Three possible implications of spacetime discreteness

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We analyze the possible implications of spacetime discreteness for the special and general relativity and quantum theory. It is argued that the existence of a minimum size of spacetime may explain the invariance of the speed of light in special relativity and Einstein's equivalence principle in general relativity. Moreover, the discreteness of spacetime may also result in the collapse of the wave function in quantum mechanics, which may provide a possible solution to the quantum measurement problem. These interesting results might have some important implications for a complete theory of quantum gravity.

Key words: spacetime discreteness; speed of light; gravity; equivalence principle; wavefunction collapse; quantum gravity

1. Introduction

The existence of a minimum size of spacetime has been widely argued and acknowledged as a model-independent result of the proper combination of quantum mechanics (QM) and general relativity (GR) (see, e.g. Garay 1995 for a review). Moreover, the argument implies that the minimum time interval and the minimum length is respectively of the order of Planck time (T_p) and Planck length (L_p). The model-independence of the argument for the discreteness of spacetime strongly suggests that discreteness is probably a more basic feature of spacetime, and it may have a firmer basis beyond QM and GR, which are still based on continuous spacetime. Therefore, it may be appropriate to re-examine the relationship between the discreteness of spacetime and the existing fundamental theories from the opposite direction. In this paper, we will take this challenge and analyze the implications of spacetime discreteness for special relativity, general relativity and quantum theory. Since the formulations and meanings of discrete spacetime are different in the existing theories and arguments, we only resort to its minimum explanation here, namely that a spacetime interval shorter than the minimum size of spacetime (i.e. Planck scale) is physically meaningless, and it cannot be measured in principle either. For instance, a physical process can only happen during a time interval not shorter than the minimum time interval, namely Planck time.

The plan of this paper is as follows. In Sec.2 we argue that the discreteness of spacetime may result in the existence of a finite invariant speed when combining with the principle of relativity. This suggests that spacetime discreteness might provide a deeper foundation for special relativity. In Sec. 3, we further argue that the discreteness of spacetime may imply the fundamental existence of gravity as a geometric property of spacetime described by GR. In particular, the dynamical relationship between matter and spacetime holds true for microscopic particles. The argument may provide a basis for Einstein's equivalence principle. Moreover, the Einstein gravitational constant in GR can also be determined in terms of the minimum size of discrete spacetime. In Sec. 4, it is argued that the discreteness of spacetime may result in the dynamical collapse of the wave function in quantum mechanics. Besides, the minimum size of discrete spacetime also yields a plausible collapse criterion consistent with experiments. This may provide a possible solution to the quantum measurement problem. Conclusions are given in the last section.

2. Discreteness of spacetime implies the invariance of *c*

In special relativity, the speed of light in vacuum, denoted by c, is invariant in all inertial frames. This postulate is not a result of logical analysis, but a direct representation of experience. The theory itself does not answer why the speed of light is invariant. On the other hand, the suggested theory of relativity without light implies that c is not (merely) the speed of light, but a universal constant of nature, an invariant speed (see, e.g. Brown 2005; Pal 2003; Torretti 1983). Furthermore, it also suggests that the existence of an invariant speed partly results from the properties of space and time, e.g. homogeneity of space and time and isotropy of space. However, this theory is still incomplete and cannot even establish a real connection between its invariant speed with c (Brown 2005). Anyway, we need to explain exactly why there is a finite invariant speed.

Since speed is essentially the ratio of space interval and time interval, it is a natural conjecture that the existence of a finite invariant speed may result from some undiscovered property of space and time, as the existing theory of relativity without light has implied. In the following, we will argue that the property is probably the discreteness of spacetime.

