CHAOTIC SPACE – TIME

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ABSTRACT

In this paper we have shown how the consideration of a chaotic mechanics supplies a redefinition of special-relativistic space-time. In particular chaotic time means no possibility of defining temporal ordering and implies a breakdown of causality. The new chaotic transformations among "undetermined" space-time coordinates are no more linear and homogeneous. The principles of inertia and of energy-impulse conservation are no longer well defined and in any case no more invariant.

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I. Introduction

Since special relativity has appeared,[1] the hierarchy between geometry and dynamics in physical theories has been questioned and turned up down.[2] Following Klein's “Erlanger Programm”,[3] as well known, geometry can be defined by its invariance transformation group. Thus, one can define geometry as, a non- - a priori given object, but as a physical, operational structure given by the dynamical invariance transformation group. This is also in a more evident way at the ground of general relativity theory.

In recent times, Poincaré’s[4] and Born’s[5] analysis on the problem of the actual predictability and determinism of classical mechanics have been independently rediscovered and developed, poínting out the very general emergence of chaos as an invariant feature of classical and quantum mechanics.[6] Indeed, already very simple and common mechanical systems give rise to chaos.[7] Hence dynamical chaos has very fastly and relevantly modified our mechanical representafion of the world, and in some way it has found a geometrical counterpart in the idea of fractals.[8] However, this kind of relation between dynamical chaos and fractal geometry rests only on external grounds and has never affected our idea of time. Only Arecchi[9] has recently attempted a new definition of time starting from bifurcations of nonlinear systems and Prigogine has recognized as a consequence of chaos the breakdown of time reversal symmetry.[10]

In this paper we would like to show how the consideration of a general chaotic mechanics supplies a redefinition of special relativistic chrono-geometry (space-time) as a whole. In such a framework a new chaotic space-time is defined, of which Lorentz space-time are found as a limiting case corresponding to a non-chaotìc (relativistic) mechanical regime.

As well known, Lorentz trasformations can be obtained by the only requisite of the preservation of temporal ordering of events, that is relativistic causality.[11]

Chaotic time means no possibility of a local or global, absolute or relativistic, temporal ordering of events, that is a breakdown of causality; hence chaotic time implies a breakdown of Lorentz transformations. The transformations defining the new chrono-geometry become event-dependent (already for inertial reference frames, in relation to dynamical chaos). We have always to consider transformations from “undetermined” space-time coordinates $\left(x_\mu \pm \Delta x_\mu\right)$ to other "undetermined" space-time coordinates $\left(x'_\mu \pm \Delta x_\mu{}'\right)$.[12] From this point of view, we bave no longer a unique spacetime coordinatization even for a particular reference frame and no space-time ìnvariants as the metric interval.

II. Chaotic time

In the last few years, chaothic *phenomenology* has been extensively revealed in varíous disciplines and, in particular, in several domains of the physics.[13] More specifically, chaos has been observed and studied ranging from General Relativity to Quantum Relativistic systems.[14] As it is well known an important feature of chaotic mechanical problems is the sensitive dependence on initial condition of the dynamical evolution: two different trajectories starting very close rapidly diverge. The above property cause an exponential growth of initial errors.

In this paper we focus our attention on Special Relativistic systems, that exhibit non-linear chaotic behaviours. Here, we give only the general features. First of all, we would like to point out the argument to justify our new analysis: as long as we bave to recover classical mechanics as a limiting case of special relativistic mechanìcs, chaos must emerge also within special relativity. Its dynamical equations are formally equivalent to the classical ones. In this case, the total error bar, associated with one of spatial coordinates of x_μ for a moving particle, evolves, as a function of the proper time τ, as follows:

$$\varepsilon(\tau) = \varepsilon_0 2^{\lambda\tau} (\lambda > 0) \qquad (1)$$

where λ is the standard Lyapunov exponent.[15]

Equation (1) gives the τ-evolution of the initial error bar ε_0 as analytlcally determined by the mechanical laws.

