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ORIGINAL RESEARCH

Addressing two recent challenges to the factive account of knowledge

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Abstract

It is widely thought that knowledge is factive – only truths can be known. However, this view has been recently challenged. One challenge appeals to approximate truths. Wesley Buckwalter and John Turri argue that false-but-approximately-true propositions can be known. They provide experimental findings to show that their view enjoys intuitive support. In addition, they argue that we should reject the factive account of knowledge to avoid widespread skepticism. A second challenge, advanced by Nenad Popovic, appeals to multidimensional geometry to build a case where it seems intuitive that a person knows p even though p is false. In addition, Popovic argues that we should reject the factive account of knowledge because most of us would not become widespread skeptics if we discovered that ordinary objects in our world are actually four-dimensional. In this paper, we defend the factive account of knowledge against these arguments by challenging the intuitive appeal of the cases and arguing that there is no real threat of widespread skepticism for the factive account of knowledge.

Keywords Knowledge · Approximate truths · Factive · Non-factive

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1 Introduction

It is widely thought that knowledge is factive (call this view the factive account of knowledge). That is, only truths can be known. Recently, Allan Hazlett has challenged the factive account of knowledge.[1] Hazlett notes that there are contexts in which ordinary speakers judge that stating 'S knows that p' is appropriate, even though they have a true belief that p is false. For example, upon making a series of discoveries, it seems appropriate for a person to say 'everything I knew about this topic is wrong.'

Various epistemologists have since resisted Hazlett's challenge.[2] They propose that non-factive attributions of knowledge can be explained away by perspective-taking, and so are non-literal attributions of knowledge. In cases where ordinary speakers judge that saying 'S knows p' is appropriate despite believing truly that p is false, the speakers take themselves as describing the situation from S's perspective. From S's perspective, S believes or feels that she has knowledge. So, when taking S's perspective, it is appropriate to say 'S knows p.' Thus, instead of attributing *actual* knowledge to S, the speaker uses the term 'knows' non-literally; 'knows' merely indicates S's belief or feeling.[3] Experimental studies have also confirmed that non-factive knowledge attributions are due to perspective-taking in the cases in question, and thus are non-literal attributions of knowledge.[4]

There are, however, two other recent challenges to the factive account of knowledge that cannot be responded to by appealing to non-literal knowledge attributions due to perspective-taking. The first challenge appeals to approximate truths. Wesley Buckwalter and John Turri argue that false-but-approximately-true propositions can be known. The second challenge is advanced by Nenad Popovic, who appeals to multidimensional geometry to build a case where a person knows p even though p is false.[5]

In this paper, the authors defend the factive account of knowledge against both these challenges. We lay out the first challenge in Sect. 2 and address it in Sect. 3. In Sect. 4, we lay out the second challenge, and we address it in Sect. 5. We maintain that all these challenges fail to show that knowledge is non-factive.

[1] Hazlett 2010, 2012.

[2] See Holton 1997, Turri, 2011, Tsohatzidis, 2012, Hannon, 2013, Buckwalter, 2014, and Domaneschi & Paola 2019.

[3] Similarly, the utterance 'everything I knew is wrong' would be time indexed to the past agent.

[4] Buckwalter 2014, and Domaneschi & Paola 2019.

[5] Popovic 2020.

2 Challenge one: approximate truths and the quasi-factive account of knowledge

Buckwalter and Turri reject the factive account of knowledge by arguing that people can know propositions that are *approximately* true, even though such propositions are not *strictly* true.[6] They hold to a quasi-factive account of knowledge—only truths and approximate truths can be known.[7]

To support their quasi-factive account, Buckwalter and Turri first point out that people commonly rely on approximation. Take scientific study as an example.[8] Scientists at NASA's jet propulsion laboratory use '3.141592653589793' as an approximate value of pi. Scientists estimate the age of the universe to be 13.8 billion years old, and so on.

