

## The relativistic theory of gravitation beyond general relativity



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### **Abstract.**

It presents the basics of the “Relativistic theory of gravitation”, with the inclusion of original texts, from various papers, published between 1987 and 2009, by their authors: S. S Gershtein, A. A. Logunov, Yu. M. Loskutov and M. A. Mestvirishvili, additionally, together with the introductions, summaries and conclusions of the author of this paper.

The “Relativistic theory of gravitation” is a gauge theory, compatible with the theories of quantum physics of the electromagnetic, weak and strong forces, which defines gravity as the fourth force existing in nature, as a static field equipped with the transmitter particles of the virtual gravitons of spins 2 and 0, within the spirit of Galilei's principle of relativity, in his generalization of Poincaré's Special Relativity that allowed the authors to universalize that the physical laws of nature are complied with regardless of the frames of reference where they apply, integrated into the Grossmann-Einstein Entwurf theory, in its further development, by those authors, therefore, this theory preserves the conservation laws of energy-impulse and angular impulse of the gravitational field jointly to the other material fields existing in nature, in the Riemann's effective spacetime, through its identity with Minkowski's pseudo Euclidean spacetime.

## Introduction.

The science of physics is a factual science, supported in the formal science of mathematics and geometry, whose object of study is the properties of matter-energy and spacetime and their mutual interactions. Its evolution regarding the nature of gravity is anchored in the early twentieth century, in the Grossmann-Einstein-Hilbert equations, since, in what was once known as the "General theory of relativity", today lacks its three principles, for Einstein having abandoned early the Mach principle, and the others two was refuted, demonstrated by great philosophers of physics, among others Norton and Earman, as is the general principle of equivalence, since in a restricted sense it applies only to homogeneous gravity or the covariance, a property of tensors, when applied in Riemann geometry, mistakenly assimilated as a general principle of relativity.

From the equations, a metric explanation of gravity is given, with a medieval stink since matter, that is, physical fields curve the spacetime, a metric field, that is, a metaphysical field that, inexplicably, determines how matter is move. How these phenomena occur remains without a physical explanation. But, "General relativity" constitutes the current paradigm on gravity, which as such, refuses to finish its cycle, despite its crucial anomalies, by virtue of its self-protection, stemming from the normal science that self-sustains and reproduces it. This is how the alleged finding of gravitational waves is presented, that although are quadrupolar waves are not gravitational waves but some type of material waves, which Einstein tactically resorted to between 1916 and 1918, to defend himself against Lorentz who rebuked the physical unreality of his "General theory of gravity" and forced him to call the static gravitational field as relativistic ether, inadmissible from gravity as a metric effect, but as a kind of ether it should have speed. Previously, Max Born had asked what it was such speed, when Einstein, in his Entwurf theory, considered the static gravity as a material field similar to the static electromagnetic field. Once Lorentz passed away and his influence in the scientific community was extinguished, Einstein honestly renounced, first in 1937, although, impeded by the influence of Harvey Robertson, professor at Princeton, and definitely in 1938, to the inadmissible gravitational waves, from his mathematical model giving a metric gravity, since, in its equations, on one side of equality it describes the geometry of spacetime represented by the metric tensor and on the other side the matter represented by the energy-impulse tensor; obviously, gravity as a metric phenomenon lacks of speed, since the dynamism of the spacetime is nothing more than its geometric reconfiguration as an effect of the dynamism of matter, so the reconfiguration of the matter itself produces material waves that propagate with determined speeds and additionally produces the reconfiguration of the spacetime its intrinsic structural geometric property (author thesis [1]). Or from the spectacular photo, largely generated with algorithms, where they undoubtedly had to use Einstein's famous equations, of a darkened star, by the action of its material gravity (the true gravity), which prevents electromagnetic waves, trapped by it, from escaping, the existence of which was predicted based on the corpuscular theory of light and Newton's equations, in 1783, by John Michell, but which is presented to world viewers, people of the common, assaulting them in their innocence and manipulating them in their absence of specialized knowledge, such as proof of the existence of black holes, also, theoretically obtained from those powerful Einstein-Grossman-Hilbert equations, in addition, chasing a new Nobel prize, to add it to the one that was granted for the alleged detection of the gravitational waves.

But, the story inevitably continues and in the perennial rise of science to higher knowledge, seven decades later, Anatoli Logunov and his team accomplished what Albert Einstein and Marcel Grossmann attempted, in "The Entwurf theory", as it was to formulate a theory in which gravity is a physical field like all the others existing in nature, subject to the fact that the physical laws are fulfilled independently of the frames of reference where they are applied, under the conservation of the energy-impulse and angular impulse of the gravitational field together with all the other material fields, and, furthermore, as static gravitational field equipped with the bosonic transmitter of the virtual graviton of spins 2 and 0. In 1913, in Zurich, when Einstein carried out his best scientific work: "The Entwurf theory", Einstein, the physicist, to preserve the laws of conservation of energy and momentum, he chose the tensors applied to Minkowski's Galilean spacetime, before the other option offered by Grossman, the mathematician, specialist in the then brand new absolute differential calculus, of applying the tensors to the general Riemann spacetime, for which Einstein had to renounce the general covariance, but which allowed him to propose an explanation of the static gravitational field as physical, in concordance with electromagnetism. However, "The Entwurf theory" failed because its equations could not give Mercury's anomalous orbit and, furthermore, violate the principle of correspondence, under conditions of the minimum limit of weak gravity, to integrate with Newton's equations. By expand these authors "The Entwurf theory", in the "Relativistic theory of gravitation", overcoming the historical limitations of Grossman-Einstein, and integrating it with Poincaré's special relativity and theories of quantum field physics, the masterful work on gravity by Logunov and his companions far exceeds the "general relativity". But after three decades of its release and after a century of Einstein, the normal science only recognizes it as an alternative theory.

In 1987, in the spirit of Poincaré's Special relativity, Entwurf theory and Quantum field physics theories, Logunov, Loskutov, and Mestvirishvili presented the "Relativistic theory of gravitation", later revised, together with the participation of Gershtein, in the following terms:

"Within the framework of SRT (Theory of Special Relativity), which describes phenomena in inertial and non-inertial frames of reference, with the help of the geometrization principle that reflects the universality of gravitational interaction with matter, and with the introduction of the mass of graviton, we succeed in unifying Poincaré's (1904) idea of the gravitational field as a physicist in the spirit of Faraday-Maxwell with Einstein's idea of the Riemannian geometry of spacetime. It is this principle of geometrization that helps us find a gauge group without displacements that will allow us to construct the Lagrangian density of the appropriate gravitational field. All this has led us to the Relativistic theory of gravitation (RTG), 1989, which has all the laws of conservation, as occurs in all physical theories."

"In this theory, due to geometrization, the total energy-impulse tensor of matter and the gravitational field is the source of the gravitational field, just what Einstein wanted when building the theory of gravitation (in Entwurf theory). In what follows we will see that the rather general physical requirements lead us to an unequivocal construction of the complete system of equations for a massive gravitational field. The equations in this theory differ considerably from the Hilbert-Einstein equations, as it preserves the

notion that inertial coordinate systems, and gravitational forces differ from the inertia principle, as they are caused by a physical field."

"The Relativistic theory of gravitation with mass of the graviton is a field theory to the same extent as classical electrodynamics, so it could be called classical gravidynamic.

"According to this theory, the homogeneous and isotropic Universe develops in a series of alternating cycles, from high to low density, etc., and can only be flat. The theory predicts the presence of a large latent mass of matter in the Universe and prohibits the existence of "black holes". Furthermore, the theory explains all the observable events known so far that occur in the solar system" [2].

"The theory is generally covariant and invariant with respect to the Lorentz group." [3].

### **1. The approaches of Henri Poincaré and Albert Einstein.**

Poincaré's contributions to the relativistic conception were more universal than Einstein's, since he was the one who introduced relativistic mechanics, formulated the principle of relativity for all physical phenomena and found the Lorentz transformation group, which he could extend to all the forces of nature and introduced spacetime with its invariant  $ds^2$ , specified by Minkowski in relative space and relative time, allowing to deduce that non-inertial reference systems can exist together with inertial ones, therefore, as different systems, contribution made by Logunov, seven decades later. Since when formulating: "all physical phenomena occur in spacetime, whose geometry is pseudo-Euclidean", he allowed the introduction of the metric tensor  $\gamma_{\mu\nu}(x)$  in the Minkowski space in arbitrary coordinates and made possible introduce the gravitational field, separating the forces inertia of gravity.

For his part, Einstein restricted relativity, in his theory of special relativity, TRE, to the constancy of the speed of light in the inertial frames, making it impossible to reach non-inertial frames of reference and, therefore, unable to drive to spacetime of Minkowski of pseudo-Euclidean geometry. In TRE, spacetime is Minkowski's Galilean. Einstein, starting from the equality of the inertial and gravitational masses, was convinced that the forces of inertia and gravity are the same thing, elevating it as the basis to build the GTR.

Meanwhile, Poincaré conceived that the forces of any origin, of course, even the force of gravity, under the Lorentz transformations, behave like the electromagnetic forces, instead Einstein identified the components of the gravitational field with the metric field, obtained in a semi riemanniano space, with the help of the transformation of coordinates, consequently without physical foundation. Thus, Einstein renounced that the gravitational field, like the Faraday-Maxwell field, has an energy-impulse density. This was the path that led him to the GTR, to the introduction of the gravitational field pseudo tensor and to gravitational energy that is not locatable.

Einstein, for an extra scientific cause, of a personal, emotional, psychological and social nature, as was the strong pressure, exercised by the best German mathematician of the time, David Hilbert, who caused him a deep crisis, in hard competition, between July and November 1915. Although, as social man, Einstein emerged well rid but cost him to sacrifice the scientist. Hilbert built a mathematical model of gravity, from the geometry of Riemann, some a few days

before that Einstein, an option that he as head of the project of the Entwurf theory stubbornly prevented Grossmann, since Einstein was the one who always decided.

The mathematical model of Hilbert was consistent with the astronomical evolution of Mercury, the deflection of electromagnetic waves in the vicinity of the Sun and the supposed integration, at the limit of weak gravity, with Newton's equations, although, several decades later, accused by Anatoli Logunov (1986) and Tom Van Flandern (1998) [4], of course, in violation of the correspondence principle, but due to limitations of the epoch, this error was unnoticed. That is, the Hilbert model overcome the anomalies of astronomical order, which were known at the time, and the limitation of the mathematical method used in Einstein's previous work of Entwurf theory, by applying the tensors in the Galileo Minkowski spacetime and not in the pseudo-Euclidean Minkowski spacetime.

Einstein, with no other alternative, was also forced to adopt, in November 1915, Hilbert's model, although in its elegant formulation,  $G_{\mu\nu} = kT_{\mu\nu}$ , which he achieved five days later, so he had to give up his concept of theory Entwurf, his best work made together with Marcel Grossmann, of the Gravitational field as a physical field, of course, represented, in Entwurf, by the energy-impulse tensor  $t_{\mu\nu}$ , impossible to fade it by a coordinate transformation and absent in GTR, by not bear it the Riemann's spacetime.

But, it should be noted that even, in GTR, the identity between inertial forces and gravity is unattainable, since the gravitational field when characterized by Einstein's curvature tensor,  $G_{\mu\nu}$ , whether it differs from zero, then the gravitational field, although not as a physical field, but as a metric field, it exists and cannot be annihilated by a choice of frame of reference, even locally.

Both Newton and Einstein, with their equations on gravity, guessed right as far as they work, it is true, with greater precision the equations of Einstein, but neither of them knew why. In the case of Einstein, although due to Hilbert, he ended up convincing himself that was important was the results, without being ware that when they are left in complete freedom, mathematicians measuring the universe, doing so through their super abstract models that they build, behind the back of conceptually understand their study objects. Thus, paradoxically, they end up blurring the materiality of the universe.

“Since construction of the relativistic theory of gravity (RTG) is based on special relativity theory (SRT), we shall deal with the latter in greater detail and in doing so we shall examine both the approach of Henri Poincaré and that of Albert Einstein. Such an analysis will permit a more profound comprehension of the difference between these approaches and will make it possible to formulate the essence of relativity theory.

In analyzing the Lorentz transformations, H. Poincaré showed that these transformations, together with all spatial rotations, form a group that does not alter the equations of electrodynamics. Richard Feynman wrote the following about this: “Precisely Poincaré proposed investigating what could be done with the equations without altering their form. It was precisely his idea to pay attention to the symmetry properties of the laws of physics”. H. Poincaré did not restrict himself to studying electrodynamics; he discovered the equations of relativistic mechanics and extended

the Lorentz transformations to all the forces of Nature. Discovery of the group, termed by H. Poincaré the Lorentz group, made it possible for him to introduce four-dimensional spacetime with an invariant subsequently termed the interval

$$d\sigma^2 = (dX_0)^2 - (dX_1)^2 - (dX_2)^2 - (dX_3)^2 \cdot (\alpha)$$

Precisely from the above it is absolutely clear that time and spatial length are relative.

