How to Be Humean about Symmetries

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Abstract

I describe three extant attempts to identify the global external symmetries within a Humean framework with theorems of some or other deductive systematization of the world, respectively, the best system, a meta-best system and a maximally simple system. Each has merits, but also serious flaws. Instead, I propose a view of global external symmetries as consequences of the structure of Humean-consistent world-making relations.

1 Introduction

A symmetry is a transformation that leaves some central characteristic of a system (for example, its dynamics) unchanged. Symmetries may be categorized in a number of ways. What may be called the 'global external symmetries' are those that leave characteristics of systems unchanged under transformations in spacetime. A prime example are those of the Lorentz group, which concern the invariances of a system's dynamics under boosts and orientations in Minkowski spacetime.

As we will see, global external symmetries are not idle observations. They are widely considered fundamental to our understanding of the physical world. It is essential, therefore, that any metaphysical framework aiming to describe the underlying ontology of physical theory should accommodate their potentially idiosyncratic features. In what follows, I focus on the prospects of one particular framework: Humeanism. As it turns out, their task is not trivial. A natural inclination for the Humean is to treat symmetries as laws and so to assimilate them within their favoured view of the latter (namely, the best systems account). This, I will argue, is a mistake. It has already been acknowledged that they cannot be mere laws in this sense, but attempts to find a sufficiently distinct yet analogous interpretation of symmetries as lawlike only raises further issues. I will offer an alternative interpretation according to which symmetries are not laws at all.

In Section 2 I'll lay out the basic characteristics of the Humean framework and outline what we should expect from its treatment of global external symmetries. After that I'll move on to assess the merits and demerits of three Humean strategies that identify global external symmetries with theorems of some or other systematization of the world: in Section 3, with theorems of the so-called best system (which also outputs the dynamical laws); in Section 4, with theorems of a best meta-system (suggested by Lange); and in Section 5, with theorems of a maximally simple system (suggested by Hicks). Drawing on the lessons learned along the way I'll argue in Section 6 that the Humean should understand global external symmetries as consequences of the structure of the world's world-making relations. Under such a view, and in contrast with the others considered, symmetries are not theorems of a system and so not technically laws or even lawlike. Section 7 considers various objections. Section 8 concludes.

Before starting, a word on generality is in order. Providing a plausible account for global external symmetries is my purpose here. How much my conclusions generalize, for example to local and internal symmetries, depends on the similarities among all the various symmetries that I will not remark on. For all I will say, it may turn out that one of the law-based accounts considered in Section 3–5 better captures some symmetry that is not global and external (for example, CPT symmetry, or the diffeomorphism invariance of general relativity) than the alternative account I provide in Section 6. Nevertheless, a focus on global external symmetries seems precisely what the key interlocutors of the below discussion (namely, Lange and Hicks) have had in mind. Moreover, there is some dispute over the genuine reality of local and internal symmetries (see, for example, Ismael and van Fraassen [2003]), which might limit the impetus for Humeans to seek a genuine account for them (one which considers them to be anything other than redundant representational features of a physical theories).[1] By contrast, global external symmetries—'symmetries' from hereon except where explicitly otherwise—are relatively uncontroversially understood to reflect something about the objective world.[2]

2 What Must a Humean Account of Symmetries Do?

Humeans believe the world is one vast mosaic of local matters of fact (Lewis [1986b], p. ix). Each of the world's sub-regions and its occupants can be wholly intrinsically typed and freely recombined with the occupants of any other set of possible sub-regions to form a possible world (though there my be limits on size).

The view is hugely popular, if also controversial. Our interests especially lie with Humean comprehension of scientific theory. Naïvely speaking, though sufficient for our purposes, a scientific theory is a set of statements, perhaps expressed in some idealized scientific language, some of which will be universal generalizations. The word 'law' (alternatively 'principle', 'rule') is often applied either to such generalizations or to the worldly states of affairs that make the generalization true (I'll continue to indulge in that ambiguity). For the Humean, who eschews primitive lawhood, non-trivially modal properties and higher-order universals, those worldly states of affairs will just be regularities. Nevertheless, to avoid well-known troubles (Armstrong [1983], Chapters 2–5), Humeans will likely wish to restrict the laws to a certain class of all true generalizations. Predictably, the relevant class will likely involve only reasonably natural and purely qualitative predicates. They are also expected to be fairly comprehensive or at least derivable from some laws that are comprehensive while not being too complex.

These expectations have widely led Humeans to favour a best system analysis of laws (Lewis

[1] Presumably that is not a plausible interpretation of diffeomorphism invariance. Nevertheless, my discussion will not aim to capture the nature of this either.

[2] 'Some invariances have obvious physical meaning. It's not hard to imagine two reference frames in the same problem and translating between them [...] Guage invariance is different. It's not about transformations of coordinates. It's a *redundancy of the description*' (Susskind and Friedman [2018], p. 267).

[1973], [1983], [1994]; Loewer [1996]; Cohen and Callender [2009]). In its most basic formulation (again, sufficient for our purposes) this defines laws to be those generalizations that are theorems of the best system: that deductive system that best describes the world's history according to a trade-off (or 'balance') of simplicity and comprehensiveness (or 'strength').[3]

What, then, of symmetries? Symmetries are clearly comprehensive regularities, most naturally described by generalizations of the behaviour of physical systems (see Section 4). These generalizations can also be expressed extremely succinctly. In combination they say what changes to expect either under various (passive) transformations of co-ordinates, or (active) boosts and reorientations of systems, and that is no change at all, or at least no change that is empirically distinguishable. I take it that such generalizations are at the very least not accidental generalizations. As will become increasingly evident below, they are just too central to physical theory for that. For one thing, symmetries clearly have the sort of modal robustness typical of physical necessity under counterfactual reasoning. But are these generalizations of the right sort to be called laws?