Buckwalter and Turri also point out that people use approximations in their everyday lives when representing, observing, and measuring things.[9] For example, I may say 'I know my mailbox is 2ft from the street' despite it actually being 1.95ft away; and I may say 'I know the current time is 9:03,' when it is actually 9:02:58. These representations merely approximate truth – they are not *strictly* true. Despite their falsity, the quasi-factive account of knowledge allows these false-but-approximately-true representations to count as instances of knowledge, as long as they adequately represent the truth. Exactly what counts as 'adequate' will vary according to context. For example, taking 3.14 as the value of pi might be adequate in a math class and thus count as knowledge, but not at a scientific laboratory.[10]

Besides appealing to the fact that people commonly use approximations in domains that people take themselves to have knowledge, Buckwalter and Turri offer two more arguments for their view. The first argument is a skeptical challenge for the factive account of knowledge.[11]

1. If the factive account of knowledge is right, approximately true beliefs do not amount to knowledge.
2. Many of our beliefs are approximately true beliefs.
3. Therefore, if the factive account of knowledge is right, many of our beliefs do not amount to knowledge.

The factive account of knowledge thus entails a form of widespread skepticism—many of our beliefs would not amount to knowledge because many of our everyday beliefs are mere approximations. On the factive account, I lack knowledge that my mailbox is 2ft from the street, I lack knowledge that the current time is 9:03, and so forth. This widespread skepticism is problematic because people ordinarily judge

[6] Buckwalter & Turri 2020a, b.

[7] The notion of quasi-factivity is introduced in 2015, though Shaffer also anticipates the quasi-factive account of knowledge in Shaffer 2011 and 2012.

[8] Buckwalter & Turri 2020a: 93.

[9] Buckwalter & Turri 2020a>: 94.

[10] Buckwalter & Turri 2020a>: 97.

[11] Buckwalter & Turri 2020a.

that people have knowledge in these cases involving approximation. The factive account of knowledge would "undermine much of what we ordinarily take ourselves to know." [12]

The second argument Buckwalter and Turri have advanced is based on an experiment they conducted.[13] In their experiment, participants were given different short scenarios involving a protagonist – Drew. These scenarios manipulate whether Drew's answer was true or false (truth-value), and whether Drew's answer was practically adequate (practical adequacy). The following outlines the scenario with the truth-value and adequacy manipulations shown in brackets:

> Drew is a civil engineer designing a structure. To fit, he must use the length of an existing foundation, to [the exact inch/within 10 inches] of its total length. After thinking carefully, Drew uses [9905/9910] inches. The actual length of the foundation was 9910 inches.[14]

In each variant, participants were asked to judge whether the length Drew used was technically false, whether the length Drew used was adequate enough to fit, and whether Drew knows the length of the foundation. These three questions focus on each of the following factors respectively: truth, practical adequacy, and knowledge.

The most important results are the judgments from participants in the case where Drew had a false belief, but his belief was practically adequate for the task at hand. Consider,

> *Drew-Wrong-adequate*: Drew is a civil engineer designing a structure. To fit, he must use the length of an existing foundation, to within 10 inches of its total length. After thinking carefully, Drew uses 9905 inches. The actual length of the foundation was 9910 inches.

Many participants judged that Drew's belief that the actual length of the foundation is 9905 is false. However, they also judged that Drew knows the length of the foundation. So, ordinary people's intuitions suggest that practically adequate approximately true beliefs can amount to knowledge.[15] These results suggest that the factive account of knowledge is false and instead, provides support for the quasi-factive account of knowledge.

Buckwalter and Turri note that one might worry that the results are due to perspective-taking, and so the knowledge attributions are non-literal. Hence, they ran a second experiment where participants were further asked to choose between the following options: "knows" or "only thinks he knows". Most participants chose "knows" – thus, they conclude that perspective-taking is not at play and that the

[12] Buckwalter & Turri 2020a: 95.

[13] Buckwalter & Turri 2020b.

[14] Buckwalter and Turri 2020b.

[15] Buckwalter & Turri 2020b.

knowledge attributions by participants should be taken as literal.[16] Thus, their challenge cannot be addressed in the same way as Hazlett's challenge.

3 Addressing challenge one

Michael Shaffer has since responded to Buckwalter and Turri's quasi-factive account of knowledge by showing how the quasi-factive account of knowledge results in problematic consequences.[17] However, Shaffer does not offer any explanation of where Buckwalter and Turri's skeptical challenge goes wrong, nor does he offer any explanation of their experimental results. We want to offer an explanation for both. Our paper thus advances a new objection to the quasi-factive account.

We start by distinguishing between two types of beliefs in Sect. 3.1. The distinction addresses the skeptical challenge as well as explains why knowledge is attributed in cases involving approximation. In Sect. 3.2, we further show that the factive account of knowledge can explain Buckwalter and Turri's experimental results.

