

# A note on the possibility of incomplete theory.

Han Geurdes

*Geurdes Datascience, KvK 64522202,  
C vd Lijnstraat 164, 2593 NN, Den Haag Netherlands  
han.geurdes@gmail.com*

Koji Nagata

*Department of Physics, Korea Advanced Institute of Science and Technology, Daejeon, Korea*

Tadao Nakamura

*Department of Information and Computer Science,  
Keio University, 3-14-1 Hiyoshi, Kohoku-ku, Yokohama, Japan*

Ahmed Farouk

*Computer Sciences Department, Faculty of Computers and Information, Mansoura University and  
Scientific Research Group, Egypt*

In the paper it is demonstrated that Bell's theorem is an unprovable theorem.

Keywords: Negation Incompleteness, Bell's theorem, LHV model.

## INTRODUCTION

Let us start our paper with a quote from professor Friedmann's last lecture [? ]. (cit ...) Most [mathematicians] intuitively feel that the great power and stability of some "rule book for mathematics" is an important component of their relationship with mathematics. The general feeling is that there is nothing substantial to be gained by revisiting the commonly accepted rule book ... .

In 1964, John Bell wrote a paper [? ] about the possibility of hidden variables [? ] causing the entanglement correlation  $E(a,b)$  between two particles. In the present paper, an inconsistency similar to concrete mathematical incompleteness [? ], will be demonstrated from his theorem. The argument for mathematical incompleteness is to proof *and* refute with known concrete mathematical axioms the mathematical statement of Bell's theorem. The author is aware of the scepticism this may raise with certain readers. However, scepticism is simply not enough to push our proof of inconsistency aside and do "business as usual" with Bell's formula.

Bell, based his hidden variable description on particle pairs with entangled spin, originally formulated by Bohm [? ]. Bell used hidden variables  $\lambda$  that are elements of a universal set  $\Lambda$  and are distributed with a density  $\rho(\lambda) \geq 0$ . Suppose,  $E(a,b)$  is the correlation between measurements with distant A and B that have unit-length, i.e.  $\|a\| = \|b\| = 1$ , real 3 dim parameter vectors  $a$  and  $b$ . The basic physics experiment is as follows: Suppose on the A side we have measurement instrument A with parameter vector  $a$ . On the B-side we have measurement instrument B with parameter vector  $b$ . There is a (Euclidean) distance  $d(A,B) > 0$  between instruments A and B which can be large if necessary. In between the two instruments there is a source  $\Sigma$  generating particle pairs. We have,  $d(\Sigma, A) = d(\Sigma, B) = \frac{1}{2}d(A, B)$ . One particle of the pair is sent to A the other particle of the pair is sent to B. The physics of the two particles of the pair is such that they are entangled, [? ],[? ].

Then with the use of the  $\lambda$  we can write down the classical probability correlation between the two simultaneously measured particles. This is what we will call Bell's formula.

$$E(a,b) = \int_{\lambda \in \Lambda} \rho(\lambda) A(a,\lambda) B(b,\lambda) d\lambda \quad (1)$$

Note that if  $\ell$  is the short-hand notation for the random variable(s), the  $E(a,b)$  simply is the expectation value of the product of two  $\{-1,1\}$  functions,  $A(a,\lambda)$  and  $B(b,\lambda)$ . It can be written as  $E(a,b) = E_{\ell}(A(a,\ell)B(b,\ell))$ . In fact we are looking at a special case of covariance computation [? ] with the use of functions A and B, depending on parameters  $a$  and  $b$  and random variables captured with  $\ell$ , projecting in  $\{-1,1\}$ .

In (??) we therefore must have  $\int_{\lambda \in \Lambda} \rho(\lambda) d\lambda = 1$ . The integration  $\int_{\lambda \in \Lambda}$  can be over, as many as we please, variables and over ditto spaces  $\Lambda$ . The density  $\rho \geq 0$  also has a very general form.

### Proof

From (??) an inequality for four setting combinations,  $a, b, c$  and  $d$  can be derived as follows

$$E(a, b) - E(a, c) = \int_{\lambda \in \Lambda} d\lambda \rho(\lambda) A(a, \lambda) B(c, \lambda) A(d, \lambda) B(c, \lambda) - \int_{\lambda \in \Lambda} d\lambda \rho(\lambda) A(a, \lambda) B(b, \lambda) A(d, \lambda) B(b, \lambda) + \int_{\lambda \in \Lambda} d\lambda \rho(\lambda) A(a, \lambda) B(b, \lambda) - \int_{\lambda \in \Lambda} d\lambda \rho(\lambda) A(a, \lambda) B(c, \lambda) \quad (2)$$

because,  $\{B(c, \lambda)\}^2 = \{B(b, \lambda)\}^2 = 1$ . From this it follows

$$E(a, b) - E(a, c) = \int_{\lambda \in \Lambda} d\lambda \rho(\lambda) A(a, \lambda) B(b, \lambda) \{1 - A(d, \lambda) B(b, \lambda)\} + \int_{\lambda \in \Lambda} d\lambda \rho(\lambda) (-A(a, \lambda) B(c, \lambda)) \{1 - A(d, \lambda) B(c, \lambda)\} \quad (3)$$

Hence, because  $1 - A(x, \lambda) B(y, \lambda) \geq 0$  for all  $x, y$  with  $\|x\| = \|y\| = 1$  and  $A(a, \lambda) B(b, \lambda) \leq 1$  together with  $-A(a, \lambda) B(c, \lambda) \leq 1$ , it can be derived that

$$E(a, b) - E(a, c) \leq 2 - E(d, b) - E(d, c) \quad (4)$$

Or,

$$S(a, b, c, d) = E(a, b) + E(d, b) + E(d, c) - E(a, c) \leq 2. \quad (5)$$

Note, no physics assumptions were employed in the derivation of (??). It is pure mathematics. Suppose, further, that if we select for  $a, b, c$  and  $d$

$$a = \frac{1}{\sqrt{2}} (1, 0, 1), \quad d = \left( \frac{1}{2}, \frac{1}{\sqrt{2}}, -\frac{1}{2} \right) \\ b = (1, 0, 0), \quad c = (0, 0, -1) \quad (6)$$

then  $E(x, y)$  cannot be the inner product of the two vectors because,  $a \cdot b = \frac{1}{\sqrt{2}}, d \cdot b = \frac{1}{2}, d \cdot c = \frac{1}{2}$  and  $a \cdot c = -\frac{1}{\sqrt{2}}$ . Hence,

$$S(a, b, c, d) = (a \cdot b) + (d \cdot b) + (d \cdot c) - (a \cdot c) = \frac{1}{\sqrt{2}} + \frac{1}{2} + \frac{1}{2} - \left( -\frac{1}{\sqrt{2}} \right) = 1 + \sqrt{2} > 2$$

In [?] Peres gives supporting argumentation to the form,  $S(a, b, c, d) \leq 2$  derived here. So we can be sure (??) and  $S(a, b, c, d) \leq 2$ , are a generally valid expression for *all* possible models under the umbrella of (??).

### COUNTER PROOF

In this section we will demonstrate that  $E(x, y)$  can arbitrarily close approximate  $x \cdot y$ . As a reminder, both  $x \in \mathbb{R}^3$  and  $y \in \mathbb{R}^3$  are unit length parameter vectors, hence,  $E(x, y) \in [-1, 1]$ . Although the physical details are unimportant, they can be verified to be within the bounds of applicability of Bell's formula (??).

