If A, then B too, but only if C: a reply to Varzi

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Dear Professor,

I missed your test, but my classmates passed it on to me, as well as their answers and your discussion (Varzi 2005). I have a different answer though, and I would like to submit it to you.

The problem is how to symbolize the following sentence in the language of sentential logic.

(*') If Alf went to the movie (A) then Beth went (B) too, but only if she found a taxi-cab (C).

Student 1 says this is not a conditional, but a conjunction of two conditionals, ‘but’ being logically equivalent to ‘and’. I agree with her in this. However, she treats the two conditionals as unrelated, which they are not. (Her answer is \((A \rightarrow B) \& (B \rightarrow C)\).)

The second conditional in (*) establishes C as a necessary condition for B, so A can only be sufficient for B when associated with C. Accordingly,
we should symbolize the first conditional as: \((A \& C) \rightarrow B\). However, this symbolic sentence only means that \(A \& C\) is sufficient for \(B\), it does not explicitly state that \(C\) is a necessary condition for \(B\) (given \(A\)). We need to add a conjunct to do this. The antecedent of ‘only if she found a taxi-cab’ is ‘Beth went too’, which is in turn the consequent of ‘If Alf went to the movie’. So the relation of \(C\) being a necessary condition for \(B\) falls under the scope of the supposition \(A\): ‘If Alf went to the movie’. Consequently, (*) only says that \(C\) is a necessary condition for \(B\) when \(B\) is associated with \(A\). The second conditional should thus be symbolized as \((A \& B) \rightarrow C\).

The whole sentence can be re-written as:

\[
(*) \text{ If Alf went to the movie and Beth found a taxi-cab, then Beth went to the movie too, and if Alf went to the movie and Beth went too, then Beth found a taxi-cab.}
\]

Accordingly, the symbolization is:

\[
(8) \ ((A \& C) \rightarrow B) \ & (A \& B) \rightarrow C).
\]

Student 4 thinks that the clause ‘but only if \(C\)’ adds a restriction on the sufficiency of \(A\) for the obtaining of \(B\). ‘If \(A\) holds, and if the additional clause \(C\) is satisfied, then the speaker is committed to the truth of \(B\’) (226). I fully agree, but I think that this only leads to the first conditional of my interpretation, with the conjunction of \(A\) and \(C\) in its antecedent. Note that the quotation from Student 4 above is itself a conditional that has \(A\) and \(C\) in its antecedent and \(B\) in its consequent.

A similar justification is adequate for the second conditional of my interpretation. Saying that Beth went to the movie (\(B\)) only if she found a taxi-cab (\(C\)) is saying that her having gone to the movie is a sufficient condition for the truth of her having found a cab. However, the previous clause ‘If Alf went to the movie’ (\(A\)) adds a restriction on this sufficiency. If \(B\) holds, and if the additional clause \(A\) is satisfied, then the speaker is committed to the truth of \(C\).

It should be noted that the answer given by Student 4, \(A \rightarrow (B \leftrightarrow C)\), is tautologically equivalent to mine. It could be understood as saying that Alf’s having gone to the movie is a sufficient condition for Beth’s having found a cab being a sufficient condition for her having gone to the movie, and for Beth’s having gone to the movie being a sufficient condition for the truth of her having found a cab – a more complicated way of putting things.

I disagree with Students 5 and 6 on the role of the word ‘too’ in this sentence. I think ‘too’ has no logical function here (though it has a linguistic one) and can be left out of the symbolization. Its linguistic function is to indicate that the consequent has something in common with
the antecedent. Its lack of logical function is attested by the strict logical equivalence between ‘If A, then B’ and ‘If A, then B too’. For example, ‘If Mary stayed, Bob stayed’ and ‘If Mary stayed, Bob stayed too’ have the same truth conditions.¹

For the sake of the argument, however, let us suppose that ‘too’ has a logical role. As Student 5 says, ‘Beth went too’ can only be true if Alf actually went (228). This is to say that ‘Beth went too’ means in this context ‘Beth went and Alf went’. But this is innocuous, since If A, then B is tautologically equivalent to If A, then (A & B). In fact, I can see no logical difference between:

(*) If Alf went to the movie then Beth went too, but only if she found a taxi-cab.

(***) If Alf went to the movie then Alf went to the movie and Beth went to the movie, but only if she found a taxi-cab.

(****) If Alf went to the movie then Beth went to the movie, but only if she found a taxi-cab.