After using eq. (1), one can easily calculate the uncertainty $\Delta[u]$ associated wíth i-th chaotic component of u_μ, directly from kinematic definitíon of the four-velocity

$$u_\mu \equiv \frac{dx_\mu}{d\tau} \qquad (2)$$

$$\Delta[u_i] = \mathrm{K} 2^{\lambda\tau}. \qquad (3)$$

In the classical physical-mathematical framework of continuum space-time the more preservative and consistent posítion ís one to assume ε_0 as ìnfinitesimal. Consequently it seems correct to consíder K as a finite constant.[16]

The first three components of the four-velocity are related to the three-vector $\vec{v} \equiv d\vec{x}/dt$ by the well-known relation:

$$u_i = \gamma v_i \qquad (4)$$

In equation (4):

$$\gamma \equiv \frac{1}{\left(1 - V^2/c^2\right)^{1/2}} \qquad (5)$$

We have to note that, in order to relate a generic coordinate time to proper time, the quantity *V*, which we must consider in eq. (5), is nothing else than *v*. Thus, the error on particle velocity is an error also on the velocíty associated to the motion of proper reference frame. Here, we stress that when we study accelerated motions Special Relativìty can be formulated only for istantaneous inertial reference frames. A simple algebraic calculation of error propagation permits to write explicity the total uncertainties

$\Delta[u^2]$, $\Delta[v^2/c^2]$, $\Delta\left[\left(1 - v^2/c^2\right)\right]^{1/2}$ and $\Delta[\gamma]$ induced by $\Delta[u_i]$ on u_i^2, v^2/c^2, $\left(1 - v^2/c^2\right)^{1/2}$ and γ respectively:

$$\Delta\left[u_k^2\right] = \begin{cases} 2u_k \Delta[u_k] & |u_k| > \dfrac{\Delta[u_k]}{2} \\ |u_k|\Delta[u_k] + \left(\dfrac{\Delta[u_k]}{2}\right)^2 + u_k^2, & |u_k| < \dfrac{\Delta[u_k]}{2} \end{cases}, \qquad (6a)$$

$$\Delta\left[v^2/c^2\right] = \frac{2\left(2u^2 + c^2\right)\Delta\left[u^2\right]}{\left(u^2 + c^2\right)^2 - \Delta^2\left[u^2\right]}, \qquad (6b)$$

$$\Delta\left[\left(1-v^2/c^2\right)^{1/2}\right]=\frac{2\Delta\left[v^2/c^2\right]}{\left[1-v^2/c^2+\Delta\left(v^2/c^2\right)\right]^{1/2}+\left[1-v^2/c^2-\Delta\left(v^2/c^2\right)\right]^{1/2}} \qquad (6c)$$

$$\Delta[\gamma]=2\frac{\Delta\left[\left(1-v^2/c^2\right)^{1/2}\right]}{\left(1-v^2/c^2\right)^{1/2}-\Delta^2\left[\left(1-v^2/c^2\right)^{1/2}\right]} \qquad (6d)$$

Hence, in the chaotic hypotesis, we can exactly extrapolate that, running τ, the error bar on γ diverges as $\Delta\left[u^2\right]^{1/2}$:

$$\Delta[\gamma]_{\tau\to\infty}=\left(\Delta\left[u^2\right]\right)^{1/2}. \quad (7)$$

From equations (3) and (6a) we thus obtained:

$$\Delta[\gamma]\gtrsim A2^{\frac{\lambda}{2}\tau} \qquad (8)$$

Finally, standard equations of Special Relativity permit to resolve t as a

function of τ:

$$t=\int_{\tau_0}^{\tau}\gamma d\tau. \qquad (9)$$

Equation (9) provides a direct route in order to calculate error bar $\Delta[t]$:

$$\Delta[t]=\int_{\tau_0}^{\tau}\left\{\left(\gamma+\Delta[\gamma]\right)-\left(\gamma-\Delta[\gamma]\right)\right\}d\tau \qquad (10)$$

From this formula, we can see that there is no reference frame for which the error on time is zero: also in the case v=0 (proper time) the error is not zero, that is also proper time as evolution variable is chaotic. Combining equations (8) and (9) it is immediate to demonstrate that the uncertainty on the time t(τ) results:

$$\Delta[t] \gtrsim B 2^{\frac{\lambda}{2}\tau}. \qquad (11)$$

On the basis of the above concluding result we can assert that: even an infinitesimal initial error, which affects one of the spatial coordinates, induces a finite error on the relative time t. Moreover, in the chaotic hypotesis, the proper-time evolution of $\Delta[t]$, as analytically governed by mechanical laws, diverges very rapidly wìth, at least, an exponentìal growth.