### Preliminaries

The model to be developed here follows the basic physical requirements of a local model. The requirements follow from looking at the physics experiment. In instrument A a set of hidden variables is supposed. Similarly, a set of hidden variables is supposed to reside in instrument B. The instruments are, as in the previous section, represented in the formulae by functions  $A(x, \lambda_I, \chi)$  and  $B(y, \lambda_{II}, \chi)$ . The (arrays of) hidden variables  $\lambda_I$  and  $\lambda_{II}$  are independent. A third set of hidden variables, denoted here by  $\chi$ , are carried by the particles. The  $\chi$  have a Gaussian density. The  $\chi$  variables are independent of  $\lambda_I$  and  $\lambda_{II}$ . Moreover,  $\lambda_I$  and  $\lambda_{II}$  are independent. Looking at (??) we see that  $\lambda = (\lambda_I, \lambda_{II}, \chi)$ . Hence, looking at (??),  $A(a, \lambda) = A(a, \lambda_I, \chi)$ ,  $B(b, \lambda) = B(b, \lambda_{II}, \chi)$  and  $\rho(\lambda) = \rho(\lambda_I, \lambda_{II}, \chi)$ . This is

a local and physically possible situation. Although the proof we deliver here is about a flaw in Bell's argumentation, hence is purely mathematics, the necessary basic physical requirements are fulfilled in the model.

It must be stressed that, in anticipation of a more detailed definition below, the mathematical form of the probability density  $\rho(\lambda_I, \lambda_{II}, \chi)$  remains *fixed* all the time. This can be easily verified in the section below devoted to the probability density. Clearly, the argumentation of that the inequality *cannot* be violated is then invalid. In the first place we show that clinging on to the inequality is merely attaching believe to one branch of the incompleteness which is demonstrated below. This believe is unfounded. The argumentation "the model is unphysical" is also broken because the basic requirements of a physical model are obeyed. Secondly, we already stated that the probability density remains fixed. Therefore it is possible to rightfully claim a genuine case of rejection of the *validity* of Bell's argumentation. It can be verified that we use a model that perfectly fits the physics requirements behind Bell's formula (??).

To wrap it up. There is no violation of locality in our model. There is no breach in the constancy of probability density form. The basic physics behind Bell's formula are fulfilled. The breakdown of Bell's argumentation is purely mathematical.

The opponent has to deliver proof why  $\lambda_I$  in instrument A and  $\lambda_{II}$  in instrument B that are independent and independently distributed variables and the independent and independently distributed  $\chi$  variables, carried by the particles, is not complying to physical realistic locality.

It will be shown that the argument of Bell is based on negation incompleteness. In other words, we will show  $S(a, b, c, d) > 2$  from the same formula that with the same physical requirements gives, along the branch of Bell's argumentation,  $S(a, b, c, d) \leq 2$ . Hence, we will show that Bell's formula supports negation incompleteness of the use of statistics in physics experimentation. If readers think otherwise then proof is the route to go. Believe, whoever is expressing it, should be -and actually is- worthless in scientific debate.

### Probability density

Let us in the first place define a probability density  $\rho$  based upon two separate  $\lambda$ 's and on  $(\chi_1, \chi_2, \chi_3)$ . Suppose,  $\alpha$  is a variable to indicate the two separate systems of hidden variables. Let us denote them with  $I$  and  $II$ , i.e.,  $\alpha \in \{I, II\}$ . Then,

$$\lambda_\alpha = (x_\alpha, \mu_{1,\alpha}, \mu_{2,\alpha}, \mu_{3,\alpha}, \tau_\alpha, n_\alpha) \in \mathbb{R}^6 \quad (7)$$

For  $\lambda_I$  we define a density  $\rho_I = \rho_I(\lambda_I)$  and for  $\lambda_{II}$  a density  $\rho_{II}(\lambda_{II})$ . The roman indices refer to the two different wings of the Bell experiment.

### The $\chi$ variables

For  $\vec{\chi} = (\chi_1, \chi_2, \chi_3) \in \mathbb{R}^3$  let us define the Normal Gaussian density

$$\rho_{Norm} = \rho_{Norm}(\chi_1, \chi_2, \chi_3) = \left(\frac{1}{2\pi}\right)^{3/2} \exp\left[-\frac{1}{2} \sum_{k=1}^3 \chi_k^2\right] \quad (8)$$

The integration of the normal density is,  $\int_{-\infty}^{\infty} d\chi_1 \int_{-\infty}^{\infty} d\chi_2 \int_{-\infty}^{\infty} d\chi_3$  and is denoted with brackets,  $\langle \cdot \rangle_{Norm}$  such that e.g.  $\langle \rho_{Norm} \rangle_{Norm} = 1$ . This enables us to formally write the total density as

$$\rho(\lambda_I, \lambda_{II}, \vec{\chi}) = \rho_I(\lambda_I) \rho_{II}(\lambda_{II}) \rho_{Norm}(\vec{\chi}) \quad (9)$$

The density defined in (??) should fulfill the requirements alluded to in the previous section devoted to the requirements of the physics behind the model. The  $\chi$  are mutually independent and are independent of the "instrument variables"  $\lambda_I$  and  $\lambda_{II}$ . Subsequently, let us turn to the use of the  $\chi$  variables in the model.

Let us, firstly, define the Heaviside function  $H(x) = 1 \Leftrightarrow x \geq 0$  and  $H(x) = 0 \Leftrightarrow x < 0$ . In the second place let us define a sign function from the Heaviside,  $\text{sign}(x) = 2H(x) - 1$ . Because of the symmetry of the Gaussian in (??), we have in the angular notation of integration for  $i, j = 1, 2, 3$  that

$$\langle \text{sign}(\chi_i) \text{sign}(\chi_j) \rho_{Norm} \rangle_{Norm} = \left(\frac{1}{2\pi}\right)^{3/2} \int_{-\infty}^{\infty} d\chi_1 \int_{-\infty}^{\infty} d\chi_2 \int_{-\infty}^{\infty} d\chi_3 \text{sign}(\chi_i) \text{sign}(\chi_j) \exp\left[-\frac{1}{2} \sum_{k=1}^3 \chi_k^2\right] = \delta_{i,j} \quad (10)$$

with,  $\delta_{i,j} = 1 \Leftrightarrow i = j$  and  $\delta_{i,j} = 0 \Leftrightarrow i \neq j$ .

*Definition of  $\rho_\alpha(\lambda_\alpha)$ ,  $\alpha \in \{I, II\}$*

Here we turn to the densities,  $\rho_\alpha(\lambda_\alpha)$ ,  $\alpha \in \{I, II\}$ . The  $\rho_\alpha(\lambda_\alpha)$  is a product of five factors,  $\rho_\alpha^r$ ,  $r = 0, 1, 2, 3, 4$ . We have, for  $T \in \mathbb{N}$  and  $T \gg 16$

$$\begin{aligned} \rho_\alpha^0 &= \frac{1}{16T \left(1 - \frac{4}{T}\right)}, \\ \rho_\alpha^1 &= \rho_\alpha^1(x_\alpha) = H\left(\frac{1}{4} + x_\alpha\right) H\left(-\frac{1}{T} - x_\alpha\right) + H\left(\frac{1}{4} - x_\alpha\right) H\left(-\frac{1}{T} + x_\alpha\right), \\ \rho_\alpha^2 &= \rho_\alpha^2(\vec{\mu}_\alpha) = \prod_{k=1}^3 H(1 + \mu_{k,\alpha}) H(1 - \mu_{k,\alpha}), \\ \rho_\alpha^3 &= \rho_\alpha^3(\tau_\alpha) = H(T + \tau_\alpha) H(T - \tau_\alpha), \\ \rho_\alpha^4 &= \rho_\alpha^4(n_\alpha) = 1 \Leftrightarrow n_\alpha \in \{0, 1\} \ \& \ \rho_\alpha^4 = \rho_\alpha^4(n_\alpha) = 0 \Leftrightarrow n_\alpha \notin \{0, 1\}. \end{aligned} \quad (11)$$

Hence, using (??) we then define  $\rho_\alpha = \prod_{r=0}^4 \rho_\alpha^r$ .

