Quantum mechanical EPRBA covariance and classical probability

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Received 11 January 2011

Key words EPR, Bell theorem, Locality

Contrary to Bell's theorem it is demonstrated that with the use of classical probability theory the quantum correlation can be approximated. Hence, one may not conclude from experiment that all local hidden variable theories are ruled out by a violation of inequality result.

1 Introduction

In the debate of the foundations of quantum theory (QM), Bell's theorem [1] is an important milestone. Based on Einstein's criticism of completeness [2], Bell formulated an expression for the correlation between distant spin measurements such as described by Bohm [3]. In Bell's expression, hidden variables to restore locality and causality to the theory (LHV's) are introduced through a probability (mass) density function and through their influence upon the elementary measurement functions in the two separate wings (denoted by the A- and the B-wing) of the experiment. Many experiments and theoretical developments arose from Bell's original paper. The most important experiment was performed by Aspect [4].

Aspect's results were interpreted as a confirmation of the completeness of quantum mechanics. From that point, QM was considered a non-local theory. In a previous paper [5] the present author argued that there was insufficient ground for this conclusion. The author would like to point that merely an appeal to the simplicity and apparent logical truth of Bell's theorem is advanced to support the exclusive non-locality interpretation. Beyond that, there is no proof that experimental results must exclusively be interpreted in this manner. According to the author the inequalities of Bell were given the status of theorem without sufficient supportive evidence save simplicity. In this paper it will be demonstrated that such a conclusive supportive argument of Bell's theorem does not exist because an approximate classical model is possible. The principles of the proposed model can also be expressed in numerical terms. This enables numerical simulation of the experiments on a computer.

Let us shortly describe a typical idealized Bell experiment. In such an experiment, from a single source, two particles with opposite spin are sent into opposite directions. For instance, we could think of a positron and an electron arising from para-Positronium that are drawn apart by dipole radiation. Subsequently, in the respective wings of the experimental set-up, the spin of the individual particle is measured with a Stern-Gerlach magnet. The measurements are found to be correlated with the, unitary, parameter vectors of the magnets, $\mathbf{a} = (a_1, a_2, a_3), \mathbf{b} = (b_1, b_2, b_3)$. It is well-known that the QM correlation for singlet state electron and positron is equal to,

$$P_{QM}(\mathbf{a}, \mathbf{b}) = -(\mathbf{a} \cdot \mathbf{b}) = -\sum_{k=1}^{3} a_k b_k. \tag{1}$$

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Bell's theorem states that local hidden variables cannot recover the quantum correlation, $-(\mathbf{a}\cdot\mathbf{b})$, because all local hidden distributions run into inequalities similar to the one stated below. Vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}$ and $\mathbf{d}$ are unitary.

$$|P_{LHV}(\mathbf{a},\mathbf{b}) - P_{LHV}(\mathbf{a},\mathbf{d})| + |P_{LHV}(\mathbf{c},\mathbf{b}) + P_{LHV}(\mathbf{c},\mathbf{d})| \leq 2 \tag{2}$$

2 Preliminary remarks

In order to study locality and causality in a quantitative manner, Bell wrote the following general expression for $P_{LHV}(\mathbf{a},\mathbf{b})$ based upon general, probability theoretical, assumptions about the classical distribution of hidden substance that was supposed to explain the quantum correlation between A and B measurements of spin. We have

$$P_{LHV}(\mathbf{a},\mathbf{b}) = \int d\lambda \rho(\lambda) A(\mathbf{a},\lambda) B(\mathbf{b},\lambda) \tag{3}$$

Here, λ represents the hidden variables. We are allowed to take as many LHVs, λ_m, with, $m = 1, 2, 3, ...$ as we like. Moreover, $\rho(\lambda)$ represents the probability density of the hidden variables. From this expresion, equation 2 is derived.

Because a classical explanation is sought for the quantum correlation, $\rho(\lambda)$ has to follow classical probability laws. This means the intuitive clear conception of probability and the associated variable space [6]. The measurement functions, $A(\mathbf{a},\lambda)$ and $B(\mathbf{b},\lambda)$ project in the set $\{-1,+1\}$, agreeing with the discrete result of measuring spins.

The author apologizes beforehand for not having mentioned many other important researches on Bell's theorem. His main concern in the paper was to obtain the proper mathematics. Two studies need to be mentioned however. In the first place the author would like to refer to the model of Hess and Phillip [7] and to a study of de Raedt [8]. The reader should also note that, although measurement functions and variables inside particles and measuring instruments are given, there is no physical explanation given. In this sense it is a mathematical model. Use will be made of some basic facts of physics, like energy quantization, in order to justify some choices in the model. Note that the hidden variables and their ranges will be defined in the probability density section. Moreover, it is at this stage of the model extremely difficult to provide a physical interpretation of the hidden variables.

