9

Parts of Propositions

CODY GILMORE

Introduction

Russellian propositions are said to have *constituents*. An 'atomic' Russellian proposition has, as constituents, the objects the proposition is about and the property or relation it predicates of them. For example, the Russellian proposition that Etna is higher than Vesuvius has, as constituents, Etna, Vesuvius, and the relation <u>being higher than</u>.

Are the constituents of a Russellian proposition *parts* of it? There are respectable views that say 'No',[1] but I think the default answer should be 'Yes'.[2] If we can make this answer work without too much strain, we should. Can we?

Here is one reason for thinking that we can't. If Russellian propositions have their constituents as parts, they generate apparent counterexamples to plausible and widely accepted mereological principles. As Frege noted (1980, p. 79), Russellian propositions would seem to violate the transitivity

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[1] It is hard to find examples of those who explicitly endorse the negative answer, though I suspect that the view is widespread among contemporary friends of Russellian propositions. Crimmins (1992, p. 114, n. 2) distinguishes constituency from what he calls 'physical parthood'. Grossmann (1992, p. 76) and Armstrong (1986) explicitly defend an analogous view: viz., that *facts* and/or *states of affairs* have (proper) constituents but not (proper) parts. See McDaniel (2009) for discussion and further references.

[2] I say this mainly because I find constituency hard to understand unless it's identified with parthood or analyzed as a restriction on it. Parthood, on the other hand, strikes me as well-understood and plausibly topic-neutral. Russell and Frege both seem to assume that Russellian propositions (if there were such things) would have their constituents as parts. (See section 4 of this chapter.) Others who endorse this view include Cappelen and Hawthorne (2009, p. 2, n. 4), and especially Tillman and Fowler (2012), who defend it at length. King (2007, p. 120, n. 42) endorses the view that propositions are certain kinds of facts that have their constituents as parts. Caplan, Tillman, and Reeder (2010) defend an analogous view about singleton sets: viz., that each of them has its member as a part.

of parthood. As William Bynoe (2010) has noted, they would seem to violate certain 'supplementation' principles. (Bynoe actually makes this point about facts, but it carries over in an obvious way.)

I offer a unified solution to these problems. One key ingredient in the solution is the view, defended in Gilmore (2009), that parthood is a four-place relation.[3] Another key ingredient is the view that the semantic contents of predicates and sentential connectives have 'slots' or 'argument positions' in them.

The plan for the chapter is as follows. In sections 2 and 3, I set up a very simple formal language and state Russellianism as a thesis about the propositions expressed by the sentences of this language. In sections 4 and 5, I present the problems for Russellianism, and in sections 6 and 7, I propose a solution. Two appendices offer independent arguments for certain components of that solution.

1. Preliminaries

Three points are worth making before we get started.

(1) There is a dispute about how many (fundamental) parthood relations there are.[4] *Compositional monists* (Lewis 1986a; Sider 2007) say that there is exactly one; *compositional pluralists* (Grossmann 1992; McDaniel 2004) say that there are at least two, presumably associated with different ontological categories and perhaps governed by different principles. I will assume that compositional monism is true, mainly for simplicity. This assumption does real work only in those parts of the chapter that deal with Frege's worry about the transitivity of parthood, and it may be dispensable even there. In the rest of the chapter I continue to operate under that assumption, but only for convenience. Thus, even if the Russellian is antecedently convinced that parthood for propositions (parthood$_p$) is different from parthood for material objects (parthood$_m$), he should still find the chapter interesting.

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[3] Similar views are independently developed and discussed in Kleinschmidt (2011).
[4] Following McDaniel (2009), say that a fundamental parthood relation is a parthood relation that is not analyzed in terms of some *more natural* (roughly in the sense of Lewis 1986b) parthood relation.

(2) Russellianism faces one obvious mereological problem that I *won't* discuss in this chapter: it's in tension with the Uniqueness of Composition, the principle that no things compose more than one entity.[5] Russellians will say that

> (i) Etna, Vesuvius, and <u>being higher than</u> compose the proposition expressed by 'Etna is higher than Vesuvius', and those same three entities also compose the proposition expressed by 'Vesuvius is higher than Etna'.

But everyone, Russellians and non-Russellians alike, will say that

> (ii) the proposition expressed by 'Etna is higher than Vesuvius' ≠ the proposition expressed by 'Vesuvius is higher than Etna'.

Taken together, (i) and (ii) entail that Uniqueness is false. Hence the tension.

I have three reasons for not wanting to discuss Uniqueness here. The first reason is that it would make an already long chapter much longer. Given what I will end up saying about the transitivity and supplementation problems, I could say similar things in response to the problem about Uniqueness. But setting out all the details is a complicated affair. The second reason is that the problem about Uniqueness isn't a special problem for Russellianism; it applies equally to Fregean accounts of propositions. (Fregeans will say that the two propositions mentioned above are both composed of the same three senses.) The third reason for not discussing Uniqueness is that the pay-off would be limited. Uniqueness is, after all, extremely contentious as mereological principles go. A great many philosophers already reject it for independent reasons. (They say: the statue and the lump of clay are composed of the same particles but are not identical, given the differences in their modal properties.) True, there are competing pressures in favor of Uniqueness, but on the whole, the principle is not sacrosanct. If Russellianism and Fregeanism both conflict with it, a respectable reaction is 'so much the worse for Uniqueness'. There are other mereological principles that are almost universally accepted and much less negotiable. If Russellianism conflicts with one of these, so much the worse

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[5] The expression 'xx compose y' is typically defined as 'each of xx is a part of y, and each part of y overlaps at least one of xx', where 'x overlaps y' is defined as 'some z is a part of both x and y'.

for Russellianism. I take the transitivity and supplementation principles to have this status.

(3) This chapter deals with *propositions*. But a very similar chapter could have been written about *facts* or *states affairs* or even certain *complex universals*. There are Russellian theories of facts (e.g.) that confront the same mereological problems that I discuss here, and the solution that I offer is equally applicable (with some minor and fairly obvious adjustments) in those cases. I focus on propositions rather than facts or states of affairs partly because Frege's transitivity objection was framed in terms of propositions. But otherwise the decision was mostly arbitrary. In any event, it will be good to keep in mind that even if one's favorite theory of propositions avoids the problems that I will discuss, one's favorite theory of facts or states of affairs might not, in which case there might still be some use for the solution that I offer. (See King 2007 for a package of views to which these remarks may be relevant.)

2. A Formal Language

In this section I introduce a formal language, L_R. The vocabulary of L_R contains just

- four predicates:
    - the monadic predicates: '**Tall**' and '**Property**'
    - the dyadic predicates: '**HigherThan**', and '**IdenticalTo**'
- seven names: '**b**', '**e**', '**h**', '**i**', '**p1**', '**p2**', '**v**'
- brackets: '**(**' and '**)**'
- the comma: '**,**'
- two sentential connectives:
    - the monadic connective, '¬'
    - the dyadic connective '**&**'

The language contains no quantifiers, variables, or term-forming operators. The notion of a *sentence* of L_R is specified by the following formation rules:

(1) If $\tau_1,...,\tau_n$ are names of L_R and Π is an n-adic predicate of L_R, then $\ulcorner \Pi (\tau_1,...,\tau_n) \urcorner$ is a sentence of L_R.

(2) If $\varphi_1,...,$ and φ_n are sentences of L_R and K is an n–adic connective of L_R then $\ulcorner K(\varphi_1,..., \varphi_n)\urcorner$ is a sentence of L_R.

(3) Only those expressions formed by repeated application of (1) and (2) are sentences of L_R.

It will also be convenient to make some standard assumptions about the *interpretation* of L_R:

(4) Each sentence of L_R expresses exactly one entity, a proposition.

(5) Each n–adic predicate of L_R expresses exactly one entity, an n–adic universal.

(6) Each name of L_R refers to exactly one entity.

Finally, some specific assumptions about the semantic contents of the predicates and names of the language:

(7) '**Tall**' expresses the property <u>being tall</u>, '**HigherThan**' expresses the relation <u>being higher than</u>, '**IdenticalTo**' expresses the relation of identity, and '**Property**' expresses the property <u>being a property</u>.

(8) '**b**' refers to the property <u>being a property</u>, '**e**' refers to Etna (the mountain), '**h**' refers to the relation <u>being higher than</u>, '**i**' refers to identity (the relation), '**v**' refers to Vesuvius (the mountain), and '**p1**', and '**p2**' refer to certain propositions to be specified later.

This is all we need to know about L_R for now.

3. Russellianism

Russellianism, as I will understand it here, is a claim about the mereological structure of propositions. It will be important for our purposes that Russellianism not prejudge questions about the adicity of parthood. This constraint makes it hard to give a precise and simple statement of that view. So I will state it loosely, by analogy with a more precise and less neutral view, which I will call *Russellianism$_{2P}$*. I will state Russellianism$_{2P}$ as the conjunction of two theses. The first is:

2P   Parthood is a two-place relation that can be expressed by the predicate 'x is a part of y'.

To state the second conjunct of Russellianism$_{2P}$, we need to define 'overlaps' and 'fuses'. We will do this in terms of set-membership and a primitive two-place predicate for parthood:

> D1   x overlaps y =df. $\exists z$(z is a part of x & z is a part of y)
>
> D2   x fuses s =df. $\exists y$ y$\in$s & $\forall y$[y$\in$s $\rightarrow$ y is a part of x] & $\forall y$[y is a part of x $\rightarrow \exists z$(z$\in$s & y overlaps z)]

Informally, to fuse a set is to be composed of that set's members: a thing x fuses s iff (i) s is non-empty, (ii) each member of s is a part of x, and (iii) each part of x overlaps at least one member of s. A thing can fuse a set without itself *being* a set and without having any sets as *parts*.

The second conjunct of Russellianism$_{2P}$ can now be stated as a thesis about the mereological structure of the propositions expressed by atomic sentences of L$_R$:

> Atomics$_{2P}$ If φ is a sentence of L$_R$, if Π is an n-adic predicate of L$_R$, and if $\tau_1,..., \tau_n$ are names of L$_R$ such that $\varphi=\ulcorner\Pi(\tau_1,..., \tau_n)\urcorner$, then the proposition expressed by φ fuses the set {x: either Π expresses x or τ_1 refers to x,..., or τ_n refers to x}.

Informally, this says that the proposition expressed by an atomic sentence of L$_R$ is composed of the universal expressed by the predicate of that sentence and the things referred to by the names in the sentence. Russellianism$_{2P}$, then, is the conjunction of 2P and Atomics$_{2P}$. Note that Russellianism$_{2P}$ says nothing about the mereological structure of propositions expressed by non-atomic sentences of L$_R$. It wouldn't be hard to extend Russellianism$_{2P}$ in a fairly natural way to cover these other cases, but there is no need to do it here.

Russellianism$_{2P}$ is a precise thesis, but it's obviously not neutral on the question of the adicity of parthood. Here is a version of Russellianism that *is* neutral on that question:

> Russellianism$_N$ If parthood is two-place, then Atomics$_{2P}$ is true, and if parthood is n-place, where n$\neq$2, then an 'n-place analogue' of Atomics$_{2P}$ is true.

This is a loose, intuitive thesis. It relies on the notion of an 'n-place analogue' of a given thesis, which is not precise. The idea is that if parthood is

n–place, where n≠2, then Atomics$_{2P}$ (and the associated definitions) can be restated in terms of a primitive n–place parthood predicate, and the resulting thesis will come out true.

Despite its looseness, the notion of an n–place analogue of a given thesis is familiar and well enough understood for our purposes. Consider

Transitivity$_{2P}$ $\forall x \forall y \forall z[(x$ is a part of y & y is a part of $z) \rightarrow x$ is a part of $z]$

Philosophers disagree about the adicity of parthood. But virtually everyone agrees that it obeys Transitivity$_{2P}$ *or some properly restated variant of that principle*. In particular, virtually everyone agrees that *if* parthood$_m$ is two–place, then it obeys Transitivity$_{2P}$, and *if* parthood is a three–place relation that holds between objects and instants, then it obeys

Transitivity$_{3P}$ $\forall x \forall y \forall z \forall w[(x$ is a part of y at z & y is a part of w at $z) \rightarrow x$ is a part of w at $z]$.

There are other principles framed in terms of a three-place parthood predicate that have some claim to being analogues of Transitivity$_{2P}$, but I take it that Transitivity$_{3P}$ stands out as the most natural, given the relevant assumptions.

Thus, the thought behind Russellianism$_N$ is that just as Transitivity$_{2P}$ has a clear 'three-place analogue', Atomics$_{2P}$ will have a clear *n-place* analogue, for any n that might plausibly be thought to specify the adicity of parthood.

4. The Transitivity Argument

The first problem for Russellianism$_N$ comes from Frege. In the following passage, he discusses the proposition (thought) expressed by the sentence 'Etna is higher than Vesuvius':

> Now that part ['teil'] of the thought which corresponds to the name 'Etna' cannot be Mount Etna itself; it cannot be the meaning of this name. For each individual piece of frozen, solidified lava which is part of Mount Etna would then also be part of the thought that Etna is higher than Vesuvius. But it seems to me absurd that pieces of lava, even pieces of which I had no knowledge, should be parts of my thought. (Undated letter to Jourdain, in Frege 1980, p. 79)

Frege offers a similar argument, this time in terms of facts, in a letter to Wittgenstein dated 28 June 1919:

> The part of a part is part of the whole…Then it appears that constituents of Vesuvius must also be constituents of this fact; the fact will therefore also consist of hardened lava. That does not seem right to me. (Dreben and Floyd 2011, p. 53)

Let o be a small rock deep in the interior of Etna, and suppose that Frege has no knowledge of or acquaintance with o. Further, let p_e be the proposition expressed by the sentence of L_R '**HigherThan(e, v)**'. Finally, suppose that parthood is a two-place relation. Then Frege's argument can be reconstructed as follows:

TA

T1	$\forall x \forall y \forall z[(x$ is a part of y & y is a part of $z) \rightarrow x$ is a part of $z]$
T2	o is a part of Etna
T3	o is not a part of p_e

T4	So, Etna is not a part of p_e
T5	But if Russellianism$_{2P}$ is true, then Etna is a part of p_e.

T6	So, Russellianism$_{2P}$ is not true.

As it stands, the argument is no threat to versions of Russellianism that are formulated in terms of constituency rather than parthood. Those versions will not say that Etna is a *part* of p_e; rather they will say that Etna is a *constituent* of p_e. This latter claim is perfectly consistent with the conjunction of T1–T3. Likewise, the argument is no threat to versions of Russellianism that posit two different parthood relations, one which (parthood$_m$) is taken to hold between o and Etna but not between Etna and p_e, the other of which (parthood$_p$) is taken to hold between Etna and p_e but not between o and Etna. Advocates of this form of Russellianism will see the argument as equivocating on 'part of' or as having a false premise.

I'm not hostile to these maneuvers, provided that they can be independently motivated. What I will try to show here is that Russellians don't *need* these maneuvers to escape the argument. There is a different escape route that, in my view, is independently motivated.

Moreover, as I point out at the end of Appendix II, it's not at all clear that these maneuvers actually solve the basic problem. For it would seem

that constituency (or parthood$_p$) ought to be transitive as well, and yet there are structurally similar cases (involving singular propositions about other propositions) that seem to be counterexamples to such a principle. So even if one distinguishes between parthood and constituency, the underlying problem seems to remain.

Finally, it's clear that Frege himself was presupposing that the very same relation held both between a proposition and its constituents (be they ordinary objects or senses) and between a mountain and the chunks of rock within it.[6] Frege's anti-Russellian conclusion would have been a complete *non sequitur* otherwise. So, for the purposes of this chapter, I will set aside the suggestion that we multiply parthood relations.

