**Failure of *classical theory of Knowledge* in Quantum Mechanics and beyond**

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 “Nature is earlier than man, but man is earlier than natural science.”

 Von Weizsacker

 “ What we observe is not nature itself, but nature exposed to our method of questioning. ”

 W. Heisenberg

*We will argue here, that one of the salient messages of standard quantum formalism implies a failure of Classical Theory of Knowledge (hereafter CTK) which is primarily based on the presumption that, our knowledge is predetermined by Nature. This failure provides pointers against the classically held ontological definiteness of future. Quantum formalism admits interpretation , that suggests, that there is no such thing as ‘universal global present’ in the sense of being able to confirm a complete set of facts of matter about it, unless this is coupled with a specific observation recipe.*

 **Key Words**: Realism, Phenomenalism, Born rule, traditional epistemology, matrix mechanics, dualism

 **1. The Classical Theory of Knowledge is fundamentally about a passive Dualism -**

 In our ordinary linguistic discourses, dualism comes into play, along with the way sense data, following the process of observation (perception in a wider sense), are finally expressed as meaningful statements. In fact, any *meaningful statement* is an outcome of the *data* *neurally processed* to be coupled (we still don't know *how*) with what *they are about*. This calls for an inevitable commitment to an innate network of belief in individual or *re-identifiable permanence* as its prime component. And this ensures, as a consequence, a natural commitment to a *category difference or dualism* within the final form of sentence - predicates and what they are *about (Subject*).

In that way, ordinary linguistic statements can be seen, for all practical purposes, as a *description of relation between our sense experiences admitting inherent dualism.* The statement , for example , ' *I see a Tree* ' can be described as an expression of relation between our sense *experience( raw data ) of Tree transformed into an immediate neural recipe ,* on the basis of which we conclude about a *Tree out there* . So the point is that, no statement is an entirely *observation statement* so far its cognitive import is concerned. Put otherwise, cognitive import calls for an heuristic ontological posit (*Tree* here) which can't entirely be reduced into sense data.[1]

So *final* outcome of process of observation on the basis of our standard means of cognition is *translated* in terms of faithful neural recipe creating characteristic scope of (invariant) semantics, that is, what this ‘recipe’ is about.

**1.1: From ordinary Language to the Language of Natural science within the ‘same’ scope of Semantics -**

The same scope of semantics can be recognized to have been extended to the *Language of Natural (Human) science* so long this language is intended to talk about the domain of our ordinary experiences.

Natural science, spelled out in the existing Language of mathematics - based on standard set theory , can be easily identified to have been endorsed with this very token of commitments to different ontological posits of common sense origins .

 However, as the role of observer (process of observation or measurement) remains *passive* in classical physical language, we are almost free to opt for any of the two choices (for all practical purposes) about the status of the *ontological posits in a Law -*

*Either*

The posits are *real* in the sense that, what stands for them in language / formalism is distinct from what they are about

*Or*

The heuristic posits are mere *constructs* in conventional sense and have no *rationally* justifiable ontological status.[2]

Any choice between *these two options* is almost *irrelevant* for a physical situation, classical Physics usually deals with - slowly varying at the moderate temperature of the Earth we are used to. No operationally distinguishable preference between *Realism* and *Phenomenalism / Nominalism* can be justified so far it is a question within the framework of classical Physical context. This very irrelevance *o*f choice is just a restatement of no *requirement* to provide hostage of an *explicit formal recipe of the role of observer in theory,* and this finds adequate expression in terms of commutative algebra.

Information about the dynamical evolution of a classical system can be expressed exhaustively in terms of a commutative algebra of observables which is equivalently algebra of subsets in phase space confirming *Boolean lattice structure*. As differential calculus is an aspect of commutative algebra (in fact all the aspects of differential calculus and consequently the theory of differential geometry, differential topology can be constructed over an arbitrary commutative algebra), it is not a problem to talk about a wide range of classical situations in terms of differential calculus with the *implicit* role of observer translated formally as commutative algebra within a *single* theoretical framework with no conceptual divide. This is also synonymous to say *NO t*o any preferred logic, for all practical purposes, for expressing the classical physical situation – using standard subject–predicate logic or any non-standard one, for example like, logic of empty subject term. Choice is at our disposal of rather than being Nature's imposed compulsion.[3]

**2.** **The ‘formal recipe’ of the *role of observer* in Quantum mechanics is not compatible with commutative framework**