Later, a further development in this direction was made by Herman Minkowski, who introduced the concepts of timelike and spacelike intervals. Following H. Poincaré and H. Minkowski exactly, the essence of relativity theory may be formulated thus: all physical phenomena proceed in space–time, the geometry of which is pseudo-Euclidean and is determined by the interval ( $\alpha$ ). Here it is important to emphasize, that the geometry of spacetime reflects those general dynamic properties, that represent just what makes it universal. In four-dimensional space (Minkowski space) one can adopt a quite arbitrary reference frame

$$X_\nu = f_\nu(x_\mu),$$

realizing a mutually unambiguous correspondence with a Jacobian differing from zero. Determining the differentials

$$dX_\nu = \partial f_\nu / \partial x_\mu dx_\mu,$$

and substituting these expressions into ( $\alpha$ ) we find

$$d\sigma^2 = \gamma_{\mu\nu}(x) dx_\mu dx_\nu,$$

$$\text{where } \gamma_{\mu\nu}(x) = \partial f_\sigma / \partial x_\mu \partial f_\sigma / \partial x_\nu, \quad \partial f_\sigma = (1, -1, -1, -1) (\beta)$$

It is quite evident that the transition undergone to an arbitrary reference system did not lead us beyond the limits of pseudo-Euclidean geometry. But hence it follows that non inertial reference systems can also be applied in SRT. The forces of inertia arising in transition to an accelerated reference system are expressed in terms of the Christoffel symbols of Minkowski space. The representation of SRT stemming from the work of H. Poincaré and H. Minkowski was more general and turned out to be extremely necessary for the construction of SRT, since it permitted introduction of the metric tensor  $\gamma_{\mu\nu}(x)$  of Minkowski space in arbitrary coordinates and thus made it possible to introduce in a covariant manner the gravitational field, upon separation of the forces of inertia from gravity.

From the point of view of history it must be noted that in his earlier works, “The measurement of time” and “The and future of mathematical physics”, H. Poincaré discussed in detail issues of the constancy of the velocity of light, of the simultaneity of events at different points of space determined by the synchronization of clocks with the aid of a light signal. Later, on the basis of the relativity principle, which he formulated in 1904 for all physical phenomena, as well as on the work published by H. Lorentz the same year, H. Poincaré discovered a transformation group in 1905 and termed it the Lorentz group. This permitted him to give the following essentially

accurate formulation of the relativity theory: the equations of physical processes must be invariant relative to the Lorentz group. Precisely such a formulation was given by A. Einstein in 1948: "With the aid of the Lorentz transformations the special principle of relativity can be formulated as follows: The laws of Nature are invariant relative to the Lorentz transformation (i.e. a law of Nature should not change if it is referred to a new inertial reference frame with the aid of the Lorentz transformation for  $(x, y, z, t)$ ".

The existence of a group of coordinate-time transformations signifies that there exists an infinite set of equivalent (inertial) reference frames related by the Lorentz transformations. From the invariance of equations it follows, in a trivial manner, that physical equations in the reference frames  $x$  and  $x'$ , related by the Lorentz transformations, are identical. But this means that any phenomenon described both in  $x$  and  $x'$  reference systems under identical conditions will yield identical results, i.e. the relativity principle is satisfied in a trivial manner. Certain, even prominent, physicists understood this with difficulty not even long ago, while others have not even been able to. There is nothing strange in this fact, since any study requires certain professionalism. What is surprising is the following: they attempt to explain their incomprehension, or the difficulty they encountered in understanding, by H. Poincaré allegedly "not having taken the decisive step", "not having gone to the end". But these judgements, instead of the level of the outstanding results achieved by H. Poincaré in relativity theory, characterize their own level of comprehension of the problem.

Precisely for this reason W. Pauli wrote the following in 1955 in connection with the 50-th anniversary of relativity theory: "Both Einstein and Poincaré relied on the preparatory works performed by H. A. Lorentz, who was very close to the final result, but was not able to take the last decisive step. In the results, obtained by Einstein and Poincaré independently of each other, being identical I see the profound meaning of the harmony in the mathematical method and analysis performed with the aid of thought experiments and based on the entire set of data of physical experiments".

Detailed investigation by H. Poincaré of the Lorentz group invariants resulted in his discovery of the pseudo-Euclidean geometry of spacetime. Precisely on such a basis, he established the four-dimensionality of physical quantities: force, velocity, momentum, current. H. Poincaré's first short work appeared in the reports of the French Academy of sciences before A. Einstein's work was even submitted for publication. That work contained an accurate and rigorous solution of the problem of electrodynamics of moving bodies, and at the same time it extended the Lorentz transformations to all natural forces, of whatever origin they might be. Very often many historians, and, by the way, physicists, also, discuss priority issues. A very good judgement concerning this issue is due to academicians V. L. Ginzburg and Ya. B. Zel'dovich, who in 1967 wrote: "Thus, no matter what a person has done himself, he cannot claim priority, if it later becomes known that the same result was obtained earlier by others".

A. Einstein proceeded toward relativity theory from an analysis of the concepts of simultaneity and of synchronization for clocks at different points in space on the basis of the principle of constancy of the velocity of light.  $\ddot{}$ Each ray of light travels in a

reference frame at “rest” with a certain velocity  $V$ , independently of whether this ray of light is emitted by a body at rest or by a moving body  $ij$ . But this point cannot be considered a principle, since it implies a certain choice of reference frame, while a physical principle should clearly not depend on the method of choosing the reference frame. In essence, A. Einstein accurately followed the early works of H. Poincaré. However, within such an approach it is impossible to arrive at non-inertial reference frames, since in such reference frames it is impossible to take advantage of clock synchronization, so the notion of simultaneity loses sense, and, moreover, the velocity of light cannot be considered constant.

In a reference frame undergoing acceleration the proper time  $d\tau$ , where

$$d\sigma^2 = dt^2 - s_{ik}dx_i dx_k, \quad d\tau = \gamma_{0\alpha} dx_\alpha / \sqrt{\gamma_{00}}, \quad s_{ik} = -\gamma_{ik} + \gamma_{0i}\gamma_{0k} / \gamma_{00}$$

is not a complete differential, so the synchronization of clocks at different points in space depends on the synchronization path. This means that such a concept cannot be applied for reference frames undergoing acceleration. It must be stressed that the coordinates in expression (β) have no metric meaning, on their own. Physically measurable quantities must be constructed with the aid of coordinates and the metric coefficients  $\gamma_{\mu\nu}$ . But all this remained misunderstood for a long time in SRT, since it was usual to adopt A. Einstein’s approach, instead of the one of H. Poincaré and H. Minkowski. Thus, the starting points introduced by A. Einstein were of an exclusively limited and partial nature, even though they could create an illusion of simplicity. It was precisely for this reason that even in 1913 A. Einstein wrote: “In usual relativity theory only linear orthogonal transformations are permitted”. Or somewhat later, in the same year, he writes: “In the original relativity theory the independence of physical equations of the specific choice of reference system is based on postulating the fundamental invariant  $ds^2 = Pdx^{2i}$ , while now the issue consists in constructing a theory (general relativity theory is implied – A.L.), in which the role of the fundamental invariant is performed by a linear element of the general form  $ds^2 = Pdx^{2i}$ ”.

A. Einstein wrote something similar in 1930: “In special relativity theory only such coordinate changes (transformations) are allowed that provide for the quantity  $ds^2$  (a fundamental invariant) in the new coordinates having the form of the sum of square differentials of the new coordinates. Such transformations are called Lorentz transformations”.

Hence it is seen that the approach adopted by A. Einstein did not lead him to the notion of spacetime exhibiting a pseudo-Euclidean geometry. A comparison of the approaches of H. Poincaré and A. Einstein to the construction of SRT clearly reveals H. Poincaré’s approach to be more profound and general, since precisely H. Poincaré had defined the pseudo-Euclidean structure of spacetime. A. Einstein’s approach essentially restricted the boundaries of SRT, but, since the exposition of SRT in the literature usually followed A. Einstein, SRT was quite a long time considered valid only in inertial reference systems. Minkowski space was then treated like a useful geometric interpretation or like a mathematical formulation of the principles of SRT within the approach of Einstein. Let us now pass over to gravity. In 1905 H. Poincaré



wrote: "... that forces of whatever origin, for example, the forces of gravity, behave in the case of uniform motion (or, if you wish, under Lorentz transformations) precisely like electromagnetic forces". This is precisely the path we shall follow. A. Einstein, having noticed the equality of inertial and gravitational masses, was convinced that the forces of inertia and of gravity are related, since their action is independent of a body's mass. In 1913 he arrived at the conclusion that, if in expression (α) "... we introduce new coordinates  $x_1, x_2, x_3, x_4$ , with the aid of some arbitrary substitution, then the motion of a point relative to the new reference frame will proceed in accordance with the equation

$$\delta \int ds = 0, \quad ds^2 = g_{\mu\nu} dx_\mu dx_\nu$$

and he further pointed out: "The motion of a material point in the new reference system is determined by the quantities  $g_{\mu\nu}$ , which in accordance with the preceding paragraphs should be understood as the components of the gravitational field, as soon as we decide to consider this new system to be "at rest". Identifying in such a manner the metric field, obtained from (α) with the aid of coordinate transformations, and the gravitational field is without physical grounds, since transformations of coordinates do not lead us beyond the framework of pseudo-Euclidean geometry". [5].

## 2. The metric work of Einstein.

From equations  $G_{\mu\nu} = 8\pi G/c^4 T_{\mu\nu}$  of Einstein-Grossmann-Hilbert (1915) results a metric description of gravity as a geometric property of spacetime resulting of Minkowski's mathematical model (1908) that combine space and time, not defined in terms of the science of physics but in philosophy as a special substance (Substantialism) or a simple relational property of the matter (Relationalism). With the alleged discovery of gravitational waves spacetime would be absurdly a dynamic material substance.

The Einstein tensor  $G_{\mu\nu}$ , as the metric field  $g_{\mu\nu}$ , is also the same gravitational field, devoid of the tensor impulse-energy  $t_{\mu\nu}$  that should generate it because in the tensor  $T_{\mu\nu}$  of impulse-energy the corresponding tensor  $t_{\mu\nu}$  does not exist, violating the conservation law of the energy impulse of matter and field gravitational, taken together. "There is no gravitational field and no gravitational force; the gravitational field is at best a geometric not a physical field, and as such it does not possess any energy" [6]. "As pointed out 90 years ago by Hilbert (1917), Einstein (1918), Schrodinger (1918) and Bauer (1918) within the focus of geometric gravity (general relativity) there are no tensor characteristics of impulse-energy for the field of gravity" [7].

The Einstein tensor,  $G_{\mu\nu} = R_{\mu\nu} + \frac{1}{2} g_{\mu\nu} R$ , it presents as the sum of the tensor of Ricci,  $R_{\mu\nu}$ , that partially determines the curvature of spacetime, and the metric tensor,  $g_{\mu\nu}$ , that determine all the geometric and causal structure of spacetime, multiplied by the scalar of curvature,  $R$ , that in each point of spacetime measure the intrinsic geometry of spacetime, near that

point. Therefore, Einstein tensor represents only geometric objects referred to spacetime as a mathematical model, in itself without any physical content, in particular, as in modern general relativity, the gravitational field,  $g_{\mu\nu}$ , it interprets as the curvature of a Lorentzian manifold (Riemann spacetime of positive curvature), which Wheeler called geometrodynamics, or rather in Einstein as the gravitational potentials: “ $g_k = -(\text{grad } \Theta)_k$ ,  $k = x_1, \dots, x_4$  that is,  $g_{\mu\nu} \leftrightarrow \Theta \Gamma_{\alpha\beta}^k \leftrightarrow g^k$ ” [8].

“From our point of view, it is not permitted to consider such a metric field as the gravitational field, since this contradicts the very essence of the concept of a field as a physical reality. Therefore, it is impossible to agree with the following reasoning of A. Einstein: *ij*The gravitational field “exists” with respect to the system  $K'$  in the same sense as any other physical quantity that can be defined in a certain reference system, even though it does not exist in system  $K$ . There is nothing strange, here, and it may be readily demonstrated by the following example taken from classical mechanics. Nobody doubts the “reality” of kinetic energy, since otherwise it would be necessary to renounce energy in general. It is clear, however, that the kinetic energy of bodies depends on the state of motion of the reference system: by an appropriate choice of the latter it is evidently possible to provide for the kinetic energy of uniform motion of a certain body to assume, at a certain moment of time, a given positive or zero value set beforehand. In the special case, when all the masses have equal in value and equally oriented velocities, it is possible by an appropriate choice of the reference system to make the total kinetic energy equal to zero. In my opinion the analogy is complete*ij*.

As we see, Einstein renounced the concept of a classical field, such as the Faraday–Maxwell field possessing density of energy-momentum, in relation to the gravitational field. Precisely this path led him up to the construction of GRT, to gravitational energy not being localizable, to introduction of the pseudotensor of the gravitational field. If the gravitational field is considered as a physical field, then it, like all other physical fields, is characterized by the energy-momentum tensor  $t^{\mu\nu}$ . If in some reference frame, for instance,  $K'$ , there exists a gravitational field, this means that certain components (or all of them) of the tensor  $t^{\mu\nu}$  differ from zero. The tensor  $t^{\mu\nu}$  cannot be reduced to zero by a coordinate transformation, i.e, if a gravitational field exists, then it represents a physical reality, and it cannot be annihilated by a choice of reference system. It is not correct to compare such a gravitational field with kinetic energy, since the latter is not characterized by a covariant quantity. It must be noted that such a comparison is not admissible, also, in GRT, since the gravitational field in this theory is characterized by the Riemann curvature tensor. If it differs from zero, then the gravitational field exists, and it cannot be annihilated by a choice of reference system, even locally.

Accelerated reference systems have played an important heuristic role in A. Einstein’s creative work, although they have nothing to do with the essence of GRT. By identifying accelerated reference systems to the gravitational field, A. Einstein came to perceive the metric spacetime tensor as the principal

characteristic of the gravitational field. But the metric tensor reflects both the natural properties of geometry and the choice of reference system. In this way the possibility arises of explaining the force of gravity kinematically, by reducing it to the force of inertia. But in this case it is necessary to renounce the gravitational field as a physical field. "Gravitational fields (as A. Einstein wrote in 1918) may be set without introducing tensions and energy density". But that is a serious loss, and one cannot consent to it. However, as we shall further see, this loss can be avoided in constructing RTG. Surprisingly, even in 1933 A. Einstein wrote:  $||$ In special Relativity theory — as shown by H. Minkowski — this metric was quasi-Euclidean, i.e. the square "length"  $ds$  of a linear element represented a certain quadratic function of the coordinate differentials. If, on the other hand, new coordinates are introduced with the aid of a linear transformation, then  $ds^2$  remains a homogeneous function of the coordinate differentials, but the coefficients of this function ( $g_{\mu\nu}$ ) will no longer be constant, but certain functions of the coordinates. From a mathematical point of view this means that the physical (four-dimensional) space possesses a Riemannian metric $||$ .

