The Boundary Stones of Thought, by Ian Rumfitt. Oxford: Oxford University Press, 2015. Pp. xiv + 345.*

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1 Introduction

In his book *The Boundary Stones of Thought*, Ian Rumfitt considers five arguments in favor of intuitionistic logic over classical logic. Two of these arguments are based on reflections concerning the meaning of statements in general, due to Michael Dummett and John McDowell. The remaining three are more specific, concerning statements about the infinite and the infinitesimal, statements involving vague terms, and statements about sets.

Rumfitt is sympathetic to the premises of many of these arguments, and takes some of them to be effective challenges to Bivalence, the following principle:

(Bivalence) Each statement is either true or false.

However, he argues that counterexamples to Bivalence do not immediately lead to counterexamples to (the Law of) Excluded Middle, and so do not immediately refute classical logic; here, Excluded Middle is taken to be the following principle:

(Excluded Middle) For each statement A, $\ulcorner A \vee \neg A \urcorner$ is true.

Much of the book is devoted to developing and assessing the most compelling versions of the five arguments Rumfitt considers. Overall, Rumfitt argues that each of these challenges is ineffective against classical logic.

As a preliminary to his exploration of the five arguments against classical logic, Rumfitt discusses a general problem for advancing a debate between proponents of competing logics. The problem is this: If in arguing for the logic you favor, you appeal to inferences which you accept but your opponent rejects, you will fail to *convince* them, even if you succeed in *justifying* your position. Rumfitt's solution to this difficulty consists of two steps: The first, which is independent of the particular dispute at hand, is an account of logic in terms of truth; this is developed in Part I of the book. The second step is specific to a given dispute: Consider a particular case which an opponent considers to constitute a counterexample to the logic you favor. Develop a theory of the truth of

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statements which covers the alleged counterexample (although it need not cover all statements) which you and the opponent agree on. Using the term loosely, call such a theory a *semantic theory*. Then show, using only inferences acceptable to both parties in the dispute, that this semantic theory, together with the account of logic developed previously, entails that the alleged counterexample to your favored logic is not a genuine one. Assuming that the opponent agrees with the proposed account of logic, this allows a rational discussion between proponents of competing logics to advance. Rumfitt carries out this second step for several disputes in his discussion of the five arguments against classical logic; this is done in Part II of the book.

The general methodological proposal just sketched is set out in chapter 1 of the book. In the remainder of this review, I will go through each of following chapters of the book, summarizing and commenting on its main contributions. Rumfitt's development of the different arguments against classical logic, the corresponding semantic theories, and their assessment, is complex and subtle. I will therefore comment in more detail on those parts of the book which are general, namely Part I on the nature of logic and a concluding chapter on Bivalence, and less on those chapters of Part II which are specific to particular challenges against classical logic.

2 The nature of logic

The terms 'statement' and 'true/false (statement)' are central to Rumfitt's discussion of the nature of logic. Since the details will matter, let me quote the passage in which he introduces these technical terms:

> Let us consider those ordered pairs whose first element is a meaningful, indeed disambiguated, declarative type-sentence, and whose second element is a possible context of utterance; by a possible context of utterance, I mean a determination of all the contextual features which can bear upon the truth or falsity of a declarative utterance. Some of these ordered pairs will be such that, were the declarative type-sentence that is the first element uttered in the context that is the second element, a single complete thought would then be expressed: the resulting utterance would say *that such-and-such is the case*. As I shall use the term, a *statement* is an ordered pair that meets this condition. [...] When a statement expresses the thought that such-and-such is the case—or more briefly, says that such-and-such—the statement is true if and only if such-and-such really is the case, and false if and only if such-and-such is not the case. (pp. 20–21)

A footnote adds that this account of will have to be refined for certain statements expressing multiple thoughts; this becomes important in the final chapter on Bivalence.