III. Chaos and Lorentz trasformations

Let us now consider how temporal ordering and causality are violated. Temporal ordering of events can be defined as:

$$x_\mu < y_\mu \, iff \, x_0 < y_0 \rightarrow \Delta s^2 = c^2 \Delta t^2 - \Delta|\vec{x}|^2 > 0 \,, \qquad (12)$$

here $x_0 \equiv ct_x$ and $y_0 \equiv ct_y$.

Thus, if we have a $x_0 \pm \Delta^{\pm}[x_0]$, with $\Delta^{+}[x_0] + \Delta^{-}[x_0] = c\Delta[t_x]$ non-negligible total error bar, we in general can write:

$$x_0 \pm \Delta^{\pm}[x_0] \gtrless y_0 \pm \Delta^{\pm}[y_0] \rightarrow \Delta s^2(x_\mu, y_\mu) \gtrless 0 \qquad (13)$$

If we now perform new Lorentz transformations

$$t' \pm \Delta^{\pm}[t'] = \left(t \pm \Delta^{\pm}[t] - (x \pm \Delta^{\pm}[x])(v \pm \Delta^{\pm}[v])/c^2\right)(\gamma \pm \Delta^{\pm}[\gamma]) \quad (14a)$$
$$x' \pm \Delta^{\pm}[x'] = \left(x \pm \Delta^{\pm}[x] - (v \pm \Delta^{\pm}[v])(t \pm \Delta^{\pm}[t])/c^2\right)(\gamma \pm \Delta^{\pm}[\gamma]) \quad (14b)$$

we have also that $x_0 < y_0$ does not ímply $x'_0 < y'_0$. So temporal ordering cannot even be defined and in any case it is not an ínvariant feature of the world of events. Thus, of course, we also find as obvious consequences Prigogine's result of the breakdown of time reversal: irreversibìlity. The principles of inertiae and of energy-impulse conservation are no longer well defined in correspondence with the error $\Delta[u]$ and they are however no more ínvariant. In fact these new transformations must be used to define a chaotic space-time: they are no more linear and homogeneous and they change hypothetical-inertial in non-inertial reference frames. As it could be derived directly from the uncertainty on $\Delta[u]$, it is no more possible to distinguish between inertial and non-inertial reference frames.

IV. Conclusions and prospects

In this paper we have shown that it is enough only an infinitesimal error on the initial condition (due, for example, to an experimental measure performed with an ideal infinitesimal precision), in order that the analytical chaotic dynamics of the system is affected by finite and rapidly increasing error bars $\Delta[u]$ and $\Delta[t]$.

In the previous section we have already discussed as the implications of the above results break down the usual mechanical special-relativistic theory, involving a new chaotic space-time.

These preliminary considerations anticìpate a systematic analysis about the way the presence of initial error imposes a revision of the physical-mathematical framework of relativistic and classical mechanics and paves the way for a new finite or, equìvalently, probabìlistic perspective.

Therefore, we could or should at least introduce, as it was done in quantum (relativistic) physics and how it was suggested by Born[18] even for classícal mechanics, probability distributions for the space-time and the other related physical variables, which can no longer be considered as actual physical variables, by changing to an intrinsic event representation where the event-"fields" themselves are the physical variables.

It can be shown as this is the case also for Galilei transformations and classical mechanics, because we have to consider a chaotic neo-newtonian space-time.

The general features of this new theoretical framework, imply the need to link operatíonal definitions of physical quantities with error theories, to which the chaotic phenomenology has given big importance.

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