This is certainly wrong, since a pseudo-Euclidean metric cannot be transformed into a Riemannian metric by transformation of the coordinates. But the main point, here, consists in something else, namely, in that in this way, thanks to his profound intuition, A. Einstein arrived at the necessity of introducing precisely Riemannian space, since he considered the metric tensor  $g_{\mu\nu}$  of this space to describe gravity. This was essentially how the tensor nature of gravity was revealed" [5].

### **3. The bimetric theories.**

Prior to RTG, in 1940, at Physical Review, Nathan Rosen, between 1934 and 1936, assistant of Einstein, published "General Relativity and Flat Space. I" and, a bit later, "General Relativity and Flat Space. II", managing to endow of GTR gravitational field with physical reality from a massive graviton, by taking the Riemann and Minkowski metrics together, but leaving undetermined, among the many that turn out, which scalar density to choose, to construct a theory of the gravitational field. Of course, it's about using two metric tensors instead of Einstein's sole tensor closely related to massive gravity structured in the ends of 1930s with foundation in the theory of the spin-2 field propagating on a Minkowski spacetime developed by Markus Fierz, author in 1939 of "Über die relativistische Theorie kräftefreier Teilchen mit beliebigem Spin", Helv. Phys, and Wolfgang Paul with whom Fierz worked as his assistant. Spin was introduced in 1925 by Ralph Kronig and independently by George Uhlenbeck and Samuel Goudsmit as an intrinsic property of massive elementary particles while Fierz also extended it to non-massive particles in his thesis on relativistic fields with arbitrary spins for his habilitation degree, in 1939, as international university teaching. Today, there are several metric theories like MOND, etc. In 1961, Robert Dicke and Carl Brans' theory of scalar tensor

emerged, based, among others, on a work by Pascual Jordan carried out in 1959; they replaced the constant  $G$  with a scalar field. Below, Logunov presents Rosen's bimetric theory:

"About 50 years ago, Rosen demonstrated in his work *Phys. Rev.*, v57, 1940, that if the Minkowski  $n_{uv}$  metric were introduced in addition to the Riemannian  $g_{uv}$  metric, then one could always construct a scalar Lagrangian density of the gravitational field with respect to transformations of arbitrary coordinates, which would contain derivatives of the order not greater than one. In particular, he constructed the Lagrangian density that led to the Hilbert-Einstein equations. This is how bimetric formalism arose. However, this approach immediately complicates the construction of the gravitational theory, since using the  $n_{uv}$  and  $g_{uv}$  tensors can write a fairly large number of scalar densities with respect to arbitrary coordinate transformations, making it completely uncertain which density Scalar should be chosen as the Lagrangian density when constructing the theory of gravitation.

Following this direction, Nathan Rosen chose different densities of the scalar for the Lagrangian density and built on his basis several theories of gravitation that, in general, produced quite different predictions for these or those gravitational effects " [2].

#### **4. The relativistic theory of gravitation.**

The relativistic theory of gravitation, RTG, is obtained from the relativity of Poincaré, not from the special relativity of Einstein, however, in any case, within the relativistic conception, originated originally in Galileo Galilei, with its formulation of the principle of relativity, according to which all physical phenomena happen in the same way, in the inertial frames, that is, not subject to forces, because the movement in them is illusory since the states of rest and movement are coordinate effects, dependent on observers.

In GTR, the Galilei principle of relativity is generalized to all frames, too to the non-inertial, since according to Einstein's principle of equivalence, inertial and gravitational forces are the same. Thus, all kinds of movement: inertial, accelerated and gravitational are illusory as they are the effect of coordinates of the observers. But, the equivalence between all kinds of frames is only achievable in Riemann geometry and only for homogeneous gravity, that is, when the  $G_{uv}$  cancel and not in extended gravity, consequently, only when gravity acts as quasi acceleration almost without attraction, that is, in regions of weak gravity, where it tends to die out.

In RTG, although non-inertial frames are applicable together with inertial ones, it does not, however, constitute the generalization of the relativity of inertial motion to all kinds of frames, since gravitational motion is different from inertial, and, of course, gravitational forces are separated from inertial forces. The generalization of the Galilei principle is that all physical phenomena, including gravitational ones, occur in the same way, both in the inertial frame of reference and in the non-inertial frame, since it is not possible to distinguish between them, since there is identity between the Minkowski pseudo Euclidean spacetime and the effective Riemann spacetime.

From the above, it is clearly established that the difference between GTR and RTG, is that in the first the Galilei principle is generalized, with the false foundation, that all types of movement are the same, while in the second, with the true basis, that all frames of reference are the same, provided that the identity between the Minkowski and Riemann geometries is fulfilled.

Both Einstein and Hilbert, in creating the supposed general relativity, did so by breaking with special relativity, so they had to renounce the conservation laws of energy-momentum and angular momentum of gravity in conjunction with the other fields of nature, giving rise to non-physical ideas related to the non-location of gravity energy, such as the impossible gravitational radiation by a metric field, the spacetime collapse of matter in the singularity of the black hole or the origin of the Universe in the Big Bang.

Such absence of conservation laws, without there being any evidence in their favor in nature, was precisely the reason that Logunov and his collaborators had to dispense with the so-called general relativity.

The relativistic theory of gravitation, based on STR, but not Einstein's but Poincaré's, assumes Minkowski's spacetime as primary, but with pseudo-Euclidean geometry, that is,  $\gamma^{uv}$  metric, so it supports all physical fields, of course, even gravity, preserving conservation laws as universal laws. The gravitational field describes it by a rank two symmetric tensor,  $\Phi^{uv}$ , as a physical field, for having an energy-impulse density, a zero mass at rest, and the two and zero spin states.

The interaction of the gravitational field with the other fields of nature occurs, in view of its universality, adding the gravitational field,  $\Phi^{uv}$ , to the  $\gamma^{uv}$  metric tensor, of the Minkowski spacetime, and interacting with the energy-impulse density of the other fields (matter), according to the principle of geometrization, due to which, the motion of matter under the action of a gravitational field,  $\Phi^{uv}$ , in the primary Minkowski spacetime, with a  $\gamma^{uv}$  metric, is equivalent to motion in a spacetime of Riemann, with a  $g^{uv}$  metric, from which the effective Riemann spacetime of RTG results.

For its part, the energy-impulse density of the gravitational field depends on the metric tensor  $\gamma^{ik}$  and the gravitational field  $O^{ik}$ , so it totally differs from GTR, where the density depends only on the metric tensor  $g^{ik}$  of the Riemann spacetime and, consequently, is completely geometrized, whereas in RTG no. From this distinction it turns out that while for the GTR the gravitational field is a metric field, instead for RTG the gravitational field is a physical field, composed of virtual spin-2 and 0 gravitons, with impulse-energy, as the electromagnetic field and the rest of fields of nature.

“In constructing the relativistic theory of gravitation, RTG, we will base our reasoning entirely on the special theory of relativity, which we call simply the relativity theory because physically there can be no other relativity. Although the name "general theory of relativity" does exist, it refers only to gravitation and not to some sort of general relativity. Long ago Fock, 1939, 1959, clarified this.

Now we briefly discuss the essence of the theory of relativity, touching especially on how Einstein interpreted the theory. This will not only be of historical interest; chiefly, it will give the reader a deeper understanding of the starting point of Einstein's reasoning which led to the creation of GTR. Minkowski, following Poincaré reasoning, developed the idea of the pseudo-Euclidean geometry of four-dimensional spacetime. The line element in this geometry has the form:

$$ds^2 = cdt^2 - dx_1^2 - dx_2^2 - dx_3^2$$

Poincaré was the first to introduce such a quantity; he demonstrated that  $ds^2$  is invariant under Lorentz transformations. He was also the first to introduce the concept of a Lorentz group and the idea of a four-dimensional space.

Even to this day many scientists believe that Minkowski provided a mathematical interpretation for the theory of relativity that formally simplified the theory. But there is more to it than this. In Logunov, 1985, it is demonstrated that the concept of a four-dimensional spacetime developed by Poincaré and Minkowski makes it possible to extend the theory of relativity from inertial reference frames to accelerated frames. In an arbitrary accelerated reference frame the line element  $ds^2$  has the form:

$$ds^2 = \gamma_{ik}(x) dx^i dx^k$$

where  $\gamma_{ik}(z)$  is the metric tensor of the Minkowski spacetime.

For the Minkowski spacetime, in view of the existence of ten Killing vectors, there are always transformations:

$$x^i = f^i(x')$$

that do not change the metric coefficients, that is,

$$ds^2 = \gamma_{ik}(x') dx'^i dx'^k$$

It is this that enables us to generalize the Poincaré relativity principle (Poincaré, 1904, 1905) and formulate the generalized relativity principle thus (Logunov, 1985): No matter what physical reference frame we take (inertial or non inertial), there always exists an infinite number of other reference frames in which all physical phenomena (including gravitational phenomena) occur in the same manner as in the initial reference frame, so that we do not have and cannot have any experimental means to distinguish in which of this infinite number of reference frames we are positioned.

Thus, noninertial reference frames occupy an equal status with inertial reference frames in the theory of relativity. It is this fundamental fact that was not clear to Einstein (see Einstein and Grossmann, 1913): In the ordinary theory of relativity only linear orthogonal transformations are admitted.

Also (see Einstein, 1913), in the initial theory of relativity, the independence of the physical equations of the special choice of reference frame is based on the postulation of the fundamental invariant quantity

$$ds^2 = \sum (dx_i)^2,$$

Whereas now we are speaking of constructing a theory (GTR — The author) in' which a linear element of a more general nature plays the role of the fundamental invariant quantity

$$ds^2 = \sum g_{ik} dx_i dx_k,$$

These passages show that at that time Einstein had yet to penetrate deep into the essence of the theory of relativity. In the special theory of relativity we are speaking not of the postulation of the line element in the form

$$ds^2 = \sum (dx_i)^2,$$

contrary to Einstein's belief, but of the pseudo-Euclidean geometry of spacetime defined by the line element

$$ds^2 = \gamma_{ik} dx^i dx^k$$

with a metric tensor  $\gamma_{ik}$ , for which the Riemann-Christoffel curvature tensor  $R_{inlm}$ , vanishes. Hence, in the special theory of relativity the law of energy-momentum conservation can be written in the general covariant form, but this fact was not understood by Einstein. What has been said found its reflection in GR, in the construction of which Einstein was guided to a great extent by the elegant formal apparatus of Riemannian geometry and his idea of the equivalence of forces of inertia and gravity (the principle of equivalence).

According to the ideology of GTR, the relativity principle cannot be applied to gravitational phenomena. It was on this central idea that Einstein and Hilbert, in creating GTR almost 70 years ago, departed basically from the special theory of relativity, which in turn led to a rejection of the laws of conservation of energy-momentum and angular momentum, to the emergence of nonphysical ideas concerning the non localization of gravitational energy, and to many other aspects not related to gravitation. These two great scientists abandoned the wonderfully simple world of the Minkowski spacetime, which possesses the maximum possible (ten-parameter) group of motions on the spacetime, and entered the jungle of Riemannian geometry, which bogged down subsequent generations of physicists studying gravitation.

Thus, will we assume that GTR is a meaningful theory we must reject both the fundamental laws of conservation of energy-momentum of matter and gravitational field and the concept of a classical field. This, however, is too great a loss, and it would be very thoughtless to agree to this without proper experimental proof. So far there is not a single experimental fact that, directly or indirectly, challenges the validity of conservation laws in the macro- and micro-worlds. Theory is only one conclusion then: we must discard GTR, giving it credit as a stage in the development of our ideas of gravitation.

In Denisov and Logunov, 1980a, 1980b, 1982b, 1982d, Logunov and Folomeshkin, 1977b, and Vlasov and Denisov, 1982, it is demonstrated that since GTR does not, and cannot, have laws of conservation of the energy-momentum of matter and gravitational field taken together, the inertial mass as defined in Einstein's theory has no physical meaning, the gravitational-wave flux as defined in GTR can always be destroyed by the proper selection of reference frame, and, hence, Einstein's quadrupole formula for gravitational waves is not a corollary of GTR. Basically it does not follow from GTR that a binary system loses energy in the form of gravitational waves. GTR has no classical Newtonian limit and, consequently, does not satisfy one of the most fundamental principles of physics, the correspondence principle. This is what the absence in GR of energy-momentum conservation laws leads to if one rejects dogmatism and ponders on the essence of the problem and makes a detailed analysis.

All this points to the fact that GTR is not a satisfactory physical theory. Hence, it is urgent to construct a classical theory of gravitation that will satisfy all the demands made of a physical theory. At the base of the suggested relativistic theory of gravitation (see Logunov, 1986, Logunov and Meslvirishvili, 1984, 1985a, 1985b, 1986b, Vlasov and Logunov, 1984, and Vlasov, Logunov, and Meslvirishvili, 1984), which completes the development of the ideas proposed in Denisov and Logunov, 1982d, we place the following physical requirements (see Logunov, 1986):

(a) The Minkowski spacetime ( $z''$ ), that is, spacetime equipped with pseudo-Euclidean geometry, is a fundamental space that incorporates all physical fields, including the gravitational. This statement is general because it is necessary and sufficient for the validity of the laws of conservation of energy-momentum and angular momentum for matter and gravitational field taken together. In other words, the Minkowski spacetime reflects the dynamical properties common for all types of matter. This guarantees the existence of universal characteristics for all forms of matter and gravitational field. Discussing the structure of the geometry of real spacetime, Einstein, 1921, noted: ...the question whether this continuum has a Euclidean, Riemannian, or any other structure is a question of physics proper which must be answered by experience, and not a question of a convention to be chosen on grounds of mere expediency.