Rumfitt sets out his views on logic in chapter 2. He argues that in successful deductive reasoning, the conclusion does not always follow *logically* from the premises. His main example concerns electrical circuits; he argues that in contexts in which such circuits are the relevant subject matter, sound deductions may take the laws of electrical circuits for granted, without taking these to be tacit – sometimes called *enthymematic* – premises. He calls the relations which are operative in the application of deductive capacities 'implication relations', and argues that they satisfy the familiar constraints on consequence relations often attributed to Tarski (reflexivity, monotonicity and cut) and truth-preservation. Thinking of arguments as pairs of sets of statements and statements, all of which share the same context, one can think of implication relations as sets of arguments; the members of such a relation are the arguments which are sound according to it.

A thinker's logical competence is now identified as a capacity to obtain, from arguments which are sound according to a given implication relation, further arguments which are sound according to the same relation. Such a capacity is underwritten by a corresponding law. E.g., a thinker's capacity to reason dilemmatically is underwritten by the Law of Dilemma, which Rumfitt states as follows, where X and Y are variables for sets of statements, and A, B and C are variables for statements:

> Whatever implication relation R may be, if X, A stand in R to C, and Y, B stand in R to C, then X and Y together with any disjunction of A with B also stand in R to C. (p. 54, (2))

What are the logical laws in general? Rumfitt answers this question (p. 67) by stating that they come in two kinds: the structural principles, such as the one which states that every implication relation is monotonic, and the laws for particular logical notions, leaving it somewhat open what exactly the logical notions are.

As a by-product of the development so far, one can give an account of the more familiar relation of logical consequence: let this be the set of arguments which are sound according to *all* implication relations. As Rumfitt notes (p. 56), the situation is reminiscent of the difference between conceiving of (a) logic as constituted by the logical truths and conceiving of it as a relation of logical consequence. Just as in the latter case, the logical truths can be seen as by-product of an underlying relation of logical consequence, so Rumfitt considers the relation of logical consequence to be a by-product of a relation of soundness-preservation.

Rumfitt's discussion is in many ways compelling, and full of careful and subtle argumentation, such as the distinction between deduction and (deductive) inference. Nevertheless, it leaves a number of crucial matters open. One of them concerns the logical laws: Rumfitt says virtually nothing about them apart from their falling into the two categories mentioned above.

Another surprising gap in Rumfitt's discussion of the nature of logic concerns the status of formal languages. Any *mathematical* study of logics such as classical first-order predicate logic operates not with statements in Rumfitt's sense but with formulas of a formal language. The details of these syntactic matters are often left somewhat underspecified, but it is usually presupposed that a rigorous syntactic theory could be developed within a standard background theory such as ZFC set theory. The availability of such a mathematical treatment of syntax is important for many theorems about various logics, such as the Löwenheim-Skolem theorem of classical first-order predicate logic, in which set-theoretic reasoning about infinite cardinalities is applied to the syntax of the language. Rumfitt's discussion of logic makes no mention of such formulas, being couched solely terms of statements. This is partly obscured by his use of letters $A, B, \ldots$

and notation adopted from sequent calculi, but these letters are clearly intended to stand for statements (although Rumfitt himself slips occasionally into taking them to stand for formulas, as on p. 53).

Any connections between Rumfitt's notions, such as his relation of logical consequence among statements, and familiar notions from the mathematical study of logic, such as the consequence relation of classical first-order predicate logic among formulas, are left to the reader to draw. Doing so is far from straightforward. In fact, some of the ways Rumfitt uses logical symbols to talk about statements, which might suggest such a connection, are even hard to understand on their own. For example, Rumfitt uses strings like '$\neg A$' or '$A \vee B$', where 'A' and 'B' are taken to stand for statements. Given that statements are pairs of sentence-types and contexts satisfying the conditions stated in the passage quoted above, it is not clear what these complex terms are meant to stand for. E.g., what if A is a statement whose first element is a sentence in a language without anything resembling the English phrase 'it is not the case that...'? What if A and B are statements of different languages? These difficulties are also illustrated by Rumfitt's talk of reading connectives 'classically' or 'intuitionistically' (p. 111); it is unclear to me what this means.