3 Symmetries as Laws

If global symmetries are laws, then for the Humean they must be either axioms or 'mere' theorems of the best system. It is immediately clear, however, that they could not be axioms. The dynamical laws—for example, the special and total force laws in classical mechanics—together entail (along with the proviso that they are the only dynamical laws) that the dynamics of all physical systems obey the symmetries of a world. One can see this in advanced mechanics by observing that the Lagrangian for any closed system does not depend on the invariant properties of each symmetry (for example, location, orientation, four-velocity). Consequently, the strength of a deductive system is not increased by adding specifications of the global symmetries as additional axioms.

But if symmetries are laws, so theorems of the best system, and yet are not the axioms, then they must be mere theorems. Yet this seems unlikely too. Global symmetries play a foundational role in physics. They 'underlay', and 'provide a structure and coherence to' the laws

[3] Additional parameters have also been suggested, including statistical 'fit' (Lewis [1994]), truth (Braddon-Mitchell [2001]), and cognitive utility (Jaag and Loew [2020]). Not much will turn on the status of these here.

(Gross [1996]). This is especially evident in their employment in explanations of the conservation laws (Hamilton [1834]; Wigner [1954]; Feinberg and Goldhaber [1963]; Landau and Lifshitz [1976]; Gross [1996]; Lagrange [1997]; all cited in Lange [2007]), which have been attributed an exalted status of their own (Feynman [1967]). Clearly, any regularities that explain the conservation laws will need to be very exhaulted indeed. Yet if the global symmetries were merely consequences of the axioms of the best system (for example, the dynamical force laws plus totality proviso) then they would presumably fail to posses this explanatory power. For instance, we could say no more than that it is a coincidence that the dynamical laws all possess the same symmetries (compare with remarks in Yang [1964], p. 394).

The foundational role of the global symmetries suggests that they will be more modally stable than the dynamical laws (Lange [2007], [2009], [2011]). Not only do we retain them when entertaining the failure of some force law, but also when considering the possibility of new forces. This intuition is born out by physical reasoning. For instance, Wigner ([1954], pp. 437–38) describes a toy model in which the equations of motion are such that the gradient of the force-potential determines velocity rather than acceleration. In this model, he remarks, 'two coordinate systems which are in uniform motion with respect to each other are not equivalent any more, and there is no way to introduce a theory of relativity [...] However, the symmetry properties of space alone remain preserved, its homogeneity and isotropy, and so does the homogeneity of time'.

Lange ([2007]) quotes a further passage from (Wigner [1972], p. 13) highlighting the derivative impact of symmetries' robustness on conservation laws:

> [...] for those [conservation laws] which derive from the geometrical principles of invariance it is clear that their validity transcends that of any special theory—gravitational, electromagnetic, etc.—which are only loosely connected.

Lange surmises that 'a conservation law explained by spacetime symmetries would still have held, had those "special theories" been different. It is a requirement, not a byproduct (i.e., a coincidence of the "loosely connected" force laws)' (Lange [2007], p. 474).

The lesson seems to be that symmetries cannot be axioms nor mere theorems of the best system. Rather, they deserve a different kind of treatment, arguably one that confers on them a superior explanatory status and counternomic stability to those of the best system's laws.

No doubt the debate over this status is not completely settled by the foregoing remarks. And Humeans might find a way to maintain the symmetries' modal robustness while keeping them as theorems of the best system. However, we might be understandably sceptical that this can be done in a way that reflects the actual reasoning of physicists as opposed to just being an *ad hoc* strategy. Humeans typically and honourably pride themselves on providing an account of laws that reflects the virtues of theory-building actually invoked in physics (see, for example, Lewis [1994]). My project will therefore be to see what the Humean can say by granting that a more radical approach needs to be taken with symmetries.

4 Symmetries as Meta-laws

Lange takes the foregoing observations to suggest that global symmetries are meta-laws. Whereas laws directly concern 'physical phenomena', the symmetries concern the laws themselves. The idea resonates with numerous physicists' remarks. Einstein ([1954], p. 329) suggested 'the content of [special] relativity theory can [. . .] be summarized in one sentence: all natural laws must be so conditioned that they are covariant with respect to Lorentz transformations'. And Wigner ([1967], p. 24) took a more general view concerning all symmetries (not just those global and external):

> From a very abstract point of view, there is a great similarity between the relation of the laws of nature to the events on one hand, and the relation of symmetry principles to the laws of nature on the other.

Lange ([2007], p. 479) suggests how this idea might be incorporated within Humeanism: 'would the meta-laws [and hence symmetries] be the members of the best system of truths about the first-order laws (i.e., about the best system of truths about the Humean mosaic)?'. Such a move would keep first-order laws much as BSA dictates, but then posits another competition of systems to be held with the first-order laws as input. Once again, considerations of strength and simplicity may be relevant. Humeans will want a system grounded in axioms that are reasonably comprehensive without being too costly in their simplicity. The generalizations that are theorems of this 'best meta-system' would be the meta-laws. And we might suppose that such theorems would inherit exactly the analogous explanatory power over the first-order laws that Humeans are willing to attribute to the physical phenomena in the mosaic (see the discussions in Lange [2007]; Yudell [2013]).