Basically, of course, this statement of Einstein's is completely correct. But the essence of the matter is much deeper. The main thing here is to understand what physical properties of matter determine the geometry of spacetime. Indeed, let us assume that if we determine the physical geometry on the basis of studies of the propagation of light and the movements of test bodies, we will establish the Riemannian structure of the geometry of spacetime. But does this mean that this geometry must be placed at the base of the theory? No, it does not, because assuming this would mean rejecting automatically the fundamental laws of conservation of energy-momentum and angular momentum, since this geometry does not possess the maximum group of motions on spacetime. And all this happened in GTR.



Thus, once we have discovered on the basis of experiments involving the propagation of light and the movements of test bodies that Riemannian geometry is valid, we must not hasten to draw conclusions about the structure of the geometry of spacetime that must be laid at the base of physics. We must first establish whether these experimental facts are primary and universal or of secondary origin and partial interest. In establishing the structure of the geometry of the physical spacetime we must proceed not from the nature of light propagation and test-body movements but from the most general dynamical properties of matter, the conservation laws, since it is not the particular physical manifestations of the motion of matter that determine the structure of the physical geometry lying at the base of physics but the general universal dynamical properties of matter.

In our theory, RTG, the physical geometry of spacetime is determined not on the basis of studies of the propagation of light and the movements of test bodies but on the basis of general dynamical properties of matter, the conservation laws, which are not only of fundamental importance but can be verified experimentally.

Requirement (a) sets RTG entirely apart from the general theory of relativity.

(b) A gravitational field is described via a symmetric second-rank tensor  $\Phi^{uv}$  and constitutes a real physical field characterized by an energy-momentum density, a zero rest mass, and spin states 2 and 0. This aspect also basically distinguishes RTG from GTR.

(c) We introduce the geometrization principle, according to which the interaction of a gravitational field with matter is achieved, in view of the universality of this interaction, by "adding" the gravitational field  $\Phi^{uv}$  to the metric tensor  $\gamma^{uv}$  of the Minkowski spacetime in the Lagrangian density of matter according to the following rule:

$$L_M(\hat{\gamma}^{uv}, \Phi_A) \rightarrow L_M(\hat{g}g^{uv}, \Phi_A)$$

Where

$$\hat{g}g^{uv} = \sqrt{-g} g^{uv} = \sqrt{-\gamma} \gamma^{uv} + \sqrt{-\gamma} \Phi^{uv} \equiv \hat{\gamma}^{uv} + \hat{\varrho}^{uv}$$

and  $\Phi_A$  are the material fields. By matter we mean all of its forms except gravitational fields. According to the geometrization principle, motion of matter under the action of a gravitational field  $\Phi^{uv}$  in the Minkowski spacetime with a metric  $\gamma^{uv}$  is equivalent to motion in an effective Riemann spacetime with a metric  $g^{uv}$ . The metric tensor  $\gamma^{uv}$  of the Minkowski spacetime and the gravitational-field tensor  $\Phi^{uv}$  in this spacetime are primary concepts, while the Riemann spacetime and its metric  $g^{uv}$  are secondary concepts, owing their origin to the gravitational field and its universal action on matter through  $\Phi_A$ . The effective Riemann spacetime is literally of field origin, thanks to the presence of the gravitational field. Einstein was the first to suggest that the space-time is Riemannian rather than pseudo-Euclidean. He identified gravitation with the metric tensor of the Riemann space-time. But this line of reasoning as much as led to rejection of the gravitational field as a physical field possessing an energy-momentum density

and to the loss of fundamental conservation laws. The geometrization principle, based on the notions of the Minkowski spacetime and a physical gravitational field, introduces the concept of an effective Riemann spacetime, and in this Einstein's idea of a Riemannian geometry finds its indirect reflection.

According to RTG ideology, since the Minkowski spacetime ( $x^m$ ) forms its base, there are standard temporal and spatial scales that do not explicitly depend on the gravitational interaction. In view of the geometrization principle, the entire dependence of the line element in the effective Riemann spacetime,

$$ds^2 = g_{ik}(x) dx^i + dx^k$$

on the gravitational field lies in the metric coefficients  $g_{ik}(x)$ . Transition to any other coordinates in RTG, say, to the proper coordinates, will result in a situation in which the proper spacetime variables will depend both on the coordinates  $x^m$  in the Minkowski spacetime and on the gravitational constant  $G$ . Hence, proper time and spatial characteristics will depend on the gravitational field. It is only in RTG that one can completely determine the effect of a gravitational field on the passage of proper time and on the variation of the distance between points.

(d) The scalar Lagrangian density of a gravitational field is a bilinear form of the first covariant derivatives,  $Dp^m g_{mn}$ , with respect to the Minkowski metric. Basically there is no way to construct a scalar Lagrangian density of such a form in GTR.

Using the concept of the Minkowski spacetime and the geometrization principle as a basis, we can write the Lagrangian density in the following form:

$$L = L_g(\dot{y}^{ik}, \dot{\phi}^{ik}) + L_M(\hat{g}^{ik}, \Phi_A)$$

In our theory the gravitational-field Lagrangian density  $L_g$  depends on the metric tensor  $y^{ik}$  and the gravitational field  $O^{ik}$ . Hence, this theory differs fundamentally from GTR, where the Lagrangian density depends only on the metric tensor  $g^{ik}$  of the Riemann spacetime. Thus, the gravitational-field Lagrangian density in our theory is not fully geometrized, whereas in GTR it is. As will be demonstrated later, the notion of a gravitational field possessing an energy-momentum density and spin states 2 and 0 combined with the geometrization principle provides the possibility of constructing an unambiguous relativistic theory of gravitation. Such a theory changes the stereotype of spacetime developed under the influence of GTR and in spirit agrees with the modern theories in elementary particle physics. It implies that Einstein's general relativity principle is devoid of any physical meaning or content (Fock, 1939, 1959). In the exposition of a number of problems we follow Denisov and Logunov, 1982d" [9], [10].

## 5. The slowdown in the lapse of a physical process.

Since the Lorentz transformation replaced the Galilei transformation in STR mechanics (1905), the time lapse depends on the relative speed, being a simple consequence of the change of coordinates in the inertial reference frames but real in the accelerated frames reference. In GTR, the time lapse really depends on the position with respect to a gravitational field. Said lapse-dependence of speed or gravitational field has been experimentally demonstrated. However, when the lapse dependency is real and not the effect of the coordinate change, it violates the relativistic principle of equivalence, formulated by Einstein, between all kinds of motion: inertial, accelerated, and gravitational. Also, the cause is not physically explained.

As a consequence that, in RTG, the static gravitational field is physical, compound of gravitons with non-zero mass, similar to static electromagnetic field, existing in the pseudo-Euclidean spacetime, therefore, endowed with the force of inertia, the reduction of physical processes in space and time due to gravitational action, produces a repulsive gravitational force that makes it impossible for matter to collapse into a singularity, as derived from the Schwarzschild equations, from the Einstein-Hilbert metric gravitational field. Thus, black holes, not the black stars that are possible in RTG, are another impossible effect coming from relativistic physics based on gravity as a non-material phenomenon, that is, as a metric phenomenon.

“The rest mass of the graviton arises unavoidably in the theory because only in this way one can consider the gravitational field as a physical field in the Minkowski space whose source is the total conserved energy-momentum tensor of all matter. And this is a non-zero mass of the graviton that changes completely the picture both of the collapse process and the evolution of the Universe.

When A. Einstein in 1913 related the gravitational field to the metric tensor of the Riemannian space it appeared that such a field caused a slowing down of the lapse of a physical process.

This slow down can be illustrated, in particular, in the case of the Schwarzschild solution, if to compare the lapse of time in the presence of the gravitational field with the lapse of time for a distant observer. However, generally, only the metric tensor of the Riemannian space takes place in the GTR and therefore any trace of inertial time of the Minkowski space is absent from the Hilbert–Einstein equations. Due to this reason the universal property of the gravitational field to slow down the lapse of time in comparison with the inertial time could not get a further development in the framework of the GTR.

The rise of the effective Riemannian space in the field theory of gravitation with preservation of the Minkowski space as a basic space gives a special importance to the property of the gravitational field to exert a slowing down influence on the lapse of time. Only in this case can one argue truly about the slowing down of the lapse of time when making the comparison of the lapse of time in the gravitation field with that in an inertial frame of the Minkowski space in the absence of gravitation.

All this is realized in RTG because the metric tensor  $\gamma_{\mu\nu}$  of the Minkowski space enters explicitly the full system of its equations.

To demonstrate that the change of the lapse of time implies the appearance of a force we turn to the Newton equation

$$m d^2x/dt^2 = F$$

If one passes formally from the inertial time to a time  $\tau$  with

$$d\tau = U(t)dt$$

then it is easy to obtain

$$m d^2x/d\tau^2 = 1/U^2 [F - dx/dt d/dt \ln U]$$

One can see from this that the change of the lapse of time defined by the function  $U$  results in the appearance of an effective force. All this bears a formal character here as in this case there is no physical reason which would change the lapse of time. But this formal example shows that if a process of slowing down of the lapse of time occurs in Nature then it unavoidably generates effective field forces, and so it is necessary to take them into account as something absolutely new and surprising. The physical gravitational force changes both the lapse of time and parameters of the space quantities in comparison with the same quantities in an inertial system of the Minkowski space without gravitation.

The field approach to gravitation excludes the concept of black holes and explains the evolution both of massive bodies and the Universe on the basis of more profound insight into the physical properties of the very gravitational field.

This confirms the deep intuition of A. S. Eddington who said at the session of the Royal Astronomical Society 11 January 1935: "The star has to go on radiating and radiating and contracting and contracting until, I suppose, it gets down to a few km. radius, when gravity becomes strong enough to hold in the radiation, and the star can at last find peace. . . . I felt driven to the conclusion that this was almost a reduction ad absurdum of the relativistic degeneracy formula. Various accidents may intervene to save the star, but I want more protection than that. I think there should be a law of Nature to prevent a star from behaving in this absurd way!".

It appears that in the framework of the field formulation of gravitation such a Law of Nature is contained in the physical property of the gravitational field to stop the process of the slowing down of the lapse of time and hence to limit its potential. This stops the process of compression.

Below, taking as examples the collapse and the evolution of the homogeneous and isotropic Universe, we will see in what way the self-restriction of the gravitational field potential arises which stops both the process of the slowing down of time and the process of the substance compression." [3].

## 6. Gravitational forces of repulsion.

In RTG, the gravitational field produces forces that are both attractive and repulsive. However, the known ones are only attractive. But, during the performance of physical processes in the extreme conditions of space contraction and time dilation, repulsive forces will appear, also of a gravitational nature.

“There is a common belief that the gravitational field provides only forces of attraction. It is seen, for example, from the fact that the physical velocity of a test body increases when it is approaching the gravitating body. However this is not quite so for strong fields. Let us consider this later. When A. Einstein had connected gravitational field with the Riemannian space metric tensor in 1912 it was found that such a field was slowing down the rate of time for a physical process. This slowing down can be demonstrated, in particular, by the example of Schwarzschild solution, if we compare the rate of time in the gravitational field with the rate of time for a distant observer. Nevertheless in general case there is only Riemannian space metric tensor in General Relativity and there are not any indication of the inertial time of Minkowski space. Just for this reason the universal property of gravitational field to slow down the time rate in comparison to the inertial time could not be further developed in General Relativity. The situation is rather opposite in the Relativistic Theory of Gravitation (RTG) as it is a field theory. In this approach the gravitational field is treated as a physical field of Faraday-Maxwell type which is developing in Minkowski space in line with all the other physical fields.

The source of universal gravitational field is the total conserved energy-momentum tensor of all matter including the gravitational field. Therefore the gravitational field is a tensor field with spins 2 and 0. Just this fact leads to geometrization: the effective Riemannian space arises but with trivial topology. This leads to the following situation: the motion of a test body in Minkowski space under the action of gravitational field is equivalent to the motion of this body in the effective Riemannian space created by this gravitational field. The arising of the effective Riemannian space in the field theory side by side with preserving the role of Minkowski space as the fundamental space gives a special meaning to the property of the gravitational field to slow down the time rate. Just in this case it is only possible to speak about the slowing of time in full, by comparing the time rate in the gravitational field with the time rate in inertial frame of reference of Minkowski space in the absence of gravitation. And all this is realized in RTG because the metric tensor of Minkowski space enters into the full system of its equations. But this general property of the gravitational field to slow down the time rate leads in the field theory to a remarkable conclusion: the slowing down of the physical process time rate in a strong field generates effective field forces of the gravitational nature. These effective forces in gravitation occur to be repulsive. To demonstrate that a change of the time rate leads to arising of a force let us consider Newton equation:

$$d^2x/dt^2 = F.$$

If we formally transform this equation in order to change the inertial time  $t$  for time according to the rule:

$$d\tau = U(t) dt,$$

then we easily get

$$d^2x/d\tau^2 = 1/U^2 (F - dx/dt d/dt \ln U).$$

It is seen from here that a change in time rate determined by function  $U$  leads to arising of the effective force. But all this is of purely formal character because there are no any physical reason in this case that could change the time rate. But just this formal example demonstrates that when a process of slowing down time takes place in nature it inevitably generates effective field forces, and therefore it is necessary to account for them in the theory as something rather new and surprising.

And just here we are to turn to gravitation. The physical gravitational field changes both the time rate and parameters of spatial quantities in comparison to their values given in inertial system of Minkowski space when gravitation is absent. Just for this reason all this should be taken into account in the gravitational field equations. In RTG metric tensor of Minkowski space appears unambiguously in these equations due to graviton mass introduced into them. Just this tensor provides the opportunity to account for effective field forces created by change of the time rate under action of the gravitational field. Here the graviton mass realizes a correspondence of the effective Riemannian space to the basic Minkowski space. Though the graviton mass is rather small nevertheless the influence of mass term becomes decisive because of great slowing down of the time rate under the action of gravitational field" [11].

