Heisenberg quantum mechanics, numeral set-theory and quantum ontology without space-time.

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Abstract. In the paper we will employ set theory to study the formal aspects of quantum mechanics without explicitly making use of space-time. It is demonstrated that von Neuman and Zermelo numeral sets, previously efectively used in the explanation of Hardy's paradox, follow a Heisenberg quantum form. Here monadic union plays the role of time derivative. The logical counterpart of monadic union plays the part of the Hamiltonian in the commutator. The use of numerals and monadic union in the classical probability resolution of Hardy's paradox [1] is supported with the present derivation of a commutator for sets.

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INTRODUCTION

In the paper the possibility of a set-theoretical foundation of particle physics is explored. We show that numeral sets from foundational set theory can be employed in physical theory. In fact numeral sets can behave quantum like in set operator commutators without the presupposition of an existing space-time. Perhaps that the use of sets without space-time but equiped with a quantal structure will provide insight into the creation of space-time in the big bang.

FORMALISM

Numeral sets & operators

The numeral sets of von Neuman and of Zermelo are the foundation of the analysis. The basis of those sets is the empty set $\emptyset = \{\}$. The von Neuman numerals are defined by $C_0 = \emptyset$ and $(\forall : n = 0,1,2,3,....)\, C_{n+1} = \{C_0,....,C_n\}$. The Zermelo numerals are defined by $D_0 = \emptyset$ and $(\forall : n = 0,1,2,3,....)\, D_{n+1} = \{D_n\}$. In a previous paper we explained the Hardy paradox with classical measure theory using the set $\omega = C_3 \cup D_3$ and the monadic union. Note that the cardinality for von Neuman numerals $|C_n| = n$ is different from the cardinality of Zermelo numerals $|D_n| = 1$, for, $n = 2,3,....$

Let us for an arbitrary set Z introduce some operators. Firstly, the monadic union on Z is defined by

$$\cup(Z) = \{x \,|\, (\exists : y \in Z)\, (x \in y)\} \tag{1}$$

Secondly, the logical counterpart of monadic union is

$$\nabla(Z) = \{x \,|\, (\forall : y \in Z)\, (x \notin y) \wedge (x \in Z)\} \tag{2}$$

Thirdly an extended counterpart of monadic union, v_0 is employed. We define

$$v_0(Z) = \cup(\{x \,|\, (\forall : y \in Z)\, (x \notin y) \wedge (x \in Z) \wedge (|x| > 1)\}) \tag{3}$$

In the present study of quantal structure in numeral sets, we will be in need of the two operators but 'indexed by' sets. Suppose we inspect a set C_2. Generally the Ω sets can be described as a structure that is build around the sets

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C_0 and D_0, both equal to the empty set, $\emptyset$. Let us inspect the operation ∇ on C_2. From the definition in (2) and $C_2 = \{C_0, C_1\}$, with, $C_1 = \{C_0\}$ it readily follows that $\nabla(C_2) = \{C_1\}$. We subsequently define ∇_T as follows: perform the operation ∇ on the, $C_0 = \emptyset$ based, set and substitute in the resulting non-empty set, $C_0 = T$. If a $\emptyset$ results, then *no* $C_0 = T$ substitution takes place. In case of $\nabla_T(C_2)$ we have $\nabla_T(C_2) = \{C_1\}|_{C_0=T}$. Now, because of $C_1 = \{C_0\}$ we find $\nabla_T(C_2) = \{\{T\}\}$.

Subsequently, let us also define sequences of operators by illustrating their activity on numeral sets. E.g. let us inspect $\cup_T \nabla_T(C_3)$. According to the previous general rule illustrated in the next example, we have $\cup_T \nabla_T(C_3) = (\cup(\nabla(C_3)))_{C_0=T}$. Because, $C_3 = \{C_0, C_1, C_2\}$ we find $\nabla(C_3) = \{C_2\}$ which follows from $(\forall : X \in C_3)\, C_2 \notin X$. Subsequently, it follows that $\cup(C_2) = \{C_0\} = C_1$, hence, $\cup_T \nabla_T(C_3) = \{C_0\}|_{C_0=T} = \{T\}$.

