A Reconstruction of Aristotle's Theory of Syllogism as a Theory of Sets

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This paper attempts to illustrate Aristotle’s logic mainly its concepts of genus, differentia and species and, more importantly, its theory of deduction as a theory of sets. If our articulation of them as sets is consistent, we might be allowed to conclude That what Aristotle had in mind in his logic was indeed a theory of sets.

1) Syllogism definition

When two predications are arranged in a way that, without the addition of any other term or relation, a new predication necessarily follows from them, we have a syllogism. Aristotle believes that ‘any form of persuasion’ is formed by syllogism. (PrA., B, 23, 68b9-18)

A syllogism includes two premises. (PrA., A, 23, 41a4-7; PrA., A, 25, 42a32-35) A premise is a settlement, or rejection of a settlement, between two terms. (cf. PrA., A, 24a28-30) Aristotle also speaks of immediate premise (πρότασις ἄμεσης), a proposition that has no other proposition prior to it. He calls such a premiss the principle (ἀρχή) of demonstration. (PsA., A, 2, 72a7-8) He draws a distinction between dialectical and demonstrative premiss:

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1 ‘A syllogism is a discourse in which, certain things being stated, something other than what is stated follows of necessity from their being so. I mean by the last phrase that they produce the consequence, and by this, that no further term is required from without in order to make the consequence necessary.’ (PrA. A, 24b18-22; cf. To., I, 1, 100a25-)
2 Abbreviations used in this paper: PrA. Prior Analytics PsA. Posterior Analytics
'If it is dialectical, it assumes either part indifferently; if it is demonstrative, it lays down one part to the definite exclusion of the other because that part is true.’ (PsA., A, 2, 72a8-11) Every syllogism includes three terms (cf. PrA., A, 25a30-31): two extremes, a major and a minor, and one middle. (cf. PrA., A, 4, 25b35-37; 26a21-23) The middle element, which is the element that is related to both of the extremes and is stated in both premises (PrA., A, 32, 47a30-34), is the element that makes a third relation and thereby a syllogism possible. Therefore, no syllogism is possible without a middle. (PrA., A, 23, 40b40-41a4 and a7-13PrA, A, 23, 84b19-27; PrA, A, 32, 47b8-9; PrA., A, 23, 68b30-37) The number of middles, however, does not make a problem because no matter how many they are, the figures will be the same. (PrA., A, 23, 41a18-20) It is indeed the middle about which the question arises. (PsA, B, 2, 89b37-90a11)

2) Peculiar, general, necessary and essential syllogism

An essential syllogism is a syllogism that its conclusion includes an essential relation. Here, the predicate in the conclusion belongs to the essence of its subject. A peculiar syllogism is a syllogism that its conclusion is a peculiar relation, which means the predicate is ‘peculiar’ to the subject. An essential predication in the conclusion does not happen unless both of the premises include an essential and universal predication; nor does a peculiar conclusion results without peculiar predication in both premises. Therefore, we have an essential syllogism when the predicates of the premises are essentially and universally to their subjects and belong to their essence. (PsA., B, 4, 91a18-25 and b7-11) Consequently, an essential conclusion does not result if only one of the premises has an essential relation. (PsA., B, 4, 91a21-23; 91b1-7) The same can be said about peculiar conclusion. (PsA., PsA., B, 4, 91a16-18; 8, 93a11-12)

A general syllogism is a syllogism its conclusion includes a general relation. A general relation between a subject and a predicate is a relation in which though the predicate is not predicated of all the subjects ‘always and in every case,’ it is predicated of most of the
cases or/and most of the times. Like an essential and a peculiar syllogism, a general syllogism also results only out of two general relations in the premises. (PsA., B, 12, 96a8-19)

A necessary conclusion, however, does not need both of the relations in premises to be necessary. The reason is that if A has a necessary relation to B even when B has a simple full inclusion relation with C, A must be necessarily related to C. (PrA., A, 9, 30a17-20)

3) **Whole-part relationship**

The whole-part relationship of the terms in syllogism can be proved both in the articulation of the figures of syllogism and in Aristotle’s own words.

a) **Whole-part relationship in Aristotle’s articulation of figures**

Based on the arrangement of the terms and the positions they take in these arrangements, there are three figures.