## **7. The cosmological consequences of repulsive gravitational forces.**

According to RTG, repulsive gravitational forces, which appear in strong gravity, make it impossible for matter to collapse into singularities and, although, at first, they generate an accelerated expansion of the Universe, due to the mass of the graviton, at some point further such expansion should end.

This expansion was discovered in 1998, alternatively, and in a standard way, dark energy has been proposed as its cause.

In RTG, the accelerated expansion of the Universe must cease due to the inevitable passage from the region of strong gravity to the region of weak gravity, and, consequently, to the domain of the inertial force present in Minkowsky's pseudo-Euclidean spacetime. Likewise, the energy of the vacuum must evolve cyclically, between a maximum and a minimum density, causing the phenomenology of the Universe to be oscillating, therefore, without Big Bang or Big Crunch.

The cosmological consequences of repulsive gravitation are:

1. Absence of the cosmological singularity.

“Thus, due to the graviton mass and, hence, to the presence of the gravitational forces related with the change of the lapse of time the cosmological singularity is eliminated, and the expansion of the Universe starts from a finite value of the scale factor\*. Specifically, here a surprising property of the gravitational field is manifested: an ability to create in the strong fields the repulsive forces which stop the process of the compression of the Universe and then provide its accelerated expansion” [3].

\* Tells us how the Universe itself evolves as a function of time.

## 2. Impossibility of unlimited "expansion of the Universe".

“Considering the gravitational field  $\phi^{\mu\nu}$  as a physical field in the Minkowski space, one has to require the fulfilment of the causality principle. This means that the light cone in the effective Riemannian space has to lie inside the light cone of the Minkowski space, i.e. for  $ds^2 = 0$  the requirement  $d\sigma^2 > 0$  holds. Writing down  $d\sigma^2$  in the spherical coordinate system

$$d\sigma^2 = c^2 dt^2 - (dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2)$$

and determining the spatial part of the interval from the condition  $ds^2 = 0$ , we have

$$d\sigma^2 = c^2 dt^2 (1 - a^4)/\beta^4 \geq 0,$$

i. e.

$$(a^4 - \beta^4) \leq 0$$

Thus the scale factor ( $a$ ) is bounded by the condition  $a \leq \beta$  and so it would be natural to assume its maximum value as

$$a_{\max} = \beta .$$

With such a choice of  $a_{\max}$  the rate of the lapse of time  $d\tau_g$  in the moment of the stop of the Universe expansion becomes equal to the rate of the lapse of the inertial time  $t$  in the Minkowski space, though the second derivative  $\ddot{a}$  and, hence, the scalar curvature  $R$  are non-zero. This is this point from which the slowing down of the rate of the lapse of time under the action of the attractive forces will proceed up to the point of the stop of compression, when under the action now already of repulsive forces the opposite process of the acceleration of the rate of the lapse of time up to the rate of the inertial time  $t$  of the Minkowski space starts. Exactly all these physical consequences require necessarily the condition  $a_{\max} = \beta$  to be held. As we will see further, the value of the quantity  $\beta$  is determined by the integral of motion. Condition does not admit an unlimited growth of the scale factor with time  $\tau$ , i.e. an unlimited “expansion” of the Universe (in the above-indicated sense) which is provided by the dynamical evolution equation of the scale factor  $a$ . Let us note, besides, that the very Universe is infinite because the radial coordinate is defined in the range  $0 < r \leq \infty$ ” [3].

## 3. Need for quintessence with $v > 0$ .

“As was already mentioned, when considering the gravitational field as a physical field in the Minkowski space, it is necessary to require the fulfilment of the causality principle. This requirement, applied to the Universe evolution, leads to inequality according to which the scale factor is bounded by the inequality  $a \leq a_{\max} = \beta$ . In other words, according to RTG, the unlimited expansion of the Universe is impossible. The mathematical apparatus of RTG automatically provides the fulfilment of this condition in the case when the matter density decreases with increase of the scale factor”.

“The field theory of gravitation appears incompatible with the existence of the constant cosmological term leading to an unlimited expansion of the Universe”.

“Thus the only possibility to explain, in the framework of RTG, the accelerated expansion of the Universe observed at the present time is the existence of a quintessence with  $\nu > 0$  or some other substance the density of which decreases with increase of the scale factor (but not faster than  $\text{const}/a^2$ ). RTG excludes a possibility of the existence both of the constant cosmological term ( $\nu = 0$ ) and the “phantom” expansion ( $\nu < 0$ )” [3].

4. The evolution of the universe is oscillatory.

“The time corresponding to the end of the accelerated expansion and the beginning of deceleration leading to the stop of expansion strongly depends on parameter  $\nu$ ”.

“The expansion up to the maximum value of the scale parameter and its consequent compression lead to the oscillatory character of the Universe evolution. The idea of the oscillatory character of the Universe evolution was repeatedly advanced earlier proceeding mainly from the philosophical considerations. Such a regime could, in principle, be expected in the closed Friedmann model with  $\Omega_{\text{tot}} > 1$ . However, firstly, the insurmountable difficulty related to the passage through the cosmological singularity and, secondly, considerations related to the growth of entropy from cycle to cycle do not allow this.

It is necessary to emphasize that in the framework of the Hilbert–Einstein equations the flat Universe cannot be oscillatory. These difficulties for the infinite Universe are eliminated in RTG. Since singularities are absent from RTG the Universe could exist an infinite time during which the interaction occurred among its domains and this led to homogeneity and isotropy of the Universe with some structure of inhomogeneity which we did not take into account for the sake of simplicity”.

“The attractiveness of the oscillatory evolution of the Universe is mentioned in the recent paper. The oscillatory regime is realized by the price of introducing a scalar field interacting with the substance and use of the extra dimensions. Some important consideration were advanced that the phase of the accelerated expansion promotes the entropy conservation in the repeating cycles of the evolution. In RTG the oscillatory character of the Universe evolution is achieved as a result of introducing of the only massive gravitational field as a physical field generated by the total energy-momentum tensor in the Minkowski space” [3].



## 8. Is Minkowski's space observable?

According to RTG equations, and the observations of the trajectories of the particles and the photon, in the effective Riemann spacetime, that is, where the gravitational field exists, the pseudo-Euclidean Minkowski spacetime tensor can be determined and by means of coordinate transformation, arriving at a coordinate system of a Galilean inertial frame, making the Minkowski spacetime observable, from which the Euclidean plane geometry of the Universe is inferred. Also, therefore, in RTG there is neither a Big Bang, nor a Big Crunch nor a Big Rip, but fluctuations between maxima and minima of energy density, causing the phenomenology of the Universe to oscillate, always subject, on one side, to the absence of absolute vacuum and, on the other hand, to the identity between the effective Riemann spacetime and that of the pseudo Euclidean Minkowski, since the former arises from the physical processes, subject to gravity, that occur in the second.

"Now we ask a question: if the Minkowski space is observable, at least in principle?

To answer it we write the equations in the form:

$$m^2 / 2 \gamma_{\mu\nu} = 8\pi G (T_{\mu\nu} - 1/2 g_{\mu\nu}T) - R_{\mu\nu} + m^2 / 2 g_{\mu\nu}.$$

It is seen from here that on the right side of the equation there are only geometric characteristics of the effective Riemannian space and the quantities that define the distribution of the substance in this space.

Now let's make use of the Weyl-Lorentz-Petrov theorem, according to which:

"If you know... The equations of all geodetic time lines and all isotropic geodesics it is possible to determine the metric tensor to a constant multiplier."

From this it follows that, with the help of the experimental study of particles and the photon in Riemann space, the effective metric tensor  $g_{\mu\nu}$  of Riemann space can in principle be determined. By further substituting  $g_{\mu\nu}$  into the equation, the Minkowski spatial metric tensor can be determined. After that, with the help of coordinate transformations, a step to an inertial Galilean coordinate system can be provided. Thus, Minkowski's space is, in principle, observable.

It is appropriate to quote here the words of V. A. Fock:

"How is the straight line defined: as a ray of light or as a straight line in Euclidean space in which the  $x_1, x_2, x_3$  harmonic coordinates are used as Cartesian coordinates? We believe that the second definition is the only correct one. We actually used it when we said that the ray of light near the Sun has a hyperbola shape,"

and additionally in this regard:

"... considering that the straight line, like a ray of light, is more directly observable, it does not matter: what is decisive in the definitions is not its direct observability, but rather a correspondence with Nature, although this correspondence it is established by an indirect deduction".

The inertial coordinate system, as we see, is related to the distribution of substances in the Universe. Therefore, RTG gives us, in principle, the opportunity to determine the inertial coordinate system. The equations of the evolution of the scale factor.

The system of equations of RTG leads unequivocally, in contrast to GTR, to a unique solution, which is the spatial Euclidean plane geometry of the Universe.

The so-called "expansion of the Universe", observed with the displacement of red, is not caused by the movement of the substance, but by the change of the gravitational field with time. This observation must be borne in mind when using the established term "expansion of the Universe".

The evolution of the empty Universe does not occur and the effective Riemannian space coincides with the Minkowski space" [3].

## **9. The geometry of spacetime.**

RTG's conception with respect to space-time is that it is a relational category, that is, as a relational property of matter, contrary to the substantialism that it considers to exist in itself. Consequently, space-time does not exist, before, in its absence and independent of matter.

However, formally, that is, in the abstract, spacetime is treated as a mathematical structure, which can even be presented as the container of matter, where its physical processes occur, which according to an express statement by RTG, is due to independence of the spacetime of the form of matter and not, rather, the consequence of its conceptual elaboration, in its passage from the material to being part of the structures of the formal science of mathematics.

From this view of spacetime as a mathematical structure but linked with matter, it results that the geometry of spacetime is necessarily pseudo Euclidean, because, contrary to Riemann geometry, this is the only known geometry that preserves the conservation laws of Impulse-energy and angular impulse-energy, formulated as fundamental physical principles, from the generalization of existing experimental data, which reveal the general dynamic characteristic properties of all forms of matter, and allow the quantitative description of the transformation of some forms of matter in others. But, because the physical processes that occur, in the pseudo Euclidean spacetime, because they are subject to the universal forces of gravity, originate the effective Riemann spacetime, which is nothing more than the expression, when the pseudo Euclidean spacetime is positively curved of Minkowski, in a related geometry such as that of Riemann.

"In Chapter II, "Space and Time", in his book "Recent Ideas", H. Poincaré wrote:

"The principle of physical relativity can serve to define space. It can be said that it provides us with a new instrument for measurement. I am going to explain how can a solid body serve to measure or, to be more correct, build a space? The point is this: When transferring a solid body from one place to another, we note that it can be applied first to one figure, then another, and we conventionally agree to consider these figures equal to each other. Geometry originated from this convention.

Geometry is nothing more than a science of the mutual interrelationships between such transformations or, speaking in mathematical language, a science of the structure of the group formed by these transformations, that is, of the group of movements of solid bodies. Now consider another group, the group of transformations that do not alter our differential equations. We come to a new path.

To define equality between two figures. We no longer say: two figures are the same, if one and the same solid body can be applied to one or the other. We will say: two figures are the same, when one and the same mechanical system are so far from their neighbors that it can be considered isolated, thus placing itself first in its material points, reproducing the first figure, and then reproducing the second figure, behaving in the second case precisely as in the first. Do these two approaches differ in essence? No. A solid body represents a mechanical system, like any other. The only difference between the old and new space definitions is that the latter is broader, as it allows the substitution of any mechanical system for the solid body. Furthermore, our new convention defines not only space, but also time. It provides us with an explanation of what two equal time intervals are or what is represented by a time interval twice as long as another. "

Precisely in this way, when discovering the group of transformations that do not alter the Maxwell-Lorentz equations, H. Poincaré introduced the notion of four-dimensional spacetime that exhibits pseudo-Euclidean geometry. This concept of geometry was later developed by H. Minkowski.

We have chosen the pseudo-Euclidean geometry of spacetime as the basis of the relativistic theory of gravity currently under development, since it is the fundamental Minkowski space for all physical fields, including the gravitational field.

Minkowski's space cannot be considered to exist a priori, since it reflects the properties of matter and therefore cannot be separated from it. Although formally, precisely because the structure of space is independent of the form of matter, it is sometimes treated abstractly, separate from matter.

In the Galilean coordinates of an inertial reference system in Minkowski space, the interval that characterizes the structure of geometry and that is invariable by construction, has the form

$$d\sigma^2 = (dx_0)^2 - (dx_1)^2 - (dx_2)^2 - (dx_3)^2$$

Here  $dx_v$  represent differential co-ordinates. Despite the fact that the interval  $d\sigma$ , as a geometric feature of spacetime, is independent of the choice of the reference system, which is due to its own construction, it can still be found in modern textbooks on theoretical physics "proofs" that the interval is the same in all inertial reference systems, although it is an invariant and is independent of the choice of the reference system.

Even an outstanding physicist like L. I. Mandelstam wrote in his book:

"... the theory of special relativity cannot answer the question, how does a clock behave when it moves with acceleration and why does it slow down, because it is not a reference system in motion with acceleration".

Incorrect assertions can be explained by the Minkowski space that many people consider to be just a formal geometric interpretation of SRT within A. Einstein's approach, rather than a revelation of spacetime geometry. Topics of such limited concepts as the constancy of the speed of light, the timing of clocks, the speed of light independent of the movement of its source became the most discussed topics. All this reduced the scope of the SRT and delayed the understanding of its essence. And its essence actually consists only in that the geometry of spacetime, in which all physical processes occur, is pseudo-Euclidean.

In an arbitrary frame of reference the interval assumes the form

$$d\sigma^2 = \gamma_{\mu\nu}(x) dx_\mu dx_\nu,$$

$\gamma_{\mu\nu}(x)$  is the metric tensor of the Minkowski space.