Let's turn to the argument. In my view, if parthood is two-place, then the argument is probably sound. I'll discuss the premises one by one, in order of decreasing obviousness-to-me.

T5. Given the set-up of the case together with our statement of Russellianism$_{2P}$, it's hard to see how this claim could fail to be true. The proposition p_e is expressed by the sentence '**HigherThan(e, v)**', and the name '**e**', which occurs in that sentence, refers to Etna. According to Russellianism$_{2P}$, then, p_e fuses a set of which Etna is a member and hence p_e has Etna as a part.[7]

T2. This premise shouldn't be controversial; volcanoes have rocks as parts. Of course, some philosophers have unorthodox views about parthood. Compositional nihilists say that, fundamentally speaking, nothing is a part of anything else (Dorr 2005). These philosophers lie beyond the reach of Frege's argument. But presumably they would not have been tempted by Russellianism in the first place. Others (van Inwagen 1990b) say that the only things that have proper parts[8] are *living organisms*. In the context of that view, Frege's argument should just be restated using a

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[6] Russell makes his own views on this topic less clear, but he gives no indication of wanting to distinguish between the relations in question. In a response to Frege's claim that "Mont Blanc with all its snowfields is not itself a component part ['Bestandtheil'] of the thought that Mont Blanc is more than 4000 metres high" (letter to Russell, 13 November 1904 (Frege 1980, p. 163)), Russell writes, "I believe that in spite of all its snowfields Mont Blanc itself is a component part ['Bestandtheil'] of what is actually asserted in the proposition 'Mont Blanc is more than 4000 metres high'" (letter to Frege, 12 December 1904 (Frege 1980, p. 169)).

[7] Frege of course holds that the senses, not the referents, of the names 'Etna' and 'Vesuvius' are parts of p_e.

[8] 'x is a proper part of y' means 'x is a part of y and x≠y'.

different example. Those of us with more commonsense-friendly views about parthood will think that this aspect of Frege's argument is fine as it stands.[9]

T1. There is a controversy as to whether parthood obeys a uniqueness of composition principle. There is a separate controversy as to whether parthood obeys a principle of unrestricted composition. But virtually everyone on both sides of both of these disputes agrees that parthood obeys a *transitivity* principle (or a properly restated variant). Indeed, many would endorse

Transitivity$_N$ Each fundamental parthood relation is such that: (i) if it's two-place, then it's transitive, and (ii) if it's not two-place, then it's governed by some 'adicity-appropriate analogue' of a transitivity principle.[10]

For present purposes, however, it suffices to note that, given 2P, Transitivity$_{2P}$ is highly plausible.

T3. This says, of a certain smallish rock, o, that it is not a part of p_e, the proposition that Etna is higher than Vesuvius. I find this claim highly attractive on its own (as did Frege, apparently), but if an argument for it is wanted, it can be motivated in at least two ways.[11]

First, one might endorse a principle of acquaintance according to which one can grasp a given proposition only if one is acquainted with each of its parts. (There are variants of this principle that do the same work while avoiding the notion of acquaintance. See note 12 below.) Given such a principle, T3 is inevitable. For surely one can grasp p_e—the proposition that Etna is higher than Vesuvius—without being acquainted with every obscure little rock inside Etna. This was Frege's situation; he grasped p_e but had no acquaintance with o. Perhaps one needs to be acquainted with *Etna itself* (and not just the sense of 'Etna') to grasp p_e; but regardless of

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[9] Even temporal parts theorists ought to grant that o is a part simpliciter of Etna. I assume that o lies within Etna throughout o's entire career.

[10] It is consistent with Transitivity$_N$ that there are non-fundamental, defined relations that count as parthood relations but that fail to be transitive. Perhaps *immediate proper parthood* is one such relation, where 'x is an immediate proper part of y' is defined as 'x is a proper part of y, and there is no z such that x is a proper part of z and z is a proper part of y', and 'x is a proper part of y' is defined as 'x is a part of y and x≠y'. It is also consistent with Transitivity$_N$ that there are multiple fundamental parthood relations, although for convenience I mostly ignore this possibility. See Varzi (2006) for a defense of Transitivity$_{2P}$.

[11] King (2007, p. 120, n. 42) would reject T3.

what acquaintance amounts to, one needn't be acquainted with *the rock o* to grasp the given proposition. The proposition is, after all, about Etna and Vesuvius, and it predicates <u>being higher than</u> of them, in that order. This proposition has nothing to do with o! Call this the *acquaintance argument*. [12] (To be sure, it can be resisted by those who are willing to replace the principle of acquaintance with something weaker. One might reject it in favor of the principle that one can grasp a proposition only if one is acquainted with each of its 'immediate' or 'privileged' parts, where Etna, Vesuvius, and <u>being higher than</u> are privileged parts of p_e but o is not. But this strikes me as somewhat *ad hoc* and artificial.)

Second, one might think that abstract entities such as propositions have their parts essentially.[13] This seems to be in the spirit of traditional Platonism, anyway. Suppose that this essentialist principle is true. Further, suppose that p_e is abstract. Then, if o is a part of p_e, it's not possible that p_e exist without having o as a part. But this clearly *is* possible. After all, it's possible that Etna and Vesuvius both exist though neither of them even overlaps o. (Maybe o could have been a part of Lassen Peak instead; maybe it could have failed to exist; maybe it could have existed and failed to be concrete.) But I take it that, necessarily, o is a part of p_e only if o overlaps$_m$ either Etna or Vesuvius. When it comes to concrete objects like o, there are at most 'three ways in' to p_e—via Etna, via Vesuvius, and *perhaps* via some fusion of the two. So p_e *could have* existed without having o as a part. Together with the essentialist

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[12] Here is a similar argument that avoids the notion of acquaintance:

   (i) If subject s grasps [entertains, believes, knows, desires,...] proposition p, then for any part p* of p, s either grasps p* or is engaged in *de re* thought about p*.
   (ii) Frege grasps p_e.
   (iii) Frege neither grasps o nor is engaged in *de re* thought about o.
   (iv) Therefore, o is not a part of p_e.

   I don't know whether Frege himself would be willing to endorse this argument, but perhaps some contemporary friends of Russellian propositions would be. Thanks to Lucas Halpin for discussion here.

[13] More carefully, one might think that if an entity e is not possibly concrete, and if e* is a part of e, then it's necessary that if e exists, e* is a part of e. Even those (e.g., Linsky and Zalta 1996 and Williamson 2002) who accept contingently non-concrete entities will presumably want to accept this principle. Let c be such an entity: c is abstract but is possibly such that it is concrete (and my younger brother, say). Were c concrete, it would presumably have parts that it does not actually have, but I doubt that there is any x such that: (i) x is actually a part of c but (ii) possibly, x is not a part of c. But nothing turns on this. Since propositions are abstract and non-contingently so, we could get by with a weaker principle—one according to which non-contingently abstract entities have their parts essentially.

   A final point. Fine (1994) draws attention to a notion of essence that plausibly cannot be reduced to modal notions. Although I am sympathetic to Fine's notion of essence, I do not mean to appeal to it here.

principle, this will give us the result that p_e in fact *doesn't* have o as a part. Call this the *essentialism argument*.[14]

Would the Russellian's prospects improve if he shifted from 2P to '3P' and, in particular, if he adopted the view that parthood is a three-place relation that can hold between a part, a whole, and an instant of time?[15] They would not. For in that case, we could simply restate the argument accordingly,[16] and it would remain forceful. Moreover, the shift to a three-place, time-relative parthood relation would generate new puzzles of its own.[17]

This leaves us with a general problem about transitivity: How can the adicity-neutral version of Russellianism (Russellianism$_N$) be combined with the adicity-neutral version of transitivity (Transitivity$_N$)?

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[14] On the assumption that facts and states of affairs are abstract, a parallel argument can be used to support the claim that o is not a part of the *fact* that Etna is higher than Vesuvius or the *state of affairs* of Etna's being higher than Vesuvius.

[15] Begin by defining 'x overlaps y at z' as '∃w[w is a part of x at z & w is a part of y at z]' and 'x fuses s at z' as '∃yy∈s & ∀y[y∈s → y is a part of x at z] & ∀y[y is a part of x at z → ∃w(w∈s & w overlaps y at z)]'. This lets us state Atomics$_{3P}$: if φ is a sentence of L_R, if Π is an n-adic predicate of L_R, and if $τ_1,...,τ_n$ are names of L_R such that φ=⌜Π($τ_1,...,τ_n$)⌝, then the proposition p expressed by φ is such that: for any t, if t is an instant, then p fuses the set {x: either Π expresses x or $τ_1$ refers to x,..., or $τ_n$ refers to x} at t. Russellianism$_{3P}$ could then be identified with the conjunction of 3P and Atomics$_{3P}$.

[16] If we let t_a be an instant at which the rock o is inside of Etna, we can say: (i) [Transitivity$_{3P}$], (ii) o is a part of Etna at t_a, and (iii) o is not a part of p_e at t_a, from which it follows that, contrary to Russellianism$_{3P}$, (iv) Etna is not a part of p_e at t_a. Premise (iii) can be motivated by analogues of the acquaintance and essentialism arguments discussed earlier. Acquaintance: one can grasp a proposition only if one is acquainted with everything that is ever a part of it. Frege grasps p_e but is not acquainted with o. So o is never a part of p_e. Essentialism: if an entity y is abstract, then for any x, if x is ever a part of y, then y is essentially such that x is always a part of y. The proposition p_e is abstract, but it isn't essentially such that o is always a part of it. Admittedly, it is not always clear how we should restate the given principles in 3P-appropriate terms. Perhaps the resulting arguments are *somewhat* less decisive than the originals. But if so, surely this is because it's difficult to think of propositions or other essentially abstract entities as having their parts in a time-relative way. (See the next note.) So even if the case for (iii) is a bit muddy, this is hardly a reason for the Russellian to embrace time-relative parthood. On the contrary, it's a reason to shy away from it.

[17] One new puzzle arises from the following:

Indexing	If Russellianism$_{3P}$ is true, then ∃t[t is an instant & Etna is a part of p_e at t]
Presence	∀x∀y∀z[x is a part of y at z → (x is present at z & y is present at z)]
Transience$_{Etna}$	∃t[t is an instant & Etna is not present at t]
Constancy$_{pe}$	¬∃t∃t*∃x[t is an instant & t* is an instant & p_e is present at t & p_e is present at t* & x is a part of p_e at t & x is not a part of p_e at t*]
	(*Propositions do not 'persist through mereological variation'.*)
Incorruptibility$_{pe}$	¬∃t∃t*[t is an instant & t* is an instant & p_e is present at t & p_e is not present at t*]
	(*Propositions are either atemporal or present at all times.*)

Given 3P, these claims are all highly plausible, but taken together, they entail that Russellianism$_{3P}$ is not true. This argument deserves further exploration, but I lack the space for it here.

5. The Supplementation Argument

The *Weak Supplementation Principle* says that if a thing has a proper part (a part with which the thing is not identical), then that thing has a second proper part that doesn't overlap the first. In terms of the two–place predicate 'x is a part of y', this comes to

WSP_{2P} $\forall x \forall y[(x$ is a part of y & $x \neq y) \rightarrow \exists z(z$ is a part of y & $\neg z$ overlaps x)]

WSP_{2P} (or some adicity-appropriate analogue) is common ground among those who disagree on much else.[18] It is endorsed by 2P-ers[19] and by 3P-ers,[20] by compositional monists[21] and compositional pluralists,[22] by friends[23] and foes[24] of unrestricted composition, and by friends[25] and foes[26] of uniqueness.

But there is a problem. When Peter Simons originally introduced Weak Supplementation, he motivated it with the following remark: "surely if a universe is complex (i.e. has proper parts at all), then at least two of these parts will be disjoint" (1987, p. 27). I regard this as one of the least negotiable claims in part–whole theory, at least when framed in adicity-neutral terms. But I deny that it is properly formalized as WSP_{2P}. Instead, the proper two-place formalization would seem to be *Quasi-Supplementation*:

QS_{2P} $\forall x \forall y[(x$ is a part of y & $x \neq y) \rightarrow \exists z \exists w(z$ is a part of y & w is a part of y & $\neg z$ overlaps w)]

In words, this says that if a thing has a proper part, then the thing has parts that are disjoint from (fail to overlap) *each other*, though not necessarily from *the first proper part*.

[18] Though its status as sacrosanct has been challenged (e.g., by Donnelly 2011), and the principle has been denied (e.g., by Smith 2009).
[19] Lewis (1991), Sider (2007), Effingham and Robson (2007), Eagle (2010).
[20] Simons (1987), van Inwagen (1990b), Koslicki (2008). These philosophers all take parthood to be somehow 'time-indexed' or 'spatiotemporally relativized'; they do not all accept 3P exactly as I've formulated it.
[21] Examples here include Lewis (1991), Sider (2007), and Koslicki (2008). Van Inwagen accepts principles that entail a temporally relativized analogue of WSP_{2P}.
[22] McDaniel (2009).
[23] Lewis (1991), Sider (2007), Varzi (2009).
[24] Simons (1987), van Inwagen (1990b).
[25] Lewis (1991), Sider (2007), van Inwagen (1990b), Koslicki (2008), Varzi (2008).
[26] Simons (1987), Lowe (2001, p. 140). The principle that Lowe dubs 'Weak Supplementation' is weaker than WSP_{2P}, but together with other principles that Lowe accepts, it entails WSP_{2P}.

QS_{2P} is significantly weaker than WSP_{2P}. To take just one case in which this difference makes a difference, consider Judith Jarvis Thomson's (1998) views about the case of the clay statue, Goliath, and the lump of clay, Lump1, that 'constitutes' Goliath. Thomson rejects Uniqueness. She holds that there is a set s of particles such that: [27]

(i)   Lumpl fuses s
(ii) Goliath fuses s
(iii) Lumpl≠Goliath,
(iv) Lumpl is a part of Goliath, and
(v)   Goliath is a part of Lump1.

WSP_{2P} is inconsistent with (i)–(iv) together with the claim that parthood is transitive.[28] QS_{2P} avoids this inconsistency, as can be seen by considering the directed graph below.[29] (Arrows represent immediate proper parthood. So, e.g., a is a proper part of Lumpl. We assume that x is a part of y iff either x = y or x bears the transitive closure of immediate proper parthood to y, so parthood is guaranteed to be reflexive and transitive.)

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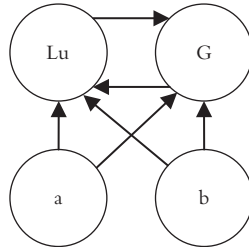
[27] In fact Thomson takes parthood to be a three-place, time-relative relation and accepts time-indexed analogues of (i)–(v). Since nothing turns on this, for simplicity I'll ignore it. See also Thomson (1983).
[28] To see this, suppose that (i)–(iv) and WSP_{2P} are all true. From the conjunction of (iii), (iv), and WSP_{2P} it follows that (vi) Goliath has a part, call it *GP*, that is not identical to Goliath and that doesn't overlap Lumpl. Given (vi) and the definition of 'fuses', we can conclude that (vii) some member of s, call it m, *overlaps* GP, i.e. (given the definition of 'overlap'), that there is some object, call it o, that is a common part of both m and GP. Now since o is a part of m (by (vii)), since m is a part of Lumpl (by (vii), (i), and the definition of 'fuses'), and since parthood is transitive, we can conclude that (viii) o is a part of Lumpl. But now we have the result that (ix) o is a part both of Lumpl and of GP, hence (given the definition of 'overlap') that (x) Lumpl and GP overlap. This contradicts a claim that follows from (vi), namely that (xi) GP does not overlap Lumpl.
We can avoid this problem by replacing WSP_{2P} with QS_{2P}. The latter entails not that Goliath has a part that doesn't overlap Lumpl, but merely that Goliath has a pair of parts (e.g., its foot and its head) that don't overlap each other.
[29] See Cotnoir (2010, p. 399) on the use of directed graphs (as opposed to Hasse diagrams) to specify models of the parthood relation in which parthood fails to be anti-symmetric.