We have the *first figure* when the arrangement of the terms is such: major-middle and middle-minor. In this figure, ‘the last is contained in the middle as in a whole, and the middle is contained, or excluded from, the first as in or from a whole.’ (PrA., 4, 25b32-34)

In this figure, ‘the middle is contained in the major and contains the minor.’ (PsA., A, 4, 25b35-36) The order of terms based on largeness of size or extension is the basic order in which the name of terms matches their sizes: major-middle-minor. The first figure, thus, indicates the relations between the biggest and the smallest class through a medium class.

We have the *second figure* when the arrangement of the elements in relations are such: major-middle and minor-middle. In this figure, the belonging of the same term to other two terms is the case and the middle is that which is predicated of both of the extremes and stands outside them. (PrA., A, 5, 26b34-27a3) The middle is, thus, the largest in extent. Though there must supposedly not be an absolute distinction between major and minor, Aristotle calls that term which is near the middle the major and that which is far away from
the middle the minor. In fact, since the middle in this figure is of the largest extent, what is posited nearer to it will be of a larger inclusion. The order of terms based on the largeness of size or extension is: middle-major-minor. Here the major is only bigger than the minor. The second figure thus indicates the relation between two subclasses of a class.

We have the third figure when the arrangement of the elements in relations are such: middle-major and middle minor. In this figure, the predication of two terms of the same subject is the case and the middle is that of which both of the extremes are predicated and is, thus, the smallest in size. That extreme which is farthest from the middle, and is thence the larger one, is called the major while the other, which is nearer to the middle, and is thence the smaller one, is called the minor. (PrA., A, 6, 28a10-15) The third figure thus indicates the relation between two classes of a subclass.

We have thus three ways of connecting extremes by the middle. While the middle in the first figure stands between the extremes, predicating of one and being predicated of the other, it stands outside in the second figure predicating of both extremes but inside in the third figure of which both extremes are predicated. (PrA, A, 23, 41a13-18; 32, 47b1-5)

In all three figures, Aristotle calls that which is the bigger among the extremes the major and that which is smaller the minor. If we have the rule that the predicate is wider in extent than its subject in mind, we can see how whole-part relationship is the basis of the articulation of syllogisms in figures. It is the size of the term that is implied in its position. Thereupon, that which is predicated of both of the other terms in the first figure is called the major because it is bigger and that of which both of the other terms are predicated is called the minor. In the second figure, since the middle is predicated of both of the extremes, it is in fact the largest and the major here is bigger only than the minor. Thus, Aristotle calls that which is near the middle, and thus the bigger, the major while that which is far away than the middle, and thus smaller, the minor. In the third figure, since both of the extremes are predicated of the middle, it is the smallest one. Thus, the minor here is smaller only than the major. It is for this reason that Aristotle calls the major and the minor
in the opposite way, comparing with the second figure: that which is nearer to the middle, and thus the smaller, is called the minor while that which is far away from the middle, and thus the bigger, the major.

\[b) \textit{Aristotle conceding whole-part relationship}\]

First of all, Aristotle repeatedly uses \textit{en holoi} in Prior Analytics (e.g. 25b33, 30a2, 47a13, 53a21, 58b27-29, 66b15-16 and 79a36-40)) cf. B202, 19, n.12) and \textit{en merei} or \textit{kata meros} (e.g. 24a16-17, 25a5-6, 42a10, 42a16, 49b37, 64a17, 64b12, 69a14).\(^3\) Moreover, the whole-part relationship of elements in syllogism is conceded in Aristotle’s words when describing the relations between terms. The formation of a syllogism, Aristotle says, thoroughly depends on whole-part relation: ‘For in general, if two things are not related as whole to part (ὡς ὅλον πρῶς μέρος) and part to whole, the prover does not prove from them, and no syllogism is formed.’ (PrA., A, 41, 49b37-39)

At 266.20-30 Alexander interprets Aristotle as not saying simply that every deduction must have a universal premise but that every deduction must include a premise universal in relation to the subject term of the conclusion to be proved. Robin Smith\(^4\) criticizes Alexander because this means, he says, that ‘every deduction must include a premise which affirms or denies something universally of its minor term, then it is simply false, as evidenced by Darii, Ferio, Baroco, Festino, and the entire third figure.’ He believes, however, that this must be read in line with Aristotle’s inclination to talk as if every deduction were a first-figure universal deduction.