(In written comments on a draft of this review, Rumfitt explained that '$\neg A$' is meant to stand for any statement which is a negation of A, where one statement is a negation of another if the two conform to the semantic theory of negation developed in section 7.1. One might worry that this renders the theory trivial, but such triviality can be avoided if one adds that the English 'it is not the case that' can be used to form negations of statements containing English sentences, as I expect Rumfitt would.)

Consequently, it does not become clear what exactly Rumfitt takes classical or intuitionistic logic to be, or what the correctness of classical or intuitionistic logic amounts to. This does not mean that Rumfitt does not say enough about them for a reader to be able to tell in later chapters whether a putative counterexample to classical logic in fact counts as violating classical logic. It is clear enough that any failure of the Law of Excluded Middle, stated above, is taken to be a failure of classical logic. But for that, no in-depth discussion of the nature of logic is needed, even if it is of independent interest. Nevertheless, apart from clarifying Rumfitt's stance, an explicit account of the relation between statements and formulas – and the relevant relations among them – might also lead to fruitful new formal questions. E.g., his suggestion that logical consequence is a by-product of a more general relation of soundness-preservation among arguments could lead to an interesting formal study of a correspondingly generalized relation among sequents composed of formulas.

Finally, Rumfitt's discussion leaves wide open which relations among statements count as implication relations. He mentions that each implication relation can be obtained as the relation of preservation of truth at a possibility of some, possibly restricted, space of possibilities. To support this, he gives a detailed presentation of a theorem which states that any relation satisfying the Tarskian constraints on consequence relations can be described as the relation of preservation of truth at a possibility for some space of possibilities and some notion of truth-at. Of course, the possibilities in this theorem are just set-theoretic constructs, and the notion of truth-at is a purely set-theoretic relation. This is therefore really a negative result: it shows that unless more is said about possibilities, claiming that each implication relation is obtained from a space of possibilities imposes no constraints on implication relations that were not already imposed by the Tarskian constraints.

Rumfitt goes on to develop the connection between implication relations and modality further in chapter 3; unfortunately, I found this very hard to understand. Starting from an implication relation $\Rightarrow$, he considers a corresponding space of possibilities $\Pi_{\Rightarrow}$. As in the formal result mentioned in the previous paragraph, these possibilities are set-theoretic constructions based on statements (p. 75). Rumfitt then starts to use a symbol $\Box_{\Rightarrow}$, and proceeds to formulate claims about $\ulcorner\Box_{\Rightarrow}A\urcorner$, where A designates, presumably, a statement, stipulating that $\ulcorner\Box_{\Rightarrow}A\urcorner$ is true if and only if A is true at every member of the space of possibilities $\Pi_{\Rightarrow}$. I have a hard time understanding how to read the string $\ulcorner\Box_{\Rightarrow}A\urcorner$. Presumably, this is supposed to designate a statement, but which? All Rumfitt has stipulated is whether it is true or not. Rumfitt seems to concede this, noting that he has only begun to *introduce* $\Box_{\Rightarrow}$, so presumably, $\Box_{\Rightarrow}$ is meant to be added as a sentential operator to the language of A. However, just stipulating whether the statement is true or false does not suffice to ensure that it expresses a (unique) thought, as required by definition for $\ulcorner\Box_{\Rightarrow}A\urcorner$ to be able to form a statement. Furthermore, Rumfitt proposes to define a binary relation R on $\Pi_{\Rightarrow}$, which can roughly be understood as playing the role of accessibility relations in Kripke models. The definition of R is as follows: 'Rxy if and only if A is true at y whenever [i.e., for every A such that] $\ulcorner\Box_{\Rightarrow}A\urcorner$ is true at x' (p. 75). But how is it supposed to be settled whether $\ulcorner\Box_{\Rightarrow}A\urcorner$ is true at some possibility x? All that has been said about the statement $\ulcorner\Box_{\Rightarrow}A\urcorner$ is whether it is true; as noted, this leaves open what thought it is supposed to express, or in general anything else that would settle whether it is true *at the set-theoretic construct* x. Rumfitt then claims that various results, of a kind familiar from the study of Kripke semantics in modal logic, can be proven, such as the claim that 'we have $\Box_{\Rightarrow}A \Rightarrow A$ if and only if R is reflexive' (p. 75). But whatever these underlying formal results are, how they are supposed to relate to statements and $\Box_{\Rightarrow}$ is at best left seriously incomplete.