Lange's suggestion faces a number of challenges. One issue noted by Lange himself is that the typical Humean treatment of counterfactuals posits 'miracles' in close possible worlds; that is, small violations from the actual laws that differentiate the laws in that world just enough to render a counterfactual's antecedent true. Lange points out that such miracles are liable to violate meta-laws as well as first-order laws that, he argues, is counterintuitive given the resilience meta-laws otherwise have under many counterlegal antecedents and their foundational use in scientific reasoning. Another problem Lange raises concerns the fact that there is no obvious way to prohibit a trivially strong and simple best meta-system. To avoid a similar issue for locating the law-giving best system Lewis restricted the available predicates to those that refer to the 'perfectly natural' properties.[4] But, Lange claims, an equivalent notion of naturalness cannot be made sense of at this meta-level.

I think these problems are serious but not insurmountable for the Humean.[5] The concern over violations of law really concerns the success of the semantics of counterfactuals offered by Lewis ([1973]). But both the best system analysis of laws and best meta-system analysis of symmetries are perfectly compatible with a semantics for counterfactuals in which laws and/or meta-laws need not be so easily violated in counterfactual reasoning. For example, meta-laws would not be violated if they had similar modal characteristics to laws in the semantics advised by Roberts ([2008]), who suggests that we understand the modal profile of laws to satisfy the condition of 'nomic preservation' wherein the closest world in which counterfactual antecedents consistent with the actual laws are true are those in which the actual laws would still be laws. Moreover, the concern with natural properties is not one that a best meta-system essentially faces so long as some method of predicate-choice other than naturalness is accepted. The move away from perfectly natural properties to a more pragmatic individuation of predicates is now commonplace among BSA advocates (see especially Earman [1986]; Loewer [1996]; Cohen and Callender [2009]) and nothing in these suggestions seems to prohibit our taking a

[4] Those that 'carve nature at the joints, they are intrinsic, they are highly specific [...] there are just enough of them to characterize things completely and without redundancy' (Lewis [1986c], p. 60).

[5] Yudell ([2013]) engages in a thorough discussion of whether symmetries will be sufficiently explanatory for the Humean under this approach. His verdict is mixed.

similar approach with regard to the best meta-system.

Nevertheless, there are more serious problems. One issue is that symmetries do not have sufficient predictive strength to be theorems of a best meta-system. Great successes of twentieth-century physics have involved the derivation (under certain plausible assumptions) of entire theories (for example, quantum electrodynamics) from local internal symmetries (such as those of the U(1) group). But global external symmetries are different in that they themselves can only be used to predict the conservation laws (in conjunction with Hamilton's principle via Noether's theorem). Insofar as these symmetries are the theorems—presumably axioms—of the best meta-system, we should expect them to be employable to predict a substantial number of first-order laws just as the first-order laws can analogously do the same for physical events. But in fact they cannot do this. The Humean could put this down to a lack of 'kindness' from the world. But the unkindness would be extreme. Given that the correspondence between symmetries and predictable first-order laws is in fact one-one (a consequence of Noether's theorem and its converse), it rather seems like strength has had no influence whatsoever.

In conjunction with Lange's claim that symmetries are meta-laws the foregoing problem would seem to present a *reductio* for the best-systems approach: if Humeanism is correct then because symmetries are meta-laws they must be theorems of a meta-system superlative in its pithy systematization of the first-order system of worldly events; but symmetries are not theorems of a pithy system because they lack the required strength; so Humeanism is not correct. The Humean might try to weasel out of this by tinkering with the desiderata for what makes a meta-system best (perhaps strength is not so crucial, perhaps simplicity dominates, and so on). But this in the end this could be unrewarding toil. For it's just not clear that symmetries really are appropriately viewed as meta-laws.

One telling remark comes from Wigner ([1967], p. 25) himself, who contrasts global external ('geometrical') symmetries with local ('dynamical') symmetries:

> The geometrical principles of invariance, though they give a structure to the laws of nature, are formulated in terms of the events themselves. Thus, the time-displacement invariance, properly formulated, is: the correlations between events depend only on the time intervals between the events, not on the time at which the first event takes place.

Wigner's remark explains why it is that, as Schroeren ([2021], p. 314) puts it, 'whether a given state-space symmetry [concerning transformations in position and momentum] is also a dynamical symmetry [concerning transformations in position and momentum that obey the dynamical laws] plays no role in the relevant symmetry-based inferences'.[6]

Hicks ([2019]) has provided further criticism of the idea that symmetries are meta-laws based on scientific and philosophical consensus. On the former, the fact symmetries are about 'the events themselves', he argues, is revealed through their employment in theory-change. When physicists find themselves looking for a new theory to replace another (for example, Newtonian mechanics with relativity) Hicks maintains that global symmetries have often been assumed in the formulation of those new theories. Regarding philosophical consensus, Hicks points out that philosophers of physics have typically adopted one of two views about global symmetries. One view, which we might call the 'numerical identity view of symmetry-related states', argues that 'two symmetry-related solutions to physical equations are simply different descriptions of the same situation' (Hicks [2019], p. 1286). Clearly, numerical identification can only hold if the symmetry-related states are entire worlds, but granting this, the view claims that if the only difference between two worlds is one of transformation in one of the symmetry-preserving variables (time, space, orientation, and so on) then the two worlds are one and the same. Greaves and Wallace ([2014], p. 60) suggest this view alone enjoys 'widespread consensus'. But Hicks also acknowledges that some nevertheless hold that two symmetry-related worlds might be numerically distinct but empirically indistinguishable. We might therefore call this the 'qualitative identity view of symmetry-related states' (compare with the view recommended in Belot [2003]). As Hicks ([2019], pp. 1288–89) reasons, however, what's important is what understanding of symmetries both of these views share:

> On either view, symmetry principles provide a constraint on first-order facts. They tell us either what systems are empirically indistinguishable or what systems are qualitatively identical. [...] rather than being higher-order principles to which the laws must adhere, the symmetry principles are generalizations about the first-order facts of the world. They are justified because they encode information about the structure of space-time.