3 Measurement functions

Here, we first introduce the model's measurement functions in terms of local hidden variables. Before doing that, let us postulate the variables that reside in the particles that propagate to the respective measurement instruments, A and B.

$$\sigma_{\{^A_B} = \sum_{k=1}^{3} \{^{a_k}_{b_k} sign(y_k) \tag{4}$$

This compact 'stacked' notation will be used throughout the paper. It must be read as

$$\sigma_A = \sum_{k=1}^{3} a_k sign(y_k),\ \sigma_B = \sum_{k=1}^{3} b_k sign(y_k) \tag{5}$$

The y_k,with, $k = 1, 2, 3$ in the previous formulae are standard normal Gaussian distributed variables, $N(0, 1)$, whereas the a_k and b_k arise from the two Stern-Gerlach instruments unitary parameter vectors.

The stacked notation implicitly denotes the similarity between the two measurement instruments. Most of the time, only the A version of the measurement functions and probability density will be given and it is implied that B follows the same mathematical structure. In this way the somewhat cumbersome stacked notation can be avoided in discussions of the mathematics. In any case and using the stacked notation here, it is clear that

$$|\sigma_{\{^A_B}| \leq \sqrt{3} \tag{6}$$

An additional set of definitions is necessary to better understand the model parameters. Let us define an ϵ_A, positive 'small' and similarly an ϵ_B such that

$$N_A = \frac{1}{\epsilon_A},\ T_A = \sqrt{\frac{\log(N_A)}{\sqrt{3}-1}} \tag{7}$$

Sometimes it is handy to either use N_A, or ϵ_A or T_A in the presentation of the model. The reader should note that they are related as presented in the equation above.

Basing ourselves upon the definitions of the previous sums that contain parameter vector components and variables carried by the particles and remembering the A and B structural equivalence, we may now write the first component of the measurement function A as

$$A_1 = sign[\frac{\sigma_A}{\sqrt{3}}\theta(t_A - \frac{1}{N_A^2})\theta(\frac{1}{N_A^2} + \frac{4}{N_A} - t_A)\sin(\alpha_A t_A) - \mu_A] \tag{8}$$

thereby introducing the hidden variables, α_A, t_A and μ_A. Here, the θ function is defined as $\theta(x) = 1$, when, $x \geq 0$, while,$\theta(x) = 0$, when, $x < 0$.

Perhaps, not necessary to say but a similar set for B, α_B, t_B and μ_B, is thought to exist that follow the same structure as A_1, in the construction of B_1. The distributions of the hidden variables or the values of hidden parameters will be given later. At the moment note that, $sign(x) = 1$ for,$x > 0$, while, $sign(x) = -1$ for, $x \leq 0$. Moreover it is illustrative to write the following integration for a certain $\xi \in [-1, 1]$

$$\begin{aligned}&\tfrac{1}{2}\int_{-1}^{1}\ sign(\xi - x)dx = \\ &\tfrac{1}{2}\int_{-1}^{\xi} dx\ -\ \tfrac{1}{2}\int_{\xi}^{1} dx = \tfrac{1}{2}(\xi - (-1)) - \tfrac{1}{2}(1-\xi) = \xi\end{aligned} \tag{9}$$

Because of the inequality on $|\sigma_A|$, we may conclude that $\frac{\sigma_A}{\sqrt{3}}\theta(t_A - \frac{1}{N_A^2})\theta(\frac{1}{N_A^2} + \frac{4}{N_A} - t_A)\sin(\alpha_A t_A) \in [-1, 1]$. Hence an integration like in the previous is allowed, when $\mu_A \in [-1, 1]$.

In addition, a second component of the measurement function A is to be defined as

$$A_0 = \begin{cases} 1,\ \text{when } \nu_A \in [-N_{\epsilon_A}, 1) \\ sign(\varphi_A - \nu_A), \text{when } \nu_A \in [1, N_{\epsilon_A}] \end{cases} \tag{10}$$

Here we have introduced the hidden variable $\nu_A \in [-N_{\epsilon_A}, N_{\epsilon_A}]$. The N_{ϵ_A} is derived from the previously introduced N_A as

$$N_{\epsilon_A} = \frac{N_A}{1 - \frac{1}{1+(N_A/\epsilon_A)^2}} \geq N_A \tag{11}$$

If, $\varphi_A \in [-N_{\epsilon_A}, N_{\epsilon_A}]$, then we may write a similar integration as

$$\frac{1}{2}\int_{-N_{\epsilon_A}}^{N_{\epsilon_A}} sign(\varphi_A - \nu_A)d\nu_A = \frac{1}{2}\int_{-N_{\epsilon_A}}^{\varphi_A} d\nu_A\ -\ \frac{1}{2}\int_{\varphi_A}^{N_{\epsilon_A}} d\nu_A = \varphi_A \tag{12}$$