Letting L be the relation of logical consequence (which counts as sound those arguments which are sound according to all implication relations), Rumfitt then works with the corresponding symbol $\Box_L$, for which he also writes 'logical necessity'. Due to the problems with $\Box_{\Rightarrow}$ for an arbitrary implication relation $\Rightarrow$ mentioned in the previous paragraph, which apply in particular to $\Box_L$, this discussion is hard to follow. Surprisingly, Rumfitt talks about $\ulcorner\Box_L A\urcorner$, where A is a 'well-formed formula of the language of first-order logic' (p. 76), and about letting '"$\Rightarrow_L$" [...] signify logical consequence (in the classical propositional calculus)' (p. 77); it does not seem straightforward how to match this up with his earlier explicit theorizing in terms of statements rather than formulas. Much of Rumfitt's general approach here is appealing, such as explicit theorizing about different kinds of necessities, singling out a maximally strong such kind, and comparing it to the familiar notions of apriority and metaphysical necessity (sections 3.4–5); this made it especially frustrating for me not to be able to engage with the discussion.

3 Five attacks on classical logic

Chapter 4 presents an early argument of Dummett's against classical logic. Rumfitt argues that this argument fails, but that Dummett's discussion suggests a promising general semantic theory, for which Rumfitt uses the label 'exclusionary semantics'. The theory works with possibilities, roughly 'way[s] things—some things, anyway—might be or might have been' (p. 112), which are ordered by an order of determination, where a possibility determines another if it is logically necessary that the latter obtains if the former obtains. The set of possibilities excluded by a statement is claimed to be closed under determination: any possibility determined by a possibility which is excluded by a statement must also be excluded by this statement. Rumfitt notes that such sets form a topological space, and consequently employs standard tools from topology to develop exclusionary semantics. The sets of possibilities excluded by statements formed using conjunction, disjunction and negation are characterized in terms of the sets of possibilities excluded by their constituent statements, and a definition of a relation of logical consequence is put forward, following earlier ideas about truth-preservation at all possibilities; Rumfitt refers to earlier work of his own for a proof that 'when logical consequence is defined in this way, the valid sequents are precisely those of the classical propositional calculus of "$\wedge$", "$\vee$", and "$\neg$"' (p. 120). These proofs are noted to employ classical inferences which are not intuitionistically acceptable. Nevertheless, Rumfitt argues that exclusionary semantics can be used to argue against intuitionistic logic: employing only intuitionistic principles, he argues that this semantic theory entails that A is a logical consequence of $\neg\neg A$. This is the first concrete application of Rumfitt's sophisticated method of advancing a debate between adherents of competing logics in the book: based on a Dummettian semantic theory – intended to defend intuitionistic logic – it is shown using only inferences acceptable to the intuitionist that their theory of logical consequence is false.

The mathematical development and results appealed to in this chapter are set out somewhat informally. This is intentional (see p. 27), and in many ways welcome. However, the informality also makes certain aspects harder to follow. For example, the mathematical results on exclusionary semantics are presumably obtained for a formal syntax, while much of the development of exclusionary semantics in this chapter is explicitly formulated in terms of statements; as in Part I, no explicit discussion of the relationship between statements and formulas is provided. The chapter also makes essential use of set theory (although Rumfitt expresses confidence that talk of sets could be replaced by plural quantification; p. 114, fn. 18). No explanation is given what kind of set theory is presupposed. An unsuspecting reader may expect this to be ZFC, but as it turns out in chapter 9, Rumfitt advocates a radically weaker set theory.