Hicks surmises that 'if symmetry principles are first-order generalizations, then they are not about the laws at all'. It is, then, no particular issue for the Humean that symmetries are

[6] Confusingly, Schroeren's use of 'dynamical' is not that of Wigner's. The former kind of symmetry is supposed to be a sub-class of state-space symmetries that include the global external symmetries. The latter is Wigner's contrast class with global external symmetries.

unlikely candidates for theorems in a best meta-system, since the intuition that motivates that approach is anyway the wrong way to think about symmetries. Justifiably, then, Hicks looks elsewhere for a Humean account of symmetries.

5 Symmetries as Maxi-laws

Since global symmetries can't be laws (Section 3) or meta-laws (Section 4) Hicks puts forward an alternative view consistent with the Humean framework, of symmetries as 'maxi-laws'. The parameters of strength and simplicity are involved in a trade-off to determine the 'best' deductive systematization.[7] But exactly what weight to attribute to each parameter has been left open. The Humean effectively assumes that there is some balance of the parameters such that the system it picks out is the law-giving one. But there's nothing to stop the Humean also believing that there is some different balance of the parameters such that the system it picks out returns theorems with other benefits. In particular, Hicks suggests that a trade-off that gives maximum weight to simplicity will pick out a maximally simple system whose theorems—the 'maxi-laws'—are the global symmetries.

The view has the benefit that the symmetries will concern physical phenomena directly rather than the laws. Hence, the foregoing concerns with treating symmetries as meta-laws don't apply. Yet Hicks also suggests that the view retains an explanation of the modal robustness of symmetries over the first-order laws. To show this Hicks draws on the 'Ramsey test' for counterfactual stability (after Ramsey [1990]), which proceeds, roughly, by assessing the impact of counterfactual supposition on our beliefs. Because, for the Humean, laws are theorems in an appreciably simple yet comprehensive system, when we suppose them to be false we jettison beliefs that are central to the justification of many others. It is just this centrality in our reasoning that tells us why we should hold fixed up to inconsistency the laws in counterfactual reasoning. By capturing symmetries in a similar way (as the theorems of a deductive system of the world) much the same can be said for their stability too, or so Hicks reasons.

Despite the apparent benefits, taking symmetries to be maxi-laws faces serious problems.

[7] Hicks ([2018]) has elsewhere suggested these desiderata should be amended for the best system. One might therefore wonder whether he is able to maintain the parallel between the maximally simple system and best system required to draw an appropriate connection between both presumed sets of generalizations.

First, Hicks's justification for symmetries' modal robustness is unconvincing, since the analogy between theorems of the best system and theorems of the maximally simple system is weak. Insofar as we can accept the Ramsey-style approach to explaining counterfactual stability (Hicks admits it is fallible) the theorems of the best system are counterfactually robust because they can be used to infer a lot, presumably in conjunction with initial conditions. They do this because they are the theorems of a system for which comprehensiveness (predictive strength) has played a significant role. But that's exactly not the case for the theorems of the maximally simple system. Lewis ([1973], p. 74) himself maintained that the limit case of trade-off between strength and simplicity that affords all the weight to simplicity would return only the laws of pure logic. That means that the trade-off for maxi-laws certainly shouldn't be interpreted as giving maximum weight to simplicity, since the symmetries are clearly more informative than pure logic. But more generally, the closer the maxi-laws come to pure logic the less they can be used to infer anything.

A second issue is still more problematic. Symmetries characterize features of systems that constrain the look of the dynamical laws. That is why it has seemed so natural (for Einstein, Wigner, and Gross) to characterize them as constraints on the dynamical laws. This characteristic of symmetries is perhaps most vividly shown by the fact that they tell us what sort of changes the Lagrangian for physical systems is invariant under, and that they are connected by inference under Noether's theorem. But if symmetries are the theorems of a system entirely distinct from the best system then there can be no guarantee that this will be the case. One can think about it like this. A systematization is like an attempt to compress the data of the world (Braddon-Mitchell [2001]; Wheeler [2016]). Compressions of data need not agree on the fundamental patterns (see the discussion in Dennett [1991], pp. 35–36); the patterns may 'cross-cut'. So there is no reason to suspect that the maximally simple system (which is supposed to output the symmetries) and the best system (supposed to output the dynamical laws) will agree on any regularities of the world at all. It will therefore remain a mystery under Hicks's view why the dynamical laws imply the symmetries, or why the symmetries constrain the dynamical laws in the precise way that they do.

Related to this concern is a third: that global symmetries (and symmetries in general) assume a prior distinction among the laws and initial conditions. Houtappel *et al.* ([1965], p. 596) express this idea as follows:

> The invariance [symmetry] principles apply only [. . .] to the so-called laws of nature. Only these are the same in equivalent coordinate systems. The laws of nature describe the further fate of a system, once the initial conditions are given. Hence, an invariance principle holds if two systems with the same initial conditions in two equivalent coordinate systems develop, from the point of view of respective coordinate systems, in the same way. We see that invariance principles can be formulated only if one admits the existence of two types of information which correspond in present-day physics to initial conditions and laws of nature. It would be very difficult to find a meaning for invariance principles if the two categories of our knowledge of the physical world could no longer be sharply separated.

At first glance this claim might seem to be in tension with Wigner's earlier comment about the precise formulation of symmetries being in terms of the physical events themselves (as opposed to the laws). Wigner's point was that even though the symmetries constrain the laws they needn't be formulated in terms of any specific laws. That's why the idea of symmetries as meta-laws seems so implausible. But notice that Wigner still made reference to correlations in his 'proper formulation' of time-displacement symmetry, where such correlations are obviously not mere accidents. This in fact supports the point made here by Houtappel et al. that symmetries are essentially constraints on what the laws can look like. In turn, this tells us that Hicks's idea of symmetries being theorems of a systematization operating on the same data as that which outputs the dynamical laws is likely implausible as well. For such an idea would not give us any reason to suppose the symmetries are conceptually bound up with first-order lawhood.

