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Article

Achilles' To-Do List

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Abstract: Much of the debate about the mathematical refutation of Zeno's paradoxes surrounds the logical possibility of completing supertasks—tasks made up of an infinite number of subtasks. Max Black and J.F. Thomson attempt to show that supertasks entail logical contradictions, but their arguments come up short. In this paper, I take a different approach to the mathematical refutations. I argue that even if supertasks are possible, we do not have a non-question-begging reason to think that Achilles' supertask is possible. The justification for the possibility of Achilles' supertask lies in the possibility of him completing other supertasks of the same kind, and the justification for the possibility of him completing these other supertasks lies in the possibility of him completing yet more supertasks ad infinitum.

Keywords: zeno; paradox; supertask

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1. Introduction

For the past century, much of the debate about Zeno's paradoxes of motion has centered around the concept of a supertask—a task made up of infinitely many subtasks. In order for Achilles to catch up to the tortoise in their footrace, he must perform an infinite number of catching-ups, but fortunately for Achilles, each task is easier than the last. With the mathematical discovery of a method for summing infinite geometric series, we can calculate how long it will take Achilles to catch up to the tortoise. Since the series representing the times it will take Achilles to accomplish each task is convergent, it sums to a finite number. Despite the fact that Achilles has an infinitely long to-do list, there is some reason to think that he can complete it in a finite time. But wait! Mathematics is fine on its own, but Achilles has to actually complete those infinite tasks. It is one thing to calculate when a supertask will be completed and another to actually do it. This distinction comes out of Max Black's [1] and J.F. Thomson's [2] attempts to show that completing a supertask is self-contradictory. In this paper, I join Black and Thomson in doubting the mathematical refutation of Zeno's paradoxes, but I do not claim that it is impossible to complete a supertask or that completed supertasks entail contradictions. Rather, I argue that the mathematical refutation of Zeno's paradoxes begs the question against Zeno, and so proponents of the approach fail to show that Achilles can actually finish his to-do list.

2. The Tortoise and the Dichotomy

For the purposes of this paper, I will only be concerned with two of Zeno's paradoxes of motion: Achilles' race with the tortoise and the dichotomy. Suppose that Achilles is in a footrace with a tortoise and that Achilles moves 10 times faster than the tortoise. The tortoise starts 10 m ahead of Achilles. Will Achilles ever pass the tortoise? We can break the race down into a series of points in time. Let t_0 be the start of the race. At t_1, Achilles has traveled 10 m and arrived at the tortoise's starting position. Of course, the tortoise has moved along the course to the 11 m point. At t_2, Achilles has traveled 11 m, but the tortoise is at the 11.1 m point. At t_3, Achilles reaches the 11.1 m point, and the tortoise is 0.01 m ahead of him. At each time, the tortoise is ahead of Achilles. Since there are an infinite number of these time points, Achilles will never catch up to the tortoise. Achilles has an infinite number of tasks to complete, but no matter how fast he is, his to-do list is just too long.

Not only can Achilles not catch the tortoise, he can not even get started. Suppose that he wanted to move from point A to point B. Before he can reach B, he must first move halfway. Before he can move halfway, he must move an eighth of the way—before that, a 16th, and so on. In Achilles' journey from A to B, there is no first task for him to complete because for any task he must complete, there is an infinity of other tasks. Similarly, there is no last task that Achilles must complete. If he moves halfway, reaching point C, which is between A and B, then he will have to reach point D, which lies halfway between C and B. He will also have to reach point E, which lies halfway between D and B, and so on. Achilles' to-do list is infinite in both directions.

The issue at hand in both paradoxes of motion is that if space is infinitely divisible, the number of tasks left to be completed is always infinite. If Achilles can continue getting closer and closer to the tortoise, then he can keep running into new time points where he has yet to catch up. Of course, if space is discrete instead of continuous, then there will be a t_n such that the distance covered by the tortoise between t_n and t_{n-1} is 0. As such, Achilles will catch up to the tortoise at t_n and pass them at t_{n+1}. Think about a race between Mario and Luigi in the Nintendo game Super Mario Bros. Mario and Luigi are separated by a distance in pixels. They can only ever move a discrete number of pixels, and so if one is moving faster than the other, the faster will at some moment pass the slower. With the dichotomy paradox, if spacetime were discrete, then we could not keep cutting the distance between A and B in half. So, if Achilles wants to move from A to B, then there are a finite number of "pixels" he must pass over. His to-do list would be finite.