Subsequently, let us also introduce the angle notation for integration of  $\alpha$  densities similar to what we wrote for the Normal density. We have,  $T \gg 16$

$$\langle \rho_\alpha \dots \rangle_\alpha = \frac{1}{2^4 T \left(1 - \frac{4}{T}\right)} \left( \int_{-\frac{1}{4}}^{-\frac{1}{T}} dx_\alpha + \int_{\frac{1}{T}}^{\frac{1}{4}} dx_\alpha \right) \prod_{k=1}^3 \int_{-1}^1 d\mu_{k,\alpha} \int_{-T}^T d\tau_\alpha \sum_{n_\alpha=0}^1 \dots \quad (12)$$

The previous leads us to  $\langle \rho_\alpha \rangle_\alpha = \frac{2}{2^4 T \left(1 - \frac{4}{T}\right)} (2^3 \times 2T) \left(\frac{1}{2} - \frac{2}{T}\right) = 1$ , and,  $T \sim$  sufficiently large number. Looking at the definition of the total density in (??), it can be derived that

$$\int_{\lambda \in \Lambda} d\lambda \rho(\lambda) = \langle \langle \rho_I \rangle_I \langle \rho_{II} \rangle_{II} \rho_{Norm} \rangle_{Norm} = \langle \rho_I \rangle_I \langle \rho_{II} \rangle_{II} \langle \rho_{Norm} \rangle_{Norm} = 1 \quad (13)$$

Hence, a valid probability density in (??) is obtained where use is made of (??) and (??). The density given in (??) is a valid fixed form density that is completely local.

*Auxiliary functions*

*The auxiliary function  $\Delta_T(y)$* : Let us in the first place define

$$\Delta_T(y) = \frac{2/\pi}{1 + T^2 y^2} \quad (14)$$

Then, because  $1 + T^2 y^2 \geq 1$  for  $y \geq 0$ , we find that  $-T \leq T \Delta_T(y) \leq T$  is valid and so,  $\text{sign}(T \Delta_T(y) - \tau_\alpha)$  can be meaningfully employed in an integration.

$$\int_{-T}^T \text{sign}(T \Delta_T(y) - \tau_\alpha) d\tau_\alpha = \int_{-T}^{T \Delta_T(y)} d\tau_\alpha - \int_{T \Delta_T(y)}^T d\tau_\alpha = (T \Delta_T(y) - (-T)) - (T - T \Delta_T(y)) = 2T \Delta_T(y) \quad (15)$$

This is true for arbitrary real  $y$ . Hence, also for  $y = x_\alpha^2 - \frac{1}{T^2}$  the previous is true.

*Elements of the measurement functions*: In the second place let us define

$$\begin{aligned} \sigma_a &= \sum_{k=1}^3 a_k \text{sign}(\chi_k) \\ \sigma_b &= \sum_{k=1}^3 b_k \text{sign}(\chi_k) \end{aligned} \quad (16)$$

It is easily demonstrated that  $|\sigma_a| \leq \sqrt{3}$  and  $|\sigma_b| \leq \sqrt{3}$ .

*Indicators:* In the third place, let us define three disjoint partitions of the real interval  $[-\sqrt{3}, \sqrt{3}]$ .

$$\begin{aligned} I_1 &= \{x \in \mathbb{R} \mid -\sqrt{3} \leq x < -1\} \\ I_2 &= \{x \in \mathbb{R} \mid -1 \leq x \leq 1\} \\ I_3 &= \{x \in \mathbb{R} \mid 1 < x \leq \sqrt{3}\} \end{aligned} \quad (17)$$

Clearly,  $I_1 \cap I_2 = \emptyset$  together with  $I_2 \cap I_3 = \emptyset$  and  $I_3 \cap I_1 = \emptyset$ . With the use of the three disjoint intervals we may employ the following auxiliary function  $\iota_k(x) = 1 \Leftrightarrow x \in I_k$  and  $\iota_k(x) = 0 \Leftrightarrow x \notin I_k$ . If  $\iota_1(x) = 1$ , then,  $\iota_2(x) = \iota_3(x) = 0$ . If  $\iota_2(x) = 1$ , then,  $\iota_1(x) = \iota_3(x) = 0$ . If  $\iota_3(x) = 1$ , then,  $\iota_2(x) = \iota_1(x) = 0$ .

*Auxiliary functions in measurement functions:* Let us fourthly also define functions that will be employed together with the  $\iota_k(x)$ , with,  $k = 1, 2, 3$ .

$$\begin{aligned} s_{1,\alpha}(z_\alpha, \lambda_\alpha) &= \{n_\alpha \text{sign}(z_\alpha + 1 - \mu_{1,\alpha}) - \delta_{0,n_\alpha}\} \text{sign}(T\Delta_T(f_\alpha(x_\alpha)) - \tau_\alpha) \\ s_{2,\alpha}(z_\alpha, \lambda_\alpha) &= \text{sign}(z_\alpha - \mu_{2,\alpha}) \\ s_{3,\alpha}(z_\alpha, \lambda_\alpha) &= \{n_\alpha \text{sign}(z_\alpha - 1 - \mu_{3,\alpha}) + \delta_{0,n_\alpha}\} \text{sign}(T\Delta_T(f_\alpha(x_\alpha)) - \tau_\alpha) \end{aligned} \quad (18)$$

The  $z_\alpha$  is a short-hand and follows,  $z_I = \sigma_a$  and  $z_{II} = \sigma_b$ , with  $\alpha \in \{I, II\}$ . The  $\sigma_a$  and  $\sigma_b$  are defined in (??). It is quite easily verifiable that  $s_{k,\alpha}(z_\alpha, \lambda_\alpha) \in \{-1, 1\}$ , with  $k = 1, 2, 3$ . Note,  $n_\alpha \in \{0, 1\}$ . In (??) the short-hand,  $f_\alpha(x_\alpha) \equiv x_\alpha^2 - \frac{1}{T^2}$  is employed.

### Measurement functions

With the use of the previous definitions we are now able to define the measurement functions A and B.

$$\begin{aligned} A(a, \lambda_I, \vec{\chi}) &= \sum_{k=1}^3 \iota_k(\sigma_a) s_{k,I}(\sigma_a, \lambda_I), \\ B(b, \lambda_{II}, \vec{\chi}) &= \sum_{k=1}^3 \iota_k(\sigma_b) s_{k,II}(\sigma_b, \lambda_{II}) \end{aligned} \quad (19)$$

Because the  $\iota_k(x)$ ,  $k = 1, 2, 3$  only have one of them unequal to zero, i.e. the  $I_k$  of (??) are disjoint, and the  $s$  of equation (??) are in  $\{-1, 1\}$ , we have that both  $A(a, \lambda_I, \vec{\chi}) \in \{-1, 1\}$  and  $B(b, \lambda_{II}, \vec{\chi}) \in \{-1, 1\}$ . Hence the measurement functions in (??) are valid in a Bell correlation  $E(a, b)$  such as given in (??). No deeper physics assumption hides behind this because one simply may select functions that project in  $\{-1, 1\}$ . Bell's formula is general. The A and B are called measurement functions but that is totally unimportant to the mathematics to be developed here.

Clearly, we can conclude that our definitions comply to the basic physical requirements of a local model. Hence, the model is allowed in Bell's formula. Note that the measurement representing functions, projecting in  $\{-1, 1\}$ , also follow the basic physical requirements. The derivation of  $S(a, b, c, d) \leq 2$  in (??) is therefore possible in this case. We will show that this is just one branch of the argument.

