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Probability and quantum foundation

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Abstract: A classical probabilistics explanation for a typical quantum effect in Hardy's paradox is demonstrated.

1. Classical probability

The common claim is that a classical probability triple, $(\Omega, \mathbf{F}, P)$ cannot explain quantum effects. Here the sample space Ω is *any* non-empty set. The σ-field, $\mathbf{F}$ is obtained from the set of all subsets, $\mathbf{P}(\Omega) = 2^{\Omega}$, of Ω. $\mathbf{F}$ is called a σ-field [1] if, (i) $\Omega \in \mathbf{F}$, (ii) $E \in \mathbf{F} \Rightarrow E^c = (\Omega - E) \in \mathbf{F}$, (iii) $E, F, ... \in \mathbf{F} \Rightarrow E \cup F \cup .... \in \mathbf{F}$. The triple is completed with a probability measure P, such that, $(\forall : X \in \mathbf{F})\,(0 \leq P(X) \leq 1),\ P(\Omega) = 1$.

2. Pre-measurement characteristics, numerals and algebra

Let us inspect the possibilities of classical probability for Hardy's paradox [2] where quantum particles like electron and positron can be measured *after* mutual annihilation. This appears to reject the possibility of pre-measurement characteristics [3].

Apart from zero, unity and two the numerals of von Neuman and of Zermelo [4] are disjoint. This fact may represent mutual exclusion of electron and positron. We have, $D_0 = C_0 = \emptyset$. Von Neuman numerals are $(n = 0, 1, 2, 3, ...)$

(1) $$C_{n+1} = \{C_0, C_1, ..., C_n\}.$$

Hence, $C_1 = \{\emptyset\}$, $C_2 = \{\emptyset, \{\emptyset\}\}$, $C_3 = \{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}$, etc.

Zermelo's system is

(2) $$D_{n+1} = \{D_n\}.$$

Hence, $D_1 = \{\emptyset\}$, $D_2 = \{\{\emptyset\}\}$, $D_3 = \{\{\{\emptyset\}\}\}$, etc.

We establish mutual exclusion (annihilation) with C_3, modeling particle 1 and D_3 modeling particle 2, for instance, as $C_3 \cap D_3 = \emptyset$.

The sample space equals $\Omega = C_3 \cup D_3$, or

(3) $$\Omega = \{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}, \{\{\emptyset\}\}\}.$$

This entails the σ-field $\mathbf{F} = \mathbf{P}(\Omega) = 2^{\Omega}$. Explicitly:

$$\mathbf{F} = \{\Omega, \emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}, \{\emptyset, \{\emptyset, \{\emptyset\}\}\}, \{\emptyset, \{\{\emptyset\}\}\}, \{\{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}, \{\{\emptyset\}, \{\{\emptyset\}\}\}\} \cup \{\{\{\emptyset, \{\emptyset\}\}, \{\{\emptyset\}\}\}, \{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}, \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}\}, \{\{\emptyset\}, \{\emptyset, \{\emptyset\}\}, \{\{\emptyset\}\}\}\} \cup \{\{\emptyset, \{\emptyset, \{\emptyset\}\}, \{\{\emptyset\}\}\}, \{\{\emptyset\}\}, \{\{\emptyset, \{\emptyset\}\}\}, \{\{\{\emptyset\}\}\}\}$$

Note, $D_3, C_3 \in \mathbf{F}$. The probability measure for Ω is $P(X) = |X|/|\Omega|$, with, $|X|$ the cardinality of X, i.e. $P \sim Uniform(\Omega)$. $(\Omega, \mathbf{F}, P)$ establishes classical probability. Finally, let us introduce the 'union of a set' [5], [6] operation,

(4) $$\cup Z = \{x | (\exists : y \in Z)(x \in y)\}$$

3. Application to Hardy's physics

For set $A = \{C_2\}$ and $B = D_3$ we see $A \subset C = C_3$ and $B = D = D_3$, hence, $B \subset D_3$. Obviously, $A \cap B = \emptyset$. The A and B represent disjoint parts of the electron and positron. Now, $C_2 \in A$ and that means, $\{\emptyset, \{\emptyset\}\} \in A$. Hence (eq. 4), $x = \emptyset$ and $x = \{\emptyset\}$ in C_2 giving $\cup A = C_2 = \{\emptyset, \{\emptyset\}\} \in \mathbf{F}$. Identically, $D_2 \in B = D_3$. Hence, $x = \{\emptyset\} = D_1$, such that $\cup B = D_2 = \{\{\emptyset\}\} \in \mathbf{F}$. Now, $C_2 \cap D_2 = \{\{\emptyset\}\} \Rightarrow P(C_2 \cap D_2) \neq 0$. There exist subsets of C_3 and D_3 that, after taking the union, allows for simultaneous probability $\neq 0$. Hence, classical probability can do something similar to quantum mechanics if the $\cup$ on disjoint (sub)sets in annihilation processes is physical.

4. Conclusion

A classical probabilistics explanation for a typical quantum behavior, similar to tunneling, has been found. If $\cup$ cannot be excluded from physics it may represent a quantum physical process and establishes a classical explanation. A possible physical picture for $\cup$ can perhaps be associated to a 'dark' mirror-matter sector [7], [8], [9] that may arise as a consequence of the experimentally established weak interaction parity non-invariance [10], [11].

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