If the foregoing issue suggests that Hicks may have oversold the symmetries' independence from the laws, a final issue with understanding symmetries as maxi-laws concerns something that Hicks may not have taken sufficiently to heart. Supposedly (according to Greaves and Wallace, at least) there is widespread consensus that the numerical identity view of symmetry-related states is correct. In Hicks's own phrase, the view is that 'symmetries tell us what structure the world does not have'. There are only certain approaches to the ontology of spacetime that are consistent with this view. It is consistent with a comparativism about distance relations according to which there are no absolute positions in spacetime, only relations of distance between systems (Dasgupta [2013]). Under the approach the world simply lacks the structure to make for any kind of difference between two worlds that differ only, say, in a boost of 3ft in some direction. Similarly for so-called sophisticated substantivalist views, according to which spacetime points exist independently of the physical systems that occupy them, but are individuated only by their place in a spatial structure rather than having any intrinsic identity (Hoefer [1996]; Pooley [2006]). It is only a thoroughgoing absolutism (or 'straightforward substantivalism'; Belot and Earman [2001]) about space that would have sufficient structure. Yet of course, with such an approach effectively comes a rejection of the consensus about symmetries.

So, if the numerical identity view is true, it seems either comparativism or sophisticated substantivalism must be the correct approach to spacetime. But then it's hard to see how the symmetries could be anything like theorems of a systematization of the world. Systematizations are supposed to take some kind of input data (facts about the mosaic) and find pithy ways of expressing patterns in it. But the very idea behind the numerical identity view is that the properties in which physical systems are invariant under changes cannot be in the data because they are not really part of the world at all. A systematization of a world whose mosaic was comparativist or sophisticated substantivalist, for instance, simply wouldn't even be able to consider facts about spatiotemporal location, orientation or Lorentzian differences defined in a way appropriate for systematization since there just aren't any such facts. Contraposing, if symmetries are the theorems of the maximally simple system, then the numerical identity view must be false. Pending any further argument against the consensus, that seems like a bad result for the account of symmetries.

6 Symmetries and World-Making Relations

The criticisms raised for the foregoing approaches are likely not conclusive, but they surely justify exploration of an alternative. On that note, let's sum up the key features of symmetries we should wish to take account of as best we can. Global external symmetries:

(1) are more theoretically foundational than laws (Section 3);

(2) are counternomically robust (Section 3);

(3) explain features of the dynamical laws (for example, the invariances of the Lagrangian)

(Section 3);

(4) are most precisely formulated in terms of physical systems themselves (Section 4);

(5) apply to dynamical change under identical initial conditions (Section 5);

(6) tell us what structure the world does not have (Section 5).

The last feature (6) is, perhaps, less strongly implied by the reasoning of modern physics. Nevertheless, it suggests a new way for the Humean to think about symmetries that (apparently) enjoys widespread consensus and makes sense of the other features. To begin with, it tells us that symmetries aren't generalizations over occurrent phenomena. Rather, they're an absence of certain kinds of phenomena. So it may be wrong to assume (as both Lange and Hicks have) that Humeans must treat symmetries as laws, that is, retrieving them as theorems of a deductive system. Moreover, it tells us that symmetries may have something to do with the world's structure (specifically, about features it lacks).

Humeanism, at least in its orthodox formulation, is an attempt to get by with only one external relation: spatiotemporal distance (Lewis [1986b], [1986c], [1994]).[8] That's not only relevant to how Humeans make sense of laws (which must be somehow derivative of how the local matters of fact are so related), but also of how worlds themselves are individuated. Lewis ([1986d], pp. 71–76) countenances the idea that there might also be a *sui generis* 'world-making relation' such that two things are world-mates if and only if they share it. His preference, however, was for spatiotemporal distance relations to do that job themselves. Accordingly, two things are world-mates if and only if they are spatiotemporally related. Granting this, it remained open to Lewis whether or not there is a unique perfectly natural such relation to individuate all worlds or whether, for instance, relativistic and non-relativistic worlds would need to be individuated by different spatiotemporal relations.

Of course, Lewis was writing at a time when quantum gravity was in its infancy. Today, it may seem promising for Humeans to move away from the traditional commitment to worlds individuated by spatiotemporal relations and instead view them as emergent from or functionally

[8] In fact, however, they may have need for more. Examples include relations to number (for quantitative property values) and occupancy relations (between events or particulars to spacetime locations), see (Arntzenius [2012]).

defined in terms of some deeper relation, for example, quantum entanglement (Brown [2005]; Cao *et al*. [2017]; Knox [2018]). So long as whatever is at the deepest level still lacks necessary connections there doesn't seem any principled reason for the Humean to reject such worlds or even that ours is one of them.[9] Given the potential for multiple Humean-consistent views where spacetime relations are emergent, it may after all be less committing for Humeans to simply talk of the world's world-making relations, whether they be spatiotemporal distance relations themselves (perhaps of relativistic or some alternative specificity) or something else entirely.

Nevertheless, given that the global external symmetries of our world concern spacetime relations, the view that symmetries tell us what spatiotemporal structure the world does not have would appear to have implications for what our world-making relations are like. Specifically, they will be such that symmetry-related changes (for example, those of the Poincaré group) to the physical content of a world would not be identifiable in the structure formed by those relations. The view made available for the Humean, then, is that the global symmetries just are (descriptions of) features of the structure of whatever relations in fact make our world that individuate our world. More specifically, symmetries are those purely structural features, which hold independently of the physical contents of the world. Call this the 'world-making view of symmetries' (WMS).