Subsequently, the $\varphi_A = \varphi_A(x_A)$ function is given by

$$\varphi_A = \begin{cases} \frac{1}{x_A},\ x_A \in [\tau_A, 1] \\ (1 + T_A^2)/2,\ x_A \in [0, \tau_A) \\ 1,\ elsewhere \end{cases} \tag{13}$$

Here τ_A is defined by $\tau_A = exp[-T_A^2(\sqrt{3}-1)] = \epsilon_A = 1/N_A$, such as given previously. As is clear from the definition of φ_A, letting the hidden variable x_A run the whole real axis for the moment, this function projects in the interval $[1, N_{\epsilon_A}] \subset [-N_{\epsilon_A}, N_{\epsilon_A}]$. This is true because, φ_A attains its maximum value, N_A when $x_A = \tau_A$ and $N_A \leq N_{\epsilon_A}$. Furthermore, for x_A outside the interval $[0, 1]$, the function φ_A is equal to unity. In addition because, $\log(N_A) < (\sqrt{3}-1)N_A$ and $N_A \leq N_{\epsilon_A}$, when φ_A attains the value, $(1+T_A^2)/2$ for $x_A \in [0, \tau_A)$, it remains inside the interval $[1, N_{\epsilon_A}]$. Hence the integration over the interval,$[-N_{\epsilon_A}, N_{\epsilon_A}]$, in the former equation may be applied.

The respective A-wing and B-wing measurement functions are defined by $A = A_0A_1$ and $B = -B_0B_1$.

4 Probability model

In this section we turn the attention to the probability density, generally denoted by $\rho(\lambda)$ in Bell's formulation of the correlation. For an effective presentation the vanishing part of partial densities will be described in the text below the definition. Moreover, we only present the A-wing of the probability densities associated to the measuring instruments. The B-wing is implicitly identical to the A-wing.

Firstly, let us define the probability density for the x_A variables

$$\rho_{x_A}(x_A) = \frac{1}{1+T_A^2},\ for\ x_A \in [-T_A^2, 1] \tag{14}$$

while $\rho_{x_A}(x_A) = 0$, elsewhere, i.e. $x_A \notin [-T_A^2, 1]$. As can be easily verified, $\rho_{x_A}(x_A)$ is a classical density for

$$\int \rho_{x_A}(x_A)dx_A = \int_{-T_A^2}^{1} \frac{1}{1+T_A^2}dx_A = \frac{1+T_A^2}{1+T_A^2} = 1 \tag{15}$$

Secondly, let us define the density of t_A as

$$\rho_{t_A}(t_A) = \frac{2t_A\epsilon_A^2 N_A N_{\epsilon_A}}{(t_A^2N_A^2+\epsilon_A^2)^2},\ for\ t_A \in [0, 1] \tag{16}$$

while $\rho_{t_A}(t_A) = 0$ for $t_A \notin [0, 1]$. This density is also classical because

$$\int \rho_{t_A}(t_A)dt_A = \int_0^1 \frac{2t_A\epsilon_A^2 N_A N_{\epsilon_A}}{(t_A^2N_A^2+\epsilon_A^2)^2}dt_A. \tag{17}$$

From this it follows that, using, $\omega_A = N_A t_A$

$$\int \rho_{t_A}(t_A)dt_A = \frac{N_{\epsilon_A}\epsilon_A^2}{N_A}\int_0^{N_A} \frac{2\omega_A}{(\omega_A^2+\epsilon_A^2)^2}d\omega_A \tag{18}$$

Hence, according to the definition of N_{ϵ_A} in Eq. (11)

$$\begin{array}{l} \int \rho_{t_A}(t_A)dt_A = \\ -\frac{N_{\epsilon_A}\epsilon_A^2}{N_A}\int_0^{N_A} \frac{d}{d\omega_A}(\frac{1}{\omega_A^2+\epsilon_A^2})d\omega_A = \\ -\frac{N_{\epsilon_A}\epsilon_A^2}{N_A}\{\frac{1}{N_A^2+\epsilon_A^2} - \frac{1}{\epsilon_A^2}\} = 1. \end{array} \tag{19}$$

Thirdly, the density of ν_A can be given by

$$\rho_{\nu_A}(\nu_A) = \frac{1}{2N_{\epsilon_A}},\ for\ \nu_A \in [-N_{\epsilon_A}, N_{\epsilon_A}] \tag{20}$$

while, $\rho_{\nu_A}(\nu_A) = 0$ for $\nu_A \notin [-N_{\epsilon_A}, N_{\epsilon_A}]$. One can easily see that $\rho_{\nu_A}(\nu_A)$ is a classical density because

$$\int \rho_{\nu_A}(\nu_A)d\nu_A = \frac{1}{2N_{\epsilon_A}}\int_{-N_{\epsilon_A}}^{N_{\epsilon_A}} d\nu_A = 1. \tag{21}$$