In this model, Lump1 and Goliath are non-identical, so (iii) is satisfied. It's easy to verify that they each fuse the set {a, b}, so (i) and (ii) are satisfied. And they're parts of each other, so (iv) and (v) are satisfied. Finally, the model satisfies QS_{2P}: the only things that have proper parts are Lumpl and Goliath, and each of them has parts (namely, a and b) that are disjoint from each other.

I don't see anything obviously problematic about Thomson's package of views; I am not entirely comfortable rejecting the package. This makes me hesitant to endorse WSP_{2P}. I have no corresponding doubts about QS_{2P}. More generally, I can see no reason to doubt

> QS_N Each fundamental parthood relation is such that: (i) if it's two-place, then it's 'quasi-supplementive', and (ii) if it's not two-place, then it's governed by an adicity-appropriate analogue of QS_{2P}.

In my view, QS_N is comparable in plausibility to $Transitivity_N$. If $Russellianism_N$ conflicts with either of them, it emerges the loser.[30] (For more on quasi-supplementation, see my 2009 and forthcoming a.)

Here is an argument from QS_{2P} to the denial of $Russellianism_{2P}$. Let p_i be the proposition expressed by '**IdenticalTo(i, i)**', and assume that parthood is two-place. Then the premises of the following argument should be plausible:

SA

S1   If $Russellianism_{2P}$ is true, then: (i) identity is a part of p_i, and (ii) $\forall x[x$ is a part of $p_i \rightarrow$ overlaps identity]

S2   Identity $\neq p_i$

S3 $\neg\exists x[x$ is a part of identity & $x \neq$ identity]

S4 $\forall x \forall y[(x$ is a part of y & $x \neq y) \rightarrow \exists z \exists w(z$ is a part of y & w is a part of y & $\neg z$ overlaps w)]

S5   So, [31] $Russellianism_{2P}$ is not true.

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[30] Donnelly (2011) objects to such uses of supplementation principles. Responding to her interesting arguments would require a separate paper.

[31] To see that the argument is valid, assume for *reductio* that the premises are all true but that, contrary to the conclusion, $Russellianism_{2P}$ is true as well. Then, given $Russellianism_{2P}$ and S1, we have: (i) identity is a part of p_i, and (ii) $\forall x[x$ is a part of $p_i \rightarrow$ x overlaps identity], i.e., *each part of p_i overlaps identity*. Together with S2 and S4, (i) entails that (iii) $\exists z \exists w[(z$ is a part of p_i & w is a part of p_i & $\neg z$ overlaps w)], i.e., *p_i has parts that are disjoint from each other*. But (iii) is inconsistent with the conjunction of (ii) and S3. To see this, suppose that all three are true. In accordance with (iii), suppose that: (iv) a is a part of p_i, (v) b is a part of

As I noted earlier, one response to the Transitivity Argument is to distinguish between two different fundamental parthood relations—one for material objects and their parts, another for propositions and their parts—and to formulate Russellianism in terms of the latter. It is worth pointing out that this response offers no help at all with the Supplementation Argument. Even if there were two fundamental parthood relations, there would be no temptation to say that the Supplementation Argument equivocates between them. Rather, it is clearly concerned with just one relation throughout—viz., the one that holds between propositions and their parts. None of its premises becomes more plausible when given a different reading. So let's consider those premises, again in order of decreasing obviousness-to-me.

> **S2**. I expect no resistance here. Identity is a two-place relation, not a proposition. It is neither true nor false. By contrast, p_i is a proposition and is true; it is not a two-place relation. (Maybe it's a *zero*-place relation.) So, by the diversity of discernibles, we can conclude that p_i and identity are diverse: they are two, not one.
>
> **S4**. This is just QS_{2P}, the 2P-appropriate quasi-supplementation principle; I will say no more about it here.
>
> **S3**. This premise says that identity does not have any proper parts, i.e., that it is *simple*. One might think that *analyzable* universals—being a sister of, perhaps, or being a bachelor—have proper parts. Specifically, one might think their proper parts are the entities that figure into their analyses. But identity is unanalyzable and hence plausibly simple. What would its proper parts be? (I assume that if identity *is* analyzable, the whole argument can easily be restated in terms of some universal that is not analyzable.)
>
> **S1**. Recall that p_i is the proposition expressed by '**IdenticalTo(i, i)**'. According to Russellianism$_{2P}$, therefore, p_i fuses the set $\{x: $ '**IdenticalTo**'

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p_i, and (vi) a is disjoint from b. Together, (iv) and (ii) entail that (vii) a overlaps identity, and (v) and (ii) likewise entail that (viii) b overlaps identity. So, given the definition of 'overlaps', we can conclude that there are things, call them *a** and *b**, such that: (ix) a* is a part both of a and of identity (from (vii)), and (x) b* is a part both of b and of identity (from (viii)). But recall S3. It says that identity is *simple*, that it has no parts aside from itself. Given (ix) and (x), respectively, this means that: (xi) a* = identity, and (xii) b* = identity. Now, since identity = a* and since a* is a part of a (from (ix)), we get the result that (xiii) identity is a part of a. Likewise, since identity = b* and since b* is a part of b (from (x)), we get the result that (xiv) identity is a part of b. Together, these last two claims entail that a overlaps—and hence is not disjoint from—b. This contradicts (vi).

expresses x or '**i**' refers to x or '**i**' refers to x}.[32] Since the only entity that is expressed by '**IdenticalTo**' or referred to by '**i**' is the relation of identity, Russellianism$_{2P}$ tells us that p_i fuses the singleton set {identity} and hence, by the definition of 'fuses', that (i) identity is a part of p_i and that (ii) each part of p_i overlaps identity. This gives us SI$_{2P}$.[33]

It is worth noting here that there is something unusual about the proposition in question, p_i: it predicates a relation of that very relation. The proposition says, concerning the relation of identity, that it is identical to it. Russellians might be tempted deny the existence of such propositions, perhaps by appeal to type restrictions of some kind. The idea would be to concede that Russellianism$_{2P}$ is false as stated, since it incurs a commitment to propositions that are banned by the relevant type restrictions, but to insist that (i) these restrictions are independently motivated[34] and that (ii) a properly restated, *typed* version of Russellianism avoids any conflict with quasi-supplementation.

I don't deny (i) or (ii). But I still think that type restrictions are artificial and should be accepted only under duress. Identity is one of those many universals that can be univocally predicated of entities of all sorts. It seems perfectly intelligible, and indeed obviously true, to say that *all* entities, universals and particulars alike, are identical to themselves. Type theories apparently have to say that there is something defective about such a claim.[35]

I cannot do justice to these issues here. So for the rest of the chapter I will set type theories aside and just take it for granted that there are atomic propositions that predicate a universal of itself. If one likes, one can read the argument as a *reductio* of that assumption.

One final point before we move on. Suppose that Russellians formulated their view in terms of a three-place, time-relative parthood relation. Would this help them with supplementation problem? It would not. As in the case of the

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[32] According to Frege, p_i is composed of the sense of the predicate '**IdenticalTo**' and the sense (not the referent) of the name '**i**'. It is therefore open to Frege to say that these are non-identical—indeed, mereologically disjoint—senses. In that case, there would be no tension between what we might call 'Fregeanism$_{2P}$' on the one hand and QS$_{2P}$ on the other.

[33] Bynoe (2010) argues that, given there is such a thing as *the fact that being a property is a property*, the 'compositional view of facts' is in tension with certain supplementation principles. He does not discuss QS$_{2P}$ in particular, nor does he consider any solution to the given problem along the lines of the one that I advocate.

[34] E.g., by the need to avoid the 'property version' of Russell's paradox.

[35] See Menzel (1993) for a criticism of type theories and for a type-free approach to properties, relations, and propositions.

Transitivity Argument, we could simply restate the Supplementation Argument in 3P-appropriate terms,[36] and it would remain forceful. So again we have a general problem: How can Russellianism$_N$ be combined with QS$_N$?

6. Parthood as a four-place relation

Elsewhere I have argued that if that material objects are multi-located in spacetime (of which more later), then we should reject 2P and 3P in favor of

4P   Parthood is a four-place relation, one that can be expressed by 'x at y is a part of z at w'.

I won't repeat that argument here. But I will do two related things. First, I will put more flesh on the doctrine that material objects are multi-located in spacetime, and I will try to give the reader an intuitive sense of how a four-place relation might hold between multi-located objects and their locations in spacetime. Second, I will address the question of how to formulate 4P-appropriate analogues of (i) the reflexivity of parthood, (ii) the transitivity of parthood, (iii) and quasi-supplementation. In addressing that question, my strategy will divide into three main steps.

    First, I will suggest that questions about how to formulate 4P-appropriate analogues of the reflexivity and transitivity of parthood have relatively straight-forward answers. Second, I will note that the analogues of these two principles conform to a quite salient pattern. Third, I will suggest that we should make the 4P-appropriate analogue of quasi-supplementation conform to that pattern as well. In tackling the hard case, we should look to the easy cases for guidance.

6.1 Exact Occupation, Multi-Location, and Four-Place Parthood

Assume that we inhabit a four-dimensional spacetime and suppose that we know what it means to say that a given material object O *exactly occupies* (hence-forth just *occupies*) a given region R. Roughly, this means that O has (or has-at-R) exactly the same size and shape as R and stands (or stands-at-R) in exactly

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[36] We could say: (i) if Russellianism$_{3P}$ is true, then: (a) identity is a part of p_i at t_a, and (b) $\forall x[x$ is a part p_i at $t_a \rightarrow x$ overlaps identity at $t_a]$; (ii) identity $\neq p_i$, (iii) $\neg\exists x[x$ is a part of identity at t_a & $x \neq$ identity]; and (iv) $\forall x \forall y \forall z[(x$ is a part of y at z & $x \neq y) \rightarrow \exists w \exists w*(w$ is a part of y at z & $w*$ is a part of y at z & $\neg w$ overlaps w* at z)]; from which it follows that (v) Russellianism$_{3P}$ is not true. I omit the proof.

the same spatiotemporal relations to other things as does R. I will say that R is a *location* of O just in case O occupies R. A thing is *multi-located* just in case it has more than one location, i.e., just in case there is more than one entity that it occupies. With the notion of occupation in hand, we can define the notion of a *path*: say that a region R is the path of an object O just in case R is the sum or union of the region(s) that O occupies.

Philosophers will disagree about which regions a given material object occupies, but they will largely agree as to which region is the object's path. For example, virtually everyone will agree that a certain four-dimensional, 93-year-long region is Gerald Ford's path. But they will disagree about which region or regions he *occupies*. 'Locational perdurantists' will say that Ford occupies exactly one region—his path. 'Locational endurantists' will say that he occupies a great many regions, each of which is a temporally unextended 'slice' of that path.[37] This is the most popular form of multi-locationism, but not the only one[38] .

Multi-locationists should endorse 4P: they should say that parthood is a four-place relation that can be expressed by the predicate 'x at y is a part of z at w'. As proponents of 4P, they will find it natural to accept the following principle, which connects *parthood* with *occupation*:

The Location Principle (LP)	If x at y is a part of z at w then: x occupies y and z occupies w.
	$\forall x \forall y \forall z \forall w[P(x, y, z, w) \rightarrow [OCC(x, y) \ \& \ OCC(z, w)]]$

According to LP, parthood has one slot for a part, a second slot for a *location* of that part, a third slot for a *whole*, and a fourth slot for a *location* of that whole.

Admittedly, if parthood is a four-place relation, then it will lack the familiar formal properties—such as reflexivity and transitivity—that are often attributed to it. But this is unlikely to be seen as a decisive objection to

[37] These terms are from Gilmore (2008). See Balashov (2010) for further discussion and for arguments against locational endurantism.

[38] Hudson (2001) argues that ordinary objects are 'worms' each of which occupies many largely overlapping four-dimensional spacetime regions. McDaniel (2004) explores a form of 'modal realism with overlap' according to which at least some material objects are wholly present in many different concrete possible worlds. Such an object exactly occupies at least one different spacetime region in each different world at which it exists (but the regions themselves are 'worldbound').

the four-place view, since exactly the same complaint can be made against 3P, and it remains very popular. In both cases, the force of the objection is mitigated by the fact that even if parthood lacks the properties of reflexivity, transitivity, and so on, it might still possess natural adicity-appropriate *analogues* of those properties.

6.2 Reflexivity and Transitivity

So let us ask: what *are* the most natural 4P-appropriate analogues of reflexivity and transitivity? We can start with the former. I suggest

Reflexivity$_{4P}$ If x occupies y, then x at y is a part of x at y.
$$\forall x \forall y[OCC(x, y) \rightarrow P(x, y, x, y)]$$

Loosely put, this says that each thing is a part of itself at each of its locations. I occupy a certain human-shaped region, R. So I, at R, am a part of myself, at R. Initially one might be tempted by a stronger reflexivity-like principle: $\forall x \forall y P(x, y, x, y)$. In words, this says that for any x and any y, x at y is a part of x at y. Together with LP, though, the stronger principle yields the absurd result that everything occupies everything.

Next consider transitivity. If parthood is a four-place relation governed by LP, then it is naturally taken to obey

Transitivity$_{4P}$ If x at x* is a part of y at y*, and y at y* is a part of z at z*, then x at x* is a part of z at z*.
$$\forall x \forall x^* \forall y \forall y^* \forall z \forall z^*[[P(x, x^*, y, y^*) \,\&\, P(y, y^*, z, z^*)] \rightarrow$$
$$P(x, x^*, z, z^*)]$$

My left hand, at a certain hand-shaped region R_h, is a part of my left arm, at a certain arm-shaped region R_a, and my left arm, at that same arm-shaped region R_a, is a part of me, at the aforementioned human-shaped region R. So, given Transitivity$_{4P}$, my left hand, at R_h, is a part of me, at R. This argument is formally valid; we have

(i)   [Transitivity$_{4P}$]
(ii)   P(my left hand, R_h, my left arm, R_a)
(iii)   P(my left arm, R_a, me, R)
(iv)   So, P(my left hand, R_h, me, R)

This is a welcome result. We expect a transitivity-like principle that governs a fundamental parthood relation to combine with claims like (ii) and (iii) to yield a formally valid argument for a conclusion like (iv). If the principle *couldn't* do this, that would speak forcefully against its status as an analogue of Transitivity$_{2P}$.

6.3 Fusion and Quasi-Supplementation

So far we have confined our mereological vocabulary to a primitive predicate for *parthood*. But the 4P-er will also find it convenient to use a variety of other mereological expressions, including a defined predicate for *overlapping*. For example, in the cased depicted below, he will want to say that c at r_c overlaps d at r_d:
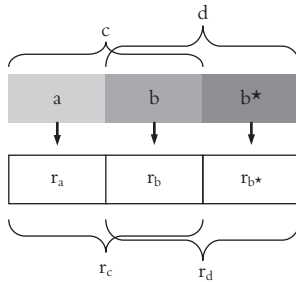
We can define this predicate via

$$D3 \qquad O(x, x*, y, y*) =df. \exists z \exists z*[P(z, z*, x, x*) \ \& \ P(z, z*, y, y*)]$$

According to D3, x at x* overlaps y at y* just in case some z, at some z*, is a part *both* of x at x* *and* of y at y*. Thus, in the case depicted above, the reason why it's true that

$$(9) \qquad O(c, r_c, d, r_d)$$

is that both of the following are also true:

$$(10) \qquad P(b, r_b, c, r_c)$$
$$(11) \qquad P(b, r_b, d, r_d)$$

In other words, b, at r_b, is a part *both* of c at r_c *and* of d at r_d.