### Evaluation

Looking at Bell's correlation in (??) let us write

$$\begin{aligned} E(a, b) &= \langle \langle \langle \rho_I \rho_{II} \rho_{Norm} A(a, \lambda_I, \vec{\chi}) B(b, \lambda_{II}, \vec{\chi}) \rangle_I \rangle_{II} \rangle_{Norm} = \\ &= \langle \langle \rho_I A(a, \lambda_I, \vec{\chi}) \rangle_I \langle \rho_{II} B(b, \lambda_{II}, \vec{\chi}) \rangle_{II} \rho_{Norm} \rangle_{Norm} \end{aligned} \quad (20)$$

Note,  $\lambda_I$  is only found in  $\rho_I$  and  $A(a, \lambda_I, \vec{\chi})$  while  $\lambda_{II}$  is only found in  $\rho_{II}$  and  $B(b, \lambda_{II}, \vec{\chi})$ . The  $\vec{\chi}$ , via the  $\sigma_a$  and  $\sigma_b$  dependence is shared between functions A and B. Note for completeness that the function  $B$  does not depend on  $a$  and  $A$  does not depend on  $b$  which is in accordance with Einstein's locality condition [? ].

In order to have a proper evaluation of the integrals in  $\langle \rho_I A(a, \lambda_I, \vec{\chi}) \rangle_I$ , and  $\langle \rho_{II} B(b, \lambda_{II}, \vec{\chi}) \rangle_{II}$  it is sufficient to look at the  $A$  side only. The  $B$  side evaluations obviously follows similar rules.

We can write explicitly for  $\langle \rho_I A(a, \lambda_I, \vec{\chi}) \rangle_I$

$$\langle \rho_I A(a, \lambda_I, \vec{\chi}) \rangle_I = \frac{1}{2^4 T (1 - \frac{4}{T})} \left( \int_{-\frac{1}{4}}^{-\frac{1}{T}} dx_I + \int_{\frac{1}{T}}^{\frac{1}{4}} dx_I \right) \prod_{k=1}^3 \int_{-1}^1 d\mu_{k,I} \int_{-T}^T d\tau_I \sum_{n_I=0}^1 A(a, \lambda_I, \vec{\chi}) \quad (21)$$

As it follows from (??), we can have three cases for  $\nu_k(\sigma_a)$ . Suppose, the selection  $a$  and the values of  $\vec{\chi}$  are such that  $\sigma_a$  is in  $I_1$ . Then  $A(a, \lambda_I, \vec{\chi}) = s_{1,I}(\sigma_a, \lambda_I)$ . Hence, with the use of (??)

$$\begin{aligned} \langle \rho_I A(a, \lambda_I, \vec{\chi}) \rangle_I &= \frac{1}{2^4 T (1 - \frac{4}{T})} \left( \int_{-\frac{1}{4}}^{-\frac{1}{T}} dx_I + \int_{\frac{1}{T}}^{\frac{1}{4}} dx_I \right) \prod_{k=1}^3 \int_{-1}^1 d\mu_{k,I} \times \\ &\int_{-T}^T d\tau_I \sum_{n_I=0}^1 \{n_I \text{sign}(\sigma_a + 1 - \mu_{1,I}) - \delta_{0,n_I}\} \text{sign}(T\Delta_T(f_I(x_I)) - \tau_I) \end{aligned} \quad (22)$$

From (??) it already follows that the  $\tau_I$  integral in (??) equals  $2T\Delta_T(f_I(x_I))$ . So let us look at the  $\mu$  integrals and the  $n_I$  sum. Before entering into more details let us note that  $(\sigma_a + 1) \in [-1, 1]$  and so

$$\int_{-1}^1 d\mu \text{sign}(\sigma_a + 1 - \mu) = \int_{-1}^{\sigma_a+1} d\mu - \int_{\sigma_a+1}^1 d\mu = 2(\sigma_a + 1) \quad (23)$$

We subsequently see, because  $\int_{-1}^{+1} d\mu_{2,I} = \int_{-1}^{+1} d\mu_{3,I} = 2$ , together with  $\int_{-1}^{+1} d\mu_{1,I} = 2$ ,

$$\begin{aligned} \sum_{n_I=0}^1 \left( \prod_{k=1}^3 \int_{-1}^1 d\mu_{k,I} \right) \{n_I \text{sign}(\sigma_a + 1 - \mu_{1,I}) - \delta_{0,n_I}\} &= 2^3 \sum_{n_I=0}^1 [n_I(\sigma_a + 1) - \delta_{0,n_I}] = \\ &2^3[-1 + (\sigma_a + 1)] = 2^3 \sigma_a \end{aligned} \quad (24)$$

Hence, if  $K_T$  is defined by

$$K_T \equiv \left( \int_{-\frac{1}{4}}^{-\frac{1}{T}} dx_I + \int_{\frac{1}{T}}^{\frac{1}{4}} dx_I \right) \Delta_T(f_I(x_I)) \quad (25)$$

then,  $\langle \rho_I A(a, \lambda_I, \vec{\chi}) \rangle_I = \frac{\sigma_a K_T}{(1 - \frac{4}{T})}$  when  $\sigma_a \in I_1$ .

Let us now suppose,  $\sigma_a \in I_3$ , i.e.  $\sigma_a - 1 \in [-1, 1]$ . Hence, only  $\nu_3(\sigma_a) = 1$  and hence,  $A(a, \lambda_I, \vec{\chi}) = s_{3,I}(\sigma_a, \lambda_I)$ . This implies,

$$\begin{aligned} \langle \rho_I A(a, \lambda_I, \vec{\chi}) \rangle_I &= \frac{1}{2^4 T (1 - \frac{4}{T})} \left( \int_{-\frac{1}{4}}^{-\frac{1}{T}} dx_I + \int_{\frac{1}{T}}^{\frac{1}{4}} dx_I \right) \prod_{k=1}^3 \int_{-1}^1 d\mu_{k,I} \times \\ &\int_{-T}^T d\tau_I \sum_{n_I=0}^1 \{n_I \text{sign}(\sigma_a - 1 - \mu_{3,I}) + \delta_{0,n_I}\} \text{sign}(T\Delta_T(f_I(x_I)) - \tau_I) \end{aligned} \quad (26)$$

In the case that  $\sigma_a \in I_3$ , we also find  $\langle \rho_I A(a, \lambda_I, \vec{\chi}) \rangle_I = \frac{\sigma_a K_T}{(1 - \frac{4}{T})}$ . Finally let us look at the case where  $\sigma_a \in I_2$ . Here we have  $\sigma_a \in [-1, 1]$ . So,

$$\langle \rho_I A(a, \lambda_I, \vec{\chi}) \rangle_I = \frac{1}{2^4 T (1 - \frac{4}{T})} \left( \int_{-\frac{1}{4}}^{-\frac{1}{T}} dx_I + \int_{\frac{1}{T}}^{\frac{1}{4}} dx_I \right) \prod_{k=1}^3 \int_{-1}^1 d\mu_{k,I} \int_{-T}^T d\tau_I \sum_{n_I=0}^1 \text{sign}(\sigma_a - \mu_{2,I}) \quad (27)$$

The result of integration in (??) is that  $\langle \rho_I A(a, \lambda_I, \vec{\chi}) \rangle_I = \frac{\sigma_a (1 - \frac{4}{T})}{(1 - \frac{4}{T})} = \sigma_a$  and  $T \sim$  sufficiently large number.