Consider a particle moving in discrete space and time[1], in which there is a minimum length, denoted by L_U, and a minimum time interval, denoted by T_U, and the ratio of minimum length and minimum time interval is the speed of light $c \equiv L_U / T_U$ [2]. According to the principle of relativity, the discrete character of space and time, in particular the minimum time interval T_U and the minimum length L_U, should be the same in all inertial frames. If the minimum sizes of space and time are different in different inertial frames, then there will exist a preferred Lorentz frame, but this contradicts the principle of relativity. In the discrete space and time, when a particle moves a minimum length L_U during a minimum time interval T_U, its speed is L_U / T_U, which equals to the speed of light c. In order to see whether this speed is invariant in all inertial frames in discrete space and time, we need to analyze its transformation in different inertial frames.

[1] Here we only consider the motion of the mass center of a particle, which can be described by a material point. For a microscopic particle moving in vacuum, its velocity can be defined as the group speed of its wave function, which may describe random discontinuous motion of the particle (Gao 2006a, 2006b, 2008). For simplicity, we always say the motion of a particle.

[2] Note that the discreteness of spacetime here does not mean that spacetime has a fixed discrete lattice, and it only means that there exists a minimum spacetime interval. A spacetime interval shorter than this minimum has no physical meaning, and it cannot be measured by any experiment either.

Suppose the particle moves in the x direction with speed c in an inertial frame S. Then its speed will be not smaller than c in another inertial frame S' with a velocity in the $-x$ direction relative to S. In other words, the speed of the particle in S' may be equal to c or larger than c. If the speed of the particle in S' is larger than c, then it will move more than a minimum length L_U during a minimum time interval T_U, and thus moving L_U will correspond to a time interval shorter than T_U during the motion. This is prohibited because T_U is the minimum time interval in discrete space and time[3]. Therefore, the speed of the particle in S' can only be c. This result also means that when the particle moves in the x direction with speed c in the inertial frame S', its speed will be also c in the inertial frame S with a velocity in the x direction relative to S'. Since the inertial frames S and S' are arbitrary, we can reach the conclusion that if a particle moves with the speed c in an inertial frame, it will also move with the same speed c in all other inertial frames. This demonstrates the invariance of c in discrete space and time.

Since time interval and space interval are primary physical quantities, while speed, which is defined as the ratio of space interval and time interval, is a secondary physical quantity, it is understandable that the properties of the characteristic speed c can be further explained by the properties of space and time. As we have argued above, the constancy of c probably results from the discreteness of space and time. By comparison, if space and time are continuous, then no characteristic space and time sizes exist, and thus it seems unnatural that there exists a characteristic speed. On the other hand, if our argument is valid, then the existence of an invariant speed c will be a firm (and maybe the first) experimental evidence of discrete space and time, in which the ratio of the minimum length L_U and the minimum time interval T_U is c.

In conclusion, the discreteness of space and time requires the existence of a finite invariant speed, which value is equal to the speed of light c. This suggests that spacetime discreteness may provide a deeper logical foundation for special relativity[4].

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[3] It seems that we can similarly argue that the motion with a speed smaller than c is also prohibited in discrete space and time. Suppose a particle moves with a speed smaller than c. Then it will move less than L_U during T_U. Since L_U is the minimum space interval in discrete space and time, the movement is also prohibited. This result obviously contradicts experience, as particles can move with a speed smaller than c in reality. However, there exist some possible ways to avoid the contradiction. First, it can be conceived that the particle moves with c during some time, and stays still during other time. Then its average speed can be smaller than c, and thus the motion can be consistent with the existing experience. Next, if motion is essentially discontinuous and continuous motion is only an approximate average display (see Gao 2006a, 2006b, 2008 for a detail analysis), then the apparent continuous motion with a speed smaller than c will not be prohibited in discrete space and time either. The reason is that an object undergoing such motion actually does not move less than L_U during T_U, as its motion is discontinuous and it can move a distance larger than L_U during T_U in a discontinuous way. Moreover, since the direction of each discontinuous movement may be forward and backward, the average velocity of the object can still be smaller than c.