The 4P-er will also want to define a predicate for fusion. To do this, we can think of fusion as a three-place relation that holds between a *thing*, a *set*, and a *location* of the thing, where the set in question is a set of ordered ⟨thing, location of that thing⟩ pairs:

D4 $F(y, s, y*) =df. \exists z(z \in s) \& \forall z[z \in s \rightarrow \exists w \exists w*[z=\langle w, w* \rangle \& P(w, w*, y, y*)]] \&$
$\forall z \forall z*[P(z, z*, y, y*) \rightarrow \exists u \exists w \exists w*[u \in s \& u=\langle w, w* \rangle \& O(w, w*, z, z*)]]$

In words, y fuses s at y* just in case: (i) s is a non-empty set, (ii) each member of s is an ordered pair whose first member at its second member is a part of y at y*, and (iii) for any z and any z*, if z at z* is a part of y at y*, then there is some ordered pair in s whose first member at its second member overlaps z at z*.

To illustrate, let m be a composite object that occupies two different regions: rm_1 and rm_2. Further, suppose that m has different parts at different locations. Finally, suppose that, at each of its locations, it is entirely composed of simples. Informally speaking, we can specify the situation as follows: at rm_1, m's simple parts are a, b, and c, whereas at rm_2, its simple parts are b, c, and d.
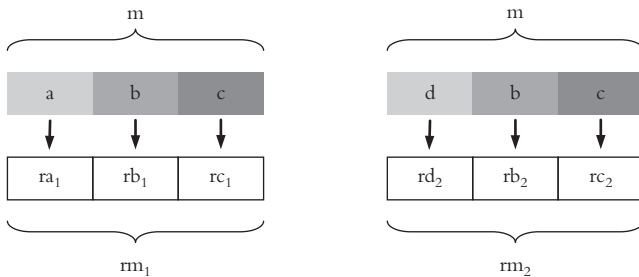
In terms of our fusion predicate, we can say:

(12) $F(m, \{\langle a, ra_1 \rangle, \langle b, rb_1 \rangle, \langle c, rc_1 \rangle\}, rm_1)$
(13) $F(m, \{\langle d, rd_2 \rangle, \langle b, rb_2 \rangle, \langle c, rc_2 \rangle\}, rm_2)$

Crucially, (12) and (13) do *not* say that m is a set or that it has sets or ordered pairs as *parts*. These sentences are all consistent with the claim that the only entities that are (anywhere) parts of m (anywhere) are *concrete material objects*. What (12) and (13) say is that m *fuses* certain sets at certain locations, where

'fuses' is a technical term defined by D4. Informally, (12) says that m, at rm_1, is composed of a at ra_1, b at rb_1, and c at rc_1.

So far I have suggested that the most natural 4P-appropriate analogues of Reflexivity$_{2P}$ and Transitivity$_{2P}$ are Reflexivity$_{4P}$ and Transitivity$_{4P}$, respectively. I take this suggestion to be relatively uncontroversial.

My next task is harder: it is to find the most natural 4P-appropriate analogue of QS$_{2P}$. My first step will be to identify a pattern in the conclusions that I've already reached regarding the reflexivity and transitivity principles. Consider the following definitions:

> D5   x is an occupation pair $=$df. $\exists y \exists z[OCC(y, z) \ \& \ x = \langle y, z \rangle]$
>
> D6   x is a pair-part of y = df. $\exists z \exists z* \exists w \exists w*[P(z, z*, w, w*) \ \& \ x = \langle z, z* \rangle \ \&$ $y = \langle w, w* \rangle]$

In words: an *occupation pair* is an ordered pair whose first member occupies its second member; and one thing is a *pair-part* of another just in case they are both ordered pairs, and the first member of the first pair, at the second member of the first pair, is a part of the first member of the second pair, at the second member of the second pair. Given LP, we can conclude that if x is a pair-part of y, then x and y are both *occupation pairs*.

Now, to see the pattern, notice two facts. First, Transitivity$_{4P}$ is equivalent to the claim that pair-parthood is transitive in the strict sense. Second, Reflexivity$_{4P}$ is equivalent to the claim that pair-parthood is reflexive over the domain of occupation pairs. I suggest that we can extend this pattern by formulating QS$_{4P}$ in such a way that it ends up being equivalent to the claim that pair-parthood is 'quasi-supplementive', at least over the domain of occupation pairs.

More generally, I suggest that the most salient pattern that emerges from Reflexivity$_{4P}$ and Transitivity$_{4P}$ is this: if a 2P-appropriate principle attributes formal property F to parthood, then the most natural 4P-appropriate analogue of that principle will be one that is equivalent to the claim that *pair*-parthood has property F, at least over the domain of occupation pairs. Here is a 4P-appropriate analogue of QS$_{2P}$ that conforms to this guideline:

> QS$_{4P}$ $\forall x \forall x* \forall y \forall y*[[P(x, x*, y, y*) \ \& \ [x \neq y \lor x* \neq y*]] \rightarrow \exists z \exists z* \exists w \exists w*[P(z, z*, y, y*) \ \& \ P(w, w*, y, y*) \ \& \ \neg O(z, z*, w, w*)]]$[39]

[39] To see that QS$_{4P}$ is equivalent to the principle, QS$_{PP}$, that pair-parthood is 'quasi-supplementive', let $\langle a, a* \rangle$ and $\langle b, b* \rangle$ be arbitrarily chosen ordered pairs, and suppose that (i) $\langle a, a* \rangle$ is a pair-part of $\langle b, b* \rangle$ and $\langle a, a* \rangle \neq \langle b, b* \rangle$. This is equivalent to the claim that (ii) $P(a, a*, b, b*)$ and $[a \neq b \lor a* \neq b*]$, given
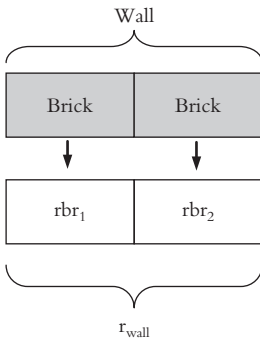
To illustrate, QS_{4P} rules out the 'David/Lumpy' case

David

Lumpy
(simple at r_L)

r_L
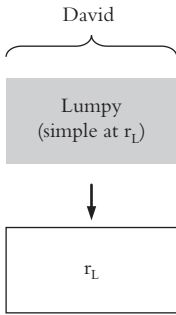
The David/Lumpy Case

(i)   David fuses $\{\langle \text{Lumpy}, r_L \rangle\}$ at r_L

(ii)   David ≠ Lumpy

(iii)   Lumpy is simple at r_L

but not the 'Brick/Wall' case.[40]

Wall

Brick Brick

rbr_1 rbr_2

r_{wall}

The Brick/Wall Case

A simple brick travels backward in time and is placed next to itself. It occupies two different regions and composes a wall that occupies the union of those regions. Thus we have:

(i)   Wall fuses $\{\langle \text{Brick}, rbr_1 \rangle, \langle \text{Brick}, rbr_2 \rangle\}$ at r_{wall}

(ii)   Wall≠Brick

(iii)   Brick is simple at rbr_1 and at rbr_2

To see why, start with 4P-appropriate definitions of 'simple', 'proper part', and 'disjoint':

D7   x is simple at x^* =df. $P(x, x^*, x, x^*)$ &
      $\forall y \forall y^*[P(y, y^*, x, x^*) \rightarrow (y = x \,\&\, y^* = x^*)]$

D8 $PP(x, x^*, y, y^*)$ =df. $P(x, x^*, y, y^*)$ & $[x \neq y \lor x^* \neq y^*]$

D9 $D(x, x^*, y, y^*)$ =df. $\neg O(x, x^*, y, y^*)$

---

the definition of 'pair-part' and the identity conditions for ordered pairs. (Also, I assume that, necessarily, entities x and x^* exist iff $\langle x, x^* \rangle$ exists.) Together with QS_{4P}, (ii) entails that (iii) $\exists z \exists z^* \exists w \exists w^*[P(z, z^*, b, b^*)$ & $P(w, w^*, b, b^*)$ & $\neg O(z, z^*, w, w^*)]$. And (iii) is equivalent to the claim that (iv) $\langle b, b^* \rangle$ has pair-parts $\langle z, z^* \rangle$, $\langle w, w^* \rangle$ that are 'pair-disjoint', i.e., that do not pair-overlap each other, i.e., that do not have a common pair-part. So QS_{4P} entails QS_{PP}. In the other direction, for arbitrary a, a^*, b, b^*, suppose that (ii) is true. Then so is (i), given their equivalence. Together with QS_{PP}, (i) entails (iv) and hence the equivalent claim (iii). This shows that QS_{PP} entails QS_{4P}.

[40] The case is adapted from Effingham and Robson (2007), who use it to argue against endurantism. Donald Smith (2009) replies, and Effingham (2010) replies to Smith. For a different response to the original paper, see Gilmore (2009, pp. 122–125).

In words: a thing x is *simple* at a location y just in case (i) x at y is a part of x at y and (ii) the only ⟨thing z, location z*⟩ pair such that z at z* is part of x at y is ⟨x, y⟩; x at x* is a *proper part* of y at y* just in case x at x* is a part of y at y* but either x is distinct from y or x* is distinct from y*; and x at x* is *disjoint* from y at y* just in case x at x* does not overlap y at y*. With these notions in hand, we can offer the following rough paraphrase of QS_{4P}: if a thing has a proper part at a region, then the thing must have disjoint proper parts at that region.

In both of the cases above, the antecedent of QS_{4P} is satisfied: roughly, David has Lumpy as a proper part, and Wall has Brick as a proper part. So, in both cases, QS_{4P} yields the result that the complex object in question must have 'disjoint proper parts' (in the 4P-appropriate sense). But while Wall does have such parts (namely, Brick at rbr_1 and Brick at rbr_2), David does not. Wall is composed of a single simple thing, Brick, *at two different locations of that thing*, and Brick at the first location does not overlap itself at the second location. This keeps the case in compliance with QS_{4P}. David, by contrast, is composed of a distinct simple thing, Lumpy, *at just one location of that thing*, and this violates QS_{4P}. [41]

(This is a very loose explanation of how QS_{4P} applies to the two cases above. When we turn to the question of how QS_{4P} applies to propositions, I will proceed more slowly and carefully. But to give a formal, full-dress discussion of the present case would be tedious and not especially useful.)

This completes my discussion of four-place parthood as it holds 'within the spatiotemporal realm'. In the next section I consider the role that this relation might play in a theory of Russellian propositions.

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[41] To see this that the David/Lumpy case violates QS_{4P}, suppose for *reductio* that (i)–(iii) from the specification of that case are all true, and that QS_{4P} is true too. Then, from the definition of 'fuses', we can conclude that (iv) P(Lumpy, r_L, David, r_L). Together with QS_{4P}, (iv) and (ii) entail that

(v) $\exists w \exists w^* \exists z \exists z^* [P(w, w^*, David, r_L) \& P(z, z^*, David, r_L) \& \neg O(w, w^*, z, z^*)]$

Suppose that a, r_a, b, and r_b are such a foursome, hence that: (vi) P(a, r_a, David, r_L), (vii) P(b, r_b, David, r_L), and (viii) $\neg$O(a, r_a, b, r_b). Given (vi) and the definition of 'fuses', we can conclude that (ix) O(a, r_a, Lumpy, r_L). Similarly, given (vii) and the definition of 'fuses', we can conclude that (x) O(b, r_b, Lumpy, r_L). Since Lumpy is *simple* at r_L, the only way for a at r_a to *overlap* Lumpy at r_L is for Lumpy at r_L to be a *part* of a at r_a. (To see this, note that (ix) entails that there is a thing, call it c, and location, call it r_c, such that: (xi) P(c, r_c, a, r_a) and (xii) P(c, r_c, Lumpy, r_L). And (xii) and (iii) entail that c=Lumpy and r_c=r_L.) Parallel remarks go for b at r_b. Thus, given (iii), (ix) yields (xiii) P(Lumpy, r_L, a, r_a) and (x) yields (xiv) P(Lumpy, r_L, b, r_b). But (xiii) and (xiv) entail that (xv) O(a, r_a, b, r_b), which contradicts (viii).

7. Applying Four-Place Parthood to Propositions

How might a four-place parthood relation hold between propositions and their constituents? Roughly put, 4P says that in order for a thing x to be a part of a thing y, there must be some location x* of x, and some location y* of y, such that x *at x** is a part of y *at y**. When the x and the y in question are two different material objects, the locations are easy to identify: they are just spacetime regions.

But in the case of propositions and their constituents, it isn't obvious what the locations might be. Spacetime regions are out of the running, since propositions and universals are abstract entities, or so I assume. What other locations might there be?

My answer is: the non-spatiotemporal 'slots' or 'argument positions' in the abstract entities expressed by predicates and connectives. Jeffrey King attributes a similar picture to Frege:[42]

> In the case of sentences like
>
>   3. Carl loves Rebecca
>
>   4. Rebecca loves Carl
>
> we have a doubly incomplete sense expressed by 'loves'. The two names express complete senses. The thoughts expressed by 3 and 4 differ in that in one case (i.e., 3), the sense expressed by 'Carl' saturates the 'first position' in the unsaturated sense expressed by 'loves' and the sense expressed by 'Rebecca' saturates the second; in the other case (i.e. 4) the reverse is true. So the unsaturated sense here both binds together the three constituents in virtue of the complete senses completing its unsaturated positions and it determines the structure of the resulting thought by having different positions that can be completed by the different complete senses. This is how the unsaturated sense both holds the thought together and allows two different thoughts to have the same constituents. Again, it should be clear how the structures of the thoughts expressed mirror the structures of the sentences. Next consider a sentence like:
>
>   5. Rebecca is strong and Lucy is shy.
>
> Here the senses of the two conjuncts are like that expressed by 2 [the sentence 'Gödel is smart'] in terms of their structures and types of constituents. The sense

---

[42] For a more detailed look at Frege's views on these matters, see Heck and May (2011).

of 'and' is doubly unsaturated, and is doubly completed by the complete senses of the two conjuncts, yielding a complete sense: the 'conjunctive thought' expressed by the whole. I assume it is easy to imagine how this account extends to other truth functional sentential connectives, including a one-place connective like 'not'. (2007, p. 13)

There are four ideas in this passage that will be important in what follows. As far as I can tell, all four can be combined with either Fregean or Russellian views about the basic ingredients of propositions.

First idea. Both predicates and sentential connectives express non-set-theoretical, mind-independent, language-independent abstract entities that can be parts of propositions. I will assume (with Russell) that predicates express *universals*. As for the entities expressed by connectives, I will call them *connectants*. Developed in this broadly Russellian way, the first idea can be dubbed *realism about universals and connectants*.

Second idea. If a predicate or connective is n-adic, then the entity that it expresses has exactly n slots in it. In the hands of the Russellian, this second idea takes the form of realism about *slots* in universals and *slots* in connectants.[43] According to this view, there really are such entities as slots: talk of slots is not to be paraphrased away. Slots are presumably abstract, and they may be ontologically dependent on the universals or connectants that host them, but it hardly follows that they don't exist or that they're unreal.