#### The integral $K_T$

In two cases of  $\sigma_a$  we have  $\langle \rho_I A(a, \lambda_I, \vec{\chi}) \rangle_I = \frac{\sigma_a K_T}{1 - \frac{4}{T}}$  and  $K_T$  is defined in (??). For the ease of notation let us write  $T = n$ . Let us repeat the definition of the  $K$  integral

$$K_n = \left( \int_{-\frac{1}{4}}^{-\frac{1}{n}} dx + \int_{\frac{1}{n}}^{\frac{1}{4}} dx \right) \Delta_n(x^2 - (1/n^2)) \quad (28)$$

The integral we want to discuss here is then re-written, using  $\Delta_n(x^2 - (1/n^2))$  defined in (??) as

$$K_n = \frac{2}{\pi} \int_{-\frac{1}{4}}^{-\frac{1}{n}} \frac{dx}{1 + n^2(x^2 - (1/n^2))^2} + \frac{2}{\pi} \int_{\frac{1}{n}}^{\frac{1}{4}} \frac{dx}{1 + n^2(x^2 - (1/n^2))^2} \quad (29)$$

Now let us take,  $y = x^2 - (1/n^2)$ . The upper limit of  $y$  is,  $\frac{1}{4^2} - \frac{1}{n^2}$ ,  $n \gg 16$ , while the lower limit is 0. Hence, for negative  $x$ , we have  $x = -\sqrt{y + \frac{1}{n^2}}$ . For positive  $x$ , we see,  $x = \sqrt{y + \frac{1}{n^2}}$ . Hence, noting  $dx = \pm \frac{dy/2}{\sqrt{y + \frac{1}{n^2}}}$ , in terms of  $y$  we can write for the two terms in  $K_n$

$$K_n = -\frac{2}{\pi} \int_{\frac{1}{4^2} - \frac{1}{n^2}}^0 \frac{dy}{1 + n^2 y^2} \frac{1/2}{\sqrt{y + \frac{1}{n^2}}} + \frac{2}{\pi} \int_0^{\frac{1}{4^2} - \frac{1}{n^2}} \frac{dy}{1 + n^2 y^2} \frac{1/2}{\sqrt{y + \frac{1}{n^2}}} \quad (30)$$

Hence,

$$K_n = \left(\frac{2}{\pi}\right) 2 \int_0^{\frac{1}{4^2} - \frac{1}{n^2}} \frac{dy}{1 + n^2 y^2} \frac{1/2}{\sqrt{y + \frac{1}{n^2}}} \quad (31)$$

Let us in the first place try to find the upper limit of  $K_n$  from the previous equation. Note, for  $n > 4$  that  $y + \frac{1}{n^2} \geq \frac{1}{n^2}$ , hence,  $\frac{1}{\sqrt{y + \frac{1}{n^2}}} \leq n$ , given  $\frac{1}{4^2} - \frac{1}{n^2} \geq y \geq 0$ . This implies

$$K_n \leq \frac{2}{\pi} \int_0^{\frac{1}{4^2} - \frac{1}{n^2}} \frac{ndy}{1 + n^2 y^2} \leq \frac{2}{\pi} \arctan \left[ \frac{n}{4^2} - \frac{1}{n} \right] \leq 1, \quad (n \sim \text{large}). \quad (32)$$

The lower limit in  $K_n$  can be found, looking at,  $1 + n^2 y^2 \leq 1 + \epsilon^2 + n^2 y^2$ , hence,

$$\frac{1}{1 + n^2 y^2} \geq \left( \frac{1}{1 + \epsilon^2} \right) \left\{ \frac{1}{1 + n^2 \left( \frac{y}{\sqrt{1 + \epsilon^2}} \right)^2} \right\}.$$

Let us, in the second place, take  $z = y/\sqrt{1 + \epsilon^2}$ , then, with  $dz = dy/\sqrt{1 + \epsilon^2}$  we can rewrite the lower limit like

$$K_n \geq \frac{2}{\pi} \frac{1}{\sqrt{1 + \epsilon^2}} \int_0^{z_{max}} \frac{ndz}{1 + n^2 z^2} \frac{1}{\sqrt{1 + n^2 z \sqrt{1 + \epsilon^2}}} \quad (33)$$

together with,  $z_{max} = (\frac{1}{4^2} - \frac{1}{n^2})/\sqrt{1 + \epsilon^2}$ . Note,  $-1 \leq \frac{2}{\pi} \arctan(x) \leq 1$  for all  $x \in \mathbb{R} \cup \{-\infty, \infty\}$ . With arctan the inverse function of the function  $-\infty \leq \tan(x) \leq \infty$ , with,  $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$  is intended. Using,  $\frac{d}{dz} \arctan(nz) = \frac{n}{1 + n^2 z^2}$  we are able to write

$$K_n \geq \frac{2}{\pi} \frac{1}{\sqrt{1 + \epsilon^2}} \int_0^{z_{max}} dz \left( \frac{d}{dz} \arctan(nz) \right) \left[ 1 + n^2 z \sqrt{1 + \epsilon^2} \right]^{-1/2} \quad (34)$$

Because  $\arctan(0) = 0$  and we have  $0 \times n^2 = 0$  when  $n \sim$  sufficiently large number, it follows that the constant factor,  $C_n$  in a partial integration treatment of the right hand of (??) looks like

$$C_n = \frac{2/\pi}{\sqrt{1 + \epsilon^2}} \left\{ \arctan \left[ \left( \frac{n}{4^2} - \frac{1}{n} \right) / \sqrt{1 + \epsilon^2} \right] \left[ 1 + n^2 z_{max} \sqrt{1 + \epsilon^2} \right]^{-1/2} - \arctan(0) [1 + (n^2 \times 0)]^{-1/2} \right\} \quad (35)$$

and  $1 + n^2 z_{max} \sqrt{1 + \epsilon^2} = 1 + \frac{n^2}{4^2} - 1$ . Hence,  $\left[ 1 + n^2 z_{max} \sqrt{1 + \epsilon^2} \right]^{-1/2} = \frac{1}{\sqrt{\frac{n^2}{4^2}}} = \frac{4}{n}$ . This implies, the constant factor

$$C_n = \frac{2/\pi}{\sqrt{1 + \epsilon^2}} \arctan \left[ \left( \frac{n}{4^2} - \frac{1}{n} \right) / \sqrt{1 + \epsilon^2} \right] \frac{4}{n} \approx 0^+$$

for,  $n \sim$  sufficiently large number.

So, under a limit,  $n \sim$  sufficiently large number, for  $z \neq 0$ , we see from  $-1 \leq \frac{2}{\pi} \arctan(nz) \leq 1$  that the extremes  $-1$  and  $+1$  are quickly approximated. In turn, the partial integration of the right hand of (??), finally looks like

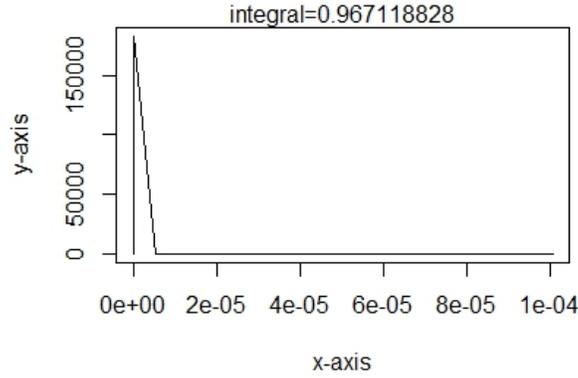
$$K_n \geq C_n - \frac{1}{\sqrt{1+\epsilon^2}} \frac{2}{\pi} \int_0^{z_{max}} dz \arctan(nz) \left( \frac{d}{dz} [1+n^2z\sqrt{1+\epsilon^2}]^{-1/2} \right) \quad (36)$$

Hence, when  $z > 0$  from  $-\frac{2}{\pi} \arctan(nz) \approx -1$  and  $C_n \approx 0^+$ , for  $n \sim$  large,

$$K_n \gtrsim -\frac{1}{\sqrt{1+\epsilon^2}} \int_0^{z_{max}} dz \left( \frac{d}{dz} [1+n^2z\sqrt{1+\epsilon^2}]^{-1/2} \right) \quad (37)$$

Note that the step from (??) to (??) is supported by the following paragraphs plus a result of numerical study represented in figure-??. In the fortran the first initial statements are there to search for a way to catch so to speak the singularity. The way it is done is given in the computer code. In paragraph -??. part D, the evidence is put together.