We note that one cannot, in principle, speak of the synchronization of clocks or the constancy of the speed of light in a non-inertial reference system. Most likely, precisely the lack of clarity about the essence of SRT led A. Einstein to conclude:

"that in the framework of the theory of special relativity there is no place for a satisfactory theory of gravity".

The free movement of a test body in an arbitrary reference system takes place along a geodetic line in Minkowski space:

$$DU_\nu / d\sigma = dU_\nu / d\sigma + \gamma_\nu \alpha\beta U_\alpha U_\beta = 0,$$

where  $U_\nu = dx_\nu / d\sigma$ ,  $\gamma_\nu \alpha\beta(x)$  are Christoffel symbols defined by the expression

$$\gamma_\nu \alpha\beta(x) = 1/2 \gamma_{\nu\sigma} (\partial\alpha\gamma\beta\sigma + \partial\beta\gamma\alpha\sigma - \partial\sigma\gamma\alpha\beta).$$

All types of matter satisfy the conservation laws of momentum-energy and angular momentum-energy. Precisely these laws, which originated from a generalization of numerous experimental data, characterize the general dynamic properties of all forms of matter by introducing universal characteristics that allow the quantitative description of the transformation of some forms of matter into others. And all this also represents experimental facts, which have become fundamental physical principles. What should be done with them? If one follows A. Einstein and retains Riemannian geometry as the basis, then they must be discarded.

That price would be too high. It is more natural to retain them for all physical fields, including the gravitational field. But, in this case, the theory must, then, be based on Minkowski space, that is, on the pseudo-Euclidean geometry of spacetime. We have adopted precisely this approach, following H. Poincaré.

The fundamental principles of physics, which reflect the many experimental facts available, indicate which geometry of spacetime is really necessary to use as the basis of the theory of gravity. Therefore, the problem of the structure of spacetime geometry is actually a physical problem, which must be solved by experiments and, from our point of view, the structure of spacetime geometry is not determined by specific experimental data in the movement of test bodies and light, but by fundamental physical principles based on the entire set of existing experimental facts. It is precisely here that our initial premises for constructing the theory of gravity differ completely from the ideas applied by A. Einstein as the basis of GTR. But they are totally consistent with H. Poincaré's ideas. We have chosen the pseudo-Euclidean geometry of spacetime as the basis of the relativistic theory of gravity, but that does not mean that the effective space is also pseudo-Euclidean. The influence of the gravitational field can be expected to lead to a change in effective space. The Minkowski space metric allows us to introduce the concepts of standard duration and time intervals, when there is no gravitational field present" [5].

### **10. The energy-impulse tensor of matter as the source of the gravitational field.**

All forms of matter, including energy as such, in Minkowski's pseudo-Euclidean spacetime are presented as physical fields, characterized by their subjection to the conservation laws of energy-momentum and angular momentum, and described by the tensor of energy-momentum of matter,  $t^{\mu\nu}$ , which due to the universality of gravity, this tensor, as a quantification of the conserved density of matter, constitutes the source of the gravitational field. A fundamental condition for this is that the movement of matter takes place in the effective Riemannian space, precisely originated in the gravitational field, therefore, from a simple topology unlike the general shape of the Riemannian space of the GTR, that for this reason, in this case, the gravitational field cannot be satisfactorily defined in Minkowski's pseudo euclidean spacetime.

In RTG, because the origin of Riemann spacetime is the gravitational field, in the Minkowski pseudo-Euclidean space, there are two types of regions of spacetime, with their metric properties, which coincide with those observed. One is where the field Gravitational tends to disappear, so it approaches with great precision to the pseudo-Euclidean space. And the other is when the gravitational field is strong, where it becomes characteristic of Riemannian space, although, without the pseudo-Euclidean geometry disappearing entirely, since the bodies still possess inertia, and it manifests as acceleration with respect to the pseudo-Euclidean space in galilean coordinates. Thus, acceleration in RTG, unlike GTR, makes absolute sense. On the other hand, in the supposed inertial frame of the "Einstein's elevator" it has been proven, experimentally and crucially, that a charge at rest emits electromagnetic waves.

Since gravity is a physical field, and therefore a component of matter, it must also be the source of the gravitational field. So, the energy-impulse tensor of matter  $t^{\mu\nu}$  is  $= t^{\mu\nu}g + t^{\mu\nu}M$ , where  $t^{\mu\nu}g$  is the energy-impulse tensor of the gravitational field and  $t^{\mu\nu}M$  is the energy-impulse tensor of the rest of the fields of matter.

"Due to the existence in Minkowski's space of Poincaré's group of ten motion parameters, for any closed physical system there are ten motion integrals, that is, the

conservation laws of energy-momentum and angular momentum are valid. Any physical field in Minkowski space is characterized by the density of the energy-impulse tensor, which is a general universal characteristic of all forms of matter that satisfies local and integral conservation laws. In an arbitrary reference system, the local conservation law is written in the form

$$D_{\mu}t^{\mu\nu} = \partial_{\mu}t^{\mu\nu} + \gamma^{\nu}_{\alpha\beta}t^{\alpha\beta} = 0.$$

Here  $t^{\mu\nu}$  is the conserved total density of the energy-impulse tensor for all fields of matter;  $D_{\mu}$  represents the covariant derivative in Minkowski space. Here and beyond, we will always treat the densities of the scalar and tensor quantities defined according to the rule

$$\sqrt{-\gamma}d^4x,$$

while an element of invariant volume in the Riemannian space is given by the expression

$$\sqrt{-g}d^4x, g = \det(g_{\mu\nu}).$$

Therefore, the principle of least action takes the form.

$$\delta S = \delta \int L d^4x = 0,$$

where  $L$  is the Lagrangian scalar density of matter. By deriving Euler's equations with the help of the principle of least action, we will automatically have to deal precisely with the variation in Lagrangian density. According to D. Hilbert, the density of the energy-impulse tensor  $t_{\mu\nu}$  is expressed through the scalar density of the Lagrangian  $L$  as follows:

$$t_{\mu\nu} = -2\delta L/\delta\gamma_{\mu\nu}, \quad (2.1)$$

$$\text{Where } \delta L/\delta\gamma_{\mu\nu} = \partial L/\partial\gamma_{\mu\nu} - \partial\sigma(\partial L/\partial\gamma_{\mu\nu, \sigma}), \gamma_{\mu\nu, \sigma} = \partial\gamma_{\mu\nu}/\partial x^{\sigma}$$

Because gravity is universal, it would be natural to assume that the conserved density of the impulse energy tensor of all fields of matter,  $t^{\mu\nu}$ , is the source of the gravitational field. Furthermore, we will take advantage of the analogy with electrodynamics, in which the conserved density of the charged vector current serves as the source of the electromagnetic field, while the field itself is described by the density of the vector potential  $\tilde{A}^{\nu}$ :

$$\tilde{A}^{\nu} = (\phi, \vec{A}).$$

In the absence of gravity, Maxwell's electrodynamic equations will have the following form in arbitrary coordinates:

$$\gamma^{\alpha\beta}D_{\alpha}D_{\beta}\tilde{A}^{\nu} + \mu^2\tilde{A}^{\nu} = 4\pi j^{\nu}, D_{\nu}\tilde{A}^{\nu} = 0,$$

Here, for generalization, we have introduced the parameter  $\mu$ , which in the unit system  $\hat{h} = c = 1$  is the resting mass of the photons.

Since we have decided to consider the conserved energy-impulse density  $t^{\mu\nu}$  to be the source of the gravitational field, it is natural to consider the gravitational field as a tensor field and describe it by the density of the symmetric tensor

$$\phi^{\mu\nu}: \phi^{\mu\nu} = \nu - \gamma\phi^{\mu\nu},$$

and in complete analogy with Maxwell's electrodynamics, the equations for the gravitational field can be written in the form

$$\gamma^{\alpha\beta} D_\alpha D_\beta \phi^{\mu\nu} + m^2 \phi^{\mu\nu} = \lambda t^{\mu\nu}, \quad (2.2)$$

$$D_\mu \phi^{\mu\nu} = 0 \quad (2.3)$$

Here  $\lambda$  is a certain constant that, according to the correspondence principle with Newton's law of gravity, should be equal to  $16\pi$ . Equation (2.3) excludes spin 1 and 0', which only retain the field polarization properties, which correspond to spin 2 and 0. The density of the energy-impulse tensor of matter  $t^{\mu\nu}$  consists of the density of the tensor of energy-impulse of the gravitational field,  $t^{\mu\nu}_g$ , and of the energy-impulse tensor of matter,  $t^{\mu\nu}_M$ . We understand that matter comprises all fields of matter, with the exception of the gravitational field,

$$t^{\mu\nu} = t^{\mu\nu}_g + t^{\mu\nu}_M.$$

The interaction between the gravitational field and matter is taken into account in the density of the energy-impulse tensor of matter,  $t^{\mu\nu}_M$ .

From equations (2.2) it follows that they will also be non-linear for the gravitational field itself, since the density of the tensor  $t^{\mu\nu}_g$  is the source of the gravitational field.

Equations (2.2) and (2.3), which we formally declare the gravity equations by analogy with electrodynamics, must be derived from the principle of least action, since only in this case we will have an explicit expression of the density of the energy tensor - impulse of the gravitational field and the fields of matter. But, for this purpose, it is necessary to construct the density of the Lagrangian of matter and the gravitational field. Here it is extremely important to carry out this construction on the basis of general principles. Only in this case can one speak of the theory of gravity. The initial scalar density of matter Lagrangian can be written in the form

$$L = L_g(\gamma_{\mu\nu}, \phi^{\mu\nu}) + L_M(\gamma_{\mu\nu}, \phi^{\mu\nu}, \phi_A),$$

Here  $L_g$  is the Lagrangian density of the gravitational field;  $L_M$  is the Lagrangian density of the fields of matter; represent  $\phi_A$  represents the fields of matter.

The equations for the gravitational field and the fields of matter have, according to the principle of least action, the form

$$\delta L / \delta \phi_{\mu\nu} = 0, \quad (2.4)$$

$$\delta L_M / \delta \phi_A = 0. \quad (2.5)$$

Equations (2.4) differ from equations (2.2), first, in that the variational derivative of the Lagrangian density is the derivative with respect to the  $\hat{\phi}^{\mu\nu}$  field, while the variational derivative in equations (2.2) is, in according to definition (2.1), taken from the Lagrangian density on the  $\gamma_{\mu\nu}$  metric. In order for equations (2.4) to be reduced to equations (2.2) for any form of matter, it is necessary to assume that the tensor density  $\hat{\phi}^{\mu\nu}$  is always present in the Lagrangian density along with the tensor density  $\hat{\gamma}^{\mu\nu}$  through a common density  $\hat{g}^{\mu\nu}$  in the shape

$$\hat{g}^{\mu\nu} = \hat{\gamma}^{\mu\nu} + \hat{\phi}^{\mu\nu}, \hat{g}^{\mu\nu} = \nu - g g^{\mu\nu}. \quad (2.6)$$

This is how the effective Riemannian space arises with the metric  $g^{\mu\nu}(x)$ . Since the gravitational field  $\hat{\phi}^{\mu\nu}(x)$ , like all other physical fields in Minkowski space, can be described within a single coordinate map, it is evident from expression (2.6) that the quantity  $\hat{g}^{\mu\nu}(x)$  it can also be fully defined in a single coordinate map. For the description of the effective Riemannian space due to the influence of the gravitational field, a map atlas is not required, which is usually necessary to describe the Riemannian space in general form. This means that our effective Riemannian space has a simple topology. In the GTR topology it is not simple. Precisely for this reason, GTR cannot, in principle, be built on the basis of ideas that consider gravity as a gravitational physical field in Minkowski space.

If condition (2.6) is taken into account, the density of the Lagrangian L assumes the form

$$L = L_g(\gamma_{\mu\nu}, \hat{g}^{\mu\nu}) + L_M(\gamma_{\mu\nu}, \hat{g}^{\mu\nu}, \phi_A).$$

It should be emphasized that condition (2.6) allows the variational derivative with respect to  $\hat{g}^{\mu\nu}$  to be replaced by the variational derivative with respect to  $\hat{\phi}^{\mu\nu}$ , and to express the variational derivative with respect to  $\gamma_{\mu\nu}$  through the variational derivative with respect to  $\hat{g}^{\mu\nu}$  and variational derivative with respect to  $\gamma_{\mu\nu}$  explicitly enters the Lagrangian L.