As I see it, the best way to think of slots is by analogy with holes in material objects. Just as the holes in a piece of cheese are not parts of the piece of cheese (Casati and Varzi 1994), the slots in a universal are (typically) not parts of that

---

[43] Many philosophers seem to be realists about slots in universals. See Gilmore (forthcoming b), King (2007, p. 41), McKay (2006, p. 13), Newman (2002, p. 148), Yi (1999, pp. 168ff.), Swoyer (1998, p. 302), Armstrong (1997, p. 121), Menzel (1993, p. 82), Zalta (1988, p. 52), Williamson (1985, p. 257), Grossmann (1983, p. 200), (1992, p. 57), and especially Crimmins (1992, pp. 99–140), who appeals to slots in the context of a detailed theory of structured Russellian propositions. Crimmins distinguishes between slots (which he calls *arguments*) in universals and certain associated entities that he dubs *roles*. Both do important work in his theory. I became aware of Crimmins's theory only after this chapter was complete, and I hope to discuss it in future work.

Russell speaks of 'positions' in facts (1956, p. 286); see note 48 for more on this. Horwich (1998, p. 91) speaks of 'positions' in *propositional structures* (which he apparently takes to be *sui generis* abstract entities) rather than in universals. Wetzel (2009, p. 134) accepts even such *sui generis* abstract entities as *places in flag types* (e.g., the position in the flag type Old Glory occupied by the third red stripe from the bottom) so I suspect that she would be sympathetic to slots in universals. See also Koslicki (2008) and Harte (2002) for discussion of a range of ideas in this neighborhood and further references.

universal.[44] Universals and connectants *host* their slots; they typically do not have their slots as parts. And just as the holes in a piece of cheese are not parts of the sandwich that has the piece of cheese as a part, the slots in a universal are (typically) not parts of propositions that have that universal as a part.[45]

Third idea. This can be expressed as two claims about the relation of occupation. First, the semantic content of a name—its *sense* for Frege, its *referent* for Russell—occupies slots in the semantic contents of predicates. In the hands of the Russellian, this yields the slogan 'referents occupy slots in universals'. Second, the semantic contents of sentences (thoughts, a.k.a., propositions) occupy slots in the semantic contents of connectives (connectants). In slogan form: 'propositions occupy slots in connectants'.[46]

Fourth idea. This is looser and more general: other things being equal, the structure of propositions should be taken to mirror the structure of the sentences that express them. Call this the *Mirroring Principle.*[47]

I will take these four ideas as guides in developing Russellianism$_{4P}$. As we'll see, they leave a number of important questions unanswered. In the next section I address some of these questions.

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[44] Here is a possible exception. If the relation of identity has two slots in it, *s.i* and *s.ii*, perhaps there is a haecceitistic property being identical with s.i. One might wish to say that s.i is both a *part of* this property (just as Obama is a part of being identical with Obama) and a *slot in* this property (just as s.i is a slot in the dyadic relation of identity from which the haecceitistic property is built up).

[45] The potential exceptions here involve those odd propositions that are 'directly about' slots: e.g., the proposition that s.i is identical to s.i. (See the previous note.)

[46] Some may object that there is no fundamental occupation relation that holds both between material objects and spacetime regions (on the one hand) and between things and slots (on the other). I have two replies.

First, if occupation were an ordinary spatiotemporal relation, such as spatiotemporal coincidence, this objection would be quite plausible: presumably nothing *spatiotemporally coincides* with a slot or indeed with any non-spatiotemporal entity. But most of us have independent reason for denying that occupation = spatiotemporal coincidence. After all, spatiotemporal coincidence is symmetric but occupation is not. (I occupy regions; they don't occupy me.) It seems to me that occupation is plausibly a 'topic-neutral' relation, much like identity and, perhaps, parthood. Surely if one is attracted the idea that the very same parthood relation can hold both between material objects and their constituents and between propositions and their constituents, one should not immediately dismiss the parallel suggestion about occupation as too bizarre to take seriously.

Second, and more tentatively, one might simply define 'OCC(x, y)' as '∃z∃wP(x, y, z, w)'. (This idea was suggested to me by Kris McDaniel.) Given that some things are parts of themselves at regions and that some things are parts of themselves at slots, this definition would give us the result that regions and slots are both occupied in the same sense.

[47] Admittedly, we will often need to depart from it. For example, it seems that the sentence '**Property(b)**' is built up from at least two distinct proper parts (a predicate and a name, not to mention the brackets), but the Russellian will presumably want to say that the proposition that the sentence expresses has just one proper part (being a property), which is referred to by '**b**' and expressed by the predicate.

7.1 Examples

Formulating Russellianism$_{4P}$ in a rigorous and general way is a compli-cated affair. Rather than lay it out all at once, we should ease our way into it by considering a series of examples, from which we can then extrapolate.

Let's focus on p_e, the proposition that Etna is higher than Vesuvius. The Russellian wants to say that, in some sense, Etna, Vesuvius, and the relation being higher than are all parts of p_e. How should this rough idea be regi-mented in terms of a four-place parthood relation?

Consider Etna and p_e. The Russellian wants to say that Etna is, somehow or other, a part of p_e. If our only parthood relation were a two-place rela-tion of parthood *simpliciter*, the Russellian's only option would be to say that Etna is a part of p_e *simpliciter*. As we've seen, that leads to problems. If our only parthood relation were a three-place, time-relative parthood relation, the Russellian would apparently be forced to say that Etna is a part of p_e *at the particular instant t_a*, or *at some time*, or *at all times*. This also leads to prob-lems. But now that we have a four-place parthood relation and an ontology of slots at our disposal, we can try something different. We can judiciously choose certain slots, x and y, and say that Etna, *at Etna's location x*, is a part of the proposition p_e, *at p_e's location y*. With any luck, this will help us avoid the problems mentioned earlier.

So let us ask: Which locations of Etna and p_e are the relevant ones? That is:

Q$_e$ Which ⟨slot x, slot y⟩ pairs are such that Etna, *at slot x*, is a part of p_e, *at slot y*?

Parallel questions can be asked about Vesuvius and about being higher than:

Q$_v$ Which ⟨slot x, slot y⟩ pairs are such that Vesuvius, *at slot x*, is a part of p_e, *at slot y*?

Q$_b$ Which ⟨slot x, slot y⟩ pairs are such that being higher than, *at slot x*, is a part of p_e, *at slot y*?

The passage from King, adapted to our Russellian assumptions, suggests a picture that will be useful in thinking about these questions.

$$p_e$$

$$\underline{\text{Etna}\ \ \text{BEING HIGHER THAN}\ \ \text{Vesuvius}}$$

s1   s2

_____CONJ __ . . . .

s3
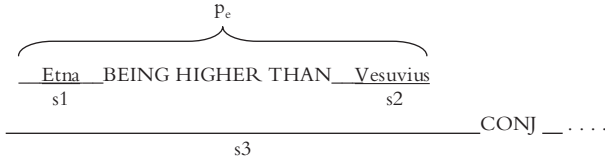
Like any proposition, p_e has a great many locations. It occupies (inter alia, perhaps) each slot in each connectant. The diagram above shows p_e as it is at just *one* of these slots: s3, which happens to be a slot in CONJ, the connectant expressed by '**&**'. But this choice is completely arbitrary. We could just as easily have depicted p_e as it is at any slot in any connectant: p_e would 'look the same' at each of those slots. (More on this later.)

Likewise, the referents of the names '**e**' and '**v**' each have a great many locations. Each of them occupies (inter alia) each slot in each universal. But again, the diagram represents each of these referents as it is at just one such location. In this case, however, the choice is not arbitrary. We are interested in p_e, the proposition expressed by '**HigherThan(e, v)**'. In that sentence, the name '**e**', which refers to Etna, occupies the first argument place in the predicate '**HigherThan**'. So the fact that Etna is shown as it is at slot s1 in being higher than is no accident.

What about being higher than? More generally, what about the semantic contents of predicates? What do they occupy? Interestingly, this question is not addressed in the King passage. As I see it, however, the natural thing to say here is that, like propositions, universals occupy (inter alia) slots in connectants. Just as an *open sentence* can be 'inserted' into the 'slot' of a one-place *connective* to form a new open sentence, the semantic content of that open sentence, a *universal*, is plausibly regarded as occupying a slot in the semantic content of the connective, a *connectant*. This picture should appeal to those who see the category of propositions and the category of universals as species of a more natural category—the category of *predicables* or *assertibles*. Propositions, on this view, are saturated or 0-adic predicables, properties are singly unsaturated or monadic predicables, dyadic relations are doubly unsaturated predicables, and so on (van Inwagen 2006b). It is in the spirit of this view to think of the slots in connectants as being occupied by predicables quite generally, regardless of their 'degree of unsaturatedness' or adicity. Specifically, it is natural to assume that *every* predicable occupies

every slot in *every* connectant. With that assumption in place, we get the result that <u>being higher than</u> occupies the slot s3 in CONJ. (More on this later, too.)

Finally, what about CONJ and its brethren? What do connectants occupy? I will assume that connectants, like propositions and universals, occupy slots in connectants. It is tempting to read Frege as embracing a similar view in the following passage from 'Compound Thoughts':

> The 'connective' in a compound thought of the fifth kind is the doubly incomplete sense of the doubly incomplete expression
>   '(not   ) and (   ).'
> Here the compound thoughts are not interchangeable, for
>   '(not B) and A'
> does not express the same as
>   '(not A) and B'.
> …Since I hesitate to coin a new word, I am obliged to use the word 'position' with a transferred meaning. In speaking of written expressions of thoughts, 'position' may be taken to have its ordinary spatial connotation. But a position in the expression of a thought must correspond to something in the thought itself, and for this I shall retain the word 'position'. In the present case we cannot simply allow the two thoughts to exchange their 'positions', but we can set the negation of the second thought in the 'position' of the first, and at the same time the negation of the first in the 'position' of the second. (Klemke 1968, p. 548)

In my terminology, we might say that '(not   ) and (   )' expresses a compound, dyadic connectant that results from putting NEG into the first slot in CONJ:
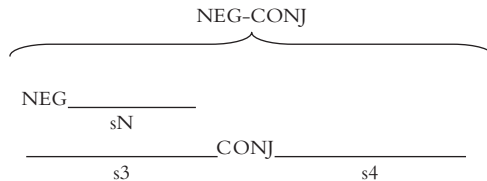
In any event, the operative assumption strikes me as natural and as harmonizing with the other views that I have set out so far.

With all of these locational assumptions in hand, we can return to questions Q_e, Q_v, and Q_b. First, Q_e: which $\langle$slot x, slot y$\rangle$ pairs are such that

Etna at x is a part of p_e at y? Without giving a full answer yet, we can at least give *one example* of a such a pair: $\langle s1, s3 \rangle$, where s1 is a slot, the 'upper' slot, in <u>being higher than</u> and s3 is a slot in CONJ. For it should now be plausible that

(14)   Etna at s1 is a part of p_e at s3; that is, P(Etna, s1, p_e, s3).

Parallel remarks apply to question Q_v: Which $\langle$slot x, slot y$\rangle$ pairs are such that Vesuvius at x is a part of p_e at y? One example is: $\langle s2, s3 \rangle$, where s2 is the *other* slot, the 'lower' slot, in <u>being higher than</u>. For it should also be plausible that

(15)   Vesuvius at s2 is a part of p_e at s3; that is, P(Vesuvius, s2, p_e, s3).

Similarly for Q_b: Which $\langle$slot x, slot y$\rangle$ pairs are such that <u>being higher than</u> at x is a part of p_e at y? One example is $\langle s3, s3 \rangle$, for

(16)   <u>being higher than</u> at s3 is a part of p_e at s3: P(<u>loving</u>, s3, p_e, s3).

Informally put, (14)–(16) say that Etna at s1, Vesuvius at s2, and <u>being higher than</u> at s3 are all parts of p_e at s3. So far, so good. But the Russellian will want to make a stronger claim. Intuitively, he will want to say not merely that these three things are parts of p_e, but also that they 'exhaust' p_e: there is 'no more' to p_e at s3 than the parts just specified.

   More precisely, he will want to say that p_e *fuses* the set $\{\langle$Etna, s1$\rangle$, $\langle$Vesuvius, s2$\rangle$, $\langle$<u>being higher than</u>, s3$\rangle\}$ at the slot s3:

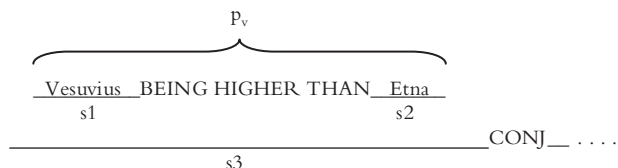(17)   F(p_e, $\{\langle$Etna, s1$\rangle$, $\langle$Vesuvius, s2$\rangle$, $\langle$<u>being higher than</u>, s3$\rangle\}$, s3)

Put a bit differently, (17) says that p_e, at its location s3, is composed of Etna at its location s1, Vesuvius at its location s2, and <u>being higher than</u> at its location s3.

   Crucially, (17) does not say that p_e is itself a set or an ordered sequence of some kind. Nor does it say that p_e has sets or ordered pairs as parts. Further, (17) does not say that p_e has any slots as parts. (17) is consistent with the view that p_e is a *sui generis* abstract entity whose only parts are Etna, Vesuvius, and <u>being higher than</u>.

So far we have merely considered p_e 'as it is at s3'. On the current view, p_e occupies other slots in other connectants and has, at those other slots, the very same semantic contents as parts. Thus, a parallel series of remarks would apply to p_e as it is at each of these other slots.

For purposes of comparison, it may help to consider another example of a proposition expressed by an atomic sentence. Let p_v be the (false) proposition expressed by the sentence '**HigherThan(v, e)**'. The following diagram represents p_v as it is at the slot s3:

$$p_v$$

Vesuvius BEING HIGHER THAN Etna
s1   s2

_____CONJ__ . . . .

s3

In light of our treatment of p_e, the diagram above makes it natural to hold that p_v, at slot s3, is composed of Vesuvius at s1, Etna at s2, and being higher than at s3. Or, more formally,

(18) $F(p_v, \{\langle Vesuvius, s1 \rangle, \langle Etna, s2 \rangle, \langle \underline{being\ higher\ than}, s3 \rangle\}, s3)$

Very loosely put, p_v and p_e are both composed at s3 of the *same things* but at *different locations* of those things.

In discussions structured propositions, facts, states of affairs, and structural universals, one often encounters similar claims. One sometimes encounters a claim to the effect that two propositions, or two states of affairs, or two structural universals, can be made up of the same constituents 'in different arrangements'.[48] One way to make precise sense out of

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[48] In 'On propositions: what they are and how they mean' (published in 1919), Russell writes that

every constituent of a fact has a position (or several positions) in the fact. For example, 'Socrates loves Plato and 'Plato loves Socrates' have the same constituents, but are different facts, because the constituents do not have the same positions in the two facts. 'Socrates loves Socrates' (if it is a fact) contains Socrates in two positions (1956, p. 286).

In a similar vein, Reinhardt Grossmann writes:

Every relation comes with distinct (non-identical) places. Consider a two-place relation like the relation of being larger than between natural numbers. This relation, R, has two distinct places, which we can indicate by writing 'R' with two slots like this: @ R #. Since the slots, the places, are different, the results will be different if the places are filled with different things: the state of affairs aRb is different from bRa. (1992: 57)

David Armstrong (1986, p. 85; 1997, p. 121) follows Grossmann in appealing to slots in the individuation of states of affairs.

such a claim is in terms of a four-place parthood relation, together with an ontology of slots.[49]

So far the only propositions that we have considered are those expressed by atomic sentences. Let's now turn to a proposition expressed by a *compound* sentence of L_R: '**&(HigherThan(e, v), HigherThan(v, e))**'. In English, this would be rendered 'Etna is higher than Vesuvius and Vesuvius is higher than Etna'. Let p_c be the proposition expressed by the given sentence of L_R. As with the previous cases, I'll depict p_c as it occurs at just one of its many locations. The location that I'll arbitrarily choose is s5, a slot in NEG. Any slot in any connectant would serve just as well.
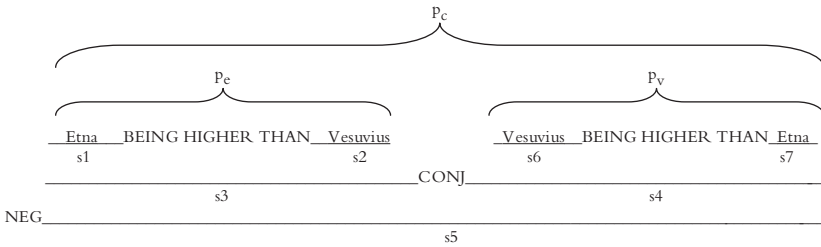
This diagram raises a hard question about slot identity. Are s1 and s2 the same slots as s6 and s7? Nothing in this chapter depends on the answer, but the question is interesting and has ramifications elsewhere, so I will touch upon it again later. (Crimmins's 1992 distinction between *slots* (which he calls *arguments*) and *roles* is relevant here. He would say that there are only two slots in being higher than, but that in p_c, each of these slots is associated with two different roles.)