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4) **Quantity: four kinds of relations and six kinds of sets**

Regarding the quantity of relation, four kinds of relations and six kinds of sets can be recognized as follows:

   *a) *Full inclusion* \((\varepsilon)\)^5

A full inclusion is a relation in which a set includes all the members of another set. In this inclusion, the including set we call **Fullset** and the included set **Demiset**. A full inclusion is thus a relation between a fullset and a demiset.

   *b) *Half inclusion* \((\varepsilon)\)

A half inclusion is a relation in which two sets overlap and each include some of the members of the other. Each of the sets in this relation we call an **Overlapping set**.

   *c) *Full exclusion* \((\varepsilon)\)

A full exclusion is a relation in which a set excludes all of the members of another set. Each of the sets in this relation is called a **Noset** of the other.

   *d) *Half exclusion* \((\varepsilon)\)

A half exclusion is a relation in which a set excludes some of the members of the other set. The set that excludes we call **Lowerlapping** set and the set that is excluded **Lowerlapped** set. Aristotle calls this relation indefinite. (PrA., A, 4, 26b14-15)

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^5 Symbols used in this paper are as follows:

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\begin{align*}
\varepsilon & \quad \text{means: all are (everyone is)} & \text{e.g.: } A\varepsilon B & \quad \text{means: every A is B} \\
\varepsilon_\varepsilon & \quad \text{means: some are} & \text{e.g.: } A\varepsilon B & \quad \text{means: some A are B} \\
\varepsilon_3 & \quad \text{means: none is} & \text{e.g.: } A3B & \quad \text{means: no A is B} \\
\varepsilon_\varepsilon & \quad \text{means: some are not} & \text{e.g.: } A3B & \quad \text{means: some A are not B}
\end{align*}
\]
A. Rules of Fruitful Multirelations

Having recognized the four kinds of relations and the eight kinds of sets, now we are to do our final duty in this paper and to show how valid Aristotelian syllogisms are indeed rules based on the relations and the sets.

1) Rule I (R1): Full extension of full inclusions to demisets

The full inclusion of a set is necessarily a full inclusion of its full inclusions. Thus, the full set of a set is necessarily the fullset of its demisets. If A is a fullset of B, it is also a fullset of every set for which B is a fullset.6

2) Rule II (R2): Half extension of full inclusion to overlapping sets

The full inclusion of a set is necessarily (at least) a half inclusion of its overlapping sets. Thus, two sets, one a fullset and the other an overlapping set, of a third set must themselves at least be overlapping. If A is a fullset of B, it is at least an overlapping set of every set for which B is an overlapping set.7

3) Rule III (R3): Full extension of full exclusion to demisets

The relation between a set and a demiset of its noset is full exclusion: neither a set can include any member of a demiset of its noset, nor a demiset of its noset can include any of its members. Thus, if A and B are nosets and C a demiset of B, neither A can include any

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6 The primary affirmative mood of the first figure that Aristotle calls the primary universal syllogism is based on this rule: ‘if A is predicated of all B, and B of all C, A must be predicated of all C.’ (PrA., A, 4, 25b37-26a1; cf. PrA., B, 2, 53b20-23) (AԑB, BԑC→AԑC) Symbols are defined in note 5 at page 5 of this paper.