3.1 A-beliefs and E-beliefs

Let us start by drawing a distinction between what we will call A-beliefs and E-beliefs. A-beliefs are beliefs whose content includes the concept of a being an approximation.[18] For example, scientists at NASA's jet propulsion laboratory believe that 'the value of pi is *approximately* 3.141592653589793,' scientists believe that 'the universe is *approximately* 13.8 billion years old,' and so forth. These are A-beliefs. E-beliefs, on the other hand, are beliefs whose content is supposed to be exact and precise. For example, scientists believe that 'the value of pi is *exactly* 22/7.' This is an E-belief.[19]

It is important to note that many claims by scientists are based on their A-beliefs. As Buckwalter and Turri note, "Human beings are limited creatures. Despite our best efforts, there are many things that we will never fully observe, represent, calculate, or measure … In these and other matters, often the best they can do is rely on approximation."[20] Thus, when scientists assert that 'the value of pi is

[16] Buckwalter & Turri 2020b.

[17] See Shaffer (2021). Roughly, he argues that the quasi-factive account of knowledge is problematic because it (a) allows people to know contradictions, (b) allows people to know all propositions, (c) allows people to have unsafe knowledge, and (d) allows it to be perfectly felicitous to assert contradictory pairs of claims. For further discussion of the problematic consequences, see Baumann 2021 and Shaffer 2022.

[18] Although most of the examples being used involve numbers and rounding off as examples of approximation, the authors do not mean to imply that rounding off is the only sense of approximation. For example, X might approximate Y by being similar or close enough to Y. Following Buckwalter & Turri (2020a: 97), we think what counts as similar or close enough will vary according to context. We suspect that in some contexts, 'approximate' depends on the subject's aims; while in other contexts, 'approximate' depends on established or accepted norms.

[19] One might wonder whether 'pi is exactly 3.14 when rounded to 2 decimal places' should be classified as an A-belief or an E-belief given our distinction. After all, it has the word 'exactly' and also the phrase 'when rounded to.' We think this should be classified as an A-belief. E-beliefs should not have any content which have to do with an approximation.

[20] Buckwalter & Turri 2020a: 93.

3.141592653589793,' or that 'the universe is 13.8 billion years old", people do not take them to have E-beliefs. It is clear that scientists do not believe that 'the value of pi is *exactly* 3.141592653589793,' or that 'the universe is *exactly* 13.8 billion years old,' and so forth. Such E-beliefs are false.

Now, notice that unlike these E-beliefs, the corresponding A-beliefs are true. Since the corresponding A-beliefs are true and people take scientists to have these A-beliefs, it is no surprise that people attribute knowledge to them. The factive account of knowledge can thus explain knowledge attributions in cases of approximation – when scientists make assertions, they often are asserting a true A-belief rather than a strictly-false-but-approximately-true E-belief.

Our suggestion extends to everyday examples. Suppose a student in class says 'the value of pi is 3.14.' Intuitively, people judge that the student has knowledge. However, notice that the student could either have an A-belief or an E-belief. The student could believe one of the following two propositions.

Approximate-pi: The value of pi is approximately 3.14.

Exact-pi: The value of pi is exactly 3.14.

It seems that people attribute knowledge to the student because people take the student as believing *Approximate-pi*, not *Exact-pi*. *Approximate-pi* is true, and so the factive account of knowledge accommodates the intuition that the student has knowledge.

Our point here is that although people commonly rely on approximations, other people usually know that those people are relying on approximations and have A-beliefs. In cases of knowledge attribution involving approximation, people take the person to have A-beliefs which are true, not E-beliefs which are strictly-false-but-approximately-true.

The distinction between A-beliefs and E-beliefs also allows us to address Buckwalter and Turri's skeptical challenge. Since people usually form A-beliefs instead of E-beliefs in cases of approximation, most of our beliefs are not strictly-false-but-approximately-true beliefs. Therefore, premise (2) of Buckwalter and Turri's skeptical challenge is false.

