory are reducible to proofs by interpretations. But this is not unrestricted. An example for this is Novak’s proof of equiconsistency between NBG and ZFC.

4 Final remark

In this article, we have insisted in Quine's strategy in ORel to show that not only ontology is relative, but the ontological reduction itself is relative. Nonetheless, instead of denying the meaningfulness of the use of the expression "the translation", we take the concept as a normative idealization. This approach allows us to come up with a more comprehensive plurality of translation methods – each of them having some (always partial) sense of translation. Eventually, as a result of this view, we may achieve some ontological comparison where it was not possible. This picture, thus, comprises the understanding that more than one method can count as partially transferring the existential requirements.

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