Chapter 5 discusses Dummett's later arguments against classical logic, which are based on a verificationist theory of meaning; these are set aside for reasons which will be familiar to many. Rumfitt then moves on to a challenge to classical logic formulated by McDowell. The challenge is that standard arguments in favor of classical logic appeal to the principle of Bivalence, for which – as McDowell and Rumfitt contend – no good argument has been given. Arguments have of course been given for Bivalence, and Rumfitt makes a case in chapter 10 that they are not convincing. In response to McDowell's argument, Rumfitt develops, over the following two chapters (6 and 7), a semantic theory which does not rely on Bivalence, but can be used to vindicate classical logic.

The semantics Rumfitt starts to formulate in chapter 6 associates with each statement a set of possibilities, as in the exclusionary semantics of chapter 4. In contrast to the earlier theory, the set associated with a statement is now taken to contain those possibilities at which the statement is true; these possibilities are called the statement's *truth-grounds*. A closure condition on truth-grounds is imposed, and the truth-grounds of a complex statement formed using conjunction, disjunction and negation are characterized in terms of the truth-grounds of its constituents. In the lattice-theoretic terminology employed by Rumfitt, the truth-grounds of a negated statement is the so-called *orthocomplement* of the truth-grounds of the negated statement. After an excursus on challenges posed by proponents of quantum logic, chapter 7 continues the development of this truth-ground semantics. A possible strengthening of it is considered, namely the claim that the truth-grounds of any statement form the orthocomplement of the truth-grounds of some statement; Rumfitt sums this up with the phrase *every statement has a back* (B).

These semantic developments lead to the second applications of Rumfitt's dialectical method: Referring to results due to Giovanni Sambin, Rumfitt states that the semantic theory just developed vindicates precisely intuitionistic logic if (B) is not included, and vindicates precisely classical logic if (B) is included; moreover, both of these claims can be demonstrated using only inferences acceptable to the intuitionist. Thus proponents of classical and intuitionistic logic which accept the semantic theory developed here can resolve their dispute by settling to their satisfaction the question whether every statement has a back. Rumfitt presents two case-studies of such disputes, involving statements which have motivated philosophers to advocate intuitionistic logic (sections 7.3 and 7.4); the first case involves the infinite, and the second the infinitesimal. In the first case, Rumfitt considers statements involving quantification over all natural numbers, which have led intuitionists in the philosophy of mathematics to reject classical logic. Rumfitt develops, on the basis of remarks by Wittgenstein, an argument that such statements are not backless. In the second case, Rumfitt considers the theory of Smooth Infinitesimal Analysis (SIA) and adapts the semantic theory of chapters 6 and 7 to it. Since SIA is classically inconsistent, he notes that statements of SIA provide counterexamples to classical logic if they make sense, while not himself taking a stand on whether they do make sense.

In chapter 8, Rumfitt considers whether vague predicates pose a threat to classical logic. He proposes a new kind of semantic theory of the extensions of color predicates. Rumfitt appeals again to topological concepts, noting that the sets satisfying the constraints of predicate extensions form a topological space. The extensions of complex predicates, formed using conjunction, disjunction and negation, are characterized in terms of the extensions of their constituent predicates. Even though negation and disjunction are not interpreted using the set-theoretic operations of complementation and union, the relevant sets form a Boolean algebra, thereby vindicating classical logic. Rumfitt notes that this argument for classical logic essentially relies on classical inferences which are not licensed by intuitionistic logic, and so is not intended to be dialectically effective in responding to intuitionists.