[4] It can be further argued that the theory of relativity should be defined in discrete space and time if space and time are indeed discrete. The new theory will be based on two postulates: (1) the principle of relativity; (2) the constancy of the minimum size of discrete spacetime, which states that the minimum time interval T_U and the minimum length L_U are invariant in all inertial frames. Note that some variants of relativity in discrete spacetime has already appeared in the research of quantum gravity (see Hagar 2009 for a general discussion). For example, doubly special relativity assumes two invariant scales, the speed of light c and a minimum length λ (Amelino-Camelia 2000, 2004; Kowalski-Glikman 2005), while triply special relativity assumes three invariant scales, the speed of light c, a mass κ and a length R (Kowalski-Glikman and Smolin

3. Discreteness of spacetime implies gravity

It is still a controversial issue whether gravity is fundamental or emergent. The solution of this problem may have important implications for a complete theory of quantum gravity. In this section we will analyze the possible implication of spacetime discreteness for gravity. It will be argued that spacetime discreteness may imply the fundamental existence of gravity as a geometric property of spacetime described by GR.

According to the Heisenberg uncertainty principle in QM we have

$$\Delta x \geq \frac{\hbar}{2\Delta p} \tag{1}$$

The momentum uncertainty of a particle, Δp, will result in the uncertainty of its position, Δx. This poses a limitation on the localization of a particle in nonrelativistic domain. There is a more strict limitation on Δx in relativistic QM. A particle at rest can only be localized within a distance of the order of its reduced Compton wavelength, namely

$$\Delta x \geq \frac{\hbar}{2m_0 c} \tag{2}$$

where m_0 is the rest mass of the particle. The reason is that when the momentum uncertainty Δp is greater than $2m_0 c$ the energy uncertainty ΔE will exceed $2m_0 c^2$, but this will create a particle anti-particle pair from the vacuum and make the position of the original particle invalid. It then follows that the minimum position uncertainty of a particle at rest can only be the order of its reduced Compton wavelength as denoted by Eq. (2). Using Lorentz transformation, the minimum position uncertainty of a particle moving with (average) velocity v is

$$\Delta x \geq \frac{\hbar}{2mc} \quad \text{or} \quad \Delta x \geq \frac{\hbar c}{2E} \tag{3}$$

where $m = m_0 / \sqrt{1 - v^2 / c^2}$ is the relativistic mass of the particle, and $E = mc^2$ is the total energy of the particle. This means that when the energy uncertainty of a particle is of the order of its (average) energy, it has the minimum position uncertainty. Note that Eq. (3) also holds true for particles with zero rest mass such as photons.

The above limitation is valid in continuous spacetime; when the energy E of a particle becomes arbitrarily large by acceleration, the uncertainty of its position Δx can still be arbitrarily small. However,

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2004). In these theories, the classical Minkowski spacetime is replaced by a quantum spacetime, such as κ-Minkowski noncommutative spacetime etc. Although these theories still have problems (e.g. energy-momentum conservation problem and composition problem) due to their extreme nonlinearity (Amelino-Camelia 2004), they are probably some in-between points on the road to a complete theory of quantum gravity (Amelino-Camelia and Smolin 2009). If the constancy of the speed of light is really a consequence of the discreteness of space and time as we have argued above, then it should not be an independent assumption, while a minimum time interval, together with a minimum length, should be the only two invariant scales in a fundamental theory.

the discreteness of spacetime will demand that the localization of any particle should have a minimum value L_U, namely Δx should satisfy the limiting relation

$$\Delta x \geq L_U \tag{4}$$

In order to satisfy this relation, the r.h.s of Eq. (3) should at least contain another term proportional to the mass or energy of the particle, namely in the first order of E it should be

$$\Delta x \geq \frac{\hbar c}{2E} + \frac{L_U{}^2 E}{2\hbar c} \tag{5}$$

This new inequality, which can be regarded as one form of generalized uncertainty principle[5], can satisfy the limitation relation imposed by the discreteness of spacetime. It means that the total uncertainty of the position of a pointlike particle has a minimum value L_U.