The distinction between A-beliefs and E-beliefs also causes a problem for the quasi-factive account of knowledge. Suppose people initially attributed knowledge to a scientist, but then they come to learn that the scientist has E-beliefs instead of A-beliefs. Intuitively, most people would judge that the scientist actually lacks knowledge. The scientist does not and cannot know that 'the value of pi is *exactly* 3.141592653589793,' because mathematically, it is proven that any finite decimal expansion is not pi. The scientist does not (and presumably, due to cognitive and scientific limitations, cannot) know that 'the universe is *exactly* 13.8 billion years old.' Similarly, if a student clarifies that they believe *Exact-pi*, not *Approximate-pi*, people would intuitively judge that the student lacks knowledge. The student does not and cannot know *Exact-pi*. People's intuitive judgments in cases involving E-beliefs suggest that knowledge is not quasi-factive. Even though E-beliefs are approximately true, people judge that such beliefs cannot amount to knowledge. In cases of approximation, people only attribute knowledge if they think a person has A-beliefs.

One reviewer raises the worry that our best physical theories would not count as knowledge on a factive account because our best physical theories are strictly false. As Jeffrey Barrett notes, our best physical theories are strictly false, but they are nevertheless in some sense approximately true.[21] Yet, we want to say that our best physical theories provide knowledge of the physical world.

In reply, we think that we need to specify whether a person believes that our best physical theories are approximately true (i.e. the person has an A-belief) or believes that our best physical theories are strictly true (i.e. the person has an E-belief). Those who have an A-belief regarding our best physical theories have knowledge. So, our best physical theories can provide knowledge of the physical world because we often form a true A-belief regarding our best physical theories.

3.2 Addressing the experimental results

Now, let us turn to address the results from Buckwalter and Turri's experiment. Recall the case where Drew had a false but practically adequate belief.

> *Drew-Wrong-adequate*: Drew is a civil engineer designing a structure. To fit, he must use the length of an existing foundation, to within 10 inches of its total length. After thinking carefully, Drew uses 9905 inches. The actual length of the foundation was 9910 inches.

Participants were asked to judge whether "Drew knows the length of the foundation."[22] Many participants agreed with the statement. This result seems to show that the quasi-factive account of knowledge has intuitive support.

However, we think it is too quick to conclude that the results support the quasi-factive account of knowledge. We think that it is important to ask what proposition participants take Drew to know. One candidate proposition is the following: <the *exact* length of the foundation is 9910 inches.> Do the participants believe that Drew knows <the *exact* length of the foundation is 9910 inches.>? Surely not, since Drew himself does not believe that the exact length of the foundation is 9910 inches. Thus, when the participants judge that Drew knows the length of the foundation, the knowledge attributed to Drew is not 'the *exact* length of the foundation is 9910 inches.' Here is another candidate proposition: <the foundation's length is *approximately* 9910 inches>. However, this runs into the same problem – Drew does not believe this proposition. So, the knowledge attributed to Drew cannot be 'the foundation's length is *approximately* 9910 inches' either.

Since Drew uses '9905 inches,' the proposition that Drew believes and knows must have '9905 inches' as part of the content, not '9910 inches.' Given this, consider the following E-belief that Drew might have: 'the exact length of the foundation is 9905 inches.' Do participants take this as the propositional knowledge attributed to Drew when they judge that Drew knows the length of the foundation? It seems unlikely. If the participants were asked whether Drew knows that 'the exact length of

[21] Barrett 2008.

[22] Buckwalter & Turri 2020b.

the foundation is 9905 inches,' plausibly, they would not attribute this as knowledge to Drew since this E-belief is false. Drew cannot know that the exact length of the foundation is 9905 because the foundation's length is not exactly 9905 inches.

Instead, we think many of the participants who judge that Drew knows the length of the foundation plausibly have in mind the following proposition.

*9905-Will-fit*The foundation length of 9905 inches will fit.

Here are four reasons for thinking that many participants who attribute knowledge to Drew have *9905-Will-Fit* in mind. First, it is clear that Drew believes *9905-Will-Fit* since he uses 9905 inches. Second, *Drew-Wrong-Adequate* is about getting a measurement that fits. Given this, when participants think about whether Drew has knowledge, it is plausible that they are wondering whether Drew knows that the measurement that he uses will fit. Third, right before asking whether Drew has knowledge, participants were asked to judge whether "The length Drew used was adequate enough to fit."[23] This question probes participants to consider whether Drew knows that *9905-Will-Fit*. So, when asked whether "Drew knows the length of the foundation," participants plausibly attribute knowledge to Drew because they believe Drew knows that *9905-Will-Fit*. Fourth, most participants judged that Drew lacks knowledge in the following variant:

Drew-Wrong-inadequate: Drew is a civil engineer designing a structure. To fit, he must use the length of an existing foundation, *to the exact inch* of its total length. After thinking carefully, Drew uses 9905 inches. The actual length of the foundation was 9910 inches.