FIG. 1. Plot of the integrand of the integral in (??). Parameters  $(n, h) = (3.5 \times 10^{17}, 5.3 \times 10^{-6})$  are given in the program parameter statements. The result is  $K_n \gtrsim 0.9671$ . The results are computed with the program referenced in the Appendix below.



part A

First let us note that

$$\frac{d}{dz} [1+n^2z\sqrt{1+\epsilon^2}]^{-1/2} = \left( \frac{-1}{2} \right) \frac{n^2\sqrt{1+\epsilon^2}}{[1+n^2z\sqrt{1+\epsilon^2}]^{3/2}} \quad (38)$$

Demonstrating the fact that  $K_n > 0$  for large  $n$ , we can ignore constants in (??). We note that there is a  $\Delta z > 0$  beyond which, given  $n$  sufficiently large, that the expression in (??) vanishes quickly for  $z_{max} \geq z > \Delta z$ . In the interval  $(0, \Delta z]$  we also may write

$$\arctan(nz) \approx nz \quad (39)$$

For only essential terms we have for the integral in (??)

$$\int_0^{z_{max}} dz \arctan(nz) \left( \frac{d}{dz} [1+n^2z\sqrt{1+\epsilon^2}]^{-1/2} \right) \propto \int_0^{\Delta z} dz \frac{n^3z}{[1+n^2z]^{3/2}} \quad (40)$$

If  $n^2z \gg 1$ , then,  $n^3z \gg 1$ . Hence, we may approximate (??) with

$$\int_0^{z_{max}} dz \arctan(nz) \left( \frac{d}{dz} [1+n^2z\sqrt{1+\epsilon^2}]^{-1/2} \right) \propto \int_0^{\Delta z} dz z^{-1/2} \propto \sqrt{\Delta z} > 0 \quad (41)$$

Because the suppressed constants in this argument, looking at (??) are positive and  $C_n$  is positive small, we may conclude that  $K_n > 0$  for large  $n$ . Interested readers can verify that numerical straightforward proof exists too which shows that the right hand of (??) is positive nonzero.

part B

The objection to substitution of  $\frac{2}{\pi} \arctan(nz) \approx 1$  in (??) is that the singularity  $\frac{n^2}{[1+n^2z\sqrt{1+\epsilon^2}]^{3/2}}$  gets too much weight in the integral. Note however also the argument in paragraph C. The "too much weight" can be countered by eliminating the singularity from the infinitesimal sum. We then may look at e.g.

$$\lim_{n \rightarrow \infty} K_n \gtrsim -\frac{2/\pi}{\sqrt{1+\epsilon^2}} \lim_{n \rightarrow \infty} \int_{\epsilon_n}^{z_{max}} dz \arctan(nz) \frac{d}{dz} [1+n^2z\sqrt{1+\epsilon^2}]^{-1/2} \quad (42)$$

This reads as

$$\lim_{n \rightarrow \infty} K_n \gtrsim -\frac{1}{\sqrt{1+\epsilon^2}} \lim_{n \rightarrow \infty} \int_{\epsilon_n}^{z_{max}} dz \left\{ \frac{d}{dz} [1+n^2z\sqrt{1+\epsilon^2}]^{-1/2} \right\} \quad (43)$$

which gives,

$$\lim_{n \rightarrow \infty} K_n \gtrsim \frac{1}{\sqrt{1+\epsilon^2}} \quad (44)$$

when  $n^2 \epsilon_n \rightarrow 0^+$ , i.e.  $\epsilon_n \propto \frac{1}{n^{2+r}}$ , with,  $r > 0$ , small, given  $n \rightarrow \infty$ .

part C

Another way to look at the problem is to approximate  $K_n$  with the use of the first mean value theorem for definite integrals. If  $f(x)$  and  $g(x)$  are non negative functions that do not change sign, in an interval  $(a, b)$  in  $\mathbb{R}$  to  $\mathbb{R}_+$  then there is a,  $c \in (a, b)$  such that

$$\int_a^b f(x)g(x)dx = f(c) \int_a^b g(x)dx$$

Furthermore we write the right hand side of (??) such that  $K_n \geq C_n + L_n$ . Here  $C_n$  is defined as in (??), while

$$L_n = -\frac{1}{\sqrt{1+\epsilon^2}} \frac{2}{\pi} \int_0^{z_{max}} dz \arctan(nz) \left( \frac{d}{dz} [1+n^2z\sqrt{1+\epsilon^2}]^{-1/2} \right) \quad (45)$$

Then obviously we may derive

$$L_n = \frac{1}{\sqrt{1+\epsilon^2}} \int_0^{z_{max}} dz f(z)g(z) \quad (46)$$

where,  $f(z) = (2/\pi) \arctan(nz)$ , continuous and positive in  $(0, z_{max})$  and

$$\forall_{z \in (0, z_{max})} g(z) = \frac{1}{2} \frac{n^2 \sqrt{1+\epsilon^2}}{(1+n^2z\sqrt{1+\epsilon^2})^{3/2}} = -\frac{d}{dz} [1+n^2z\sqrt{1+\epsilon^2}]^{-1/2} \geq 0$$

continuous and integrable (finite integral) in  $(0, z_{max})$ . We have that  $\forall_{z \in (0, z_{max})} f(z)g(z) \geq 0$ . So, we may apply the first mean value theorem for definite integration (n large but finite). This means that there is a  $\zeta \in (0, z_{max})$  such that

$$\frac{1}{\sqrt{1+\epsilon^2}} \int_0^{z_{max}} dz f(z)g(z) = f(\zeta) \frac{1}{\sqrt{1+\epsilon^2}} \int_0^{z_{max}} dz g(z) \quad (47)$$

Hence, for sufficient large but finite  $n$ , we see  $f(\zeta) = (2/\pi) \arctan(n\zeta) \approx 1$ , such that

$$L_n \approx -\frac{1}{\sqrt{1+\epsilon^2}} \int_0^{z_{max}} dz \frac{d}{dz} \left[ 1 + n^2 z \sqrt{1+\epsilon^2} \right]^{-1/2} = \frac{1 - (4/n)}{\sqrt{1+\epsilon^2}} \quad (48)$$

Hence,  $K_n \geq C_n + L_n$  gives what is described in the next paragraph. It is noted that the first mean value theorem for definite integration is based on the intermediate value theorem. In Bishops constructive analysis, there is serious doubt about the intermediate value theorem [?, introduction section]. Present developments show that a weaker version of the theorem can be maintained but without axiom of choice [?].

part D

Returning to equation (??). This then gives, using  $z_{max} = (\frac{1}{4^2} - \frac{1}{n^2}) / \sqrt{1+\epsilon^2}$ ,

$$K_n \gtrsim -\frac{1}{\sqrt{1+\epsilon^2}} \left[ \frac{4}{n} - 1 \right] \rightarrow \frac{1}{\sqrt{1+\epsilon^2}}, \quad (49)$$

under the condition,  $n \sim$  large number. Hence, we may conclude that:

$$1 \geq \lim_{n \rightarrow \infty} K_n \gtrsim \frac{1}{\sqrt{1+\epsilon^2}}.$$

This leads us to,  $K_n \approx 1$ , where  $\epsilon^2$  can be arbitrary small positive real.