3. Tasks and Supertasks

Some philosophers claim that it only seems impossible for Achilles to complete his infinitely long to-do list. A task made up of an infinity of subtasks is called a supertask. Some have argued that supertasks are completable. Suppose Achilles wishes to move from A to B, which are 10 m apart. Think of his to-do list written out.

Move halfway between A and B to point C. (5 m)
Move halfway between C and B to point D. (2.5 m)
Move halfway between D and B to point E. (1.25 m)
Move halfway between E and B to point F. (0.625 m)
...

Each task on Achilles' to-do list is easier than the last. If we suppose that Achilles runs at 10 m/s, then task 1 will take him half a second, task 2 will take a quarter of a second, task 3 will take an eighth of a second, and so on. If we add up the infinite series of times it takes Achilles to complete each subtask of the supertask, we obtain

$$\sum_{n=1}^{\infty} 2^{-n} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots = 1$$

The limit of the partial sums of the times for each subtask as n approaches infinity is 1 s. That is to say, the sum of the times it will take Achilles to complete each subtask is 1 s. This should come as no surprise. If Achilles is traveling at 10 m/s, then it should take him 1 s to travel the 10 m from A to B. While it is true that Achilles has an infinitely long to-do list, each task is easier than the last, and so the amount of time it will take Achilles to complete his to-do list is finite.

Max Black and J.F. Thomson both reject this solution to Zeno's paradoxes. Black asks us to imagine an infinity machine that moves marbles from one tray to another. Before the machine is started, there are two trays beside it. One tray, called A, contains an infinite number of marbles. The other tray, called B, is empty. Once the machine is started, it will begin moving marbles from A to B. For each marble it moves, the machine doubles its speed. So, if the first marble takes 1 min to move, the second marble will take half a minute, and so on. Summing the geometric series we obtain from this example, we should expect the machine to be finished after 2 min. In fact, Black describes a number of such machines and configurations of machines to deal with potential objections. We do not need to worry about the possibility of imagining infinities or any of the other worries Black considers. Our concern is just with the logical possibility of such a machine completing an infinite number of actions. In other words, will there ever be a moment where A is empty, and B has an infinite number of marbles in it? Black argues that such a moment cannot arrive. For each marble that is removed from A, there is another marble ready to go. Since there is no last marble to be moved, A will never be empty.

Richard Taylor [3] and John Watling [4] charge Black with question-begging in his argument against supertasks. Black, they claim, assumes that for a task to be completed, it must have a last subtask that is completed. That is to say, for the infinity machine to complete its supertask, there must be a last marble that is moved. This assumption can be jettisoned without a logical contradiction. Our intuitions tell us that there must be a last marble that is moved, but our intuitions often break down when it comes to infinities. With every day to-do lists, there is a last task on the list, but with infinitely long to-do lists, there is no last task. When completing a supertask that takes 2 s, as the timer gets closer and closer to 2 s, one gets closer and closer to finishing all of the tasks. When the timer reads 2 s, all of the tasks are completed.

Thomson provides a different approach to supertasks. Imagine a desk lamp with a switch on it. Thomson sets out to flip the switch on the lamp an infinite number of times. Each flip of the lamp's switch will take half as much time as the previous flipping. Assuming that the first flip takes 1 s, then all infinite flips will take 2 s total. Is the lamp on or off at the end of the infinite flips?