Buckwalter and Turri take these results to show that whether participants attributed knowledge to Drew depends on whether Drew's belief was practically adequate. However, our suggestion that participants have in mind *9905-Will-Fit* easily explains these results as well. In *Drew-Wrong-Adequate*, Drew has knowledge because he knows *9905-Will-Fit*. Once the scenario is modified, however, Drew does not know *9905-Will-Fit* because *9905-Will-Fit* is false. Thus, Drew lacks knowledge.

Another plausible belief that participants might attribute to Drew is an A-belief, rather than an E-belief (e.g., *9905-Will-Fit*). We suggest that some participants might take Drew to know the following true A-belief:

9905-Approximately: The approximate length of the foundation is 9905 inches.

People often assume that there is a margin of error in construction, and so it is plausible that some participants would take Drew to believe that 'the *approximate* length of the foundation is 9905 inches' or that 'the length of the foundation is *around* 9905 inches.' We suggest that whether *9905-Approximately* counts as true is based

[23] Buckwalter & Turri 2020b.

on context.[24] In the scenarios provided by Buckwalter and Turri, what counts as an approximate length of the foundation would be understood in terms of whether the measurement used is within the margin of error that would fit. In *Drew-Wrong-Adequate*, *9905-Approximately* is true, and hence Drew has knowledge. In *Drew-Wrong-Inadequate*, *9905-Approximately* is false, and so Drew cannot have knowledge.

Our suggestions that participants who judge that Drew has knowledge have either *9905-Will-Fit* or *9905-Approximately* in mind allows the factive account of knowledge to accommodate the results of Buckwalter and Turri's experiment. Therefore, their experiment does not support the quasi-factive account of knowledge. To support their quasi-factive account, Buckwalter and Turri must show that when participants attribute knowledge in *Drew-Wrong-Adequate*, participants think that Drew knows a proposition which is false-but-approximately-true. But it is far from clear that this is the case.

4 Challenge two: a four-dimensional world and the threat of skepticism

Let us now turn to the second challenge to the factive account of knowledge. Popovic appeals to multidimensional geometry to build a case where an agent knows p even though p is false.

Consider the following scenario (call this the *Tesseract Case*). Suppose that our three-dimensional world is nested within a four-dimensional space. Due to our cognitive limitations, people can only perceive three-dimensional projections of four-dimensional objects.[25] Now, suppose that there is a tesseract. The tesseract is a four-dimensional analogue of the cube (similar to how the cube is a three-dimensional analogue of the square). Suppose that this tesseract intersects with our three-dimensional space in such a way as to form a cube, and suppose that this tesseract moves exactly like how a cube would in our three-dimensional space. Thus, our observation and interaction with this tesseract is identical to that of an ordinary cube.[26]

Now, suppose Tom finds this tesseract and forms the following belief:

Object-is-a-Cube: "The object in front of me is a cube."[27]

Suppose that Tom's justification for *Object-is-a-Cube* is as strong as it can be – Tom just had a medical exam to confirm that his vision is perfect and that he is not hallucinating, and he is surrounded by several geometry experts who all agree with him that the object is a cube.[28] Popovic argues that it is reasonable to conclude that Tom possesses knowledge. However, *Object-is-a-Cube* is false because a tesseract and a

[24] The authors think that Buckwalter & Turri (2020a: 97) would agree since they think that what counts as an adequate approximation varies according to context.

[25] Popovic 2020: 1570.

[26] Popovic 2020: 1570–1571.

[27] Popovic 2020: 1571.

[28] Popovic 2020: 1571.

cube are not identical; they are different kinds of objects. Therefore, Popovic concludes that the *Tesseract Case* is a case where an agent knows that p, but p is false.[29]

To resist Popovic's conclusion, one has to either maintain that:

(A) Tom does not know that *Object-is-a-Cube*, or/and.
(B) *Object-is-a-Cube* is true.

Popovic dismisses (B) in a footnote. Popovic thinks that in order to hold that *Object-is-a-Cube* is true, one would need to postulate that "words refer to appearances".[30] Presumably, what Popovic means is that a defender of (B) would say that *Object-is-a-Cube* is actually:

> *Object-is-appearance-of-cube*: The object in front of me has the appearance of a cube.