WMS need not ascribe any unHumean dispositional or necessitating essences to the world-making relations (that is, spatiotemporal relations or to their more fundamental grounds should they have any). All that is required is that when they are in place, the structure they establish can ground some differences between worlds and not others. For instance, they will, presumably, ground spatiotemporal differences of betweeness and congruence among physical systems (see Field [1980]). But they cannot, according to WMS, ground differences between worlds whose physical contents differ only in terms of symmetry-related changes.

We saw earlier (Section 5) that such a structure will plausibly lead to either the a comparativist or sophisticated substantivalist understanding of spacetime, since the remaining view

[9] Humeans would, I expect, have to remain suspicious of the idea that the deepest level comprises primitive laws or causal interactions among fundamental entities. What they can allow is that there may be some entities with monadic intrinsic properties related by a primitive external surrogate of spatiotemporal distance whose systematization leads to theorems that describe the known quantum laws and statistics that are easily converted into the terms of emergent spacetime variables.

('straightforward substantivalism') is inconsistent with (6). For instance, if comparativist, the fundamental relations might permissibly be determinate distance relations (for example, being two light-seconds from) that take events or particulars as relata. The world-making structure would then be that of a graph with events/particulars as the nodes and distance relations (or their more fundamental correlates) as edges. If the world has this structure, then there would be no numerical differences to the world under symmetry-related changes, since the only meaningful spatiotemporal changes would be ones that change the relative distances between events/particulars. Indeed, there could be no symmetry-related changes at all. No doubt the idea that the world comprises a vast 'Humean mosaic' of local matters of fact lacks something of its metaphorical appeal under such a view of the world's fundamental world-making relations, but so long as the view avoids necessary connections and retains committed to only a single kind of world-making external relation it satisfies Humean credentials.

Alternatively, the sophisticated substantivalist Humean might permit occupancy relations of particulars or events at locations in spacetime, or rather go super-substantivalist and take the events to be properties of spacetime locations (Sklar [1974]). Crucially, however, neither sort of sophisticated substantivalist will take the locations in spacetime to have primitive identities. There has been plenty of work on the possibility of such a view in the context of general relativity (Maidens [1992]; Stachel [1994]; Hoefer [1996]; Belot and Earman [2001]; Pooley [2006]). The thought is that the points in continuously differentiable manifolds that contribute to models of the general theory of relativity (GTR) have no primitive identity. Such points will be weakly discernible, insofar as their cardinality is non-trivial and they can be individuated by their contingent metrical properties, yet worlds in which the same structure of geometrical and metrical relations (for example, of distance and collinearity) is distributed over them and their occupants would be identical. Consequently, models of GTR that are related only by a permutation of the points in the manifold (a diffeomorphism) that retains same qualitative relations in the metric and stress-energy tensors would be models of the same world. Hence, for the sophisticate about GTR, diffeomorphism invariance is a symmetry about what structure is lacking in spacetime.[10]

[10] Here I'm assuming, with Maudlin ([1990]) and Hoefer ([1996]), that spacetime is the manifold with a metric.

Of course, in the present discussion we've been focusing on the more homely invariances of global external symmetries. But the sophisticated view is just as viable here too. Indeed, being sophisticated about GTR invariances seems to imply sophistication about global external symmstries. As Hoefer ([1996], p. 20) concedes, 'this stance [sophistication about GTR] does force substantivalists to give up certain counterfactuals, such as: had God preferred, it could have placed the bodies of the world three feet East of where they actually are'. However, the inference doesn't go the other way, the Humean needn't be committed to GTR in order to instil a sophisticated substantivalism for the other symmetries. As Pooley ([2006]) points out, it would be wrong to think that the metrical and affine structures of non-GTR theories about spacetime (for example, special relativity) are significantly different in this respect. So, regardless of their favoured view about the fundamental world-making relations, the sophisticated substantivalist can view the manifold of points operated on by the geometrical and metrical relations of the metric tensor as lacking the kinds of information that would undermine the numerical identity view of symmetry-related states.

Among substantivalist views, the sophisticate gains credence in the context of GTR by showing how their substantivalism avoids the 'hole argument' (Earman and Norton [1987]). But it's worth noting that Humeans may also be inclined towards such a view anyway. Lewis ([1986c]) famously rejected haecceities which, if not the same as primitive identities, seem to require them (Hoefer [1996], p. 15). His grounds were multifarious and intertwined with other doctrines he favoured (no overlapping worlds, counterpart theory, modal realism) many of which have been taken up by Humeans subsequently. Although comparatively new to the debate over the structural properties of spacetime, sophisticated substantivalism is clearly a view that provides no specifically Humean reason for rejecting.

WMS is developed with feature (6) in mind, but it looks promising for making sense of further features (1)–(5). The fact that global symmetries concern the very world-making relations themselves and not any of the physical ongoings among their relata reasonably legitimizes their foundational role (1). The world-making structure of spatiotemporal relations specifies the structural properties of the 'canvas on which possible worlds are painted' (Earman [1986], p.24). Just as there is no painting without a surface on which to paint, there is no world's history without a spacetime structure (or a structure built from some alternative relation; see Section 6) in which to place it. Hence, the symmetries are not mere axiomatic laws describing correlations among the world's events, and they are certainly not theorems of such axioms.