Fourthly, the density for μ_A is given by

$$\rho_{\mu_A}(\mu_A) = \frac{1}{2}, \; for \; \mu_A \in [-1, 1] \tag{22}$$

while $\rho_{\mu_A}(\mu_A) = 0$ for $\mu_A \notin [-1, 1]$. This density is the well-known uniform density on $[-1, 1]$ and is classical with

$$\int \rho_{\mu_A}(\mu_A) d\nu_A = \frac{1}{2} \int_{-1}^{1} d\mu_A = 1. \tag{23}$$

For completeness the Gaussian standard normal density for three variables, denoted here with $N(0,1)(\mathbf{y})$, is written as

$$\rho_{\mathbf{y}}(\mathbf{y}) = \frac{1}{(2\pi)^{3/2}} exp[-\frac{1}{2}|\mathbf{y}|^2] \tag{24}$$

here, $|\mathbf{y}|^2$ is the Euclidean norm of the vector $\mathbf{y}$. Needless to say that $N(0,1)(\mathbf{y})$ for $\mathbf{y} \in R^3$ is classical. With the use of the previously defined A-wing densities, the total A-wing and B-wing density can be given

$$\rho_{\{^A_B} = \rho_{x_{\{^A_B}} \rho_{t_{\{^A_B}} \rho_{\nu_{\{^A_B}} \rho_{\mu_{\{^A_B}} \tag{25}$$

From the definition of the components it follows that $\rho_{\{^A_B}$ are classical. Hence, the total density $\rho_{tot} = \rho_{\mathbf{y}} \rho_A \rho_B$ is classical too. Note for completeness that ρ_{tot} plays the role of $\rho(\lambda)$ in the definition of Bell.

5 Evaluation of the model, preliminaries

Before presenting the complete evaluation of all the integrals in the model we show some essential results. First, note that integration of sums containing signs of y_k, with, $k = 1, 2, 3$, gives

$$\int_{-\infty}^{+\infty} dy_1 \int_{-\infty}^{+\infty} dy_2 \int_{-\infty}^{+\infty} dy_3 \; N(0,1)(\mathbf{y}) sign(y_k) sign(y_m) = \delta_{k,m}. \tag{26}$$

Here, $\delta_{k,m}$ is Kronecker's delta $k, m = 1, 2, 3$.

Secondly, let us evaluate the integral over the $\varphi_A(x_A)$ function. From the definitions of the measurement functions (in particular Eq. (13)) and the probability model (in particular Eq.(14)) it follows that

$$\begin{aligned} &\int_{-T_A^2}^{1} \tfrac{\varphi_A(x_A)}{1+T_A^2} dx_A = \int_{-T_A^2}^{0} \tfrac{1}{1+T_A^2} dx_A + \int_{0}^{\tau_A} \tfrac{(1+T_A^2)/2}{1+T_A^2} dx_A + \\ &\int_{\tau_A}^{1} \tfrac{1}{(1+T_A^2) x_A} dx_A \end{aligned} \tag{27}$$

Hence, from the previous it follows that

$$\begin{aligned} &\int_{-T_A^2}^{1} \tfrac{\varphi_A(x_A)}{1+T_A^2} dx_A = \tfrac{T_A^2}{1+T_A^2} + \tfrac{1}{2} exp[-T_A^2(\sqrt{3}-1)] + \tfrac{T_A^2(\sqrt{3}-1)}{1+T_A^2} = \\ &\tfrac{\sqrt{3}}{1+(1/T_A^2)} + O(\epsilon_A). \end{aligned} \tag{28}$$

Note that $O(\epsilon_A)$ is employed in the sense of Landau [9], without by necessity taken a limit.

6 Evaluation of the model, expectation value

In order to evaluate the integrations in our model for Bell's expression for the correlation, let us define the expectation values. For a general proper function F in the A-wing variables, x_A, t_A, ν_A, μ_A and the B-wing variables x_B, t_B, ν_B, μ_B we define

$$E(F) = \int \rho_{tot} F dx_A dt_A d\nu_A d\mu_A dx_B dt_B d\nu_B d\mu_B dy_1 dy_2 dy_3 \tag{29}$$

More in specific in an 'operator'-like form whith the order of integration as indicated

$$E(F) = \int_{-\infty}^{+\infty} dy_1 \int_{-\infty}^{+\infty} dy_2 \int_{-\infty}^{+\infty} dy_3 \int_{-T_A^2}^{1} dx_A \int_0^1 dt_A \int_{-N_{\epsilon_A}}^{N_{\epsilon_A}} d\nu_A \int_{-1}^{1} d\mu_A \times \int_{-T_B^2}^{1} dx_B \int_0^1 dt_B \int_{-N_{\epsilon_B}}^{N_{\epsilon_B}} d\nu_B \int_{-1}^{1} d\mu_B \{\rho_{tot} F\} \tag{30}$$