## RESULT

Returning to  $\sigma_a \in I_1$  and  $\sigma_a \in I_3$ , it is found that approximately we may write  $\langle \rho_I A(a, \lambda_I, \vec{\chi}) \rangle_I \approx \sigma_a$ , because under  $T = n$  sufficiently large,  $K_T \approx 1$ . Moreover under  $T \sim$  sufficiently large number we also see that for  $\sigma_a \in I_2$  that  $\langle \rho_I A(a, \lambda_I, \vec{\chi}) \rangle_I \approx \sigma_a$ . Hence, because a similar evaluation for  $B$  can take place

$$\begin{aligned} \langle \rho_I A(a, \lambda_I, \vec{\chi}) \rangle_I &\approx \sigma_a \\ \langle \rho_{II} B(b, \lambda_{II}, \vec{\chi}) \rangle_{II} &\approx \sigma_b \end{aligned} \quad (50)$$

Because using (??) and our previous result, we are allowed to write

$$E(a, b) = \langle \langle \rho_I A(a, \lambda_I, \vec{\chi}) \rangle_I \langle \rho_{II} B(b, \lambda_{II}, \vec{\chi}) \rangle_{II} \rho_{Norm} \rangle_{Norm} \approx \langle \sigma_a(\vec{\chi}) \sigma_b(\vec{\chi}) \rho_{Norm} \rangle_{Norm} \quad (51)$$

This implies, together with (??) that

$$E(a, b) \approx \sum_{i=1}^3 \sum_{j=1}^3 a_i b_j \delta_{i,j} = \sum_{j=1}^3 a_j b_j \quad (52)$$

The latter equation concludes the refutation part of the present paper. In the appendix, to help the reader, an algorithm in R is presented to demonstrate the numerical possibility of  $K_n \approx 1$ .

## CONCLUSION

In our paper, under locality [?, ?], we have construed a model that must, by design, not be able *in any way* to violate the  $S(a, b, c, d) \leq 2$ . We note that the local hidden variables physical picture is that variables with the index  $\alpha = I$  reside in measurement instrument A and  $\alpha = II$  reside in measurement instrument B. The  $\chi$  Gaussian variables can be seen as being carried by the particles to the respective measurement systems. This is a perfectly valid physical possibility.

### Contextuality

In the discussion with colleagues, the idea was put forward that the proposed paper did not come with an interesting new conclusion. The claim was that contextuality of the probability density used. This probability contextuality is clearly described in e.g. [? , page 7-8]. In th latter paper we find that a contextual model *must* have, mutatis mutandis, a probability measure:  $dP(\lambda, \lambda_a, \lambda_b)$ . Then looking at our definition of the density, it must be carefully noted that the probability density in section - Probability density ?? -and therefore the probability measure- is *independent* of the setting. Note that in equations (?? - ??) we see that the "lambda's" are *completely independent* of the settings vectors. We have two sets of "lambda's" that refer to two wings of the experiment i.e. to two different measurement instruments. Hence, we are justified to claim that ours is a probability measure that *does not change* in different contexts. The density is non-contextual. It must be noted as an extra that the model is valid for two dimensional setting parameters as well as for the present three dimensional setting vectors. Different boundary values are in the former case employed in the model  $\iota$  functions. This latter observation is also an indication that we are looking at a non-contextual model [? ]. The fact that the measurement functions are described with two different sets of variables and that the source sends out the  $\chi$  variables makes it a valid local hidden variables model. In addition, it is necessary to note that we obviously did *not use* conditional probability measures. Hence this part of contextuality described in [? ] also does not apply to the present formalism.

We note that, although this discussion about the physics of the model may be looked upon as important, the fact remains that it is the *mathematics* behind the theorem that breaks down in two conflicting branches.

### Bell correlation formula and its inconsistency

In some discussions we also uncovered that there are colleagues that argue as follows. The hidden variables model reproduces, in approximation, the quantum correlation. Hence, it disagrees with Bell's conclusions. Only based on that state of facts, the following philosophy of logic error is employed. Although the reader could not find an error in the counter model, the reader nevertheless decides that the model *must* be wrong. This, forgive me, stupidity is supported by the idea that Bell's theorem is "simple" and the counter model is difficult. This is going pretty close to aesthetic judgements like "this painting is more true because of its color". To save face, the reader then fully ignores the possibility of concrete mathematical incompleteness. He claims that the authors must demonstrate the error in Bell's derivation. The reader also seems to forget that the conclusion of the present authors is based on the mean value theorem for integration. This theorem is far older than Bell's mathematical exercise. In fact one can claim that Bell's work rests implicitly on the validity of the mean value theorem. Because of seniority, also an error in philosophy of logic but it illustrates the crooked argumentation, Bell's theorem must be rejected when no error is found in the counter model of Geurdes et al. Of course we remain here in non constructive, classical mathematics.

It is, of course, not forbidden to make philosophy of logic errors. It only decreases the logical content of the field of study. On the other hand let us, indeed, try to meet the "Bell theorem above all else...unless error is found" approach. Let us look at Bell's correlation formula in (??) and the  $A$  and  $B$  functions that project in  $\{-1, 1\}$ . Suppose, for the sake of argument that a very simple correlation model is looked at. Suppose we have a setting  $b$  that implies  $B(b, x) = 1$  for all values of the hidden variable  $x \in \mathbb{R}$ . Furthermore, like in the main text, the Heaviside function  $H(x) = 1 \Leftrightarrow x \geq 0$  and  $H(x) = 0 \Leftrightarrow x < 0$ . Hence, a sign function  $\text{sign}(x) = 2H(x) - 1$ . Perhaps that the hostile reader wants to see a closed expression for the definition of  $H(x)$ . We have,  $x \in \mathbb{R}$ ,

$$H(x) = \lim_{n \rightarrow \infty} \exp\left(-\frac{e^{-nx}}{n}\right)$$

Clearly,  $H(x) = 1$ , when  $x \geq 0$  and  $H(x) = 0$ , when,  $x < 0$ . Note that the  $x$  is independent of the  $n$ . So,  $A(a, x) = \text{sign}(x - a)$  is a possible measurement function with  $a \in [-1, 1] \cap \mathbb{R}$ . Suppose, for the sake of the argument that we look at a Normal density hidden variable for  $x$ . The correlation in (??) is then written as

$$E(a, b) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-x^2/2} \text{sign}(x - a) dx$$

Because,  $\text{sign}(x) = 2H(x) - 1 \in \{-1, 1\}$ , we are allowed to write,  $\text{sign}(x - a) = \frac{1}{\text{sign}(x - a)}$ . Suppose furthermore that

$$G(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-z^2/2} dz$$

It then follows that

$$E(a, b) = \int_{-\infty}^{\infty} \frac{dG(x)}{dx} \text{sign}(x - a) dx \equiv I$$

We also may write

$$E(a, b) = \int_{-\infty}^{\infty} \frac{dG(x)}{dx} \frac{1}{\text{sign}(x - a)} dx \equiv J$$

Using partial integration on  $I$  it is possible to obtain

$$I = G(x)\text{sign}(x - a)|_{-\infty}^{\infty} - \int_{-\infty}^{\infty} G(x) \frac{d}{dx} \text{sign}(x - a) dx = 1 - 2 \int_{-\infty}^{\infty} G(x) \delta(x - a) = 1 - 2G(a) < 1$$

Using partial integration on  $J$  it is possible to get

$$J = G(x) \frac{1}{\text{sign}(x - a)} |_{-\infty}^{\infty} - \int_{-\infty}^{\infty} G(x) \frac{d}{dx} \left( \frac{1}{\text{sign}(x - a)} \right) dx = 1 + 2 \int_{-\infty}^{\infty} G(x) \frac{\delta(x - a)}{\text{sign}^2(x - a)} = 1 + 2G(a) > 1$$

because,  $\frac{d}{dx} \frac{1}{f(x)} = -\frac{1}{f^2(x)} \frac{df(x)}{dx}$  and  $\text{sign}^2(x - a) = 1$ . Based on the definition of the sign function the obtained difference between  $I$  and  $J$  must make the reader aware of the fact that Bell's result may look simple but definitely is *not* simple at all.

Perhaps that a hostile reader now wants to invoke Coulombeau theory of distributions. However, we note that apparently the objective of the hostile reader is to ignore the subtlety of concrete mathematical incompleteness and not to provide any proper reason whatsoever. The main goal of the hostile reader is to maintain business as usual with Bell inequalities. We believe that there is sufficient evidence that this is no longer possible.