However, Popovic argues that this view is objectionable because our knowledge of the external world would be limited to knowledge of appearances.[31] Popovic also considers a modification to this view, which holds that words usually refer to the objects themselves except in cases where people unintentionally pick out higher-dimensional objects. That is, only when people unintentionally pick out higher-dimensional objects do words refer to appearances. However, he dismisses this modification because it is objectionably ad hoc.

What about (A)? Is it plausible to deny that Tom knows *Object-is-a-Cube*? Popovic thinks not. First, he finds it intuitive that Tom knows *Object-is-a-Cube* because of some details he mentions surrounding the case. He says,

> I would like to remind the reader about a few peculiarities of this case: it is true that [Tom] sees a cube; it is also true that [Tom's] vision is functioning properly and that [Tom] is not hallucinating. This seems like a prima facie case for knowledge. The main issue is that [Tom] can only see a three-dimensional projection of this object, due to limitations of our perception.[32]

Second, Popovic offers an argument against (A). He asks us to consider the following case (call this the *Many Four-Dimensional Objects Case*). Similar to *Tesseract Case*, our three-dimensional world is nested within a four-dimensional space, but people can only perceive four-dimensional objects as three-dimensional due to our cognitive limitations. Unlike, *Tesseract Case*, suppose that *most* of the physical objects people perceive are actually higher-dimensional objects. Many of our beliefs regarding external objects would be false. Now, suppose that one day, scientists discover

[29] Popovic 2020: 1571.

[30] Popovic 2020: 1571.

[31] Furthermore, this view faces the following objection. If a person is looking at a fake barn which has the appearance of a barn, surely, we do not want to say that the person knows that the object in front of her is a barn. However, someone who holds that words refer to appearances would have to attribute knowledge.

[32] Popovic 2020: 1572.

the true nature of our world and reveal it to everyone. People would now know that many of our beliefs regarding external objects are actually false.

Popovic argues that if one maintains that knowledge is factive, then they would have to become widespread skeptics in *Many Four-Dimensional Objects Case*. Yet, Popovic asserts that most people would not be moved to become widespread skeptics. The discovery that our world is quite different would have little impact on our epistemic practices.[33] Instead, Popovic says that "we would most likely simply continue to attribute knowledge, whilst admitting that the nature of physical objects is different than what we thought."[34] So, if someone maintains that people still have knowledge in *Many Four-Dimensional Objects Case*, then they cannot affirm (A), even though the object Tom perceives is actually a Tesseract and not a cube.

5 Addressing challenge two

Before we address Popovic's argument, it is worth noting first that Popovic's argument cannot be dismissed by appealing to the non-literal use of knowledge. If people only have knowledge in the non-literal sense (i.e., people merely believe or feel as if they know that there are three-dimensional objects), then people still do not really possess knowledge in *Many Four-Dimensional Objects Case*. So, this would still result in the problem of widespread skepticism.

Second, it is also important to note that the case Popovic raises is not a case involving approximate truth. A cube is not an estimation or an approximation of a Tesseract. If three-dimensional objects are approximations of four-dimensional objects, then there is no in-principle reason barring two-dimensional objects to be approximations of three-dimensional objects, and no in-principle reason barring one-dimensional objects to be approximations of two-dimensional objects. However, a square is not approximately a cube, just as a line is not approximately a square. So, a different response is needed to defend the factive account of knowledge. we provide two such responses in the following sub-sections.

5.1 Is it intuitive to judge that tom knows*Object-is-a-Cube*?

While Popovic finds it intuitive that Tom knows *Object-is-a-Cube*, we have the opposite intuition – Tom does not know *Object-is-a-Cube*. Can this clash of intuitions be resolved? We think that it can.

Popovic finds it intuitive that Tom has knowledge because "it is true that [Tom] sees a cube; it is also true that [Tom's] vision is functioning properly and that [Tom] is not hallucinating."[35] Thus, we suggest that Popovic's intuitions are driven by a principle like:

[33] Popovic 2020: 1572–1573.

[34] Popovic 2020: 1572.

[35] Popovic 2020: 1572.

S1: S knows that X is present if S sees X, and S's visual faculties are functioning properly.