How to understand the new uncertainty term demanded by the discreteness of spacetime then? Obviously it indicates that the momentum-energy uncertainty of a particle results in an inherent uncertainty of its position proportional to the former. The problem is how the momentum-energy uncertainty generates the position uncertainty. First, the new position uncertainty cannot originate from the quantum motion of the particle, as it is very distinct from the usual quantum uncertainty of position, which is inverse proportional to the momentum uncertainty. Next, since there is only one particle, the new uncertainty of its position cannot result from any interaction between it and other particles such as electromagnetic interaction either. Therefore, there is only one possibility left, namely that the momentum-energy uncertainty of the particle influences the spacetime where it moves and further results in its position uncertainty. This further implies that the momentum and energy of a particle will change the geometry of its background spacetime (e.g. in each momentum branch of a quantum superposition). We can also give an estimate of the strength of this influence in terms of the new position uncertainty term $\frac{L_U{}^2 E}{2\hbar c}$. This term tells us that an energy uncertainty $\Delta E \approx E$ will lead to an inherent length uncertainty $\Delta L \approx \frac{L_U T_U E}{2\hbar}$ in space. This further requires that the energy E contained in a region with size L will change the proper size of the region to

$$L' \approx L + \frac{L_U T_U E}{2\hbar} \tag{6}$$

When the energy is equal to zero or there are no particles, the background spacetime will not be changed.

The above argument may provide a deeper basis for Einstein's equivalence principle in GR. The principle is usually argued with the help of classical mechanics and Newton's law of gravity, along with the

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[5] The argument here might be regarded as a reverse application of the generalized uncertainty principle (see, e.g. Garay 1995; Adler and Santiago 1999). But it should be stressed that the existing arguments for the principle are based on the analysis of measurement process, which conclusion is that it is impossible to *measure* positions to better precision than a fundamental limit. On the other hand, in the above argument, the uncertainty of position is objective and real, and the discreteness of spacetime means that the objective uncertainty of the position of a particle has a minimum value, which is independent of measurement.

experimental evidence of the equivalence of gravitational and inertial mass. The drawback of such an argument is that it may obscure the physical meaning of GR. For example, it suggests that gravity may be merely emergent at the classical level. By comparison, the above argument based on QM and the discreteness of spacetime implies that gravity is essentially a geometric property of spacetime, which is determined by the energy-momentum contained in that spacetime, not only at the classical level but also at the quantum level.

On the basis of the equivalence principle, there are some common steps to “derive” the Einstein field equations, the concrete relation between the geometry of spacetime and the energy-momentum contained in that spacetime, in terms of Riemann geometry and tensor analysis as well as the conservation of energy and momentum etc. For example, it can be shown that there is only one symmetric second-rank tensor that will satisfy the following conditions: (1) Constructed solely from the spacetime metric and its derivatives; (2) Linear in the second derivatives; (3) The four-divergence of which is vanishes identically (this condition guarantees the conservation of energy and momentum); (4) Is zero when spacetime is flat (i.e. without cosmological constant). These conditions will yield a tensor capturing the dynamics of the curvature of spacetime, which is proportional to the stress-energy density, and we can then obtain the Einstein field equations[6]

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \kappa T_{\mu\nu} \tag{7}$$

where $R_{\mu\nu}$ the Ricci curvature tensor, R the scalar curvature, $g_{\mu\nu}$ the metric tensor, κ is the Einstein gravitational constant, and $T_{\mu\nu}$ the stress-energy tensor.