### Mathematical breakdown of the theorem

In the paper it was derived that a model with  $S(a, b, c, d) > 2$  can be obtained observing all conditions for a local model. I.e.  $E(a, b) \approx \sum_{j=1}^3 a_j b_j$ . was derived using *local* modeling. In passing we note that, using the random variable notation  $\ell$ , it follows from our model that  $E_{\ell}(A(a, \ell)) = E_{\ell}(B(b, \ell)) = 0$ . This easily derives from the symmetry of the Gaussian.

Our result is unrelated to a quantum mechanical violation of the inequality. We can make this claim because, in the first place, a local Bell formula model was used. All the requirements for a local physical model were fulfilled. The probability density has a fixed form. The objection, "the proposed model is unphysical" is clearly invalid. The reader carefully notes that all the basic physical requirements for a local model were fulfilled. In the second place, looking at the derived inequality from Bell's formula, one must mathematically *never* be able, with what kind of a model one cares to select under the umbrella of locality, to obtain  $S(a, b, c, d) > 2$ .

Subsequently, the reader is reminded that in the paper *no hidden physics assumptions* were used in any step of the derivation. The derivation was completely mathematical. The basic physical requirements are merely there to show that Bell's formula is valid physics in both branches of the argument. If the reader thinks it's otherwise he has to *demonstrate* that the mathematics provided in the model cannot be realized in a physical situation. Bell's formula is general so in our conception, this form of opposition breaks down. To be more specific, there were *no hidden physics assumptions like non-locality* in the derivation of  $E(a, b) \approx \sum_{j=1}^3 a_j b_j$ , hence,  $S(a, b, c, d) > 2$ , in our model. The reader can easily verify this.

In previous papers, the first author already pointed out that there are inconsistencies in the Bell argumentation [? ], [? ] and [? ]. The presented demonstration shows unequivocally that for fixed density and realizable physics, Bell's formula give rise to conflicting conclusions. The believe in a one-branched Bell formula, such as expressed in [? ] is unfounded because it neglects the possibility of the demonstrated negation incompleteness. Arguments quoted from [? ] in favor of CHSH or one branch interpretation of Bell's formula, are therefore invalid. We note especially the implicit claim about the necessity of a computer violation before due credit can be given to any, more theoretically oriented, criticism. This is, according to us, an unfounded and malignant expression of keeping the faith in only one preferred branch of the argument. The first author has also performed work in the field of computer violation study [? ]. To be complete, we note that the  $E(x, y) \approx x \cdot y$  for any setting  $x \in \mathbb{R}^3$  unit length and  $y \in \mathbb{R}^3$  unit length, where the form of the density of the hidden variables remain fixed as given in equation (??) above.

We demonstrated that there exists a necessity to take a closer look at the rulebook of ZFC mathematics [? ] and arguments concerning the foundations of quantum mechanics.

## REFERENCES

- 
- J.S. Bell, "On the Einstein Podolsky Rosen paradox," *Physics*, 1, 195, (1964).
  - A. Einstein, B., Podolsky, and N. Rosen, "Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?," *Phys. Rev.* 47, 777, (1935).
  - H. Friedman, "Concrete mathematical incompleteness", Lecture at the Andrzej Mostowski Centenary, Warsaw, Poland, 2013
  - H. Friedman, "Boolean Relation Theory and Incompleteness," Ohio state University, 2010.
  - D. Bohm, "Quantum Theory," pp 611-634, Prentice-Hall, Englewood Cliffs, 1951.
  - R.V. Hogg, A.T. Graig,"Introduction to Mathematical Statistics, Third edition" Prentice-Hall, Englewood Cliffs, 1995.
  - A. Peres, "Quantum Theory: Concepts and Methods," Kluwer Academic, 2002.
  - E. Bishop, "Foundations of constructive analysis", McGraw Hill, 1967.
  - M. Hendtlass, "The intermediate value theorem in constructive mathematics without choice", *An.. Pure & Appl. Logic*, 163, 1050, (2012).
  - A. Khrennikov, "Bell-Boole Inequality: Nonlocality or Probabilistic Incompatibility of Random Variables?", *Entropy*, 10(2), 19, (2008); doi:10.3390/entropy-e10020019.
  - A. Khrennikov, "CHSH inequality: Quantum probabilities as classical conditional probabilities", arXiv: 1406.4886v1, *Foundations of Physics*, 7, 711, (2015).
  - A. M. Gleason, "Measures on the closed subspaces of a Hilbert space", *J. math. and Mech.*, 6, 885, (1957).
  - H. Geurdes, "A Counter-Example to Bell's Theorem with a Softened Singularity, " *Galilean Electrodynamics*, 17, 16, (2006).
  - H. Geurdes, "A note on Bell's' expression for the QM correlation," *AIP Conf. Proc.* 1508, 365, (2012); doi: 10.1063/1.4773149
  - H. Geurdes, "A probability loophole in the CHSH," *Results in Physics*, 4, 81, (2014).
  - R.D. Gill, "No loophole in the CHSH", *Results in Physics*, 5, 156, (2015).
  - H. Geurdes, "A computational proof of locality in entanglement.", A. Khrennikov, B. Toni (eds.), *Quantum Foundations, Probability and Information, STEAM-H*, Springer, in the press, 2018.

## APPENDIX

This result can be found in figure-?? above With this particular parameters we get  $1 \geq K_n \gtrsim 0.9671$ .

```

program testAtAr
integer nmax, n, m, j, k
real*8 h, xx, funfArr, nlim, f2, ffHelp
real*8 pw
real*8 eps, beta, fact, y0, ystart, yfin
real*8 g1, g2, f, f0, res1, integral
parameter(nlim=3.5e17)
c   parameter(nlim=4.6e16)
parameter(nmax=50)
parameter(m=100)
parameter(h=5.3e-6)
c   parameter(h=9.999e-6)
real*8 ffArray(m), xxVar(m)
c output for plot
open(1,
+file='res.txt'
+,status='unknown')
open(2,
+file='xres.txt'
+,status='unknown')
open(3,
+file='yres.txt'

```

```

+,status='unknown')
c
  write(1,*)0.0
  write(2,*)0.0
  eps=1/nlim
c eps>0 as required
  f0=(nlim*nlim)*dsqrt(1.0+(eps*eps))
c determine the proper starting point given the integration
c interval h to 'catch' the singularity at a given n
  beta=-(2.6)/(3.0)
  y0=1.0/dsqrt(1.0+(eps*eps))
  pw=-(2.0)/(3.0)
  f2=(2.0)**pw
  y0=y0*(f2*(nlim**beta)-(1/(nlim*nlim)))
  ystart=y0-(9.0*h)
  yfin=y0+(20.0*h)
c the yfin is there to not waste too much iterations
  xx=ystart
  j=0
c capture the function in the ffArray
10 continue
  if(xx.gt.0) then
    j=j+1
    xxVar(j)=xx
    ffHelp=funfArr(xx,nlim)
    g1=ffHelp/dsqrt(1.0+(eps*eps))
    f=f0*xx
    g2=f0/((f+1.0)**(1.5))
    ffArray(j)=(g1*g2)/2.0
    write(1,*)ffArray(j)
    write(2,*)xxVar(j)
  endif
  xx=xx+h
c adding the next h => next input x
  if (xx.lt.yfin) go to 10
  write(*,*) 'number of iterations=',j
c integration
  res1 = integral(ffArray,h,j)
  write(*,*)'integral=',res1
  write(3,*) res1
  close(unit=3)
  close(unit=2)
  close(unit=1)
  stop
  end
real*8 function integral(ffArray,h,n)
integer i,j,k,n,m
parameter (m=100)
real*8 sum,h,ffArray(m)
sum=0
do 10 j=1,n
10   sum=sum+(ffArray(j)*h)
integral=sum
return
end
real*8 function funfArr(xx,nlim)

```

```
real*8 xx,nlim
real*8 pi,y,z
z=nlim*xx
pi=4*atan(1.0)
y=(2/pi)*atan(z)
funfArr=y
return
end
```