S1 has some intuitive appeal. However, we think that part of its intuitive appeal is due to the word 'see.' This is because people often treat 'see' as a success verb (i.e., if S sees X, then X must be present). We suggest that S1 has intuitive appeal because people treat 'see' as a success verb in S1. If people do not treat 'see' as a success verb in S1, S1 lacks its intuitive appeal. For consider the following principle, S2, where the success verb 'see' is replaced with 'has a perceptual experience of':

S2: S knows that X is present if S has a perceptual experience of X, and S's visual faculties are functioning properly.

S2 is not intuitively appealing. Consider the following case: Jane has a perceptual experience of an apple in front of her. She has also received a clean bill of health, has perfect eyesight, and knows that all her perceptual experiences are generated from her eyes (i.e., she is not hallucinating). So, Jane's visual faculties are functioning properly. However, suppose that the object before her is a fake apple that looks identical to a real apple. Does she know that there is an apple before her?

Intuitively, no. Jane does not know that an apple is present even though she has a perceptual experience of an apple and her visual faculties are functioning properly. So, S2 lacks intuitive appeal.

One might also find S1 intuitively appealing because of the phrase 'S's visual faculties are functioning properly.' Proper function is often closely related to reliability. If one's faculties are functioning properly, then they are typically treated as reliable. However, our visual faculties might not be reliable even if they are functioning properly because of limitations in our abilities. For example, a person with perfect eyesight will not be able to see objects reliably in pitch darkness even though his eyesight is functioning properly. Our visual faculty is limited and thus unreliable when there is not enough light. Similarly, Jane's visual faculty is limited in the following way: it cannot distinguish between a real apple and a realistic-looking fake apple. So, even though her visual faculties are functioning properly, she would not be able to tell that the object before her is just a fake apple. Similarly, in the *Tesseract Case*, Tom's visual faculty is limited in that it cannot distinguish between three-dimensional objects and four-dimensional objects. So, even though Tom's visual faculties are functioning properly, he would not have the ability to perceive four-dimensional objects, and is thus unreliable in perceiving four-dimensional objects. Once people understand that properly functioning visual faculties can be unreliable, S1 further loses its intuitive appeal.

5.2 How many false beliefs in *many four-dimensional objects case*?

Let us turn to address Popovic's argument that if someone maintains that knowledge is factive, then they would have to embrace widespread skepticism in *Many Four-Dimensional Objects Case*.

Popovic's argument relies on the claim that many of our beliefs would be false in *Many Four-Dimensional Objects Case*. However, we think this claim is false; in fact, many of our beliefs would still remain true. *Only* our beliefs regarding three-dimensional geometrical shapes would be false.

Consider the following two propositions: 'A cube is present here' and 'a tree is present here.' If the object were a tesseract, 'a cube is present here' would be false. However, if the tree was some four-dimensional object, we would maintain that 'a tree is present here' is still true. This is because it is not in the concept of a tree to be necessarily three-dimensional in nature.

It is not in the concept of ordinary objects that they are necessarily three-dimensional in nature. People can discover that a tree actually has four-dimensions. However, it is impossible to discover that a cube actually has four-dimensions.[36] A cube is necessarily three-dimensional because being three-dimensional is part of the concept of cube.

Therefore, if society discovers that people indeed live in a four-dimensional world, only beliefs involving three-dimensional geometrical shapes would be false. If knowledge is factive, people would only have to deny knowledge in cases where our belief involves three-dimensional geometrical shapes. People's other beliefs would still remain true, and hence could amount to knowledge. Hence, there is no threat of widespread skepticism.[37] So, Popovic's argument fails.

6 Conclusion

To sum up, we have looked at two recent challenges to the factive account of knowledge, one involving approximate truths, and the other involving multidimensional geometry. Both challenges appeal to intuitive support and a skeptical challenge. However, we have argued that all these arguments fail. The factive account of knowledge survives both of these challenges.

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Declarations

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[36] People can discover that *what they believe to be* a cube actually has four-dimensions. But people cannot discover that an *actual* cube actually has four-dimensions.

[37] Popovic might point out that if knowledge is factive, then people would lack knowledge in cases where their belief involves three-dimensional geometrical shapes. This consequence, however, does not seem costly to me. In fact, it seems right. If people discover new things about our world, surely it is expected that some of their old beliefs turn out false, and they did not actually have knowledge.

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