Student 5 believes that the correct symbolization of (*) cannot be true ‘in a scenario in which Alf did not go to the movie while Beth found a cab and went alone’ (227). Why not? He says that this ‘is not what the sentence says’. Indeed, the sentence does not say so, but neither does it exclude this possibility. The sentence only says something about what may have occurred if Alf went to the movie. It does not say anything about what may have happened in case he did not go.

Student 6 had the good idea of examining all possible conjunctions of the three conditions involved in (*):

1. A & C & B  
2. A & C & ∼B  
3. A & ∼C & B  
4. A & ∼C & ∼B  
5. ∼A & C & B  
6. ∼A & C & ∼B  
7. ∼A & ∼C & B  
8. ∼A & ∼C & ∼B

Conjunction 1 corresponds to the possibility explicitly stated in the conditional: Alf went, Beth found a taxi-cab and so Beth went too. Conjunction 2 is incompatible with (*), since, according to it, Alf went,

¹ ‘Too’ does have a relevant logical function – that of an anaphoric presupposition – when we have independent sentences in the same piece of discourse. For example: ‘Mary and Rita stayed. Sarah and Jack left. Bob and Phil stayed too.’ If we symbolize the first sentence as ‘M & R’, we should symbolize the third one as ‘B & P & M & R’, since it can only be true if Mary and Rita stayed. However, if we had one single sentence, ‘Mary and Rita stayed and Bob and Phil stayed too’, we would have the symbolization ‘M & R & B & P’ and it would be useless to include M and R twice. The symbolization would be the same if the sentence did not have the word ‘too’.
Beth found a taxi-cab, but she did not go. It violates the first of my conditionals. Conjunction 3 is also incompatible, since it says that Alf went, Beth did not find a taxi-cab, but nevertheless she went too. It violates the second conditional of my interpretation. By contrast, it is possible that Alf went, Beth did not find a cab and so she did not go (conjunction 4).

All the remaining conjunctions are compatible with (*). To see why, let’s give more flesh to these bones. Let’s suppose that Beth is interested in Alf, and that is why she wanted to meet him at the cinema. However, it was raining heavily and there was no public transport near her house, so she depended on the availability of a cab. David is interested in Beth, but for her he is only a good friend.

If Alf did not go to the movie, Beth may have invited David to go with her and she may have taken a taxi-cab to meet him at the cinema (conjunction 5). Perhaps it is also true in this case that she went to the movie only if she found a taxi-cab. But (*) does not require this. Perhaps David has a car and so she may have gone whether or not she found a cab. If she did not find a cab, David may have taken her in his car (conjunction 7). Thus, if Alf did not go, possibly it was not true that she went to the movie only if she found a taxi-cab – but only in this case. If Alf went, then finding a cab is necessary (exclusion of conjunction 3), because if she did not find one, she certainly preferred not to go (conjunction 4). This is because she would not like Alf to see her in David’s company and it would be unfair to ask David for a ride to stay with Alf!

It is also possible that Beth did find a cab but then received a phone call from David, who told her he would not be able to go. She did not want to go alone and so she decided not to go (conjunction 6). Or else neither did she find a cab nor was David available, so she did not go (conjunction 8).

The two conditionals present in (*) say something about what may have been the case, but nothing about what actually occurred. If the speaker wanted to exclude all the four possibilities in which Alf did not go (conjunctions 5 to 8), as Students 5 and 6 surmise, he would have said:

(*****) Alf went to the movie and, if Beth found a taxi-cab (and only if she did), she went too.

Thus, instead of

(8) \((A \& C) \rightarrow B\) \& \((A \& B) \rightarrow C\),

we would have:

\[ A \& (B \leftrightarrow C). \]

Summing up: According to (*), it may be that Alf went and Beth went too, but only if she found a taxi-cab. She certainly did not fail to go if Alf
went and she found a cab. She may have gone while Alf did not, only if or whether or not she found a cab. And possibly neither Alf nor Beth went to the movie. It becomes clear that (*) necessarily excludes only the two conjunctions that need to be excluded by my interpretation. Exclusion of conjunction 2 justifies $(A \land C) \rightarrow B$ and exclusion of conjunction 3 justifies $(A \land B) \rightarrow C$.

Concerning your discussion, we see that we need not ignore parsing and that the underlying logical form of (*) is not so distant from the grammatical form.

Yours sincerely,
Student 7.2

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**Reference**


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2 I am grateful to Achille Varzi and to the Editor for helpful comments on previous versions of this article.