Hence, from the previous we may conclude that $E(1) = 1$ as required. In order to have a less cumbersome presentation we may rewrite the previous total expectation as

$$E(F) = E_{\mathbf{y}}(E_A(E_B(F))) \tag{31}$$

From Eq. (30) and the definition of the total density, ρ_{tot} we may obtain that taking the Gaussian expectation is equal to evaluate a proper function f as

$$E_{\mathbf{y}}(f) = \int_{-\infty}^{+\infty} dy_1 \int_{-\infty}^{+\infty} dy_2 \int_{-\infty}^{+\infty} dy_3 \; N(0,1)(\mathbf{y}) f \tag{32}$$

Moreover, we are able to write $E_A(F_A)$ for a proper F_A as

$$E_A(F_A) = \int_{-T_A^2}^{1} dx_A \int_0^1 dt_A \int_{-N_{\epsilon_A}}^{N_{\epsilon_A}} d\nu_A \int_{-1}^{1} d\mu_A \; \rho_A F_A \tag{33}$$

and similar for B-wing expectation $E_B(F_B)$. Observe that Bell's theorem does not disallow integral ordering. Moreover, there appears to be no experiment possible that may exclude the ordering presently employed. It may be noted too that $P_{LHV}(\mathbf{a}, \mathbf{b}) = E(AB)$, which justifies the title of (quantum mechanical) covariance. When $E(A) = E(B) = 0$ and $E(A^2) = E(B^2) = 1$ the covariance and correlation coincide.

7 Evaluation of the model, approximation

Let us now inspect more closely $E_A(A)$ and note that $A = A_0 A_1$. Firstly we take a closer look at the integration of A_1.

As already explained we may write

$$\int \rho_{\mu_A} A_1 d\mu_A = \frac{1}{2} \int_{-1}^{+1} sign[\tfrac{\sigma_A}{\sqrt{3}} \theta(t_A - \tfrac{1}{N_A^2}) \theta(\tfrac{1}{N_A^2} + \tfrac{4}{N_A} - t_A) \sin(\alpha_A t_A) - \mu_A] d\mu_A \tag{34}$$

or

$$\int \rho_{\mu_A} A_1 d\mu_A = \frac{\sigma_A}{\sqrt{3}} \theta(t_A - \frac{1}{N_A^2}) \theta(\frac{1}{N_A^2} + \frac{4}{N_A} - t_A) \sin(\alpha_A t_A). \tag{35}$$

Secondly, the integration of A_0 over ν_A gives

$$\frac{1}{2N_{\epsilon_A}} \int_{-N_{\epsilon_A}}^{N_{\epsilon_A}} A_0 d\nu_A = \frac{1}{2N_{\epsilon_A}} \int_{-N_{\epsilon_A}}^{1} d\nu_A + \frac{1}{2N_{\epsilon_A}} \int_{1}^{N_{\epsilon_A}} sign(\varphi_A - \nu_A) d\nu_A \tag{36}$$

or

$$\frac{1}{2N_{\epsilon_A}} \int_{-N_{\epsilon_A}}^{N_{\epsilon_A}} A_0 d\nu_A = \frac{1}{2N_{\epsilon_A}} \{1 + N_{\epsilon_A} + \int_{1}^{\varphi_A} d\nu_A - \int_{\varphi_A}^{N_{\epsilon_A}} d\nu_A\} = \frac{\varphi_A}{N_{\epsilon_A}}. \tag{37}$$

Hence, in the evaluation of $E_A(A)$ we see that the product of $\frac{\sigma_A}{\sqrt{3}} \theta(t_A - \frac{1}{N_A^2}) \theta(\frac{1}{N_A^2} + \frac{4}{N_A} - t_A) \sin(\alpha_A t_A)$ and $\frac{\varphi_A}{N_{\epsilon_A}}$ enters the integration over t_A and x_A. As was agreed in the expectation value part of the evaluation of the model the t_A integral 'goes first'.