In chapter 9, Rumfitt returns to the threat to classical logic via infinity, now focusing on set theory rather than arithmetic. Inspired by Dummett's writings, Rumfitt supposes for the sake of argument that an opponent of classical logic is right in claiming that 'the fundamental mathematical domains are posits of our mathematical conceptions' (p. 265). In defense of classical logic, Rumfitt proposes a semantic theory for statements about set theory along the following lines: A mapping from the set-theoretic language to itself – going back to Gödel – is formulated, and a statement in this language is claimed to be true just in case it is mapped to one which is provable in intuitionistic logic from the axioms of a certain set theory. This latter set theory is chosen in accordance with the views about the philosophy of mathematics under consideration. Although this semantics vindicates classical logic, as the sentences whose translations end up being derivable – and so are true according to the semantic theory – include all theorems of first-order classical logic, the resulting set theory is rather weak, comprising only the theorems of Kripke-Platek set theory with an axiom of infinity. Most notably, this does not include ZFC's power set axiom; it does not even entail that the set of natural numbers has a power set. This is a striking conclusion to arrive at, especially given the extensive use of set theory Rumfitt makes in earlier chapters of the book. By way of example, readers following up his reference to Sambin's soundness and completeness proofs of intuitionistic and classical logic with respect to the semantics developed in chapters 6 and 7 find that he appeals to an intuitionistic type theory; unless they are experts in intuitionistic set and type theories, it will be quite hard for them to see whether there is in fact a body of set-theoretic principles which is both acceptable to Rumfitt given his later discussion and does the work he requires it to do earlier on.

One of the most interesting aspects of Part II is the use of unusual semantic theories, involving technical developments from topology and lattice theory, and making use of (possibly incomplete) possibilities rather than possible worlds. As Rumfitt notes (p. 27), such constructions deserve to be known more widely. In fact, the appeal to possibilities – or states of affairs, as one may call them – rather than possible worlds seems to be gaining some momentum in recent years, with in-depth studies being carried out by, e.g., Kit Fine and Wesley Holliday. While technically interesting, Rumfitt's use of different semantic theories of fundamentally different character, with no obvious way of unifying them, makes it in some cases hard to see the motivation for them. It is also striking that even though the thoughts expressed by statements feature prominently in the definition of statements and the final chapter, to be discussed presently, they play no part in any of the semantic theories discussed in the book.

4 Bivalence

One of the main claims of Rumfitt's book is that classical logic, including the Law of Excluded Middle, can be upheld even if Bivalence is rejected. Chapter 10 concludes the book by responding to an argument from Excluded Middle to Bivalence, which Rumfitt traces back to Aristotle. In brief, the argument goes as follows: Let u be a statement expressing the thought that P. By Excluded Middle, $P \vee \neg P$. If P, then by the definition of truth for statements, u is true; if $\neg P$, then by definition of falsity, u is false. Thus classical propositional logic allows us to infer that u is true or u is false. Rumfitt provides a formal deduction, using quantifiers binding variables in sentence position to regiment talk of thoughts

(p. 303). He argues that the argument is unconvincing, for several independent reasons.

The first reason, according to Rumfitt, is that there might be statements which express several thoughts (pp. 304–308). One of Rumfitt's proposals for an example of a statement which might express multiple thoughts involves apparently unrestricted quantification over sets 'implicitly restricted to the members of a standard model for set theory' (p. 306). Thus 'the statement "There exists a strongly inaccessible cardinal"' may expresses many thoughts, one for each standard model (ibid).

However, the term 'statement' is surely a theoretical one – after all, statements were introduced as ordered pairs of sentences and contexts. So, plausibly, the term may stipulatively be understood in the way Rumfitt introduced it in the passage quoted above (pp. 20–21), qualified as preliminary in a footnote. The original stipulation rules out that a statement expresses multiple thoughts, and so the putative counterexamples to Bivalence just mentioned. In the light of this, a rejection of Bivalence emerges as less unorthodox than it might initially seem to be, being dependent on subtleties in what is counted as a statement. Furthermore, if 'statement' is indeed such a theoretical term, then we should not expect to have firm pre-theoretic judgements about it. But it is precisely such judgements Rumfitt appeals to in order to cast Bivalence into doubt (see, e.g., p. 302).

Rumfitt's second reason for being dissatisfied with the argument from Excluded Middle to Bivalence concerns vague predicates. In this case, he finds fault with the account of truth of statements on which it relies; in some such cases, Rumfitt contends, we must reject the claim that if statement u expresses (the unique thought) that P, and P, then u is true (p. 309). This, however, is how he stipulatively defined truth of statements (preliminarily) in the passage quoted above (pp. 20–21). He returns to the issue a few pages later, granting that one may 'continue to adhere to Bivalence [by adopting] "minimalist" notions of truth and falsity' (p. 318) – presumably the notions set out some 300 pages earlier in the book. Rumfitt's reason for being dissatisfied with these notions is that theorizing in these terms gives us no way of achieving the main aim of the book, namely to advance a rational discussion between adherents of classical and intuitionistic logic. (In the relevant passage, Rumfitt in fact speaks of '*justifying* Excluded Middle' (p. 318), which, for reasons Rumfitt develops earlier, may mischaracterize the issue.)