The left thing is to determine the value of the Einstein gravitational constant κ. It is usually derived by requiring that the weak and slow limit of the Einstein field equations must recover Newton’s theory of gravitation. In this way, the gravitational constant is determined by experience as a matter of fact. If the above argument is valid, the Einstein gravitational constant can also be determined in theory in terms of the discreteness of spacetime. Consider an energy eigenstate limited in a region with radius *R*. The spacetime outside the region can be described by the Schwarzschild metric by solving the Einstein field equations:

$$ds^2 = (1 - \frac{r_S}{r})^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin\theta^2 d\phi^2 - (1 - \frac{r_S}{r}) c^2 dt^2 \tag{8}$$

where $r_S = \frac{\kappa E}{4\pi}$ is the Schwarzschild radius. By assuming that the metric tensor inside the region *R* is the same order as that on the boundary, the proper size of the region is

$$L \approx 2\int_0^R (1 - \frac{r_S}{R})^{-1/2} dr \approx 2R + \frac{\kappa E}{4\pi} \tag{9}$$

Therefore, the change of the proper size of the region due to the contained energy E is

[6] Another route to deriving the Einstein field equations is through an action principle using a gravitational Lagrangian.

$$\Delta L \approx \frac{\kappa E}{4\pi} \tag{10}$$

By comparing with Eq. (6) we find $\kappa = 2\pi \frac{L_U T_U}{\hbar}$ in Einstein's field equations. It can be seen that this formula itself also suggests that gravity originates from the discreteness of spacetime (together with the quantum principle that requires $\hbar \neq 0$). In continuous spacetime where $T_U = 0$ and $L_U = 0$, we have $\kappa = 0$, and thus gravity does not exist.

The above argument for the fundamental existence of gravity might have an important implication for quantum gravity. As we know, there exists a fundamental conflict between the superposition principle of QM and the general covariance principle of GR[7] (Penrose 1996, 1998, 2000; Rovelli 2004); QM requires a presupposed fixed spacetime structure to define quantum state and its evolution, but the spacetime structure is dynamical and determined by the state according to GR. The conflict indicates that at least one of these basic principles must be compromised in order to combine into a coherent theory of quantum gravity. But there has been a hot debate on which one should yield to the other. The problem is actually two-fold. On the one hand, QM has been plagued by the quantum measurement problem, and thus it is still unknown whether its superposition principle is universally valid, especially for macroscopic objects. On the other hand, it is not clear whether the gravity described by GR is emergent or not either. The existing heuristic "derivation" based on Newton's theory cannot determine whether gravity as a geometric property of spacetime described by GR is fundamental.

If gravity is really emergent, for example, GR is treated as an effective field theory, then the dynamical relation between the geometry of spacetime and the energy-momentum contained in that spacetime, as described by Einstein's field equations, will be not fundamental. As a consequence, different from the superposition principle of QM, the general covariance principle of GR will be not a basic principle, and thus no conflict will exist between quantum and gravity and we may directly extend the quantum field theory to include gravity (e.g. in string theory). In fact, the general covariance principle of GR has been compromised here because it is not fundamental. Note that, besides the string theory, there are also some interesting suggestions that gravity may be emergent, such as Sakharov (1968/2000)'s induced gravity (see also Visser 2002), Jacobson (1995)'s gravitational thermodynamics, and Verlinde (2010)'s latest idea of gravity as an entropic force (see also Gao 2010). On the other hand, if gravity is not emergent but fundamental as we have argued above, then quantum and gravity may be combined in a way different from the string theory. Now that the general covariance principle of GR is universally valid, the superposition principle of QM probably needs to be compromised when considering the fundamental conflict between them (Christian 2001; Gao 2006a; Penrose 1996, 1998, 2000). We will further analyze this possibility in terms of the discreteness spacetime in the next section.

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[7] This conflict between QM and GR can be regarded as a different form of the problem of time in quantum gravity. It is widely acknowledged that QM and GR contain drastically different concepts of time (and spacetime), and thus they are incompatible in nature. In QM, time is an external (absolute) element (e.g. the role of absolute time is played by the external Minkowski spacetime in quantum field theory). In contrast, spacetime is a dynamical object in GR. This then leads to the notorious problem of time in quantum gravity (Isham and J. Butterfield 1999; Kiefer 2004).