Unlike classical Physics, it seems extraordinarily difficult to retain *Realism* within the framework of existing language of quantum mechanics. The ‘formal recipe’ of the role of observer in quantum theory was first devised by young Heisenberg ( 1924 ) in terms of matrices , though this was almost an accident as Heisenberg then did not even know what a matrix was about . We need not get here into the much talked about story of how young Heisenberg was turned up with some non-commutative square arrays , not recognizing them as matrices , while working with the ailing *Atomic theory* of Bohr .[4]

Fortunately , these arrays were soon recognized as matrix by Max Born ; [and] his own hypothesis proposed a bit later , (Born Rule ’ 1926 ) to make epistemic sense of Schrodinger’s $ψ$*-function* is well known to be compatible with Heisenberg Matrix . Discussion about this *compatibility* is part of any standard textual curriculum of quantum mechanics. We need not get into that here. [5]

But it can be said (effectively) that, developments from Heisenberg (1924) to Born (1926) can be *read* as indicating difficulties to attribute unambiguous *preexisting* properties to objectpertaining to quantum mechanical domain.Let us see how …

Born's postulate can be identified as a metarule which can be described as the rule that restricts the language of *external observer* not pertaining to the level of what is being observed.

In that way, we can say, that, what follows Born's rule are part of the *meta*language of the same theory - that serves to bridge between Schrodinger's approach and others in Goottingen and Cambridge. According to Born, |ψ|2 , rather than ψ alone can be attributed an epistemic sense for *our* level of experience. ψ alone is epistemically void and can only be made sense in terms of anonymous referent in context of an ensemble. But Born's rule - though good enough to calculate the probability of outcome realized for our level of experience, epitomized the crux of a new interpretational problem of its own. In fact this is all about the difference between quantum superposition and statistical mixture, which led to the difficulties to interpret probability measure individualistically. Large majority of standard textbooks don't mention this point with sufficient stress it deserves.

Presence of the interference term like 2Re[ C1C2\* $<u\_{n}\left|ψ\_{1}><u\_{n}<u\_{n}\right| ψ\_{2}>\*$] in the full expression of probability measure suggest a straightforward difficulty in property attribution to the members of the ensemble designated by ψ . We can't simply describe the pre measurement situation as an *incoherent totality* or statistical mixture of, for example, N|C1|2 systems in state |$u\_{1} >$, N|C2|2 systems in state |$u\_{2} >$ , and in that way we are not entitled to talk any more in terms of Gibbsian ensemble within M- B counting scheme.

As is well known that very often our knowledge about any state of affair happens to be less than complete, and in this situation we usually talk in terms of chance in epistemic sense. Let us take for example the knowledge of somebody's unknown location - *where is he !!* – we can ask ..

An incoherent totality or statistical mixture of all mental replicas of possible locations of the fellow sought after constitutes an ensemble with all possible locations as members of the ensemble with characteristic probability measures.

However, this is an instance of epistemic ignorance in sharp contrast with what we understand as ontological. But quantum probability doesn’t admit any such ignorance interpretation.

It seems difficult, if not impossible, to interpret |$<u\_{n}$|ψ$>$|2 individualistically. We can’t interpret |$<u\_{n}$|ψ$>$|2 as an objective measure of *preexisting* possessed value of an individual system .This is the bottom line of the whole story after Born rule . .

But if |$<u\_{n}$|ψ$>$|2 can't be a measure characterizing an individual's preexisting possessed values, then what does it actually characterize?

Bell ( 60 years of uncertainty, A. I. Miller( Ed.) , Plenum Press, NY, 1990) was most clear to articulate that |$<u\_{n}$|ψ$>$|2 provides a measure of system's *being found there* as against *being there* which we usually understand in context of classical/ epistemic ignorance.

So, |$<u\_{n}$|ψ$>$|2 can't, in principal, be referred to system alone, but system coupled inseparably with the measuring device or experimental context.

And system alone individually ( without any reference to measuring apparatus) even loose it's ontological import as the measured value, for example, Q *now* can't be attributed to it as possessed or preexisting just before the measurement - individual quantum state can't be *retro-constructed* !

Somehow the act of measurement participates to create a reality that can't be said to *preexist*. This is precisely how *meta*language of standard formalism doesn't seem to allow a consistent space time picture at the level of ontology.

These all lead directly to a question about an arbitrary dividing line between the classical and quantum reality. Wherefrom actually the rules of quantum mechanics takes over? This is often called Heisenberg's paradox, which is enigmatically about the location of the dividing line - quantum formalism, accommodating within it the formal recipe of the role of observer can't be understood as a description of relation between the sense experience without postulating some ad-hoc switch over device or arbitrary cut. In that way the situation is about an unbridgeable dualism or epistemic Gap. This is the essential hardcore of quantum measurement problem.