> *It seems impossible to answer this question. It cannot be on, because I did not ever turn it on without at once turning it off. It cannot be off, because I did in the first place turn it on, and thereafter I never turned it off without at once turning it on. But the lamp must be either on or off. This is a contradiction.* [2] (p. 5)

Paul Benacerraf [5] argues against those whom he calls the "new Eleatics". His focus is on Thomson's argument that completed supertasks produce contradictions. Regarding the lamp, Benacerraf claims that Thomson has failed to give us enough information to determine the status of the lamp when the supertask is completed. We cannot determine from the description of the supertask itself what will be the case after an infinite number of switch flips. Without a complete description, we cannot make a determination about the status of the lamp, and so we cannot say, as Thomson does, that it cannot be on and cannot be off.

> *True, for each n the value of the lamp after n switchings is accurately represented by its partial$_n$ sum; but what reason is there to believe that its value after all the switchings will be accurately represented by the sum of all the terms, i.e., by the limit of the partial$_n$ sums? None.* [5] (p. 771)

Black's argument against supertasks fizzles out because it makes unnecessary assumptions about the nature of tasks. Thomson's argument fails for a similar reason. He assumes that a description of a supertask also provides a description of the end state after the supertask. The assumptions in both Black's and Thomson's arguments are reasonable with every day, finite to-do lists, but we are not logically required to carry those assumptions over to infinitely long to-do lists. In the next two sections, I provide a different way to reject the mathematical refutations of Zeno's paradoxes. Unlike other modern Eleatics, I treat completing supertasks as logically possible.

4. Nested Supertasks

The mathematical refutation of Zeno's paradoxes works something like the following: Achilles' To-Do List (ATDL):

A to C
C to D
D to E
E to F

...
P1. Achilles can move from A to B if and only if he can complete (ATDL).
P2. Achilles can complete (ATDL)
C. Achilles can move from A to B.

Black and Thomson reject this argument by rejecting both P1 and P2. They reject P2 by arguing that it is impossible to complete a supertask. They then reject P1 by holding to the common sense position that motion is possible. So, if motion is possible without the possibility of completing supertasks, P1 is false.

We have seen, however, that there are some problems with Black's and Thomson's thought experiments. So, if one wishes to hold the line against supertasks, another route ought to be taken. Luckily, just such a route is available. Even if supertasks are possible, this does not prove that supertasks related to continuous motion are possible.

Suppose that Achilles wanted to carbo-load before his race with the tortoise, and so he wants to cook some pasta. He finds a recipe that lists out the individual tasks necessary to cook pasta. Assuming for the moment that Achilles can move, can he cook the pasta? Of course, he can, but one necessary condition on his ability to cook the pasta is that he can perform each of the necessary tasks listed in the recipe. If there are some necessary subtasks he cannot complete, then he cannot complete the task itself.

In the same way, if Achilles cannot complete the journey from C to D, then he cannot complete the journey from A to B. We need to know that each of the tasks on Achilles' To-Do List is possible before we can say that completing the supertask is possible. But, each task on (ATDL) is the same kind of task as the whole of (ATDL); that is to say, each task Achilles must complete is a supertask. He has a supertask of supertasks on his plate.

Note that the assumption here is different from the one Taylor and Watling charge Black with. I am not assuming that one must complete a first or a last subtask in order to complete a task. Instead, I am making what I hope is the less controversial assumption that one must complete all necessary subtasks to complete a task.

5. The Begged Question

If we assume that Achilles can complete the supertask of moving from A to B, then we must conclude that he can complete each of the necessary subtasks of (ATDL). Note that the sense of completion here is the sense of there being a time at which all of the tasks are complete, not the sense that would require Achilles to perform the first or the last of the tasks. That is to say, there must be a time at which all of the subtasks for the supertask of moving from A to C are completed, and similarly for C to D, and so on.

But herein lies the problem. The argument listed above claimed that Achilles could move from A to B because he could complete the supertask (ATDL). How do we know that he can complete (ATDL)? The mathematics of limits and sums of infinite geometric series do not give us enough evidence. After all, the sum described above requires Achilles to move from A to C in half a second. In order for us to conclude that he will be completed in 1 s, we must know that he can traverse the distance from A to C in halfa second. That is to say, in order to know that he can move from A to B, we have to first know that he can move from A to C.

We could construct a similar argument for the movement from A to C. Let ($ATDL_{sub}$) be the supertask for moving from A to C.