Indeed,

$$\delta L / \delta \hat{\phi}^{\mu\nu} = \delta L / \delta \hat{g}^{\mu\nu} = 0, \quad (2.7)$$

$$\delta L / \delta \gamma_{\mu\nu} = \delta^* L / \delta \gamma_{\mu\nu} + \delta L / \delta \hat{g}^{\alpha\beta} \cdot \partial \hat{g}^{\alpha\beta} / \partial \gamma_{\mu\nu}. \quad (2.8)$$

The asterisk in formula (2.8) indicates the variational derivative of the Lagrangian density with respect to the metric  $\gamma_{\mu\nu}$  that is explicitly present in L. According to (2.1), formula (2.8) can be written in the form

$$t^{\mu\nu} = -2\delta L / \delta \hat{g}^{\alpha\beta} \cdot \partial \hat{g}^{\alpha\beta} / \partial \gamma_{\mu\nu} - 2\delta^* L / \delta \gamma_{\mu\nu}.$$

Taking into account equation (2.7) in the previous expression we obtain

$$t^{\mu\nu} = -2\delta^* L / \delta \gamma_{\mu\nu}. \quad (2.9)$$

Comparing equations (2.9) and (2.2) we obtain the condition



$$-2\delta^*L/\delta\gamma_{\mu\nu} = 1/16\pi [\gamma^{\alpha\beta}D_\alpha D_\beta \phi^{\mu\nu} + m^2 \phi^{\mu\nu}] \quad (2.10)$$

that, if it is fulfilled, it allows to derive the equations of the gravitational field, (2.2) and (2.3), directly from the principle of minimum action. Since the matter fields are not present on the right side of (2.10), this means that the variation in the Lagrangian density of matter,  $L_M$ , with respect to the explicitly present metric  $\gamma_{\mu\nu}$  must be zero. So that no additional constraints on the motion of matter determined by Eqs. (2.5) arise, it follows directly that the tensor  $\gamma_{\mu\nu}$  does not explicitly enter the expression for the Lagrangian density of matter  $L_M$ . Expression (2.10) then assumes the form

$$-2\delta^*L_g/\delta\gamma_{\mu\nu} = 1/16\pi [\gamma^{\alpha\beta}D_\alpha D_\beta \phi^{\mu\nu} + m^2 \phi^{\mu\nu}]. \quad (2.11)$$

Therefore, it all comes down to finding the Lagrangian density of the gravitational field itself,  $L_g$ , that would satisfy condition (2.11). At the same time, from the above arguments we reach the important conclusion that the density of the Lagrangian of matter,  $L$ , has the form

$$L = L_g(\gamma_{\mu\nu}, \hat{g}^{\mu\nu}) + L_M(\hat{g}^{\mu\nu}, \phi_A). \quad (2.12)$$

Therefore, from the requirement that the density of the energy-impulse tensor of matter be the source of the gravitational field, it naturally follows that the motion of matter must take place in the effective Riemannian space. This statement has the character of a theorem. Therefore, it is clear why the Riemannian space arose, instead. Precisely, this circumstance gives us the possibility of formulating the gauge group, and then constructing the density of the Lagrangian condition (4.24) that satisfies (2.11), according to (B.20).

$$L_g = 1/16\pi \hat{g}^{\mu\nu} (G^\lambda_{\mu\nu} G^\sigma_{\lambda\sigma} - G^\lambda_{\mu\sigma} G^\sigma_{\nu\lambda}) - m^2/16\pi (1/2 \gamma_{\mu\nu} \hat{g}^{\mu\nu} - \nu \cdot g - \nu \cdot \gamma) \quad (4.24)$$

$$-2\delta^* L_g / \delta \gamma_{\mu\nu} = 1/16 \pi (-J^{\mu\nu} + m^2 \Phi^{\mu\nu}) \quad (B.20)$$

An interesting image emerges that is that the movement of matter in Minkowski space with the metric  $\gamma_{\mu\nu}$  under the influence of the gravitational field  $\phi^{\mu\nu}$  is identical to the movement of matter in effective Riemannian space with the metric  $g_{\mu\nu}$ , determined by the expression (2.6). We call such an interaction of the gravitational field with matter known as the geometrization principle. The geometrization principle is a consequence of the initial assumption that a universal characteristic of matter, the density of the energy-impulse tensor, serves as the source of the gravitational field. Such a Lagrangian density structure of matter indicates that a unique possibility is realized for the gravitational field to join within the Lagrangian density of matter directly to the density of the tensor  $\hat{\gamma}^{\mu\nu}$ .

The effective Riemannian space literally has a field origin, due to the presence of the gravitational field. Therefore, the reason that the effective space is Riemannian, and not another, lies in the hypothesis that a conserved universal quantity, the density of the energy-impulse tensor, is the source of gravity. We will explain this fundamental property of gravitational forces by comparing them with electromagnetic forces. In the case of a homogeneous magnetic field, a charged particle in Minkowski space is known

to undergo, due to the Lorentz force, a motion along a circle in the plane perpendicular to the magnetic field. However, this motion is far from identical even for charged particles, if their charge-mass ratio is different. Also, there are neutral particles, and their paths in a magnetic field are just straight lines. Therefore, due to the non-universal character of electromagnetic forces, their action cannot be reduced to the geometry of spacetime. Gravity is another matter. It is universal, any test body moves along identical paths given identical initial conditions. In this case, due to the hypothesis that the energy-impulse tensor of matter is the source of the gravitational field, it is possible to describe these trajectories using geodetic lines in the effective Riemannian spacetime due to the presence of the gravitational field in space from Minkowski. In those regions of space, where there is a small gravitational field present, we have metric properties of space that closely approximate the actually observed properties of pseudo-Euclidean space. On the other hand, when the gravitational fields are strong, the metric properties of the effective space become Riemannian. But in this case, too, pseudo-Euclidean geometry does not disappear without a trace; it is observable and manifests itself in that the movement of the bodies in the effective Riemannian space is not free of inertia, but is accelerated with respect to the pseudo-Euclidean space in Galilean coordinates. Precisely for this reason, acceleration in RTG, unlike GTR, makes absolute sense. Consequently, the "Einstein elevator" cannot serve as an inertial frame of reference. This is manifested in that a charge at rest in the "Einstein elevator" will emit electromagnetic waves. This physical phenomenon must also testify in favor of the existence of the Minkowski space. Furthermore, the Minkowski space metric can be defined from studies of the distribution of matter and the motion of test bodies and light in effective Riemannian space.

The equation of motion of matter does not include the metric tensor  $\gamma_{\mu\nu}$  of the Minkowski space. Minkowski's space will only influence the movement of matter using the metric tensor  $g_{\mu\nu}$  of Riemann's space, derived, as we will see later, from the gravity equations, which contain the metric tensor  $\gamma_{\mu\nu}$  of Minkowski's space. Since the effective Riemannian metric arises on the basis of the given physical field in the Minkowski space, it follows that the effective Riemannian space has a simple topology and is presented on a single map. If, for example, matter is concentrated in an island-type region, then, in Galilean coordinates of an inertial reference system, the gravitational field  $\phi^{\mu\nu}$  cannot decrease slower than  $1/r$ , but this circumstance imposes a strong restriction on the asymptotic behavior of the effective Riemannian geometry  $g_{\mu\nu}$  metric

$$g_{\mu\nu} = \eta_{\mu\nu} + O(1/r), \text{ aquí } \eta_{\mu\nu} = (1, -1, -1, -1). \quad (2.13)$$

If, on the other hand, one simply takes the Riemann metric as a starting point, without assuming that it originated from the action of a physical field then such restrictions do not arise, since the asymptotic metric  $g_{\mu\nu}$  even depends on choosing three-dimensional spatial coordinates. However, the physical quantity, in principle, cannot depend on the choice of three-dimensional spatial coordinates. RTG imposes restrictions on the choice of the reference system. The reference system can be arbitrary, if it only makes a one-to-one correspondence between all the points of the inertial reference system in

the Minkowski space and provides the following inequalities, necessary to introduce the concepts of time and spatial length, which must be satisfy:

$$\gamma_{00} > 0, dl^2 = s_{ik} dx^i dx^k > 0; i, k = 1, 2, 3,$$

where

$$s_{ik} = -\gamma_{ik} + \gamma_{0i}\gamma_{0k}/\gamma_{00}.$$

In our theory of gravity, the geometric characteristics of Riemannian space arise as field quantities in Minkowski space, and for this reason, their transformation properties become tensor properties, even if this was not the case from the point of view conventional. Thus, for example, the Christoffel symbols, given as field quantities in Galilean coordinates of Minkowski space, become tensors of the third rank. Similarly, ordinary derivatives of tensor quantities in the Cartesian coordinates of Minkowski space are also tensor.

The question may arise: why is a metric division, like (2.6), not performed in GTR by introducing the concept of gravitational field in Minkowski space? The Hilbert-Einstein equations only contain the quantity  $g_{\mu\nu}$ , therefore, it is impossible to say unambiguously with the help of which metric  $\gamma_{\mu\nu}$  of the Minkowski space we must define, according to (2.6), the gravitational field. But the difficulty is not only in the above, but also in that the solutions of the Hilbert-Einstein equations are generally not found on a map, but in an entire map atlas. Such solutions for  $g_{\mu\nu}$  describe a Riemannian space with a complex topology, while Riemannian spaces, obtained by representing the gravitational field in Minkowski space, are described on a single map and have a simple topology. It is precisely for these reasons that field representations are not GTR compliant, as they are extremely rigorous. But this means that there can be no GTR field formulation in Minkowski space, in principle, no matter how much someone and who wants this to happen. The apparatus of Riemannian geometry leans towards the possibility of introducing covariant derivatives in the Minkowski space, which we take advantage of when constructing RTG. But to implement this, it was necessary to introduce the Minkowski space metric into the gravitational equations, and therefore it became possible to perform the functional relationship of the Riemannian space metric,  $g_{\mu\nu}$ , with the Minkowski space metric,  $\gamma_{\mu\nu}$ " [5].

## **11. Conservation of the energy moment and the angular momentum with respect to the geometry of spacetime.**

The geometry of spacetime mainly determines the power to obtain conservation laws for the impulse-energy and the angular impulse, from a closed system of interacting fields. Likewise, it allows establishing the existence of the homogeneity and isotropy properties of a type of spacetime.

There are three types of four-dimensional space geometries that have the properties of homogeneity and isotropy, which allow the introduction of the 10 motion integrals for a closed system. These are the Lovachevski negative or hyperbolic curvature space, the zero or Euclidean curvature space, and the constant or elliptic Riemann positive curvature space.

However, in order to have the largest number of conserved quantities, it is necessary to reject Riemannian geometry in its general form, and because the existing experimental data on strong, weak and electromagnetic interactions suggest that the natural geometry of spacetime is pseudo Euclidean it is necessary to choose the Minkowski pseudo Euclidean spacetime, for all fields, including the gravitational one. This is one of the main theses of RTG on the gravitational interaction. In this way, the laws of conservation of energy-momentum and angular momentum are fulfilled and they ensure the existence of the ten motion integrals for a system consisting of the gravitational field and the other material fields.

The reason why Minkowski's pseudo-Euclidean spacetime guarantees the ten integral laws of conservation of momentum-energy and angular momentum, in a closed system of interacting fields, is to admit a group of ten motion parameters, consisting of a subgroup of translation of four parameters and a subgroup of six parameters of rotation, and have the corresponding Killing vectors.

Minkowski's pseudo-euclidean spacetime, being its metric tensor  $\gamma^{\mu\nu}$ , invariant in shape under the translations, is homogeneous, so its properties do not depend on the position of its origin. And because it is also under rotation it is isotropic, so all directions have the same status. Only in this space are there separate conservation laws for the energy-momentum and the angular momentum of a closed system.

The gravitational field in RTG, like all other physical fields, is characterized by an energy-impulse tensor that contributes to the total energy-impulse tensor of the system. This constitutes the main difference between RTG and GTR.

On the other hand, when acting on matter, a gravitational field changes the geometry of matter. Then, the movement of material bodies, in the pseudo-Euclidean spacetime, under the action of the gravitational field is not distinguished from their movement in an effective Riemann spacetime.

Since the experimental data also points out that the action of a gravitational field on matter is universal, it causes RTG to also formulate the principle of geometrization, whereby the effective Riemann spacetime will be universal for all forms of matter.

Thus, the effective Riemann spacetime is an energy-impulse carrier. The amount of energy used to generate this spacetime is exactly equal to the amount contained in the gravitational field; therefore, the propagation of curvature waves in Riemann spacetime occurs through gravitational waves in pseudo-Euclidean spacetime, with transfer of energy, like all other waves existing in nature. Of course, in RTG, the existence of curvature waves in Riemann spacetime originates directly from the existence of gravitational waves in the Faraday and Maxwell sense, waves carrying an energy-impulse density.

“The geometry of spacetime largely determines the possibility of obtaining conservation laws for a closed system of interacting fields. As is well known (see Bogoliubov and Shirkov, 1979, and Novozhilov, 1975), a theory for any physical field can be constructed on the basis of the Lagrangian formalism, in this case the physical field is described by a function of coordinates and time, known as the field function,

and the equations for determining this function can be found by employing the variational principle of least action. Besides producing field equations, the Lagrangian approach to constructing a classical theory of wave fields makes it possible to derive a number of differential relationships known as differential conservation laws (see Noether, 1918). These relationships follow from the invariance of the action integral under transformations of the spacetime coordinates and link the local dynamical characteristics of the field with the respective covariant derivatives in a geometry that is natural in relation to these characteristics.

At present in the literature two types of differential conservation laws are distinguished: strong and weak. Usually a strong conservation law is a differential relationship that holds true solely owing to the invariance of the action integral under coordinate transformations and does not require the existence of equations of motion for the field. A weak conservation law can be obtained from a strong conservation law by allowing for the equations of motion for a system of interacting.

It must be stressed that notwithstanding their name, differential conservation laws do not require conservation of some quantity either locally or globally. They are simply differential identities linking the different characteristics of a field and are valid because the action integral does not change under coordinate transformation (i.e. is a scalar). Their name was given by analogy with the differential conservation laws in pseudo-Euclidean spacetime, in which differential conservation laws may lead to integral laws. For example, writing the law of conservation for the total energy-momentum tensor of a system of interacting fields in a Cartesian system of coordinates of the pseudo-Euclidean spacetime, we get

$$\partial/\partial x^0 t^{0i} + \partial/\partial x^\alpha t^{\alpha i} = 0$$

Integrating over a certain volume and employing the divergence theorem, we get

$$d/dx^0 \int t^i_0 dV = - \int t^{\alpha i} dS_\alpha$$

This relationship means that the variation in the energy-momentum of a system of interacting fields inside a certain volume is equal to the energy-momentum flux through the surface enclosing the volume. If this flux is zero, or  $\int t^{\alpha i} dS_\alpha = 0$ , we arrive at the conservation law for the total 4-momentum of an isolated system, where

$$\partial/\partial x^0 P^i = 0 \text{ and } P^i = 1/c \int t^{0i} dV$$

Similar integral relationships in the pseudo-Euclidean spacetime can be obtained for angular momentum as well.

However, in an arbitrary Riemann spacetime the presence of a differential covariant conservation equation does not guarantee the possibility of obtaining a respective integral conservation equation. The possibility of obtaining integral conservation laws in an arbitrary Riemann spacetime is totally dependent on the geometry of the spacetime and is closely linked with the existence of Killing vectors in the given spacetime or, as is sometimes said, with the existence of a group of motions in the

Riemann spacetime. Let us dwell on this in more detail, since the formalism developed here can be used to obtain integral conservation laws in arbitrary curvilinear systems of coordinates of the pseudo-Euclidean spacetime as well.