Notice the difference between the set selection in v_0 as $v_0(C_3 \cup D_3) = \cup(\{C_2\})$, because, $|C_2| = 2 > 1$ and in $\nabla(C_3 \cup D_3) = \{C_2, D_2\}$, because for ∇ no restrictions on the cardinality of the selected set is introduced.

Special symbolism

Now that the operators on numeral sets and possible sequences of operators are defined, we turn to the commutator. In quantum theory a commutator for quantum operators, A and B and written as $[A,B]_-$ is defined by $[A,B]_- = AB - BA$. Here we seek to inspect a similar form for sets. In order to make the formalism somewhat easier to read, the usual set theoretical elementary operations as union or difference between sets are written in an alternative symbolism when necessary. We write, $\otimes$ for $\cap$; $\oplus$ for $\cup$; $\ominus$ for the 'difference' between sets. Hence, when X and Y sets we may use when necessary, $X \oplus Y = X \cup Y$; $X \otimes Y = X \cap Y$ and $X \ominus Y = X - Y$. The $\otimes$ will obtain some special characteristic when combinations of set and set operator are under study.

Similarly, we can construe 'sums' or 'differences' from operator sequences. For our purpose we write an operator commutator, $[\cup, \nabla]_\ominus$ on a certain numeral set, denoted here with, ω, as $[\cup, \nabla]_\ominus(\omega) = (\cup\nabla \ominus \nabla\cup)(\omega) = \cup\nabla(\omega) \ominus \nabla\cup(\omega)$. The use of $\ominus$ is twofold, namely difference of operator sequences and of sets, but that is a not very important detail at this moment.

In the study of quantum forms and numeral sets, we will make use of the product of a set and a set operator. E.g. we use $t \otimes \cup_T$. This $\otimes$ sequence of a set t and an operator $\cup_T$ is itself a new operator. If we for instance are interested in a new sequence of ∇_T and $t \otimes \cup_T$ the $\otimes$ symbolic sequence operator separates the sets from the set operators. We have, $\nabla_T(t \otimes \cup_T) = t \otimes (\nabla_T \cup_T)$, etc. Note that on a set ω we see

$$\nabla_T(t \otimes \cup_T)(\omega) = t \otimes (\nabla_T \cup_T)(\omega) = t \cap (\nabla(\cup(\omega)))_{C_0=T} \tag{4}$$

is intended. This concurs with the symbol definitions given previously but also illustrates the special role of $\otimes$. Note that the sets t and T are related to each other as

$$t = \{T\} \tag{5}$$

Moreover, it is assumed that a unity element, 1 exists such that $1 \otimes X$, with X a set, leads to $1 \otimes X = 1 \cap X = X$.

PHYSICAL MODELLING

Heisenberg's commutator

With the use of set theory it is attempted to mimic the following Heisenberg relation

$$[v,H]|\psi\rangle = i\hbar \frac{dv}{dt}|\psi\rangle \tag{6}$$

Here, $v = v(t)$ represents the operation of the measurement of the particle's velocity and H is the Hamiltonian while $|\psi\rangle$ is the state-vector of the system. This is a textbook case of Heisenberg's form of quantum mechanics.

Let us now try to see if the numeral sets introduced in the previous sections that were effectively used in the explanation of Hardy's paradox in terms of classical probability, can be mould into a similar form. First let us introduce the 'velocity' set operator.

$$v = (1 \otimes v_0) \oplus (t \otimes \cup_T) \tag{7}$$

It should be noted that, $\frac{dv}{dt} = a(t) + t\frac{da}{dt}$. If we take $\omega = \cup(\Omega)$ and $\Omega = C_4 \cup D_4$, a form similar to: $\cup_T^2$ occurs in $(t \otimes \cup_T)(\omega) = t \cap (\cup_T(\cup(\Omega)))$. When the non empty set results on the right hand side of $\otimes$, the substitution $C_0 = T$ will be made for the $t \otimes \cup_T^2$ operation on Ω as well as for the $(t \otimes \cup_T)$ operation on ω. The two forms give the same result. Now this explains why $\cup_T$ can be seen as a kind of time derivative such as in (7). Of course, then the sequence of v_0 and $\cup$ leading to $v_0\cup$ must somehow represent an initial velocity when operated on $\Omega = C_4 \cup D_4$. This is then identical to v_0 on $\omega = \cup(\Omega)$.