Right now, however, I want to comment on the mereological structure of p_c. Given what we've said about p_e and p_v, the natural thing to say about p_c is that, at s5, it is composed of p_e at s3, p_v at s4, and CONJ at s5. In terms of our fusion predicate, this comes to

(19) $F(p_c, \{\langle p_e, s3\rangle, \langle p_v, s4\rangle, \langle CONJ, s5\rangle\}, s5)$

---

[49] In my terminology, something of the neighborhood of the Russell-Grossmann principle can be formalized as:

U_G $\forall x\forall x*\forall y\forall z[[F(x, y, z) \,\&\, F(x*, y, z)] \rightarrow x{=}x*]$

This says that it never happens that two different things fuse the same set (of ordered $\langle$thing, location$\rangle$ pairs) at the same location. Since p_e and p_v presumably do not fuse the same set at s3, they apparently respect U_G, despite the fact that they are distinct propositions that in some sense are 'composed of the same parts'. It turns out, however, that there are more complicated examples that violate U_G unless we accept certain potentially controversial assumptions about slot identity. (In particular, we need to reject the view that I dub *Slot Constancy*, presumably in favor of the view I dub *Slot Variance*. See section 7.3.1 for these views.) It does not pay to go into the details here.

Taken together, (18) and (19) have an interesting and welcome feature. Given the definition of our fusion predicate, (18) entails

(20)   Etna at s1 is a part of p_e at s3; i.e., P(Etna, s1, p_e, s3)

and (19) entails

(21) p_e at s3 is a part of p_c at s5; i.e, P(p_e, s3, p_c, s5)`

Together with Transitivity$_{4P}$, (20) and (21) yield a formally valid argument for

(22)   Etna at s1 is a part of p_c at s5; i.e., P(Etna, s1, p_c, s5).

This is exactly the sort of conclusion we expect a transitivity-like principle to generate. Indeed, if Transitivity$_{4P}$ didn't generate such a conclusion, this would speak against its status as a natural analogue of Transitivity$_{2P}$.

7.2 A General Formulation of Russellianism$_{4P}$

By now, I hope, the reader will have a rough sense of how to generalize on the foregoing examples. My goal in this section is to be explicit about how the generalization should go.

7.2.1 Five theses about slots   I will start with five theses about universals, connectants, and slots. My official statement of Russellianism$_{4P}$ will have each thesis as a conjunct. The first thesis says that slots come in at least two incompatible kinds:

Two Kinds   There are *objectual* slots, there are *predicative* slots, and no slot is both objectual and predicative.

The idea will be that the slots in connectants are predicative slots and are occupied by things like universals, connectants, and propositions, whereas the slots in universals are objectual slots and are occupied by anything and everything whatsoever.

The next three theses make explicit the link between the adicity of a universal or connectant and the number of slots in it. They employ a primitive four-place predicate 'x is an nth slot in y at (y's location) z'. They also employ

the three-place predicate 'x is a slot in y at z' defined as 'for some n, x is an nth slot in y at z'. I start with a claim about universals:

Slots in Universals   If a universal u is n-adic, then for each predicative slot s_{pr}, u has exactly n slots in it at s_{pr}, each of these slots is objectual, and, for any positive integer m less than or equal to n: (i) there is exactly one m^{th} slot in u at s_{pr} and (ii) for any j, if j≠m, then nothing is both an mth slot in u at s_{pr} and a jth slot in u at s_{pr}.

In other words, an n-adic universal has n objectual slots in it (and no other slots) at each predicative slot, and there are facts about how these slots are ordered: one is first, another one is second,…, and yet another one is n^{th}.[50] Next comes a parallel claim about connectants:

Slots in Connectants   If a connectant c is n-adic, then for each predicative slot s_{pr}, c has exactly n slots in it at s_{pr}, each of these slots is predicative, and, for any positive integer m less than or equal to n: (i) there is exactly one m^{th} slot in c at s_{pr} and (ii) for any j, if j≠m, then nothing is both an mth slot in c at s_{pr} and a jth slot in c at s_{pr}.[51]

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[50] Even among those who are realists about slots in universals, this thesis will be controversial. Timothy Williamson (1985), e.g., is a realist about slots, but he holds that, at least in some cases, a given relation is identical to its converse. In particular, he holds that being before = being after. According to Williamson, this relation has two slots in it: a slot for the thing said to be earlier, call it s_E, and a slot for the thing said to be later, call it s_L. When we think of the relation as being before, it will be natural to treat s_E as 1st and s_L as 2nd; but when we think of the relation as being after, it will be natural to treat s_L as 1st and s_E as 2nd. But on the assumption that neither of these 'perspectives' on the relation is objectively privileged, the natural thing to say is that the slots are not ordered in the relevant way, and hence there is no fact of the matter as to which of the slots is the 1st slot and which is the 2nd. (See Crimmins 1992, pp. 166ff., Fine 2000, Fine 2007, and MacBride 2007 for discussion of related issues.)

   I suspect that it's possible to state a version of Russellianism$_{4P}$ that (i) is consistent with the claim that the slots in a universal lack any objective order and that (ii) still retains the virtues of the present account. The crucial maneuver, I think, is to replace our four-place predicate 'x is the nth slot in y at z' with the five-place predicate 'relative to predicate Π, x is the nth slot in y at z', and to restate the certain components of Russellianism$_{4P}$ accordingly. This predicate allows us to say, e.g., that *relative to the predicate 'is before'*, s_E is the 1st slot in being before at location s3, whereas *relative to the predicate 'is after'* s_E is the 2nd slot in being before at location s3.

[51] For reasons parallel to those given in the previous note, this thesis will be controversial even among those who are initially sympathetic to realism about slots in connectants. As before, I suspect that I could state a version of Russsellianism$_{4P}$ that avoids the controversial claims about order while retaining the advantages of the present account.

Neither Slots in Universals nor Slots in Connectants takes a stand on the following question: do n-adic universals and n-adic connectants carry the same n slots around with them wherever they go? (Call the view that they do *Slot Constancy*.[52]) Or do they grow a new crop of slots at each new location? (Call this rival view *Slot Variance*.[53]) As I mentioned earlier, in connection with the 'hard question about slot identity', all the main points in this chapter can be consistently combined with either answer (as far as I can tell). I think there are several considerations that make Slot Variance the more plausible view, but I don't have the space to go into this here. So I'll remain neutral for the rest of the chapter.

   Our fourth and fifth theses impose constraints on the behavior of slots. The fourth thesis says, in effect, that while universals and connectants may *occupy* objectual slots, they do not *have slots in them* at objectual slots:

   Objectual Saturation   If x is a slot in y at z, then z is a predicative slot.

According to Objectual Saturation, universals and connectants have slots in them *only at predicative slots*. Thus, if an n-adic universal occupies both objectual slots and predicative slots, then it has different numbers of slots in it at different locations: it has n slots in it at each of its predicative locations, and it has 0 slots in it at each of its objectual locations. In that case, a universal would be analogous to an enduring, multi-located piece of clay that has different numbers of holes in it at different locations. Such a piece of clay might have, say, one hole in it at one region (early in its career, when it is shaped like a doughnut) while having no holes in it at another region (later in its career, when it is shaped like a lump). I offer some considerations in support of Objectual Saturation in Appendix I.

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[52] More carefully formulated: for any s, any s*, any x, any y, and any y* if s is a slot in x at y and s* is a slot in x at y*, then s is a slot in x at y*. This says that a universal or connectant has exactly the same slots in it at each location at which it has any slots in it at all.

[53] More carefully formulated: for any s, any s*, any x, any y, and any y*, if s is a slot in x at y and s* is a slot in x at y* and y≠y* then s≠s*. This says that nothing is a slot in a given universal or connectant at two different locations of that universal/connectant. As I discovered after completing this chapter, basically the same considerations that motivate slot variance can also be accommodated in the manner of Crimmins (1992, pp. 99–140), who distinguishes between *slots* ('arguments') and *roles*. Roughly put, he takes the latter to be more 'fine-grained' than the former.

The *fifth* thesis about slots is parallel to Objectual Saturation. Just as Objectual Saturation says that nothing has *slots* at objectual slots, the fifth thesis says that nothing has *proper parts* at objectual slots:

Objectual Simplicity   If x at y is a part of z at w and w is an objectual slot, then: x = z and y = w.

According to Objectual Simplicity, each entity is simple at each objectual slot that it occupies. Thus, while propositions (e.g.) are presumably complex at predicative slots, they are simple at any objectual slots that they occupy; and while concrete particulars are often complex at spacetime regions, they too are simple at any objectual slots that they may occupy. Motivations for Objectual Simplicity are given in Appendix II.

7.2.2   Atomics4P and Compounds4P   I am now in a position to state the two core components of Russellianism$_{4P}$. The first is

Atomics$_{4P}$ If φ is a sentence of L_R, if Π is an n-adic predicate of L_R, and if $\tau_1, \ldots, \tau_n$ are names of L_R such that φ=$\ulcorner\Pi(\tau_1, \ldots, \tau_n)\urcorner$, then: there is a proposition p, there is a n-adic universal u, there is an entity $o_1, \ldots,$ and there is an entity o_n such that: p is the proposition expressed by φ, Π expresses u, τ_1 refers to $o_1, \ldots, \tau_n$ refers to o_n, and for *any* predicative slot s_{pr}:
F(p, {x: either x=⟨u, s_{pr}⟩ or x=⟨o_1, the 1st slot in u at s_{pr}⟩,..., or x=⟨o_n, the nth slot in u at s_{pr}⟩}, s_{pr})[54]

To illustrate, let φ be '**HigherThan(e, v)**', let Π be '**HigherThan**', let τ_1 be '**e**', let τ_2 be '**v**', and let s3 be a predicative slot. Then, since '**HigherThan**' is dyadic and expresses being higher than, since '**e**' refers to Etna, since '**v**' refers to Vesuvius, and since p_e is the unique proposition expressed by '**HigherThan(e, v)**', Atomics$_{4P}$ lets us conclude that

(23)   F(p_e, {⟨being higher than, s3⟩, ⟨Etna, the 1st slot in being higher than at s3⟩, ⟨Vesuvius, the 2nd slot in being higher than at s3⟩}, s3)

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[54] In accordance with the suggestion in note 50, phrases like 'the 1st slot in u at s_{pr}' could be replaced with phrases like 'the 1st slot in u at s_{pr}, relative to Π'. This would accommodate Williamson's view that the slots in a universal lack the relevant sort of 'perspective independent' ordering.

Morever, since s1=(the first slot in <u>being higher than</u> at s3), and since s2=(the second slot in <u>being higher than</u> at s3), (23) amounts to the claim that p_e, at s3, is composed of <u>being higher than</u> at s3, Etna at s1, and Vesuvius at s2.

My diagrams typically show a proposition as it is at just one of its locations. For example, in the diagram of p_e, that proposition is shown as it is at s3. I have emphasized that each proposition occupies multiple slots in multiple connectants, and I have said, rather loosely, that each proposition 'looks the same' or 'has a similar structure' at each such location. Atomics$_{4P}$ makes this precise, and it does so without taking a stand on the Slot Constancy vs. Slot Variance dispute.

So much for Atomics$_{4P}$. Here is the second core component of Russellianism$_{4P}$:

> **Compounds$_{4P}$**   If φ, ψ_1,..., and ψ_n are sentences of L_R, if K is an n-adic sentential connective of L_R, and if $\varphi=\ulcorner K(\psi_1,..., \psi_n)\urcorner$ then: there is a proposition p_φ, there is an n-adic connectant c, there is a proposition p_{ψ_1},..., and there is a proposition p_{ψ_n}, such that: p_φ is the proposition expressed by φ, K expresses c, ψ_1 expresses p_{ψ_1},..., and ψ_n expresses p_{ψ_n}, and for any predicative slot s_{pr}:
> $F(p_\varphi, \{x$: either $x=\langle c, s_{pr}\rangle$ or $x=\langle p_{\psi_1}$, the 1st slot in c at $s_{pr}\rangle$,..., or $x=\langle p_{\psi_n}$, the nth slot in c at $s_{pr}\rangle\}, s_{pr})$[55]

Compounds$_{4P}$ handles sentences of arbitrary complexity, not merely sentences that contain just a single connective.

Now, since L_R contains connectives of just two adicities (monadic and dyadic), Compounds$_{4P}$ is more general than it needs to be. But this is a virtue; it makes it clear how Compounds$_{4P}$ could be adapted to languages with a larger range of connectives, some of them non-truth-functional. (Atomics$_{4P}$ is also more general than it needs to be in this respect.)

This completes my presentation of Russellianism$_{4P}$. It is the conjunction of 4P, Two Kinds, Slots in Universals, Slots in Connectants, Objectual Saturation, Objectual Simplicity, Atomics$_{4P}$, and Compounds$_{4P}$.

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[55] Those who deny that the slots in a connectant exhibit the relevant sort of perspective-independent ordering can replace phrases like 'the 1st slot in c at s_{pr}' with those like 'the 1st slot in c at s_{pr}, relative to K'. See notes 50 and 51.

## 7.3 Russellianism$_{4P}$ and the Supplementation Argument

We're now in a position to see how Russellianism$_{4P}$ solves the problem about quasi-supplementation. Recall that p_i is the proposition expressed by '**IdenticalTo(i, i)**'; it is the proposition that says, of identity, that it is identical to it. Earlier in the chapter, I argued that if Russellianism$_{2P}$ is true, then p_i generates a counterexample to QS$_{2P}$, and that if Russellianism$_{3P}$ is true, then p_i generates a counterexample to QS$_{3P}$. The question before us now is: if Russellianism$_{4P}$ is true, does p_i generate a counterexample to QS$_{4P}$?

I will argue that it doesn't, or at least that we have no reason to think that it does. According to Russellianism$_{4P}$, there are predicative slots, and each predicative slot s_{pr} is such that:

(24) $F(p_i, \{\langle \text{identity}, s_{pr}\rangle, \langle \text{identity, the 1st slot in identity at } s_{pr}\rangle, \langle \text{identity, the} $
          $\text{2}^{nd}\text{ slot in identity at } s_{pr}\rangle\}, s_{pr})$

Let's consider p_i as it is at one arbitrarily chosen predicative slot, s5 (a slot in NEG). Since s5 is a predicative slot, and since identity is a dyadic universal, Slots in Universals lets us conclude that identity has exactly two slots in it at s5–a unique *first* slot there, and a unique *second* slot there. Let s8 be the first slot in identity at s5, and let s9 be the second slot in identity at s5. All this is captured in the diagram below:
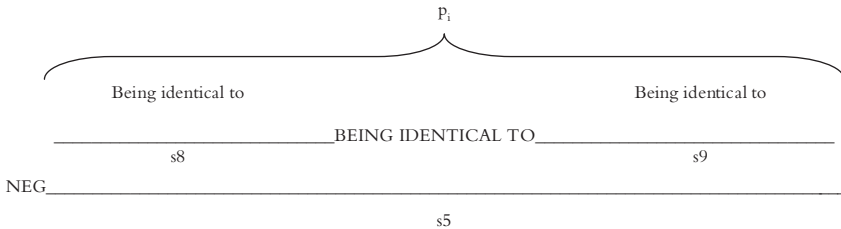
As the reader will already have guessed, QS$_{4P}$ tolerates this case for the same reason that it tolerates the Brick/Wall case. My discussion of the present case will therefore run parallel to my discussion of the Brick/Wall case.