To sum up, we have argued that the discreteness of spacetime implies that gravity as a geometric property of spacetime described by GR is fundamental. In particular, the dynamical relationship between matter and spacetime holds true not only for macroscopic objects, but also for microscopic particles. This argument may provide a deeper basis for Einstein's equivalence principle. Moreover, the Einstein gravitational constant in GR can also be determined by the minimum size of discrete spacetime. It is also suggested that the fundamental existence of gravity as argued above may have further implications for a complete theory of quantum gravity.

4. Discreteness of spacetime may result in wavefunction collapse

It is an important issue in the foundations of QM whether the wave function really collapses. This is related to the notorious quantum measurement problem. In this section, we will argue that the discreteness of spacetime may result in the collapse of the wave function, and the minimum size of discrete spacetime also yields a plausible collapse criterion consistent with experiments. This may provide a possible solution to the quantum measurement problem.

Consider a quantum superposition of two different energy eigenstates. Each eigenstate has a well-defined static mass distribution in the same spatial region with radius *R*. For example, they are rigid balls of radius *R* with different uniform mass density. The initial state is

$$\psi(x,0) = \frac{1}{\sqrt{2}}[\varphi_1(x) + \varphi_2(x)] \tag{11}$$

where $\varphi_1(x)$ and $\varphi_2(x)$ are two energy eigenstates with energy eigenvalues E_1 and E_2 respectively. According to the linear Schrödinger evolution, we have:

$$\psi(x,t) = \frac{1}{\sqrt{2}}[e^{-iE_1 t/\hbar}\ \varphi_1(x) + e^{-iE_2 t/\hbar}\ \varphi_2(x)] \tag{12}$$

and

$$\rho(x,t) = |\psi(x,\mathrm{t})|^2 = \frac{1}{2}[\varphi_1{}^2(\mathrm{x}) + \varphi_2{}^2(\mathrm{x}) + 2\varphi_1(\mathrm{x})\varphi_2(\mathrm{x})\cos(\Delta E/\hbar \cdot t)] \tag{13}$$

This result indicates that the probability density $\rho(x,t)$ will oscillate with a period $T = h/\Delta E$ in each position of space, where $\Delta E = E_2 - E_1$ is the energy difference. This has no problem when the energy difference is very small as in usual situations. But when the energy difference ΔE exceeds the Planck energy E_p [8], $\rho(x,t)$ will oscillate with a period shorter than the minimum time interval T_U that is the order of T_p [9]. This is inconsistent with the requirement of the discreteness of spacetime, according to

[8] There is no limitation on the maximum of the energy of each eigenstate in principle. For example, the energy of a macroscopic object in a stationary state can be larger than the Planck energy (cf. Penrose 1996). On the other hand, if the energy of a microscopic particle cannot be larger than the Planck energy and QM indeed fails at the energy scale larger than the Planck energy, then there will be no quantum superposition of different spacetimes (as defined later) either, which is also consistent with the latter conclusion of this section.

[9] Here we ignore the gravitational fields in the superposition, as their existence does not influence our analysis and conclusion. When the energy difference is very tiny such as for a microscopic particle, the corresponding gravitational fields

which the minimum time interval T_U is the minimum distinguishable size of time, and no change can happen during a time interval shorter than T_U. Therefore, when considering the restriction of the minimum time interval, the superposition of two energy eigenstates with an energy difference larger than the Planck energy, which results in an oscillation with a period shorter than the minimum time interval, cannot hold and must collapse into one of its energy eigenstates[10], which has no such oscillation.

We can give a further argument for this conclusion. The existence of a minimum time interval demands that no oscillation or interference effect between the two energy branches in superposition can exist when their energy difference exceeds the Planck energy. If there is no such oscillation, the above probability density will not change with time and the corresponding superposition state will become an energy eigenstate. Since the measurement result of the superposition state can only be one of the initial energy eigenstates, this resulting energy eigenstate should be also one of the initial energy eigenstates. This then means that the superposition state has collapsed into one of its energy branches[11].