Having established that (7) may represent $v = v_0 + ta(t)$ in a real physical experiment, we then turn to the commutator defined in a previous paragraph.

Computation of the set commutator

In this paragraph we turn the attention to $[v, \nabla_T]_\ominus$. Here, v is defined in (7). It is then intutively clear that we will have two compound operators to deal with, namely, $1 \otimes v_0$ and $t \otimes \cup_T$. It is intuitively clear that we may deal with the two compound operators separately, viewed over the $\ominus$ of the set commutator, when computing the ∇_T commutator with v.

The $1 \otimes v_0$ commutator with ∇_T

Let us inspect $[(1 \otimes v_0), \nabla_T]_\ominus$. We already have established that

$$[(1 \otimes v_0), \nabla_T]_\ominus = 1 \otimes [v_0, \nabla_T]_\ominus = [v_0, \nabla_T]_\ominus \tag{8}$$

It is clear that $\omega = \{C_0, C_1, C_2, D_2\}$. Hence, from the definition of v_0 and inspection of ω it follows that $v_0(\omega) = \cup(\{C_2\}) = C_2$ because $|C_2| > 1$ and $(\forall : X \in \omega)\, C_2 \notin X$. In addition, from the definition and the rule of use for ∇ in (2), it follows that, $\nabla(\omega) = \{C_2, D_2\}$. The commutator contains the $v_0\nabla_T$ and the $\nabla_T v_0$ operations of ω. For, $v_0\nabla_T(\omega)$ we find

$$v_0\nabla_T(\omega) = (v_0(\{C_2, D_2\})_{C_0=T} = C_2|_{C_0=T} = \{C_0, C_1\}_{C_0=T} = \{T, t\}. \tag{9}$$

This is so because from $|C_2| > 1$ only C_2 is selected from $\{C_2, D_2\}$. This results in a singleton set $\{C_2\}$. Using the definiton of $\cup$ from (1) the C_2 is obtained such as in (9). Because, $C_1 = \{C_0\}$ and $C_0 = T$ together with, $t = \{T\}$, the result in (9) readily follows. For, $\nabla_T v_0(\omega)$ we find

$$\nabla_T v_0(\omega) = (\nabla(C_2))_{C_0=T} = \{C_1\}_{C_0=T} = \{\{T\}\} = \{t\} \tag{10}$$

From the commutator given in (8) it then follows that

$$[(1 \otimes v_0), \nabla_T]_\ominus(\omega) = v_0\nabla_T(\omega) \ominus \nabla_T v_0(\omega) = \{T, t\} \ominus \{t\} = \{T\} = t. \tag{11}$$

In order to do justice to the idea that $\cup_T$ plays the part of temporal derivative, let us subsequently investigate, $\cup_T(1 \otimes v_0)(\omega)$ as part of a construction to arrive at a similar form as in (6). Hence, it is necesary to have

$$\cup_T(1 \otimes v_0)(\omega) = (1 \otimes \cup_T v_0)(\omega) = \cup_T v_0(\omega) \tag{12}$$

because of the role of the 1 or unity set. We already established that $v_0(\omega) = C_2$. The $\cup_T$ operation then gives $\cup_T v_0(\omega) = \cup_T(C_2) = C_1|_{C_0=T} = \{T\} = t$. This implies that

$$[(1 \otimes v_0), \nabla_T]_\ominus(\omega) = \cup_T((1 \otimes v_0)(\omega)) \tag{13}$$

which is a step into the direction of (6) when $v = (1 \otimes v_0) \oplus (t \otimes \cup_T)$ from (7) is employed.

The $t \otimes \cup_T$ commutator with ∇_T

Now we turn to the second term in the operator of (7). We can write

$$[(t \otimes \cup_T), \nabla_T]_\ominus(\omega) = t \otimes [\cup_T, \nabla_T]_\ominus(\omega) \tag{14}$$

We have $\cup_T \nabla_T(\omega) = (\cup_T(\nabla_T(\omega)))_{C_0=T}$. Now, because we already have established that $\nabla(\omega) = \{C_2, D_2\}$ and, by definition, $D_2 = \{C_1\} = \{D_1\}$, it follows that $\cup_T \nabla_T(\omega) = \{C_0, C_1\}_{C_0=T} = \{t, T\}$, when we note that from C_1 it follows that $\{T\} = t$ is introduced. Subsequently, $\nabla_T \cup_T (\omega) = (\nabla(\cup(\omega)))_{C_0=T} = (\nabla(C_2))_{C_0=T}$. If we now note that $C_2 = \{C_0, C_1\}$, it follows from (2) that $\nabla_T \cup_T (\omega) = \{\{C_0\}\}_{C_0=T} = \{t\}$, because $t = \{T\}$.