7 One mood of the first figure (AԑB, BԑC→AԑC) (PrA., A, 4, 26a17-20) and two moods of the third figure (BԑA, BԑC→AԑC) (BԑA, BԑC→AԑC) (cf. PrA., A, 6, 28b5-11 and 11-21) are based on this rule.
of the members of C, nor C any of the members of A\(^8\) where Aristotle says: ‘If then the terms are related universally a syllogism will be possible whenever the middle belongs to all of one subject and to none of another.’ (PrA., A, 5, 27a3-5; cf. PrA., A, 5, 26b34-27a3) Thus both of the moods at PrA., A, 5, 27a5-14 are based on this rule. Also cf. PrA., A, 5, 27a37-b6). This rule is asserted by Aristotle himself: ‘If A belongs to no B and to all C, we conclude that B belongs to no C. If then D is subordinate to C, clearly B does not belong to it … And yet B does not belong to E, if E is subordinate to A.’ (PrA., B, 1, 53a25-34) Aristotle does not, however, accept the extension of the rule to D and E as a syllogism.

4) Rule IV (R4): Half exclusion between a noset and an overlapping set of a set

There is half exclusion between an overlapping set of a set and its noset. Thus, an overlapping set of a set is necessarily a lowerlapped set of its noset. If B be an overlapping set and C a noset of A, it is necessary that B must be a lowerlapped set of C\(^9\) are based on this rule: ‘If the middle term is related universally to one of the extremes, a particular negative syllogism must result whenever the middle term is related universally to the major whether positively or negatively, and particularly to the minor and in a manner opposite to that of the universal statement.’ (PrA., A, 5, 27a26-32; cf. a32-36) It is also the basis of a mood of the first ((A\(\varepsilon\)B, B\(\varepsilon\)C\(\rightarrow\)A\(\varepsilon\)C) and (A\(\varepsilon\)B, B\(\varepsilon\)C\(\rightarrow\)C\(\varepsilon\)A)) (PrA., A, 4, 26a17-20) and one mood of the third figure. ((B\(\varepsilon\)A, B\(\varepsilon\)C\(\rightarrow\)C\(\varepsilon\)A) and (B\(\varepsilon\)A, B\(\varepsilon\)C\(\rightarrow\)A\(\varepsilon\)C)) (PrA., A, 6, 28b31-35)).

This rule is in strict contradiction with N5 in which it is said that there must be no necessary relation between a noset and an overlapping set of a set. One mood of the first (PrA., A, 4, 26a36-39) and one mood of the second figure (PrA., A, 5, 27b4-6) do not come to a necessary result based on this rule. Whereas it is asserted at PrA., A, 4, 26a17-20 that when

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\(^8\) This rule is a basis of a mood in the first figure (A\(\varepsilon\)B, B\(\varepsilon\)C\(\rightarrow\)A\(\varepsilon\)C) where Aristotle says: ‘If A is predicated of no B, and B of all C, it is necessary that no C will be A.’ (PrA., A, 4, 26a1-2; cf. PrA., B, 2, 54) and two valid moods of the second figure ((A\(\varepsilon\)B, C\(\varepsilon\)B\(\rightarrow\)C\(\varepsilon\)A) and (A\(\varepsilon\)B, C\(\varepsilon\)B\(\rightarrow\)C\(\varepsilon\)A)

\(^9\) Two moods of the second figure ((A\(\varepsilon\)B, C\(\varepsilon\)B\(\rightarrow\)A\(\varepsilon\)C) and (A\(\varepsilon\)B, C\(\varepsilon\)B\(\rightarrow\)C\(\varepsilon\)A)
no A is B but some B are C, there is a valid syllogism, it is asserted at PrA., A, 4, 26a36-39 that it cannot be a syllogism. It seems the problem arises out of Aristotle’s differentiation between major and minor: when the universal relation is on the side of the major we have a syllogism:

\[ A \varepsilon B, B \varepsilon C \rightarrow C \varepsilon A \]

But when the universal relation is on the side of the minor, there is no syllogism:

\[ C \varepsilon B, B \varepsilon A \rightarrow \text{No syllogism} \]

It is obvious that if we do not differentiate major from minor, there will be no difference between the two above types. But how can we differentiate them? How can we distinguish the major from the minor out of the two mentioned relations? How can we say C is a minor but A a major when we have only those two relations? Should not we determine our major and minor based on our own relations? But there is no real major or minor in such relations.