What is surprising here is not that Rumfitt takes it to a large degree to be a matter of stipulation what statements to count as true or false – this is already suggested by the highly theoretic nature of statements, and the fact that he stipulatively defines truth and falsity for statements, at least initially – and that he uses the pragmatic concern of dialectical efficacy to select between competing accounts. And indeed, I see no reason why we shouldn't allow ourselves a large degree of freedom in stipulating these semantic terms. But in the light of this, it is even more surprising that Rumfitt appeals to pre-theoretic judgements about truth and falsity of statements; indeed, it is even surprising that he makes a big deal of his rejection of Bivalence. With his highly specific notion of a statement and his notions of truth and falsity which deviate crucially from very natural ways of understanding them, it seems far from clear that Bivalence as formulated by Rumfitt is in any sense a component of any orthodox semantic theory. Of course, this depends on Rumfitt's variant characterizations of statements and their truth and falsity. Strangely, he doesn't provide any such characterizations to replace his earlier account. But if statements and their truth and falsity are not what we were told earlier they were, then what have we been talking about all this time?

(A third reason for being dissatisfied with the argument from Excluded Middle to Bivalence is given using examples from set theory. Rumfitt writes:

> I suggested in Chapter 9 that we could affirm $\ulcorner A \vee \neg A \urcorner$ whenever A involves only bounded quantification over sets. But unless $\wp(\omega)$ and $\wp(\wp(\omega))$ exist, CH is no such statement, so we cannot affirm $\ulcorner \mathrm{CH} \vee \neg \mathrm{CH} \urcorner$, and the Simple Argument does not even get started. (p. 317)

Of course, rejecting Excluded Middle is no way of showing it to be compatible with a failure of Bivalence. In written comments on a draft of this review, Rumfitt clarified that this passage is not meant to articulate his own view, but a response available to someone like Solomon Feferman who endorses a 'semi-constructive' philosophy of mathematics.)

Overall, I have not been convinced that there is a sense of Bivalence which is both reasonably counted as orthodoxy and which should be rejected. However, it is important to note that this does not cast doubt on Rumfitt's detailed development of various non-standard semantic theories in Part II. Indeed, this is a case in which the argumentation in chapter 10 seems insensitive to some of the admirably careful discussions in earlier chapters of the book (see p. 9): Any body of interesting true claims about the truth and falsity of statements may counts as a semantic theory for the purposes of resolving a dispute between proponents of competing logics. So even if Bivalence holds, and some form of classical semantics is adequate – and so dialectically unhelpful in a dispute between a proponent of classical logic and a proponent of intuitionistic logic – this does not preclude that some other semantic theory is both adequate and plays a useful role in this dispute. This is yet another reason to be skeptical of Rumfitt's rejection of his earlier definitions of truth and falsity of statements: the methodological reason he adduces against them is misguided.

5 Conclusion

Rumfitt's book is dense and rich. There is no doubt that it makes an important contribution to the debate between intuitionistic and classical logicians. Apart from its novel developments, it also presents many of the arguments in this area very carefully, different versions being considered and historic sources discussed at length, sometimes to the point of rendering the dialectic somewhat convoluted. Although the book defends classical logic, Rumfitt clearly takes the challenges posed by intuitionists seriously; this makes for a very engaging read.

The Boundary Stones of Thought has much to offer to anyone interested in the philosophy of logic or philosophical logic, such as a sophisticated methodology for advancing a debate between competing logics, a novel account of the nature of logic, and several promising semantic theories. It is to be hoped that it finds a wide readership among philosophers and logicians, within but also beyond the debate about intuitionistic logic.*

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