We obtain the following expression $I =$

$$\frac{\sigma_A}{\sqrt{3}}\frac{\varphi_A}{N_{\epsilon_A}}\int_0^1 \frac{2t_A\epsilon_A^2 N_A N_{\epsilon_A}}{(t_A^2 N_A^2+\epsilon_A^2)^2}\theta(t_A-\frac{1}{N_A^2})\theta(\frac{1}{N_A^2}+\frac{4}{N_A}-t_A)\sin(\alpha_A t_A)dt_A. \tag{38}$$

This equation can be rewritten, noting $\epsilon_A N_A = 1$, as

$$I = \frac{\sigma_A\varphi_A\epsilon_A}{\sqrt{3}}\int_{\frac{1}{N_A^2}}^{\frac{1}{N_A^2}+\frac{4}{N_A}}\frac{2t_A}{(t_A^2 N_A^2+\epsilon_A^2)^2}\sin(\alpha_A t_A)dt_A. \tag{39}$$

Therefore we employ the filter $\theta(t_A-\frac{1}{N_A^2})\theta(\frac{1}{N_A^2}+\frac{4}{N_A}-t_A)\in\{0,1\}$ in the A_1 to 'catch' one of the first few violent fluctuations. If, $N_A \gg 0$ the integration collapses to a single term approximation of calculating the area under a recangular triangle with base, $dt_A = \frac{4}{N_A}$. We have

$$\begin{aligned}&\tfrac{\sigma_A\varphi_A\epsilon_A}{\sqrt{3}}\int_{\frac{1}{N_A^2}}^{\frac{1}{N_A^2}+\frac{4}{N_A}}\tfrac{2t_A}{(t_A^2N_A^2+\epsilon_A^2)^2}\sin(\alpha_A t_A)dt_A\approx\\ &\tfrac{\sigma_A\varphi_A}{2\sqrt{3}}\tfrac{2/N_A^5}{(2/N_A^4)^2}\tfrac{1}{N_A^2}\tfrac{4}{N_A}sin(\alpha\tfrac{1}{N_A^2}).\end{aligned} \tag{40}$$

with, $t_A = 1/N_A^2$ in the integrand leading to a height of the triangle $H=\frac{1}{2}N_A^3\frac{1}{N_A^2}sin(\alpha_A\frac{1}{N_A^2})$ and $B = dt_A = \frac{4}{N_A}$ its base. The well-known formula for the triangular area is $\frac{1}{2}HB = \frac{1}{2}\frac{1}{2}N_A\frac{4}{N_A}sin(\alpha_A\frac{1}{N_A^2})$. Hence, when $\alpha = \pi N_A^2/2$ it follows that

$$I \approx \frac{\sigma_A\varphi_A}{\sqrt{3}}. \tag{41}$$

The use of the area of a triangle is justified by the form of the integrand function in the interval $[\frac{1}{N_A^2}, \frac{1}{N_A^2}+\frac{4}{N_A}]$.It is an approximation that will become better and better when N_A is selected larger and larger. Because, we take $N_A \gg 0$ it follows that $1/N_A^m > 0$, $m = 1,2,3,4$. Thus, for increasing N_A with increasing precision an approximation of the integral with a rectangular triangle with height, H, computed in $t_A = 1/N_A^2$ and a base, $4/N_A$ can be employed.

In order to justify the lower and upper bound of integration, the quantum of energy, expressed in Planck's constant (for the historical discovery of the quantum of energy in the blackbody radiation theory see e.g. [10]), h can be invoked. Physically, there is overwhelming evidence that energy is quantised and that the minimum positive amount of energy, $E > 0$, cannot be lower than Planck's constant. This may perhaps explain the minimum stepsize of $\Delta t = 4/N = h$. The granulation of the integral comes from the θ filter in the measurement function. The following relation between N, ignoring the A or B index, and h is supposed to be $N = 1/4h$. In this respect $1/N^2$ can be interpreted as the smallest possible 'form' that allows measurement. The reader is reminded that the whole mathematical exercise is merely there to explain (correlation between) measurements of spins. Hence, restriction based on the finitude of energy in measurement may be invoked to allow equation 40 as the optimal form of (collapsed) integration. It also suggests a relation between the t variables and energy processes in the measurement.

8 Correlation

From equations 28 and 41 it is then inevitable that

$$E_A(A)\approx\frac{\sigma_A}{\sqrt{3}}[\frac{\sqrt{3}}{1+(1/T_A^2)}+O(\epsilon_A)] \tag{42}$$

This previous result can be rewritten in terms of $O(1/T_A^2)$ as

$$E_A(A)\approx\sigma_A(1+O(1/T_A^2)) \tag{43}$$

A similar result obtains for $E_B(B)$, thereby keeping in mind that, $B = -B_0 B_1$. Hence,

$$E_B(B) \approx -\sigma_B(1 + O(1/T_B^2)). \quad (44)$$

Assuming that ϵ_B is small positive of the same order as ϵ_A. Because both A as well as B project in the set $\{-1, 1\}$, and we already established that $E(1) = 1$, it follows that $E(A^2) = E(B^2) = 1$. The Gaussian integration is included.