5) **Rule V (R5): Extension of half exclusion to demisets**

The half exclusion of a set is necessarily extendable to its demisets. Thus, a lowerlapped set of a set is necessarily a lowerlapped set of its demiset. If B is a demiset of A and C a lowerlapped set of it, it is necessary that C must be a lowerlapped set of B too.\(^\text{10}\)

6) **Rule VI (R6): Half inclusion between two full inclusions of a third set**

Two sets that are both full sets of a set are necessarily overlapping sets. In other words, if a set be a demiset of two sets, these two sets must overlap each other. If B be a demiset of both A and C, A and C must necessarily be overlapping sets.\(^\text{11}\)

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\(^\text{10}\) One mood of the second figure \(((A \varepsilon B, C \varepsilon B \rightarrow C \varepsilon A) \text{ and } (A \varepsilon B, C \varepsilon B \rightarrow A \varepsilon C))\) is based on this rule. (cf. PrA., A., 5, 27a37-b5; PrA., A, 5, 27a26-32)

\(^\text{11}\) One mood of the third figure is based on this rule. \((B \varepsilon A, B \varepsilon C \rightarrow A \varepsilon C \text{ and also we can say: } C \varepsilon A)\) (cf. PrA., A, 6, 28a18-22)
7) **Rule VII (R7): Half Extension of full exclusion to fullsets**

The relation between a set and a fullset of its noset is necessarily a half exclusion. Thus, the fullset of a set has necessarily half exclusive relation with its noset. In other words, If a set is a demiset of another set and at the same time a noset of a third set, it is necessary for its fullset to be a lowerlapped set of its noset: suppose B is a demiset of A and a noset of C, it is necessary for A to be a lowerlapped set of C.\(^\text{12}\)

This rule is both accepted and denied by Aristotle in one paragraph: ‘If R belongs to all S, and P to no S, there will be a syllogism to prove that P will necessarily not belong to some R. … But if R belongs to no S, P to all S, there will be no syllogism.’ (PrA., A, 6, 28a25-31) Thus:

\[
\begin{align*}
S & \in R, S \notin P \rightarrow S \notin P \\
S & \in R, S \in P \rightarrow \text{No Syllogism}
\end{align*}
\]

But what is the difference between these two? They cannot be different if there was no difference between S and P. But how can they be different when we are concluding based on only two relations? Aristotle wants the major and the minor be taken as different even when the relations do not make them different. So how can they make one different from the other? The inconsistency, the, arises from this differentiation, which is not meaningful when we take them as sets.

8) **Rule VIII (R8): Extension of half inclusion to full sets**

The half inclusion relation with a set is extendable to its fullsets. In other words, half inclusion of a set necessarily leads to the half inclusion of its fullsets. Thus, if a set is a lowerlapped set of another set, its fullset must also be at least a lowerlapped set of it. If B

\(^{12}\) One mood of the third figure ((B\(\sim\)A, B\(\in\)C\(\rightarrow\)C\(\sim\)A) and (B\(\in\)A, B\(\in\)C\(\rightarrow\)A\(\sim\)C)) is based on this rule. (cf. PrA., A, 6, 28a25-31) It seems that a mood of the first figure (A\(\sim\)B, B\(\in\)C\(\rightarrow\)C\(\sim\)A) must also be based on this rule.
is a lowelapped set of C, and A a fullset of B, it is necessary that A must be at least a lowerlapped set of C.\textsuperscript{13}

This rule is in contradiction with N11 where there is no necessary relation between a fullset and a lowerlapped set of a set. (PrA., A, 6, 28b22-23) Thus, we have two following formulae:

\[ S \in R, S_3P \rightarrow R_3P \]
\[ S \in P, S_3R \rightarrow \text{No Syllogism} \]

As we pointed out in our discussion of R4 and R7, this inconsistency is backed by Aristotle’s differentiation between major and minor while they cannot be differentiated.