So keeping this in view, Nature seems to work on the basis of two level scheme of language. Making epistemic sense of any formalism is a difficult metalinguistic issue.

So the semantic consequence of these formal developments was about *failure* to assert something quite unexpected, which can’t be fit within the standard commutative framework of CTK

* there exists no X *with preexisting property P* as we are used to in our familiar classical situations *.*

This is equivalent to admit a difficulty with a consistent space time descriptionof quantum mechanical entities leading to an apparently *unbridgeable Dualism* between the level of *our everyday reality* and that of quantum.

In fact, some philosophers of science as well as a few physicists like later Schrodinger are even ready to go to an extent of describing quantum reality as a land of *anonymity* with neither *name* nor any clear *meaning* of ‘one’ of its particular (quantum mechanical) kind - domain of quantum looks very much like mystic land with *entity but no identity !*

 So Born postulate is followed by the challenge to make a *choice* between two options -

 Is knowledge *predetermined* by Nature? (Confirming CTK)

 *Or*

Nature is *open* to be determined by the mode of observation or our method of questioning addressed to Her, providing a scope of admitting free *choice/will* ( to impose on Nature) on *our* part .( beyond CTK )

**2.1: Deviation from *CTK?***

In fact, Knowledge has been traditionally held in most of the Philosophical traditions as something which is *predetermined* by Nature, rather than by *our* means of cognition or method of enquiry/ experimentally verifiable questions (EVQ) addressed to Her. While different natural languages capture more or less invariant spectrum of metaphysical commitments in an *objective way*, history of physics is surely not a matter of semantics invariance as this history instantiates a series of changes of Theory as well as semantics. So, any means of cognition on *our* part (you may say, any process of observation/ sense perception) does not apparently seem to predeterminethe semantic content of formalism in Physics in an invariant way.

Perhaps more straightforward is to say, that, our means of cognition / EVQ is expected just to passively ensure *within* our skin boundary what is ***pre-***determined by Nature. Philosophers as well as logicians are expected only to make sense of the *conceptual* and *logical recipe of what is going on inside our skin boundary* (just as a faithful representation of what *is* there outside).

 Kant is well known to have proposed a conceptual recipe ( *Critique of Pure Reason* ,1787) , presupposing Nature as an *invariant atemporal semantic totality,* expressed faithfully in Newtonian paradigm committed to Euclidean geometry and classical Aristotelian logic *( Surely he had no reason to envisage quantum mechanics )* . Kant’s epistemicanalysis famously inspired a small group of Physicists during the late 19th century, mostly within the ideological jurisdiction of Ernst Mach in Austria, to question about the epistemic justification of the-then classical Physical Laws – whether a Law captures an objective hidden *mechanism* or is only a *description of relation of the order of our sense experience without* commitment to anything *more hidden* .[6]

This is not difficult to recognize this debate as late 19th century outgrowth of the traditional conflicts between *realists* and *phenomelalists*.But as we mentioned before, that, there is no way to *operationally* prefer between these two options so far classical physical situations are concerned. Kantian recipe famously motivated the Vienna-based Logical Positivists also (almost during the same time quantum mechanics was developing) to try to ensure that the conceptual totality of *our knowledge* (including the scientific) is a matter of standard logico-empirical constructions.

But the lessons of standard quantum mechanics can be understood as an overt deviation from the Kantian framework of understanding knowledge recipe. In fact the logical positivists were convinced that the laws of logic were synonymous with the *laws of thought.* That is precisely why they tried to develop the logical *counterpart of neural recipe* following sense experience in terms of *first order logic with ordinary set theory as proper part*. Their failure is a well known episode in the history of analytical philosophy in West, and *Gödel* is usually credited to have supplied the rigorous formal justification for their failure in his *Incompleteness Theorem (1931)*. So perhaps a purely compositional and analytic semantics will not be of much help so far it is a question of ensuring the possibility of *our knowledge* within our skin boundary! However, that is the gateway to another story of profound significance more for the Neuroscientists. We need not get into that.

This seems to pose challenge, at the same time, for the neuroscientists to make sense of a *sense organ version* of the so called *quantum measurement problem* or, in other words, of a formal counterpart of the neural recipe of the way perception ensures invariant cognition. What sort of logic can possibly capture the neural recipe *within our skin* boundary confirming awareness?.

**2.2: The Question of choice between *Realism* and *Phenomenalism* is no longer *irrelevant* after these all**

Now we can formulate the traditional question of choice between *Realism* and *Phenomenalism* in a more specific context-

Whether the concept of *Individual (Thing)* is compelled *on* us as naturally *predetermined* irreducible semantic rudiment?