In both cases, we have a composite object that, at a given location, is composed of a distinct simple object, at multiple locations. (This is a rough

way of putting it.) In both cases, the antecedent of QS_{4P} is satisfied. In the present case, we have, e.g.,[56]

(25)   P(identity, s8, p_i, s5), and
(26)   identity≠p_i ∨ s8≠s5.

In fact, both disjuncts in (26) are true. Together with QS_{4P}, (25) and (26) entail

(27)   ∃z∃z*∃w∃w*[P(z, z*, p_i, s5) & P(w, w*, p_i, s5) & ¬O(z, z*, w, w*)]

In order for (27) to be true, p_i needs to have, at s5, parts that are disjoint from each other 'in the 4P way'. More precisely, there needs to be a quadruple ⟨z, z*, w, w*⟩ such that

(i)   P(z, z*, p_i, s5)
(ii)   P(w, w*, p_i, s5)
(iii)   ¬∃x∃x*[P(x, x*, z, z*) & P(x, x*, w, w*)], that is, ¬O(z, z*, w, w*)

But it should be clear that there is such a quadruple; indeed, there are several of them. One of them is ⟨identity, s8, identity, s9⟩.[57] To see that this quadruple satisfies the four clauses above, let's consider them in turn. As for (i) and (ii), Atomics$_{4P}$ assures us that we have:

(i*)   P(identity, s8, p_i, s5), and
(ii*)   P(identity, s9, p_i, s5).

Finally, consider (iii). Is there an ⟨x, x*⟩ pair such that x, at x*, is a part *both* of identity at s8 *and* of identity at s9? Clearly there is no intuitive pressure to say there is such a pair. What would it be? This is the crucial point; this is what gives Russellianism$_{4P}$ its advantage over Russellianism$_{2P}$ vis-à-vis the supplementation problem.

However, we can go further if we like. Given our assumption that identity is simple at each of its locations (which gave rise to the puzzle in the

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[56] I say 'e.g.' because ⟨identity, s8, p_i, s5⟩ is not the only quadruple that satisfies the antecedent of QS_{4P}. It is also satisfied by the following quadruples: ⟨identity, s5, p_i, s5⟩ and ⟨identity, s9, p_i, s5⟩.
[57] Here are some others: ⟨identity, s5, identity, s8⟩, ⟨identity, s8, identity, s5⟩, and ⟨identity, s5, identity, s9⟩.

first place), we can *argue* that there is *no* pair of the type mentioned in the previous paragraph. The simplicity assumption gives us

(28) $\forall x \forall x^*[P(x, x^*, identity, s8) \rightarrow [x=identity \text{ \& } x^*=s8]]$, and

(29) $\forall x \forall x^*[P(x, x^*, identity, s9) \rightarrow [x=identity \text{ \& } x^*=s9]]$.

According to (28), the only $\langle x, x^* \rangle$ pair such that $P(x, x^*, identity, s8)$ is $\langle identity, s8 \rangle$; and according to (29), the only $\langle x, x^* \rangle$ pair such that $P(x, x^*, identity, s9)$ is $\langle identity, s9 \rangle$. But since $s8 \neq s9$, it follows that there is no single $\langle x, x^* \rangle$ pair such that x, at x^*, is a part *both* of identity at s8 *and* of identity at s9. Thus we have

(iv*) $\neg \exists x \exists x^*[P(x, x^*, identity, s8) \text{ \& } P(x, x^*, identity, s9)]$

In other words, identity at s8 is disjoint from identity at s9. The proposition p_i does have, at s5, a proper part. So the antecedent of QS_{4P} is satisfied. But p_i also has, at s5, parts that are disjoint from each other 'in the 4P way'. So the consequent of QS_{4P} is satisfied as well.

As it is at s5, the proposition p_i is composed of a simple thing (identity), at three of its locations (s5, s8, and s9). For any two of these locations, identity at the first location is disjoint from identity at the second. Moreover, as $Atomics_{4P}$ indicates, parallel remarks would apply to p_i as it is *at every other predicative slot as well*. Given Russellianism$_{4P}$, therefore, the proposition p_i doesn't seem to pose any threat to QS_{4P}.

Accordingly, there appears to be no sound 4P-appropriate analogue of the supplementation argument. As I see it, this gives the friend of Russellianism$_N$ a reason to embrace four-place parthood and an ontology of slots.

7.4 *Russellianism$_{4P}$ and the Transitivity Argument*

Recall that p_e is the proposition expressed by '**HigherThan(e, v)**'. Earlier in the chapter I argued that if Russellianism$_{2P}$ is true, then p_e gives rise to an apparent counterexample to Transitivity$_{2P}$, for in that case it appears that the rock o is a part of Etna, that Etna is a part of p_e but that o is not a part of p_e. I also argued that if Russellianism$_{3P}$ is true, p_e gives rise to a parallel problem for Transitivity$_{3P}$. Now we should ask: if Russellianism$_{4P}$ is true, does p_e give rise to an apparent counterexample to Transitivity$_{4P}$? In this section, I argue that it does not.

7.4.1   A Solution   To get the relevant sort of counterexample to Transitivity$_{4P}$ involving o, Etna, and p_e, we would need to have an ordered ⟨location x, location y, location z⟩ triple such that:

(i)   o at x is a part of Etna at y; i.e., P(o, x, Etna, y), and

(ii)   Etna at y is a part of p_e at z; i.e., P(Etna, y, p_e, z).

In particular, there would need to be a location y of Etna such that o, at some location of o, is a part of Etna at y, and Etna, *at that very same location y*, is a part of p_e at some location of p_e. But there is no intuitive pressure to say that there is such a location of Etna; indeed, the existence of such a location is highly implausible. To see this, note three points.

First, given 4P, it *is* plausible that there are ordered ⟨location x, location y⟩ pairs such that o at x is a part of Etna at y. But since o and Etna are distinct material objects, I take it that the only ordered pairs that satisfy the given description are pairs of *spacetime regions*. More generally, I assume that if m and m* are non-identical material objects, and if m at x is a part of m* at y, then x and y are both spacetime regions. In slogan form: *material objects are parts of other material objects only at spacetime regions*.

Second, given Russellianism$_{4P}$, it is plausible that there are ordered ⟨location y, location z⟩ pairs such that Etna at y is a part of p_e at z. But since p_e is a proposition, I take it that the only ordered pairs that satisfy the given description are pairs of *slots*. More generally, I assume that if p is a proposition, and if e at x is a part of p at y, then x and y are both slots. As a slogan: *things are parts of propositions only at slots*.

Third, nothing is both a slot and a spacetime region. Slots, like the universals and connectants that host them, are non-spatiotemporal entities. Spacetime regions are spatiotemporal entities.

Taken together, these three points entail that there is no ⟨x, y, z⟩ triple such that: (i) o at x is a part of Etna at y, and (ii) Etna, at that very same location y, is a part of p_e at z. Hence Transitivity$_{4P}$ never 'kicks in' and yields the absurd conclusion that the rock o, at some location, is a part of the proposition p_e, at some location. So it appears that there is no sound 4P-appropriate analogue of Frege's transitivity argument.

7.4.2   A Loose End   One crucial component of Frege's transitivity argument was the claim that the rock o is not a part of the proposition that

Etna is higher than Vesuvius. I noted that this claim could be motivated in at least two ways: by appeal to the acquaintance argument (or some variant thereof), or by appeal to the essentialism argument. But those arguments relied on principles that were formulated in terms of a two-place parthood relation. Do those principles have sufficiently close 4P-appropriate counterparts? If not, we may seem to have thrown the baby out with the bathwater. For in that case, we would not have shown that a version of Russellianism harmonizes with the intuitive data driving Frege's argument. Fortunately, the principles do have very close 4P-appropriate counterparts.

First, consider the original acquaintance principle: one can grasp a given proposition only if one is acquainted with each of its parts.[58] A natural four-place analogue of this is:

Acquaintance$_{4P}$ For any s and any p, if s grasps p and p is a proposition, then $\forall x \forall y \forall z[P(x, y, p, z) \rightarrow$ s is acquainted with x]

In other words, one grasps a given proposition only if one is acquainted with each entity that is, anywhere, a part of that proposition, anywhere. Together with the fact that Frege grasps the proposition p_e but is not acquainted with the rock o, this principle entails that o is not, anywhere, a part of p_e, anywhere. And friends of Russellianism$_{4P}$, of course, face no pressure to deny this, even with the appropriate transitivity-like principle in place. They say that 'o is a part of Etna but only at spacetime regions', whereas 'Etna is a part of p_e but only at slots'.

Next consider the original essentialist principle: abstract entities have all of their parts essentially. A natural four-place analogue is

MEA$_{4P}$ $\forall x[(\neg x \text{ is concrete}) \rightarrow \forall y[\exists x* \exists y* \ P(y, y*, x, x*) \rightarrow$ $\square (x \text{ exists} \rightarrow \exists x* \exists y* P(y, y*, x, x*))]]$[59]

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[58] Or consider the original principle about de re thought: If subject s entertains proposition p, then for any part p* of p, s either grasps p* or is engaged in *de re* thought about p*. A natural four-place analogue is:

For any s, any p, and any p*, if p is a proposition and $\exists x \exists y P(p*, x, p, y)$, then either (i) s grasps p* or (ii) s is engaged in de re thought about p*.

Together with the claim that p_e is a proposition and that Frege entertains p_e but neither grasps nor is engaged in de re thought about o, this entails that o is not, anywhere, a part of p_e, anywhere.

[59] Stronger principles are also available. One is:

$\forall x[(\neg x \text{ is concrete}) \rightarrow \forall x* \forall y \forall y*[P(y, y*, x, x*) \rightarrow \square(x \text{ exists} \rightarrow P(y, y*, x, x*))]]$.

It is not clear to me which is the closer analogue of the original principle. But since the friend of Russellianism$_{4P}$ can apparently accept either principle without getting into any trouble, we need not take a stand on this question.

In other words, if a thing x is abstract, then if a thing y, somewhere, is a part of x, somewhere, then x is necessarily such that if it exists, that same thing y is, somewhere, a part of x, somewhere. Together with the plausible claim that p_e is not concrete and is not necessarily such that if it exists, then o is, somewhere, a part of it, somewhere, MEA_{4P} yields the result that o is not, anywhere, a part of p_e, anywhere. Again, friends of Russellianism$_{4P}$ face no pressure to deny this.

7.4.3   An Objection and a Reply: Slots and Three-Place Parthood?   Objection. As soon as we help ourselves to slots, we can block Frege's transitivity argument even without appealing to four-place parthood. For suppose that parthood is a three-place relation that can hold between a part, a whole, and a *location* (perhaps a location of the part, perhaps a location of the whole). Then one might argue that there is no location L such that: (i) o is a part of Etna at L and (ii) Etna is a part of p_e at L. (Presumably o is a part of Etna only at spacetime regions, and Etna is a part of p_e only at slots.) But if there is no such location, then the relevant transitivity-like principle, Transitivity$_{3P}$, never 'kicks in' and yields the absurd conclusion that o is a part of p_e at some location. Thus it appears that, with slots in hand, 4P becomes unnecessary, so far as transitivity-based considerations go.

Reply. I concede that Frege's argument can be blocked without 4P. But 4P is still needed to give a satisfying account of certain 'transitivity phenomena' associated with propositions and their parts. Three-place parthood is not up to the task.

To respond to Frege's argument, we wanted a transitivity-like principle that *wouldn't* kick in as applied to the case Frege considered. Both the 4P-er and the 3P-er can manage this, given an ontology of slots together with the assumption that parthood is somehow 'location relative'. But there are further cases in which, intuitively, we want a transitivity-like principle that *does* kick in to secure certain expected results. It turns out the 4P-er can manage this but that the 3P-er cannot.

Recall the point I made about the compound proposition p_c. (I won't reproduce the diagram here.) I noted a welcome fact—namely, that the following argument is formally valid and has highly plausible premises, given the relevant background assumptions:

(i) $P(Etna, s1, p_e, s3)$
(ii) $P(p_e, s3, p_c, s5)$

(iii)　　[Transitivity$_{4P}$]

(iv)　　So, P(Etna, s1, p$_c$, s5)

Moreover, I suggested that if one's chosen transitivity-like principle didn't yield a similar argument for a similar conclusion, this would cast doubt on its status as an appropriate analogue of Transitivity$_{2P}$.

Now I want to argue that this point constitutes a problem for the proposal that Russellians can take parthood to be a three-place relation that holds between a part, a whole, and a location. For suppose that this view about parthood is correct. Then it appears that we will be unable to use the relevant transitivity-like principle (Transitivity$_{3P}$) to show that Etna is a part of p$_c$ at some location.

To see why, note that if parthood is three-place and 'location relative', then at least one of the following principles will be true:

LP$_{3Pa}$　　$\forall x \forall y \forall z$[x is a part of y at z $\rightarrow$ x occupies z]

LP$_{3Pb}$　　$\forall x \forall y \forall z$[x is a part of y at z $\rightarrow$ y occupies z]

According to the first, parthood holds between a part, a whole, and a location of the part. According to the second, parthood holds between a part, a whole, and a location of the whole. They both prevent us from constructing the desired sort of argument. Let me explain.

What we want is a formally valid argument whose first premise says that Etna (at some location, perhaps) is a part of p$_e$ (at some location, perhaps), whose second premise says that p$_e$ (at some location, perhaps) is a part of p$_c$ (at some location, perhaps), whose third premise is the relevant transitivity-like principle, and whose conclusion says that Etna (at some location, perhaps) is a part of p$_c$ (at some location, perhaps).

Initially, friends of LP$_{3Pa}$ of LP$_{3Pb}$ might be tempted to offer the following arguments, respectively:

Argument 1a		Argument 1b	
(a.i)	Etna is a part of p$_e$ at s1	(b.i)	Etna is a part of p$_e$ at s3
(a.ii)	p$_e$ is a part of p$_c$ at s3	(b.ii)	p$_e$ is a part of p$_c$ at s5
(a.iii)	[Transitivity$_{3P}$]	(b.iii)	[Transitivity$_{3P}$]
(a.iv)	So, Etna is a part of p$_e$ at s1	(b.iv)	So, Etna is a part of p$_c$ at s5

The premises of these arguments are plausible (given their respective background assumptions), but both arguments are invalid. In order for Transitivity$_{3P}$ to yield the relevant conclusion, the first two premises need to specify a *common location* L such that Etna is a part of p_e at L and p_e is a part of p_c at that very same location L. But in neither argument is such a common location specified. To fix this 'common location problem', the friend of three-place, location-relative parthood might propose either

Argument 2	or	Argument 3
(2.i) Etna is a part of p_e at s1		(3.i) Etna is a part of p_e at s3
(2.ii) p_e is a part of p_c at s1		(3.ii) p_e is a part of p_c at s3
(2.iii) [Transitivity$_{3P}$]		(3.iii) [Transitivity$_{3P}$]
(2.iv) So, Etna is a part of p_c at s1		(3.iv) So, Etna is a part of p_c at s3

These arguments are formally valid, but the friend of LP$_{3Pa}$ and the friend of LP$_{3Pb}$ will both say that each argument has at least one implausible premise. The friend of LP$_{3Pa}$ will deny (2.ii) on the grounds that p_e is a part of p_c at the predicative location *s3*, not at *s1*;[60] and she will deny (3.i) on the grounds that Etna is a part of p_e at *s1*, not s3. The friend of LP$_{3Pb}$ will deny (2.i) on the grounds that Etna is a part of p_e at *s3*, not s1; and she will deny (3.ii) on the grounds that p_e is a part of p_c at *s5*, not s3.