Moreover, because of the results in 43 and 44 we now can conclude from the symmetry of the Gaussian that $E(A) = E(B) \approx 0$, thereby suppressing the $1/T_A^2$ and $1/T_B^2$ error terms. Finally, from equation26 and combining equations 43 and 44 again, suppressing the $1/T_A^2$ and $1/T_B^2$ error terms and noting that $E(AB) = E_{\mathbf{y}}(E_A(E_B(AB))) = E_{\mathbf{y}}(E_A(A)E_B(B)) \approx -E_{\mathbf{y}}\sigma_A\sigma_B$ we get $E(AB) \approx$

$$-\int_{-\infty}^{+\infty} dy_1 \int_{-\infty}^{+\infty} dy_2 \int_{-\infty}^{+\infty} dy_3 \; N(0,1)(\mathbf{y}) \sum_{k=1}^{3}\sum_{m=1}^{3} a_k b_m sign(y_k) sign(y_m) \quad (45)$$

Hence

$$E(AB) \approx -\sum_{k=1}^{3}\sum_{m=1}^{3} a_k b_m \delta_{k,m} = -\sum_{k=1}^{3} a_k b_k \quad (46)$$

which is the quantum correlation between the two measurement parameters.

9 Conclusion and discussion

In the previous sections the quantum correlation $-(\mathbf{a}\cdot\mathbf{b})$ was obtained from a classical model. As can be verified, the measurement functions project in $\{-1, 1\}$. The employed densities are all positive and each density integrates to unity.

Furthermore, some of the mathematics was backed with numerical analysis to optimize the integration. The selected parameters agree with the specifications of a classical model and are, hence, allowed. It should be clear that the numerical exploration is only there to justify further elaboration of the integral. The collapse of the integral to the single term with stepsize $\Delta t = 4/N$ is backed by reference to the fundamental quantum of energy, h. In this sense, the integral over t is approximated with a sum with a stepsize that is minimally $\Delta t = 4/N$. The θ filtering reduces this to a single term. This form of approximation is completely classically and the use of a rectangular triangle can justified by the shape of the integrand. The use of a finite stepsize does not violate classicality because approximations with a finite computer are considered valid (classical) approximations and not in need of a conceptual jump to a granular form of non-locality computing. Note also that with selection of the parameters, no limit is taken in the model itself.

In addition, the mathematical operations leading to $E_A(A) \approx \frac{\sigma_A}{\sqrt{3}}[\frac{\sqrt{3}}{1+(1/T_A^2)} + O(\epsilon_A)]$ are elementary and tested many times in other theories. The problem of a possible quickly vanishing integral I for small ϵ is solved by employing θ filtering.

For completeness, $E(AB) = E_{\mathbf{y}}(E_A(E_B(AB))) = E_{\mathbf{y}}(E_B(E_A(AB)))$. Hence, because A and B are completely independent, the previous interchange of expectation values may be employed. This is also a sign of genuine locality. As can be verified, the densities plus measurement functions of the A and B wing are completely independent.

The employed integration is in a definite order. Ordered integrations are allowed in Bell's theorem. If one wants to disallow them there has to be given solid reasons based on observations in physical nature without reference to non-locality of course. Finally it must be admitted that the physical nature of the local hidden parameters is unknown. The only differentiation between hidden variables at this moment is the sign of Gaussian variables in the particles themselves on the one hand and the sets related to the A and B wing on the other. In the paper only some initial steps were taken and further research will be necessary to establish a physical picture. However, it can be concluded that classical probability is not equivalent to classical physics and Bell's manipulations on his expression of the correlation in terms of local hidden variables do not by necessity establish a theorem.

We might observe from the definition of measurement functions that $A \leq 1$. Now if $E_A(A) \approx \sigma_A$ and σ_A as defined in equation (5) then with a properly behaving expectation operation we would have expected to see: $E_A(A) \leq E_A(1)$. However, when $E_A(A) \approx \sigma_A$ this will be violated, because, $E_A(1) = 1$. Note that the complete expectation E does behave as expected and, hence, the behavior of E_A cannot be uncovered in statistical physics experiment. Because Born's probabilistic interpretation of the quantum theory also is in need of an ensemble of particles before anything can be said about the probability, the previous statement can be defensed. In addition, the most simple defense would be to state that the E_A operation does not respect the sign in $A \leq 1$ because it is not a statistical expectation that can be measured in physical reality. The model would not be able to obtain the quantum correlation from a classical probability model. Experimental tests to accomplish quantum behavior would then be largely trivial. The question is: can this curiosity of the E_A operator be neutralized by calling it an error. Below we will try to demonstrate that the curious behavior of E_A is *neither* an error *nor* a deviation from a classical probabilistic description.