 Or

 Our *mind* needs it as a convenient heuristic posit - something as a part of restricted jurisdiction of the way knowledge is ensured within our skin boundary following sense experience ?

Making a choice is no longer irrelevant now even after the preliminary lessons of quantum mechanics we ‘formally’ learnt nearly 85 years back. But making a decisive choice isundoubtedly not easy*.* But  *,* at least a token of doubt about the notion of *individual* has to be admitted – this notion, though provisionally needed, but may not *be* a part of Nature’s objective scheme.

These all press upon us to ponder over the issue of interplay between *Logic, Language and Reality*  from within a more fundamental point of view (than is usually done by the Physicists) of compelling commitments to *individual*. Almost always we commit to the first option and this commitment pervades our usual mathematical artifacts based on commutative algebra.

**2.2.1: Reality of Future is questioned as a consequence -**

 Most possibly the single upshot of the whole implications can be stated precisely about the difficulty of holding the ontological definiteness of future. But, this is *not* how we are familiar to understand reality. We are comfortable to understand reality as almost a paradigmatic instance of predetermined *value definiteness* – things around us possess definite values of their own independent of observation recipe as well as *sequence* of observations. The dynamic evolution of state properties can be faithfully captured within the framework of *Boolean property structure*.

 So a failure to affirm quantum mechanically that *there exists a distinguishable object with property P is equivalent to* a failure of possessed value definiteness . So the issue of *choice* boils down finally to a question of choice between -

 Is it a failure of the mathematical Language of quantum mechanics (fragment of functional analysis coupled with non-commutative algebra)?

 Or

 This is the best we can capture about reality within a linguistic framework …

We are now better equipped formally as well as empirically to make at least a bet for a *choice*. Nature really needs some tricks in *Her exclusive store*! And what we describe as ‘*failure’* (of our comfortable picture of Reality) seems to be needed by Nature to administer a holistic control over the far flung parts of the huge Universe. We have learnt a lot about these after Bell (1965) as well as through different further developments about the possibility of dynamical entanglements. So now we are at least in a *better* position than ever to take stance in the traditional debate between *Realism and Phenomenalism* held for a long time mainly among Philosophers.

However, Physicists so far, on an average, seem to be largely equivocal or even insensitive about the choice between the two options. [7]

 Any further details to make more sense of this *choice artifact* would be a matter of another article. So let us keep this for the next occasion.

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**Notes and References -**

[1]: The grand spell of post-Humean empiricism within *phenomenalistic* *framework* came almost to an end by 40s of the last century leaving the message that statements about physical things could not be *translated* into statements about *actual* or *possible* sense data . It was soon realized that there is no finite set of statements about sense data from which even a single physical-object statement can be deduced.

[2]: This question of *choice* is just a scientific counterpart of the traditional debate between Phenomenalism and *Realism*, heightened, in one way or another, after Hume, George Berkley...

[3] : The concern of *Empty subject term* is one of the focal issues in Analytical philosophy in West. Russell famously proposed his *Theory of description* to solve this issue. Logic of *Empty Subject term* is often discussed as a modern variant of Buddhist Logic.

[4]: For a wonderful account of the early preparatory phases of Quantum mechanics, I will strongly recommend a rather old Book - The *Story of the Quantum,* *Banesh Hoffmann* , Dover Publications , 1958

Chapter *IX* is devoted to Heisenberg’s discovery of Matrix mechanics.

[5]: See , for example , Chapter 3 , Quantum Theory of Matter , John C. Slater , 1951

 [6]: *History of Physics is* almost oblivious to the great debate between Boltzmann alone and the Macheans , rooted mainly to divergent epistemological ( “ *erkenntnistheoretisch*”) attitudes to these very issues*.* However , though remained almost very little talked about , the epistemic essence of *Mach Boltzmann debate* can be recognized to have *resurfaced* itself again in the foundational debates in quantum mechanics particularly in its early Einstein Bohr versions , with Einstein (and partly early Schrodinger) playing almost the role of Boltzmann in his 20th century version and Bohr being halfhearted ( or confused !) *Macheans* ! This recurrence is not accidental if we remember that , History of Physics in general and quantum mechanics in particular can definitely be *read* as different instantiations of  *doubts to classical theory of knowledge committed to invariant semantics* Nature seems to ensure always within a compositional and analytical framework ( simple part and whole relation ).

[7]: Except very few like later Schrodinger , Born , Langevin , Heinz Post … and contemporary mathematician like Weyl , the indication of *failure of Individuation and Identity* was hardly a popular message for the mainstream Physicists. Consequently the responses of the Physicists (following 1927 …) cannot even be qualified as *properly addressed* to this failure at least from ontological point of view.