Therefore, in the above example, when the energy difference ΔE reaches the Planck energy E_P, the superposition should collapse into one of its branches during a Planck time scale T_P, so that the probability density $\rho(x,t)$ will not oscillate with a period shorter than this minimum time scale. At first sight, it seems that this collapse criterion favors the energy-driven collapse models (e.g. Adler 2003; Fivel 1997; Hughston 1996; Percival 1994), according to which the collapse time formula is

$$\tau_{\mathrm{c}} \approx \frac{\hbar E_{\mathrm{P}}}{(\Delta E)^2} \tag{14}$$

Indeed, this formula requires that when the energy difference ΔE is about the Planck energy E_P, the collapse time is about the Planck time T_P. However, as rightly pointed out by Pearle (2004), the energy-driven collapse models cannot consistently account for the existing experiments, as well as the definiteness of macroscopic objects. In fact, in the above special example, the energy difference is equivalent to some kind of difference of spacetimes, while a collapse model based on spacetime difference may be consistent with the existing experiments and macroscopic experience (Gao 2006a; Penrose 1996). In the following, we will give a more detailed analysis.

Since there is one kind of equivalence between the difference of energy distribution and the difference of spacetimes according to GR, the above result in fact implies that the quantum superposition of two

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in the superposition are almost the same and not orthogonal, and the interference effect between the two branches or the oscillation can be detected in experiment, while when the energy difference become larger and larger such as approaching the Planck energy, the gravitational fields in the superposition are not orthogonal either, and thus the oscillation can also be detected in principle.

[10] Note that Penrose (1996)'s gravity-induced collapse argument strongly depends on the assumption that gravity is not emergent but fundamental and the general covariance principle of GR is universally valid, and thus it does not refute other theories without quantum collapse such as string theory that rejects this assumption. By comparison, the argument here only depends on the existence of a minimum size of spacetime, though, as we have argued in the last section, gravity as a geometric property of spacetime described by GR may be indeed fundamental.

[11] In a similar way, the existence of a minimum length demands that no spatial oscillation can exist for the superposition of two momentum bases when their momentum difference exceeds the Planck energy divided by the speed of light. This suggests that the existence of a minimum length will also result in the collapse of the wave function.

different spacetimes cannot exist and should collapse into one of the definite spacetimes in the superposition. In order to make this argument more precise, we need to define the difference between two spacetimes here. As suggested by the generalized uncertainty principle denoted by Eq. (5), the difference of energy ΔE corresponds to the difference of spacetime $\frac{L_U{}^2 \Delta E}{2\hbar c}$. Then as to the branch states in a quantum superposition with energy difference ΔE, the difference between the spacetimes determined by the branch states may be characterized by the quantity $\frac{L_U{}^2 \Delta E}{2\hbar c}$. The physical meaning of such spacetime difference can be further clarified as follows. Let the two energy eigenstates in the superposition be limited in the regions with the same radius *R* (they may locate in different positions in space). Then the spacetime outside the region can be described by the Schwarzschild metric denoted by Eq. (8). By assuming that the metric tensor inside the region *R* is the same order as that on the boundary, the proper size of the region is

$$L \approx 2\int_0^R (1-\frac{r_S}{R})^{-1/2} dr \tag{15}$$

where $r_S = \frac{2GE}{c^4}$ is the Schwarzschild radius. Then the space difference of the two spacetimes in the superposition inside the region *R* can be characterized by

$$\Delta L \approx \int_0^R \frac{\Delta r_S}{R} dr = \Delta r_S = \frac{2L_p{}^2 \Delta E}{\hbar c} \tag{16}$$

This result is consistent with the generalized uncertainty principle. Accordingly as to the branch states in a quantum superposition, we can define the difference of their corresponding spacetimes as the difference of the proper spatial sizes of the regions occupied by these states. Such difference represents the fuzziness of the point-by-point identification of the spatial section of the two spacetimes (cf. Penrose 1996).