**B. Cases where multirelations are not fruitful**

In the same way as the rules of fruitful multirelations, our reconstruction of relations and sets show how and why some of the syllogisms do not lead to valid results in Aristotle’s logic.

1) **N1: No necessary relation between a fullset and a noset of a set**

There is no necessary relation between a fullset of a set and its noset. If A is a fullset and C a noset of B, there is no necessary relation inferable between A and C.\textsuperscript{14}

This rule is in strict contradiction with R7, where the half extension of full exclusion to fullsets is approved, while two moods of the third figure and one mood of the first figure are accepted to result in syllogism. This contradiction is evident in one text where Aristotle both accepts R7 and denies it. (PrA., A, 6, 28a25-31)

\textsuperscript{13} One mode of the third figure ((B_3A, B_3C \rightarrow A_3C) and (B_3A, B_3C \rightarrow A_3C)) (cf. PrA., A, 6, 28b15-21)
\textsuperscript{14} One invalid mood of the first (A_3B, B_3C \rightarrow A_3C / No Syllogism) (PrA., A, 4, 25b37-26a2) and two invalid moods of the third figure ((B_3A, B_3C \rightarrow A_3C / No syllogism) and (B_3A, B_3C \rightarrow A_3C / No syllogism)) (PrA., A, 6, 28a30-31 and 28b25-31) are based on this rule.
2) **N2: No necessary relation between two full exclusions**

No relation can be inferred between two nosets of a set. If A and C are two nosets of B, none of the four relations is necessarily inferable between A and C.\(^{15}\)

3) **N3: No necessary relation between a full and a half inclusion**

The relation between a fullset of a set and an overlapping set of it can be either of the four kinds of relations and, thus, no syllogism is inferable between them. If A is a fullset and C an overlapping set of B, there is no necessary relation between A and C.\(^{16}\)

4) **N4: No necessary relation between a demiset and a lowerlapping set of a set**

The relation between a demiset and a lowerlapping set of a set is not necessarily one of the four kinds of relations. If B is a lowerlapping set of A and at the same time a fullset of C, the relation between A and C is not a necessary relation.\(^{17}\)

5) **N5: No necessary relation between a noset and an overlapping set of a set**

If A be a noset and C an overlapping set of B, no necessary relation is inferable between A and C.\(^{18}\) This is in strict contradiction with R4 in which it is necessary to be a half inclusion relation between an overlapping set of a set and its noset:

\[ \text{A} \varepsilon \text{B}, \text{B} \varepsilon \text{C} \rightarrow \text{A} \varepsilon \text{C} \]

\[ \text{and } \text{A} \varepsilon \text{B}, \text{B} \varepsilon \text{C} \rightarrow \text{C} \varepsilon \text{A} \]

\(^{15}\) One invalid mood of the first (A\varepsilon B, B\varepsilon C \rightarrow \text{No syllogism}) (PrA., A, 4, 26a9-11), one invalid mood of the second (A\varepsilon B, C\varepsilon B \rightarrow \text{No syllogism}) (cf. PrA., A, 5, 27a20-21) and one invalid mood of the third figure (B\varepsilon A, B\varepsilon C \rightarrow \text{No syllogism}) (PrA., A, 6, 28a33-34) are based on this rule.

\(^{16}\) One invalid mood of the first (A\varepsilon B, B\varepsilon C \rightarrow \text{No syllogism}) (PrA., A, 4, 26a20-21 and a30-34) and two invalid moods of the second figure ((A\varepsilon B, C\varepsilon B \rightarrow \text{No syllogism}) and (A\varepsilon B, C\varepsilon B \rightarrow \text{No syllogism})) (PrA., A, 5, 27b1-3 and b23-28) are based on this rule.