P1. Achilles can move from A to C if and only if he can complete ($ATDL_{sub}$).
P2. Achilles can complete ($ATDL_{sub}$)
C. Achilles can move from A to C.

($ATDL_{sub}$), just like (ATDL), is made up of an infinite number of subtasks that make up the points in between A and C. Of course, the same problem raises its head again. Let α be the point halfway between A and C. In order for Achilles to complete ($ATDL_{sub}$), he must complete the supertask ($ATDL_{2xsub}$) made up of the points in between A and α.

Of course, we could go on making arguments like the ones above to show that Achilles can complete each of the supertasks, but we will never reach an end. We are stuck in an infinite regress. The justification for P2 in each of the infinite arguments rests with another argument needing the same kind of justification. Without some other reason to think that Achilles can complete the special kinds of supertasks he is faced with, we never break free from the regress.

Supporters of the mathematical refutation may retort that at each stage in the regress, a justification can be given for the previous point in the regress. We know that we can keep giving the same argument infinitely, and so there is no premise in any of the infinite arguments that goes without support. It may be an infinite regress, but the supporters of the mathematical refutation may argue that it is not a vicious regress.

The issue here is that the goal of the supporter of the mathematical refutation is to show that completing a supertask of supertasks is possible. But, since this is the very question at hand in the debate, the supporter cannot use the possibility of completing a supertask of supertasks to support each layer of the regress without begging the question. At each stage in the regress, the modern Eleatics will ask for evidence that a supertask of supertasks is completable. If the only evidence that is given in response takes as a premise that completing a supertask of supertasks is possible, then we are left at an impasse. The mathematical refutation as it is now does not give the modern Eleatic a reason to accept the possibility of completing supertasks of supertasks.

The issue with Achilles' to-do lists is that they are made up of the same kinds of supertasks. The same issue does not arise with, for example, Black's infinity machines. The supertask for Black's machines is made up of discrete actions—moving individual marbles. So, assuming motion is possible, Black's machine could finish after 2 min because each of the tasks it must perform is a regular old mundane task, not a super task. So, even if we have a reason to think that supertasks are completable, that does not mean that Achilles' supertasks of supertasks are completable.

Suppose that all of the marbles that Black's infinity machine is set to move are labeled with the natural numbers. The machine follows the rule: move the marble with the lowest number on it next. So, it moves each marble in the order of its label, moving sequentially through the natural numbers. Now, suppose that whenever the machine is set to pick up a marble, a new marble is added to the tray. Suppose that before the new marble is added, the next marble to be moved is labeled n. The newly added marble's label displays the real number that is halfway between n and the label on the most recently moved marble. If no marble has been moved, then the newly added marble is labeled (½)(n).

One problem with Black's infinity machines is that they are not analogous to Achilles' supertask. Black's machines have a first task, even if they lack a last task. Each task is discrete and does not form a supertask. With this new version of the infinity machine thought experiment, to move the marble labeled '1', the machine must first move an infinite number of marbles. The same applies to moving the marble labeled '½'. Each task in the machine's supertask is itself a supertask. So, we cannot rely on the sum of the geometric series to argue that the machine can complete its supertask. After all, the same reasoning would be necessary at every level, and we would never reach any non-question-begging justification for the claim that the machine can complete its supertask.

6. Concluding Remarks

The common mathematical refutation of Zeno's paradoxes claims that Achilles can complete his supertask because such a task will take a finite amount of time. This argument, however, rests on the assumption that he can perform each of the necessary tasks that form the supertask in finite amounts of time. The tasks that form Achilles' supertask are themselves supertasks of the exact same kind. So, the argument must be applied again at the lower level to each of the tasks that make up (ATDL). The problem is that when we apply the argument again at lower levels to justify the premises of the argument at higher levels, we end up in an infinite regress. We never end up with an independent justification for the claim that Achilles can complete a supertask in a finite amount of time. As such, the only justifications that are supplied for the conclusion that Achilles can complete (ATDL) beg the question in the debate with the modern Eleatics.

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