The spacetime difference defined above can be rewritten in the following form:

$$\frac{\Delta L}{L_p} \approx \frac{\Delta E}{E_P} \tag{17}$$

This relation indicates one kind of equivalence between the difference of energy and the difference of spacetimes for the above quantum superposition of two energy eigenstates[12]. Therefore, we can also give a collapse criterion in terms of spacetime difference. If the difference ΔL of the spacetimes in the superposition is close to the minimum size L_p, the superposition state will collapse to one of the definite spacetimes in about a minimum time interval T_p. If the difference ΔL of the spacetimes in the superposition is smaller than L_p, the superposition state will collapse after a finite time interval larger than T_p. As a result, the superposition of spacetimes can only possess a spacetime uncertainty smaller than

[12] It should be stressed that they are not equivalent in general situations. It is the difference of spacetimes, not the energy difference in the superposition that results in the collapse of the wave function (cf. Pearle 2004).

the minimum size in discrete spacetime. If such uncertainty limit is exceeded, the superposition will collapse to one of the definite spacetimes instantaneously. This will ensure that quantum state and its evolution can still be consistently defined during the process of quantum collapse, as the spacetimes with a difference smaller than the minimum size can be regarded as physically identical[13] (cf. Penrose 1996). A primary analysis has shown that this collapse criterion based on the minimum size of discrete spacetime is consistent with the existing experiments and macroscopic experience (Gao 2006a, 2006b).

Two comments are in order before we conclude this section. First, it is sometimes claimed that the existence of a minimal length suggests that space should have a quantized structure at the Planck scale, analogous to the quantization of energy in QM (see, e.g. Rovelli 2004), and thus it will support the assumption of the existence of quantum superposition of spacetimes. However, this claim might go beyond the basic meaning of the minimum size of spacetime. As we have seen above, contrary to this claim, the existence of a minimal length may prevent the superposition of different spaces and thus permits no quantization of space. Next, the above result seems at odds with the most approaches to quantum gravity, which are based on continuous spacetime manifold. However, it may be not against all expectations as we already reject the continuous spacetime manifold in our analysis by resorting to the discreteness of spacetime. Indeed, in view of the existence of an absolute minimum spacetime size one may plausibly question whether any theory based on shorter distances, such as a spacetime continuum, really makes sense (Adler and Santiago 1999). At least, one should worry whether it is appropriate in quantum gravity to assume the same ‘continuum’ (i.e. manifold) structure for spacetime as that employed in both QM and GR (Isham and Butterfield 1999).

To sum up, the discreteness of spacetime may result in the collapse of the wave function and further prohibit the existence of quantum superposition of different spacetimes. This may provide a possible solution to the quantum measurement problem. Moreover, quantum and gravity may be reconciled with the help of the quantum collapse in discrete spacetime as a result. In this way, there will be no quantized gravity in its usual meaning. In contrast to the semiclassical theory of quantum gravity, however, the theory will naturally include the backreactions of quantum fluctuations to gravity (e.g. the influence of wavefunction collapse to spacetime), as well as the reactions of gravity to quantum evolution (Gao 2006b). Therefore, it might provide a consistent framework for a fundamental theory of quantum gravity. Certainly, the details of such quantum collapse and the properties of the discrete spacetime need to be further studied. Our analysis suggests that spacetime may be not a pure quantum dynamical entity, but it is not wholly classical either.

5. Conclusions

We have argued that the existence of a minimum size of spacetime may explain the invariance of the speed of light and Einstein’s equivalence principle, and thus the discreteness of spacetime may provide a

[13] Due to the universal existence of quantum fluctuations, there still exists a bit of difference between the spacetimes whose difference is smaller than the minimum size of discrete spacetime. Such difference will generate a very slow collapse of the superposition of these spacetimes. Thus, strictly speaking, the spacetimes are almost physically identical.

deeper basis for special and general relativity. Moreover, we argue that spacetime discreteness may also help to solve the quantum measurement problem in quantum mechanics. These interesting results might have some further implications for a complete theory of quantum gravity, though it is still unclear how to incorporate the discreteness of spacetime into the unified theory.

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