\(^{17}\) One mood of the first figure is based on this rule. (A\varepsilon B, B\varepsilon C \rightarrow \text{No syllogism}) (PrA., A, 4, 26a30-34)

\(^{18}\) One mood of the first (PrA., A, 4, 26a36-39) and one mood of the second figure (A\varepsilon B, C\varepsilon B \rightarrow \text{No syllogism}) (PrA., A, 5, 27b4-6; cf. PrA., B, 1, 53a34–) are based on this rule.
6) **N6: No necessary relation between a noset and a lowerlapping set of a set**

If C is a noset and A a lowerlapping set of B, there is no necessary relation between A and C.\(^{19}\)

7) **N7: No necessary relation between a fullset and a lowerlapped set of a set**

If A is a fullset and C a lowerlapped set of B, there is no necessary relation inferable between A and C.\(^{20}\)

8) **N8: No necessary relation between a full and a half exclusion**

There is no necessary relation between a noset and a lowerlapped set of a set. If A is a noset and C a lowerlapped set of B, there will be no necessary relation between A and C.\(^{21}\)

9) **N9: No necessary relation between two halfly related sets**

Two halfly related sets, whether two half inclusions, two half exclusions or one half inclusion and one half exclusion, cannot bring about a third necessary relation. None of two overlapping sets, two lowerlapping sets, two lowerlapped sets, one overlapping and the other lowerlapping/lowerlapped set or one lowerlapping and the other lowerlapped set may result in a necessary relation. Thus, if A is an overlapping or a lowerlapping or a

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\(^{19}\) One mood of the first (A\(\varepsilon\)B, B\(\varepsilon\)C→ No syllogism) (PrA., A, 4, 26a36-39) and two moods of the third figure ((B\(\varepsilon\)A, B\(\varepsilon\)C→ No syllogism) and (B\(\varepsilon\)A, B\(\varepsilon\)C→ No syllogism)) ((PrA., A, 6, 28b31-36 and 28b36-29a6) are based on this rule.

\(^{20}\) One mood of the first figure (A\(\varepsilon\)B, B\(\varepsilon\)C→ No syllogism) (PrA., A, 4, 26a36-39) is based on this rule.

\(^{21}\) One mood of the first figure (A\(\varepsilon\)B, B\(\varepsilon\)C→ No syllogism) (PrA., A, 4, 26b10-14) and two moods of the second figure ((A\(\varepsilon\)B, C\(\varepsilon\)B→ No syllogism) and (A\(\varepsilon\)B, C\(\varepsilon\)B→ No syllogism)) (PrA., A, 5, 27b12-16 and b28-31) are based on this rule.
lowerlapped set of B, we do not have a syllogism whenever C is an overlapping or a lowerlapping or a lowerlapped set of B.\(^{22}\)

10) \(N10: No\ necessary\ relation\ between\ two\ demisets\ of\ a\ fullset\)

If B is a fullset of both A and C, there will be no necessary relation between A and C except when one of A and C is the fullset of the other, which leads to a valid syllogism based on R1.

11) \(N11: No\ necessary\ relation\ between\ a\ fullset\ and\ a\ lowerlapped\ set\ of\ a\ set\)

If A is a fullset and C a lowerlapped set of B, there will be no necessary relation between A and C.\(^{23}\)

This is in strict contradiction with R8 where half inclusion to fullsets is said to be extendable. (PrA., A, 6, 28b15-21)

All figures, their moods and the rules applicable to each of them are shown in three tables in the appendix.

C. Is Aristotle’s Logic a Theory of Classification?
There are some commentators who agree that Aristotle’s logic somehow is, or at least contains, a theory of classification.

\(^{22}\) Four moods of the first ((A∈B, B∉C→ No syllogism), (A∉B, B∉C→ No syllogism), (A∉B, B∉C→ No syllogism) and (A∉B, B∉C→ No syllogism)) (PrA., A, 4, 26b21-25), four moods of the second ((A∈B, C∈B→ No syllogism), (A∈B, C∉B→ No syllogism), (A∉B, C∉B→ No syllogism) and (A∉B, C∉B→ No syllogism)) and four moods of the third figure ((B∈A, B∉C→ No syllogism), (B∉A, B∉C→ No syllogism), (B∉A, B∉C→ No syllogism), (B∋A, B∋C→ No syllogism) and (B∋A, B∋C→ No syllogism)) (PrA., A, 6, 29a6-9) are based on this rule.