In light of all this, the 3P-er might propose replacing Transitivity$_{3P}$ in Argument 1a or 1b with some stronger transitivity-like principle, such as

Transitivity$_{3Pa}$	$\forall x \forall y \forall z \forall w \forall w*[(x \text{ is a part of } y \text{ at } w \ \& \ y \text{ is a part of } z \text{ at } w*) \rightarrow$
	$x \text{ is a part of } z \text{ at } w]$, or
Transitivity$_{3Pb}$	$\forall x \forall y \forall z \forall w \forall w*[(x \text{ is a part of } y \text{ at } w \ \& \ y \text{ is a part of } z \text{ at } w*) \rightarrow$
	$x \text{ is a part of } z \text{ at } w*]$

These substitutions would make Arguments 1a and 1b formally valid, respectively. But they are too strong to be plausible, and with them in place,

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[60] The point is not that LP$_{3Pa}$, together with our background locational assumptions, *entails* the negation of (2.ii). After all, from a purely logical point of view, one can accept both (2.ii) and LP$_{3Pa}$, provided that one is willing to hold that p_e occupies the objectual slot s1, which presumably it does. The point is that, although p_e plausibly *occupies* s1, p_e is not plausibly *a part of p_c* there, given LP$_{3Pa}$. If parthood is relativized to locations of the part, then p_e is plausibly a part of p_c only at *some* of p_e's locations—more specifically, only at '1st slots' in CONJ. Parallel remarks apply to the LP$_{3Pa}$-er's reason for denying (3.i) and to the LP$_{3Pb}$-er's reasons for denying (2.i) and (3.ii). (Though in the case of the LP$_{3Pa}$-er's reason for denying (3.i), it is somewhat doubtful that Etna, or indeed any concrete particular, even occupies s3 or any other predicative slot.)

Frege's argument would be up and running again. According to the friend of LP_{3Pa}, o is a part of Etna at spacetime region r_o, say; and Etna is a part of p_e at s1. Together with $Transitivity_{3Pa}$, it follows that o is a part of p_e at r_o, which I take to be an undesirable outcome. A parallel problem arises for $Transitivity_{3Pb}$.

   In sum, it looks as though the 3P-er will be unable to formulate a plausible, reasonably natural transitivity-like principle that both *fails* to apply in Frege's case and *successfully* applies in the case I've just considered. The 4P-er has no trouble formulating such a principle.

8. Conclusion

In sections 1–5, I noted that if parthood is two-place, then $Russellianism_N$ is in tension both with $Transitivity_N$ and with QS_N. In sections 6–7, I argued that if parthood is four-place, this tension disappears. For in that context, one is apparently free to accept $Russellianism_{4P}$, $Transitivity_{4P}$, and QS_{4P}, in which case one will accept their 'adicity-neutral' counterparts as well. This gives friends of $Russellianism_N$ a reason to look favorably upon four-place parthood and the relevant doctrines about slots.
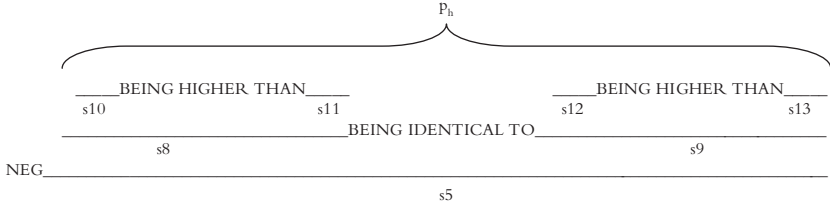
   Of course, the formal language that I considered is extremely simple. In some respects, it is clear how $Russellianism_{4P}$ could be extended to richer languages: the addition of further names, and of further predicates and sentential connectives of fixed adicity (including non-truth-functional connectives), poses no apparent problem. But in other respects, it's not at all obvious how the extension should go. What should the friend of $Russellianism_{4P}$ say about languages that contain quantifiers and variables? Term-forming operators? Multi-grade predicates? I don't know whether any natural extension of the present approach can handle these phenomena. But given how well $Russellianism_{4P}$ performs as a solution to the problems about transitivity and quasi-supplementation, I think it would be worthwhile to find out whether it can be extended. Moreover, if $Russellianism_{4P}$ is plausible as an account of the propositions expressed by the sentences of L_R, perhaps that's of some interest in its own right, whether or not the account can be extended. (One might think, e.g., that the only *facts* or *states of affairs* that exist are ones that correspond to sentences in a language like L_R, and one might think that *their* mereological structure can be captured via something along the lines of $Russellianism_{4P}$.)
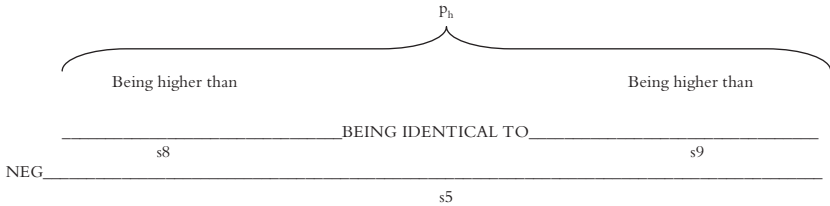
Appendix I: In Support of Objectual Saturation

To see the rationale for Objectual Saturation, recall that the language L_R contains a name, '**h**', for <u>being higher than</u>. Further, suppose that p_h is the proposition expressed by the sentence '**IdenticalTo(h, h)**'. Now consider the following diagrams:

INCORRECT

P_h

_____BEING HIGHER THAN_____   _____BEING HIGHER THAN_____
s10   s11   s12   s13
_____BEING IDENTICAL TO_____
          s8   s9
NEG_____
                                   s5

CORRECT

P_h

Being higher than   Being higher than

_____BEING IDENTICAL TO_____
          s8   s9
NEG_____
                                   s5

   In the first diagram, <u>being higher than</u> has slots in at s8 and at s9. In the second diagram, it doesn't. Only the second diagram accords with Objectual Saturation. There are three closely related reasons for preferring the second diagram.

   First, it seems to me that if the first diagram were accurate, then the proposition p_h would have objectual slots in it at s5[61] and hence it would be an unsaturated entity at that location.[62] But p_h is a *proposition* and hence

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[61] In particular, s10, s11, s12, and s13 would each be a slot in p_h at s5. If Slot Constancy is true, then s10=s12 and s11=13 (though s10≠s11 and s12≠s13), in which case p_h would have exactly *two* slots in it at s5. If Slot Variance is true, s10–s13 are all distinct, in which case p_h has exactly *four* slots in it at s5.

[62] There is a general principle at work here, viz.:

Slot Inheritance   If e_1 at s_1 is a part of e at s, if s_{e1} is a slot in e_1 at s_1, and if $\neg \exists x P(x, s_{e1}, e, s)$, then s_{e1} is a slot in e at s.

Consider an analogy. If a small block of wood, o, is a part of larger wooden structure, d, and if o has a hole, h, in it, then h will also be a hole in the larger structure d, unless d has another part (a round peg, say) that fills h.

plausibly doesn't have slots in it at any of its locations; rather, it is *saturated* at each of its locations. Call this the *saturation argument*.

Second, in Appendix II, below, I offer independent reasons for thinking that propositions, though complex at predicative slots, are simple at objectual slots. This claim does not fit well with the view that universals have slots in them at objectual slots. After all, if being higher than had slots in it at s8, wouldn't these slots be occupied by things? In particular, wouldn't s10 be occupied by Etna and wouldn't s11 be occupied by Vesuvius? And if so, wouldn't those things, at those slots, together with being higher than at s8, compose p_e at s8? But in that case p_e would be complex at s8. And this would conflict with my preferred view that propositions are simple at objectual slots. Call this the *simplicity argument*.

Third, it seems to me that the second diagram conforms more closely to the Mirroring Principle mentioned earlier than does the first diagram. In L_R, the only linguistic entities that are permitted to occupy the argument places in a predicate are *names*, which are 'slot free', i.e., devoid of argument places. And since the argument places in a predicate correspond to the (objectual) slots in the universal expressed by the predicate, the Mirroring Principle should lead us to expect that the *occupants* of objectual slots will be 'slot free' as well, at least *at the given objectual slots*. But according the first diagram, being higher than has slots in it at the objectual slots s8 and s8; it is not 'slot free' at those locations. Call this the *mirroring argument*.

Appendix II: In Defense of Objectual Simplicity

Objectual Simplicity can be motivated in two ways: first, by appeal to the Mirroring Principle, and second, inductively.

Mirroring. In L_R, the only linguistic entities that are permitted to occupy the argument places in a predicate are names, which are all syntactically simple. So, since the argument places in a predicate correspond to the (objectual) slots in the universal expressed by the predicate, the Mirroring Principle should lead us to expect that the occupants of objectual slots are also simple, at least *at those objectual locations*.
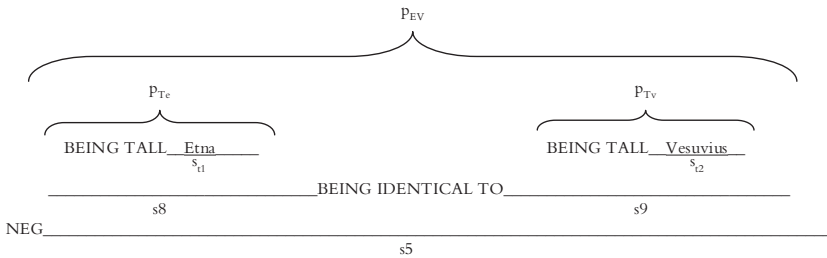
Induction. One can argue that both *material objects* and *propositions* are simple at objectual slots. I take it that this provides some inductive evidence for the more general claim that *everything* is simple at objectual slots. We can consider the two cases separately.

Material objects. I assume that if any material objects have proper parts at objectual slots, at least some of these objects have *other material objects* as proper parts at objectual slots.[63] But if considerations about transitivity and acquaintance (or *de re* thought) can be used to motivate the view that the rock o is not (at any of its locations) a part of Etna at s1, these considerations should apply quite generally, so that for *any* two material objects o1 and o2 and *any* objectual slot s, we have acquaintance-based reasons to say that o1 is not (at any of its locations) a part of o2 at s.[64]

Propositions. There are two main reasons for taking propositions to be simple at objectual slots. The first arises from considerations about de re thought and transitivity, the second from QS_{4P}.

(1) *Transitivity*. Consider the sentence of L_R '**Tall(e)**', and let p_{Te} be the proposition that it expresses. Likewise, let p_{Tv} be the proposition expressed by the sentence '**Tall(v)**'. These are different propositions: p_{Te} predicates <u>being tall</u> of Etna, not Vesuvius; p_{Tv} predicates <u>being tall</u> of Vesuvius, not Etna. Let the name '**p1**' of L_R refer to p_{Te}, and let the name '**p2**' of L_R refer to p_{Tv}. Finally, consider the sentence of L_R, '**IdenticalTo(p1, p2)**', and let p_{EV} be the proposition it expresses.

I assume that if any propositions have proper parts at objectual slots, then p_{Te} has proper parts at s8 (an objectual slot in identity), as follows:

In particular, I assume that if any propositions have proper parts at objectual slots, then

(30)   Etna at s_{t1} is a part of p_{Te} at s8; i.e., P(Etna, s_{t1}, p_{Te}, s8).

But (30) leads to potentially objectionable conclusions. Together with Transitivity$_{4P}$ and the claim that p_{Te} at s8 is a part of p_{EV} at s5 (which the friend of Russellianism$_{4P}$ will endorse), (30) entails that

(31)   Etna at s_{t1} is a part of p_{EV} at s5, i.e., P(Etna, s_{t1}, p_{EV}, s5).

And this claim can be attacked by appeal to considerations about acquaint-ance or *de re* thought. For it seems to me that one could grasp [entertain, believe,...] p_{EV} without being engaged in *de re* thought about either Etna or Vesuvius.[65] Consider the following case:

> John's favorite proposition is p_{Te}, and Bob's favorite proposition is p_{Tv}. Mary knows John and Bob but has no acquaintance with Etna or Vesuvius. She knows that John and Bob each have favorite propositions that they frequently enter-tain, but, intuitively, she doesn't know *which* propositions these are. Perhaps she knows that both John's and Bob's favorite propositions are geological in subject matter, but she doesn't know much else about them. John and Bob introduce names for their favorite propositions—'p1' and 'p2', respectively—and share them with Mary. If she sees John with a dreamy look on his face, she might say, 'Entertaining p1 again today, John?' and get the reply, 'Yes; I'm afraid so.' After several years of this sort of thing, Mary comes to believe, concerning the propositions in question, that they're identical. She still has never heard of Etna or Vesuvius.

In this case, it is plausible that Mary believes (and hence grasps) p_{EV} but that she is not acquainted with, and is not engaged in *de re* thought about, Etna or Vesuvius. Given the principles appealed to earlier, then, we should conclude that (31) is false and hence that propositions are

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[65] Similar considerations are discussed by Jason Stanley and Joshua Armstrong (2011). They attribute the idea to Timothy Williamson.

simple at objectual slots. As a result, a less misleading diagram of p_{EV} would be as follows:
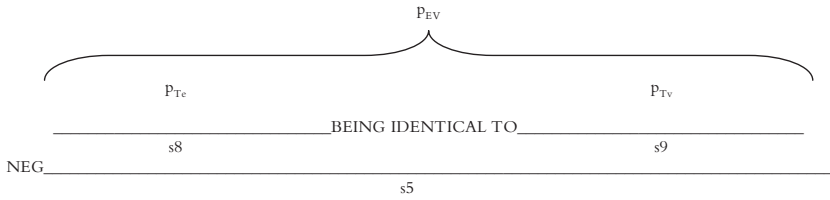
It is worth noting that this case also shows that the problem underlying the Transitivity Argument, TA, from section 4, is not solved by distinguishing between constituency and parthood (or between parthood$_p$ and parthood$_m$). It's no less plausible that constituency is transitive than that parthood is transitive. And yet, on the assumption that constituency is two-place, we seem to have a counterexample here. For if constituency is two-place, then Etna is a constituent of p_{Te}, and p_{Te} is a constituent of p_{EV}, but Etna is not a constituent of p_{EV} (since Mary believes p_{EV} but is not acquainted with or engaged in *de re* thought about Etna).

(2) *Quasi-supplementation*. As Dan Rabinoff pointed out to me, the assumption that propositions are complex at objectual slots, when combined with Slot Constancy, gives rise to counterexamples to QS$_{4P}$. (I leave it as an exercise for the reader to describe these examples. Hint: let the name '**p**' of L$_R$ refer to the proposition, p_b, expressed by the sentence '**Property(b)**' of L$_R$, and consider the sentence '**Property(p)**' of L$_R$. Let p_p be the (false) proposition that this latter sentence expresses. According to Russellianism$_{4P}$, there are slots s and s* such that p_b at s is a part of p_p at s*. What happens if p_b complex at a slot like s?) I want to retain QS$_{4P}$, of course, so this fact gives me a reason to reject either Slot Constancy or the assumption that propositions are complex at objectual slots.[66]