We can also see from this picture why the symmetries are more robust than the first-order laws (2) and why the dynamical laws are constrained and limited by the symmetries (3). The properties of the surface of a canvas puts limits on the dimensions for any painting on it while the distribution of paint does not constrain any properties of the canvas. Similarly, the world's structure sets limits on what group of dynamical changes are possible and what meaningful differences there can be between the initial conditions of distinct worlds. By contrast, the range of possible dynamic laws does not constrain the world-making structure in any meaningful sense. Indeed, even if the world were too chaotic to admit a superlative systematization at all—so that there would be no first-order laws—the symmetries would nevertheless hold.

Once we grant that a world has the same world-making structure as ours, we preclude certain kinds of laws from taking place in them. This suggests that learning of the laws can be an indication of the world-making structure and hence the symmetries. But since the symmetries are broader than our specific dynamical laws they are more properly formulated in the way Wigner suggests, in terms of transformations to systems and the resulting invariance of whichever laws (correlations) in fact hold (4). We may contrast this with symmetries' lack of constraint on initial conditions. For the structure of categorical world-making relations says nothing about how it must be populated by events, only what group of dynamical changes are possible and what meaningful differences there can be between the initial conditions of distinct worlds (5).

The superior modal robustness of the world-making relations (hence symmetries) introduces extra detail for how the Humean must think about ordering worlds. Clearly, If we are allowed to assume (a) that the spatiotemporal distance relations are themselves symmetrical, (b) that they exclusively form symmetrical structures (permitting non-trivial automorphisms), and (c) they are the only possible world-making relation, then the global symmetries will be true in all worlds and hence trivially counternomically robust. But even if neither (b) nor (c) hold of necessity (I assume (a) is true if and only if it is necessarily true), it is reasonable to suppose that one would want to hold fixed the world's world-making relation and its symmetrical structure under counterfactual reasoning.

To justify this we might draw on Hicks's reasoning involving the Ramsey test. This says that we hold fixed the symmetries in counterfactual reasoning because giving them up would be to disturb the inferential connections among our beliefs too much. I argued above that Hicks's own view of symmetries as maxi-laws cannot sustain this idea because there is no reason to suppose the theorems of a maximally simple system would supply such inferentially important beliefs. But that is no criticism of the importance of the symmetries themselves, which WMS understands via an alternative route.

Nevertheless, even if this manner of support for counterfactual robustness does not ultimately convince us we should acknowledge that while we may readily conceive of counternomic worlds, it is more of a conceptual exercise and arguably less central to much of scientific reasoning that we entertain worlds in which the world-making structure is different. We can entertain different structures when we posit them explicitly, but for most counterfactual reasoning we will hold the structure fixed. What is being suggested, then, is that counterfactual reasoning operates with something like a 'zeroth priority' for determining the similarity relation over worlds, to come before all those in Lewis's ([1986a]) original prioritization of (1) laws, (2) big matters of fact, and (3) small matters of fact:

(0) It is of zeroth importance to keep the fundamental external relations of the same type with the same structural automorphisms.

Although Lewis himself may not have offered such a priority, his interest in world-making relations suggests it is not out of keeping with broadly Humean considerations about counterfactual reasoning.

The WMS is, I think, reasonably clear. It draws on features already available to the Humean framework but in a way importantly different to the foregoing accounts that attempted to shoehorn symmetries into a systematization model. By grounding symmetries directly in the spacetime structure of the world WMS has a claim to being far more closely connected with how physicists actually think about global external symmetries. For instance, the Lorentz group and Poincaré group more generally are most naturally conceived as characterizations of (Minkowski) spacetime, not about what's going on in it. The following sort of textbook remarks are indicative:

> Translation in Space applicable to all isolated systems, is based on the assumption of *homogeneity of space*, i.e. every region of space is equivalent to every other, or alternatively, physical phenomena must be reproducible from one location to another. (Tung [2003], p. 9)
>
> [Minkowski] space can be presented in two different ways each based on the set $\mathfrak{R}^4$. First one may regard $\mathfrak{R}^4$ as a 4-dimensional real vector space together with the Lorentz inner product [...] Second, one could regard $\mathfrak{R}^4$ as a 4-dimensional smooth manifold admitting the global smooth Lorentz metric. (Hall [2004], p. 147)

Both characterizations clearly treat symmetries (spatial translation in the first, Lorentzian symmetries in the second) as definable in terms of characteristics of space (respectively, its homogeneity and its being Minkowskian). Given that such characterizations are typical it is perhaps strange that it ever seemed reasonable to think of global external symmetries as theorems of a systematization of worldly events, or a systematization of a systematization of worldly events. Symmetries in fact concern the arena in which those events play out. Curiously, Hicks himself remarks that symmetries 'encode information about the structure of space-time' (see quote in Section 4). But what this precisely means for the Humean, I suggest, is that they cannot really be laws or lawlike (by virtue of being theorems of a system) at all.

I'll end the defence of WMS by considering some objections.

7 Objections and Replies

First, a warm-up objection. WMS conceives of symmetries not as generalizations about the distribution of qualitative facts in the mosaic but as facts about what structure the world-making relations form. This, I've argued, is to render symmetries in contrast with laws, at least as the Humean understands them. But one might think that this is exactly wrong; symmetries are surely laws of nature if anything is!

I don't think the Humean should feel under any pressure to satisfy the intuition that global symmetries are laws. If their account shows how symmetries satisfy features (1)–(6), then that should be entirely sufficient. Moreover, much of the foregoing reasoning has emphasized the point that physicists tend to mark out symmetries as having a role distinct from 'mere laws'.

So what's the value in trying to retain that title for them? In fact, one might think that the very idea that they should be treated as laws in the first place is what's gotten Humeans off on the wrong foot. Symmetries are deeper than that.