The demonstration will be done by employing a model from the theory of generalized functions. Let us inspect Hadamard's finite part function. For $x \neq 0$ we have

$$P_f \frac{\theta(x)}{x} = \frac{d}{dx} \{\theta(x) \log |x|\} . \tag{47}$$

Here, $\theta(x) = 1$ when $x > 0$ and $\theta(x) = 0$ when $x \leq 0$. Let us employ x in the interval $[-e, e]$. Let us also note that $P_f \frac{\theta(x)}{x} \geq 0$ for all real x, e is the base of Napier's log function $\ln$. Let us define an E_h operator like

$$E_h(\varphi(x)) = \int_{-e}^{+e} \varphi(x) P_f \frac{\theta(x)}{x} dx \tag{48}$$

with $\varphi(x)$ an arbitrary 'well behaved' function. In this case we may write for $E_h(1)$ the following expression

$$E_h(1) = \int_{-e}^{+e} P_f \frac{\theta(x)}{x} dx = \int_{-e}^{+e} \frac{d}{dx} \{\theta(x) \log |x|\} dx = [\theta(x) \log |x|]_{x=-e}^{x=+e} \tag{49}$$

Hence, $E_h(1) = 1$. Let us subsequently note that $\theta(1-x)\theta(1+x) = 1$ when $-1 < x < 1$ and zero elsewhere. Subsequently, $\theta(1-x)\theta(1+x) \geq x\theta(1-x)\theta(1+x)$, because for $\forall : x \in (-1, 1)$ we have $1 > x$ and $\forall : x \notin (-1, 1)$ the inequality collapses to $0 = 0$. With similar reasoning in mind as for E_A one would expect $E_h\{\theta(1-x)\theta(1+x)\} \geq E_h\{x\theta(1-x)\theta(1+x)\}$.

If we then inspect the h-expectation of $\theta(1-x)\theta(1+x)$, it is easy to see using equation (49) that $E_h\{\theta(1-x)\theta(1+x)\} = 0$. For, $x\theta(1-x)\theta(1+x)$ we first note that

$$E_h\{x\theta(1-x)\theta(1+x)\} = \int_{-e}^{+e} x\theta(1-x)\theta(1+x) \frac{d}{dx} \{\theta(x) \log |x|\} dx \tag{50}$$

This then gives

$$E_h\{x\theta(1-x)\theta(1+x)\} = [x\theta(x) \log |x|]_{x=-1}^{x=+1} - \int_{-1}^{+1} \{\theta(x) \log |x|\} dx \tag{51}$$

or

$$E_h\{x\theta(1-x)\theta(1+x)\} = -\lim_{\epsilon \to 0^+} [x \log |x|]_{x=\epsilon}^{x=1} + \lim_{\epsilon \to 0^+} \int_{\epsilon}^{+1} x \frac{d}{dx} \log |x| dx \tag{52}$$

Hence, $E_h\{x\theta(1-x)\theta(1+x)\} = 1$, despite, $E_h\{\theta(1-x)\theta(1+x)\} = 0$ and $\theta(1-x)\theta(1+x) \geq x\theta(1-x)\theta(1+x)$. Hence, E_h does not respect $\leq$ but is based on a valid density $\rho_X(x) = P_f \frac{\theta(x)}{x}$ for $x \in [-e, e]$. The fact that ρ_X can play the role of a density will be explained below.

If we associate a σ field $\mathbf{F} = \{\emptyset, \Omega\}$ with $\Omega = [-e, e]$, to the density, $\rho_X(x) = P_f \frac{\theta(x)}{x}, \forall : x \in \Omega$, then it can be verified that the measure, $P(X) = 1$, when $X = \Omega \in \mathbf{F}$ and $P(X) = 0$ when $X = \emptyset \in \mathbf{F}$. This can formally be obtained from the computation in equation (49) and coincides with $E_h(1) = 1$ for $X = \Omega$. It is true because,

$$P(X) =_{def} \int_{x \in (X \in \mathbf{F})} \frac{d}{dx} \{\theta(x) \log |x|\} \, dx \tag{53}$$

For completeness, the field $\mathbf{F}$ is closed under complement, under union and under intersection and is therefore a genuine algebra[11] hence, in this case, σ field. Therefore the measure $P(X), X \in \mathbf{F}$ is a classical probability measure

The E_A operator in the correlation model shows a similar behavior as E_h. There is nothing wrong with employing this in a statistical theory as long as the complete expectation, i.e. the E of the model does behave properly, i.e. $A \leq 1$ then $E(A) \leq E(1)$. Note also that the use of illustrative arguments to a feature of a larger model is common practice in developing a theory.

In conclusion, because the E_A behavior cannot be excluded in experiments and it does not violate the 'classicality' of the density, an explanation based on local hidden variables has been found for results obtained in experiments such as Aspect's. Note that classical probability theory also has been employed to explain Hardy's paradox [12]. In addition it has been demonstrated that the mathematics that Bell used did not exclude the possibility of LHV models [13]

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