\(^{23}\) One mood of the third figure ((B∋A, B∋C→ A∋C / No syllogism) and (B∋A, B∋C→ C∋A / No syllogism)) (PrA., A, 6, 28b22-23) is based on this rule.
1. K. J. Spalding\textsuperscript{24} thinks that Plato’s Ideas are fundamentally generic and the dialectic is a process of classification. He brings forward two reasons for his claim: i) classification reduces many individuals to a unity which is above them and ii) it does so by overcoming oppositions. He appeals to \textit{Philebus} (16, V.): ‘The precise question to which the previous discussion requires an answer is how they are one and also many and not at once infinite, and what number of species is to be assigned to them before we allow them to drop into infinity.’ Spalding asserts: ‘If a process such as this was dialectic then, in requiring not merely genera but also species, it was pre-eminently classification.’

2. Gareth B. Matthews\textsuperscript{25} says: ‘Man … is said of the individual man, say Socrates. And what that means is that Socrates is classified basically and fundamentally as a man. Put the other way round, man is said of Socrates means that man classifies Socrates in a fundamental way. … Here one might wonder why we shouldn’t say that Socrates is said of Socrates… The reason seems to be that Socrates does not classify Socrates; it names him.’

3. Discoll\textsuperscript{26} believes that PrA. I, 24b26-30 suggests the interchangeability of talk about universal and classes.

4. Phil Corkum believes that the fact that Aristotle thinks in terms of collectivity does not necessarily mean that he must be thinking of sets because there is an alternative: mereology. A major difference between these two theories is that while set theory differentiates membership from inclusion, mereology does not make such a differentiation and has a unitary part relation. This explains why Aristotle does not distinguish class-membership from class-inclusion, and this objection is based on the wrong assumption that Aristotle’s theory is a set theory. He believes that Aristotle’s theory is a mereology rather than a set theory. ‘A mereology,’ he says,

\begin{flushright}
\textsuperscript{24} Spalding, K. J., On the Sphere and Limit of the Aristotelian Logic, Mind, Vol. 17, No. 66, Apr. 1908, p. 218
\textsuperscript{26} Quoted from Graham, Daniel W., Aristotle’s Two Systems, 1987, Oxford University Press, p. 32 n. 21
\end{flushright}
‘unlike a set theory, typically employs uniting part relation and not distinct membership and inclusion relations.’

The view he ascribes to Aristotle is this: ‘both subjects and predicates refer, while holding that he would deny that a sentence is true just in case the subject and predicate name one and the same thing.’ He thinks that Aristotle’s semantic is not identity but ‘the weaker relation of constitution.’ Based on this view, ‘an ordinary predication such as ‘Man is mortal’ expresses a true thought, in Aristotle’s view, just in case the mereological sum of human is a part of the mereological sum of mortals.’ Nonetheless, he insists that he is not claiming that categorical predications ‘can be reductively analyzed into mereological claims’ but only that ‘certain mereological claims provide informative truth conditions for the thoughts which are expressed by categorical predications.’

5. K. J. Spalding believes that ‘classification is the real, if implicit, basis of the Aristotelian logic.’ He accepts, however, that there are to be found theories in Aristotle’s logic being emphatically not developments of classification.’ He also admits that classification could not have been the meaning of Aristotle’s logic. What he emphasizes on is that: ‘The most important feature of the Aristotelian logic is perhaps its insistence on universality- on connexion of contents, on independence of enumeration.’

6. To resolve Velastos’ objection that Aristotle does not distinguish class-membership from class-inclusion, Daniel W. Graham thinks that it is not clear that taking secondary substances as classes is a valid assumption.

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28 Ibid., pp. 14-15
29 Ibid., p. 16
31 Ibid., p. 224
32 Graham, Daniel W., Aristotle’s Two Systems, 1987, Oxford University Press, p. 32
# Appendix – Table of Figures and Moods

## First Figure

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