A second objection, however, seeks to undermine that last thought. Isn't it entirely plausible, so the objection goes, that the world could have failed to be symmetrical in these ways. For example, had the world been more like Aristotle conceived it then our dynamical laws would have reflected a tendency for things to move to a privileged 'centre' to the universe and others to move away from it. In that case both spatial and rotational symmetries would not have held. But in describing such a possibility, it doesn't seem like we're saying anything significant about the structure of the world-making relations. Rather, it can seem like we're thinking of a world where the mosaic of events privileges a single location (over time).

As innocent as such imaginings appear to be, we ought to be more careful. For what is it exactly that is being imagined? One option is that we are imagining that there is some point in space individuated independently of any relational or occupancy facts and to which much of the content of the universe either tends towards or away from. But that is a world that cannot exist if the numerical view of symmetry-related states has things right. For under this view, there are no such independently individuated spatial or spacetime points. Since the defender of WMS takes it that this view is correct they will have to assert that the world being imagined is after all one with a structure very different from that of our own world. As noted, the WMS-defender is in good company on this view.

By contrast, it might be that what is being imagined is a world in which there are tendencies for objects to move towards or away from some identifiable and persisting object (for example, the Earth) or some location defined relative to some object(s). Such a view needn't invoke independently individuated spatial or spacetime points. Instead, the symmetries can be captured in terms of relational facts, which is consistent with the numerical view of symmetry-related states. The defender of WMS will, I think, have to accept that such worlds do fall among those, including our own, that are united by a shared structure of world-making relations. But it's also not clear that such worlds don't satisfy the symmetries. Of course, the physics will be different. In our world, there are numerous sources of all known forces. In the Aristotelian world the

Earth and heavens will establish their own unique long-range fields that compel other objects away or towards them. But I see no reason why such fields of force wouldn't be associated with dynamics that obey the global external symmetries.

A final objection I'll consider considers precisely the opposite sort of world. Rather than one where symmetries seem to be violated despite an underlying symmetrical world-making structure, the Humean might be inclined to allow for the (metaphysical) possibility that systems described by the dynamical laws could satisfy our invariances under symmetry-related changes even though the underlying world-making structure is asymmetric. The asymmetry could, for example, be a consequence of the world-making relation itself being asymmetrical. Alternatively, it could be a consequence of the structure of relations failing to satisfy non-trivial automorphisms. But if the laws don't reflect these characteristics, then they will not reflect or 'make use of' such asymmetries (by analogy one might think of a symmetrical painting on an asymmetrical canvas).

In such a world, the qualitative identity view of symmetry-related states would seem to be the more appropriate view. For even though they wouldn't be empirically distinguishable, there would be a genuine numerical difference between systems under symmetry-related changes. But here arises a potential problem. In such worlds, since the numerical-identity view would not be correct, the symmetries would not, according to WMS, have any foundational status. Moreover, from our perspective there is little empirical reason to prefer the numerical over the qualitative identity view of symmetry-related states, and indeed it might very well turn out that the world just described is the actual world. Yet in that case WMS would have failed to justify our intuitions about what symmetries are like.

I think the Humean probably should countenance the possibility of asymmetrical world-making structures. And I think the Humean therefore should grant that a world with such a structure might be empirically indistinguishable from ours. The scientists in that world may discover a set of symmetrical characteristics that correspond with each of ours, only in our world we think they reflect features of the underlying structure whereas we know theirs only to reflect general features of the laws. Those who hold the numerical identity view in their world are just wrong, even if it is the consensus there (as it is here). There are certainly the right sort of conditions here for a sceptical argument to the effect that we could never know we are right about the numerical view. But that shouldn't alarm the Humean; sceptics crop up everywhere (compare with Schaffer [2005]). What may sound more problematic is that, according to WMS, in a world where the numerical view is incorrect the symmetries cannot have most of the other features (1)–(5) either.

In response, the WMS defender should admit that under the qualitative identity view, the pervasive symmetries really would be more of a surprising coincidence. Rather than having deep modally robust features, the world just happens to be such as for its dynamical laws to be unified under some fairly simple invariances. For the reasons given in Section 4 and Section 5 it would be still wrong to maintain that they should be understood as either meta-laws or maxi-laws. More probably they should just be understood as mere laws, that is, theorems of the best system. But since we don't in fact think the world is like that we have recourse to a different understanding of symmetries, one that confers on them much more explanatory weight. To commit to WMS is, then, to admit that which of the numerical and the qualitative identity view of symmetry-related states is right has serious implications for symmetry's explanatory power. Under one view, they are rock bottom explainers, under the other they are derivative generalizations.

Summing up, WMS has Humean credentials and a fair explanation for all the characteristics we've been given reason to attribute to the symmetries. It gives them a foundational role in physics, indeed one that goes beyond the laws themselves, but it can do so without rendering apparently conceivable worlds incoherent.

8 Conclusion

When setting out to give a Humean account of global symmetries it's natural to want to treat them as laws of some kind. Humeans already have a well worked out approach to laws—the best system analysis—and it's all too tempting to use it. Here I've explored what are surely the three most obvious ways to do this: have symmetries feature in the best systematization of the mosaic itself (Section 3), have symmetries feature in a best meta-systematization of the best system (Section 4), and have symmetries feature in some alternative systematization of the mosaic with a different weightings given to strength and simplicity (Section 5). Each has its benefits, but also serious flaws. I have proposed an alternative view, of symmetries as consequences of the purely structural features of the world's world-making relations, naturally assumed by the Humean to be ones of spatiotemporal distance. This seems to have the backing of how physicists tend to think about the global external symmetries. Moreover, it makes use of a structure the Humean must anyway grant exists and, if they endorse the consensus numerical identity view of symmetry-related states, are anyway liable to believe gives rise to symmetry-related behaviour independently of any systematization of the world.

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