From the commutator in (14) we then may derive from the previous special role of $\otimes$ that

$$[(t \otimes \cup_T), \nabla_T]_\ominus(\omega) = t \cap [\{t, T\} \ominus \{t\}] = t \cap \{T\} = t \cap t = t. \tag{15}$$

Subsequently we turn our attention to $\cup_T (t \otimes \cup_T)(\omega)$. From the definitions previously given it follows that

$$\cup_T (t \otimes \cup_T)(\omega) = t \otimes \left(\cup_T^2\right)(\omega) = t \cap (\cup(C_2))_{C_0=T} = t \cap C_1|_{C_0=T} = t \cap \{T\} = t \cap t = t. \tag{16}$$

This result in (15) and (16) gives

$$[(t \otimes \cup_T), \nabla_T]_\ominus(\omega) = \cup_T (t \otimes \cup_T)(\omega). \tag{17}$$

This leads to the theorem that for physical measurement of velocity the operator $v = (1 \otimes v_0) \oplus (t \otimes \cup_T)$ and set operators can be used such that the following commutation relation resembling a quantum form is obtained

$$[v, \nabla_T]_\ominus(\omega) = (\cup_T v)(\omega). \tag{18}$$

CONCLUSION AND DISCUSSION

In the previous it was demonstrated that numeral set theory can be employed in a quantum-like format such that for measuring velocity of a particle, associated to a compound set operator expression $v = (1 \otimes v_0) \oplus (t \otimes \cup_T)$ resembling the standard physical form, a Heisenberg commutator can be obtained. This supports the use of numeral sets and monadic union in the context of Hardy's paradox. Of course, there is some heuristic arbitrariness in the choice of the set operators that mimic the Hamiltonian and the time derivative. The state vector can be mimicked by ω.

If we inspect the expression for $\frac{dv}{dt}$ then it can be concluded that $a(t)$ in equation $\frac{dv}{dt} = a(t) + t\frac{da}{dt}$ can be mimicked by $1 \otimes a$, with, $a = \cup_T v_0$ and $t\frac{da}{dt}$ with $t \otimes \cup_T^2$.

In a sense the presented study is philosophical because the logical structure of quantum theory and numeral set theory plus operators are studied and appear the same. The point is that numeral sets are without reference to space-time and the definition of the velocity is a a pure abstract expression of compound set operators. In other words there is no need of a space-time in numeral set theory to have a space-time to move in. The reasoning is 'in analogy'. However, the numeral sets have the advantage to be able to explain the Hardy paradox with conceptually reference to a hidden mirror sector. There is no space-time necessary[2] to mimic the quantum commutator forms in the Heisenberg representation and to explain Hardy's paradox with local hidden causalities inside a classical probability triple.

It can be asked if space-time is the cause of all trouble for quantum forms because an analysis without explicit use is able to solve the problem, even in terms of classical probability, without having mysterious non-localities or the influence of the mind on the measurement. In the previous paper on set theory the idea was that a hidden mirror sector [2] could assist in understanding of the hidden causality. This is a very valuable conceptual point and adds to the idea that space-time is the cause of paradox in quantum theory. Hidden causality can be a form of absence of space-time

[2] The author and reader are in need of space-time.

influence. As was explained here, quantum mechanics can be done without explicit reference to space-time and this set theoretical approach, with monadic union operator, solved the Hardy paradox in classical probabilistic terms. The absence of explicit refernce to space and time solves the paradox and enables to remain within the boundaries of quantum-like forms.

With the previous cosmological point in mind it can be claimed that the advanced set theoretical quantum theory has no paradoxes because it uses no space-time and is perhaps the ontological 'behind the scenes' of the real space-time physics that takes place in experiment. Another cosmological point could be to employ the predictions of the set theoretical quantum theory for the creation of space-time in the big bang. One can perform quantum-like manipulations on numeral sets without space-time involved in the consideration.

REFERENCES

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