In an arbitrary Riemann spacetime we have the following covariant conservation equation for the total energy-momentum tensor of the system:

$$\nabla_i T^{mi} = \partial_i T^{mi} + \Gamma^m_{ni} T^{ni} + \Gamma^i_{ln} T^{mn} = 0 \quad (5.1)$$

Let us multiply this equation by the Killing vector, that is, a vector  $\eta_m$  that satisfies Killing's equation

$$\nabla_m \eta_n + \nabla_n \eta_m = 0 \quad (5.2)$$

In view of the symmetry of the tensor,  $T^{mn} = T^{nm}$ , the equation  $\nabla_i T^{mi} = 0$  can be written thus:

$$\eta_{mi} T^{mi} = \nabla_i (\eta_m T^{mi}) = 0$$

If we employ the properties of a covariant derivative, we get

$$\nabla_i (\eta_m T^{mi}) = 0$$

Since the left-hand side of this equation is a scalar, we can multiply it by  $\sqrt{-g} dV$  and integrate over a certain volume. We then arrive at the following integral conservation law in the Riemann spacetime:

$$d/dx^0 \int \sqrt{-g} T^{0m} \eta_m dV = - \int \sqrt{-g} T^{\alpha m} \eta_m dS_\alpha \quad (5.3)$$

If the flux of the 3-vector through the surface surrounding the volume is nil, then

$$\int \sqrt{-g} T^{0m} \eta_m dV = \text{const} \quad (5.3')$$

Thus, if Killing vectors exist, then from the differential conservation equation (5.1) we can obtain the integral conservation laws (5.3) and (5.3').

Let us now establish what restrictions must be imposed on the Riemann spacetime metric so that Killing's equation (5.2) will have a solution, that is, the conditions that vector  $\eta_n$  must meet so that Eq. (5.2) is satisfied. We note, first, that Killing's equation (5.2) follows from the requirement that the Lie variations of the metric tensor of the Riemann spacetime under the infinitesimal coordinate transformations

$$x'^n = x^n + \eta^n(x) \quad (5.4)$$

vanish (here  $\eta^n(x)$  is an infinitesimal 4-vector). Indeed, under such a transformation of the coordinates the Lie variation of the metric tensor  $g_{in}$  assumes the form

$$\delta_L g_{in} = -\eta_{in} - \eta_{ni}$$

Comparing this with (5.2), we see that Killing's equation requires that the Lie variation of the metric tensor  $g_{in}$  vanish:  $\delta_L g_{in} = 0$

Thus, Killing vectors describe infinitesimal coordinate transformations that leave the metric form-invariant.

Killing's equation (5.2) constitutes a system of first-order partial differential equations. According to the general theory (see Eisenhart, 1933, Petrov, 1960, and Pontryagin, 1966), to establish the solvability conditions for a system of partial differential equations, we must reduce this system to the form

$$\partial 2\Theta^a / \partial x^i = \psi^a(\Theta^b, x^n) \quad (5.5)$$

where  $\Theta^a$  are the unknown functions;  $i, n = 1, 2, \dots, N$ ; and  $a, b = 1, 2, \dots, M$ . Then the solvability conditions for system (5.5) can be obtained from the relationship

$$\partial \Theta^a / \partial x^i \partial x^n = \partial \Theta^a / \partial x^n \partial x^i$$

by replacing the first-order partial derivatives with the right-hand side of Eqs. (5.5):

$$\psi^a_{,i} / \partial x^n + \psi^a_{,i} / \partial \Theta^b \psi^b_{,n} = \psi^a_{,n} / \partial x^i + \psi^a_{,n} / \partial \Theta^b \psi^b_{,i} \quad (5.6)$$

If the solvability condition (5.6) is met identically in view of the validity of Eqs. (5.5), the system (5.5) is said to be completely integrable and its solution will contain  $M$  parameters, the greatest possible number of arbitrary constants for the given system. But if (5.5) is not completely integrable, its solution will contain a smaller number of arbitrary constants. Let us establish the conditions in which the solution to Killing's equations (5.2) in the Riemann spacetime  $V_n$  contains the greatest possible number of parameters and find this number.

All calculations will be carried out in an explicitly covariant form, which is a generalization of the above scheme for finding the solvability conditions for the system of Killing's equations. We differentiate covariantly Killing's equations (5.2) with respect to parameter  $x^n$ . The result is

$$\eta_{i;jn} + \eta_{j;in} = 0$$

In view of this we have

$$\eta_{i;jn} + \eta_{j;in} + \eta_{i;nj} + \eta_{n;ji} - \eta_{j;ni} - \eta_{n;ji} = 0$$

Regrouping the terms in this expression, we get

$$\eta_{i;jn} + \eta_{i;nj} + (\eta_{j;in} - \eta_{j;ni}) + (\eta_{n;ji} - \eta_{n;ji}) = 0 \quad (5.7)$$

On the other hand, in view of the commutation relation for covariant derivatives we have

$$\eta_{i;jn} - \eta_{i;jn} = \eta_k R^k_{inj} \quad (5.8)$$

If we substitute this into (5.7), we get

$$2\eta_{i;jn} + \eta_k R^k_{inj} + \eta_k R^k_{jin} + \eta_k R^k_{njj} = 0 \quad (5.9)$$

Using Ricci's identity we get

$$R^k_{inl} + R^k_{nli} + R^k_{ljn} = 0 \quad (5.10)$$

which means that we can write (5.9) in the following form:

$$\eta_k R^k_{inj} + \eta_k R^k_{jin} = \eta_k R^k_{nij}$$

$$\eta_{i;jn} = -\eta_k R^k_{nij}$$

We have, therefore, arrived at the following covariant equations:

$$\eta_{i;n} + \eta_{n;i} = 0 \quad \eta_{i;jn} = -\eta_k R^k_{nij} \quad (5.11)$$

Let us transform this system of covariant differential equations into a system containing only first covariant derivatives. To this end, in addition to the  $N$  unknown components of vector  $\eta_{km}$ , we introduce an unknown tensor  $\lambda_{jm}$  that obeys the equation

$$\eta_{i;n} = \lambda_{jm} \quad (5.12)$$

This tensor contains  $N^2$  unknown components, but only  $N(N + 1)/2$  of these are independent since this tensor is antisymmetric in view of Eqs.(5.2) and (5.12):

$$\lambda_{mi} + \lambda_{jm} = 0 \quad (5.13)$$

If we allow for all this, the sought system of covariant differential equations assumes the form

$$\eta_{m;i} = \lambda_{mi}, \quad \lambda_{m;ij} = \eta_k R^k_{jim} \quad (5.14)$$

We have, therefore, reduced Killing's equations (5.2) to a system of a special type consisting of linear differential equations in first-order covariant derivatives.

This system is a covariant generalization of system (5.5), with the unknown functions  $\Theta^a$  being the  $N(N + 1)/2$  components of tensors  $\eta_m$  and  $\lambda_{mi}$

$$\Theta^a = \{\eta_m, \lambda_{mi}\}$$

The solvability condition for system (5.14) can be obtained from the commutation relation for covariant derivatives, which follows from the independence of the order in which derivatives are taken in partial differentiation. On the basis of this rule we get

$$\begin{aligned} \eta_{i;mj} - \eta_{i;jm} &= \eta_k R^k_{imj} \\ \lambda_{m;ij} - \lambda_{m;ji} &= \lambda_{ik} R^k_{mjl} + \lambda_{km} R^k_{ijl} \end{aligned} \quad (5.15)$$

Replacing the first covariant derivatives on the left-hand sides of (5.15) with their expressions (5.14) and employing (5.13), which reflects the fact that  $\lambda_{im}$  is antisymmetric, we arrive at the solvability conditions for system (5.14) in the form



$$\lambda_{im;j}\lambda_{ij;m}=\eta_k R^k_{imj} \quad (5.16)$$

$$(\eta_k R^k_{jmi})_{;l}-(\eta_k R^k_{lmj})_{;j}=\lambda_{ik}R^k_{mj}+\lambda_{km}R^k_{ijl} \quad (5.17)$$

It is easily verified that (5.16) is satisfied identically because of the validity of Eqs. (5.14) and the properties of the curvature tensor. Thus, if condition (5.17) is satisfied identically solely because of the symmetry properties of the Riemann spacetime, then system (5.14) will be completely integrable and, hence, the solution to Killing's equations (5.2) will contain the greatest possible number  $M=N(N+1)/2$  of arbitrary constants. Since the unknown functions  $\eta_m$  and  $\lambda_{ml} = -\lambda_{lm}$  entering into system (5.14) must be independent in this case, the left-hand side of (5.17) vanishes identically only if

$$R^k_{mij;l}R^k_{lij,m}=0 \quad (5.18)$$

$$\delta^n_j R^k_{iml} - \delta^k_j R^n_{jml} - \delta^n_i R^k_{jml} + \delta^k_i R^n_{jml} + \delta^n_l R^k_{mij} - \delta^n_l R^k_{mij} - \delta^n_m R^k_{lij} + \delta^k_m R^n_{lij} = 0 \quad (5.19)$$

If we contract (5.19) on  $l$  and  $n$  and allow for the relationships  $R^n_{lmn} = R_{lm}$  and  $R^n_{nml} = 0$  and for Ricci's identity (5.10), we get

$$(N-1) R^k_{mij;l} - R^k_{lij,m} = 0$$

From this it follows that

$$R_{lmij} = 1/N-1 (g_{jl} R_{mi} - g_{ji} R_{lm}) \quad (5.20)$$

Multiplying this equation into  $g^{mi}$ , we get

$$N R_{jl} = g_{jl} R$$

If we now substitute this into (5.20), we arrive at a condition in view of which (5.19) is satisfied identically:

$$R_{lmij} = R/N(N-1) [g_{jl}g_{mi} - g_{ji}g_{lm}] \quad (5.21)$$

Combining (5.21) and Eq. (5.18) results in a requirement that the scalar curvature must satisfy:

$$[\delta^k_j g_{im} - \delta^k_i g_{jm}] \partial/\partial x^l R - [\delta^k_j g_{il} - \delta^k_i g_{jl}] \partial/\partial x^m R = 0$$

If we multiply this by  $\delta^l_k g^{mi}$  we get

$$(N-1) \partial R/\partial x^j = 0$$

Since in the case considered  $N > 1$ , the above condition is met if and only if  $R$  const. Hence, the solvability conditions (5.18) and (5.19) for Killing's equations (5.2) will be satisfied identically if and only if the Riemann spacetime curvature tensor has the form

$$R_{lmij} = R/N(N-1) [g_{jl}g_{mi} - g_{ji}g_{lm}]$$

with  $R = \text{const.}$

Thus, Killing's equations have solutions containing the greatest possible number  $M = N(N - 1)/2$  of arbitrary constants (parameters) if and only if the Riemann spacetime  $V_n$  is a constant-curvature space, and if  $V$  is not a constant-curvature space, the number of parameters will be smaller than  $M$ .

Hence, mathematically speaking, the presence of integral conservation laws for energy-momentum and angular momentum reflects the existence of certain properties inherent in spacetime: its homogeneity and isotropy. There are three types of four-dimensional spaces that possess the properties of homogeneity and isotropy to an extent that allows for introducing ten integrals of motion for a closed system. These are the space with constant negative curvature (Lobachevski's space, or hyperbolic space), the zero-curvature space (pseudo-Euclidean space), and the space with constant positive curvature (Riemann space). The first two spaces are infinite, with an infinite volume, while the third is closed, with a finite volume, but has no boundaries.

Let us now find the Killing vector in an arbitrary curvilinear system of coordinates of the pseudo-Euclidean spacetime. To this end we first write Killing's equations in the Cartesian system of coordinates:

$$\partial_i \eta_n - \partial_n \eta_i = 0$$

Hence, to determine a Killing vector we have a system of ten first-order partial linear differential equations. Solving this system according to general rules, we obtain

$$\eta_i = a_i + \omega_{im}x^m \quad (5.22)$$

where  $a_i$ , is an arbitrary constant infinitesimal vector, and  $\omega_{im}$  is an arbitrary constant infinitesimal tensor satisfying the condition

$$\omega_{im} = -\omega_{mi}$$

Thus, solution (5.22), as expected, contains all ten arbitrary parameters. Since (5.22) contains ten independent parameters, we actually have ten independent Killing vectors, and (5.22) constitutes a linear combination of the ten independent vectors.

Let us establish the meaning of these parameters. Substituting (5.22) into (5.4), we get

$$x'^n = x^n + a^n + \omega_m^n x^m \quad (5.23)$$

We see that the four parameters  $a^n$  are components of the 4-vector of infinitesimal translations of the reference frame. The three parameters  $\omega_{\alpha\beta}$  are the components of the tensor of rotation through an infinitesimal angle about a certain axis (the so-called proper rotation). The three parameters  $\omega_{0\beta}$  describe infinitesimal rotations in the  $(x^0, x^\beta)$  plane, known as Lorentzian rotations. Since the metric tensor  $\gamma^{mn}$  is form-invariant under translations, the pseudo-Euclidean spacetime is homogeneous; its properties do not depend on the position of the origin in the space. Similarly, the form-invariance of the metric tensor  $\gamma^{mn}$  under four-dimensional rotations leads to the isotropy of this space, which means that all directions in the pseudo-Euclidean spacetime have equal status.

Thus, the pseudo-Euclidean spacetime admits of a ten-parameter group of motions consisting of a four-parameter translation subgroup and a six-parameter rotation

subgroup. The existence of this group of motions and the corresponding Killing vectors guarantees ten integral laws of conservation of energy-momentum and angular momentum in a system of interacting fields. Indeed, allowing for the fact that in the Cartesian system of coordinates  $v\text{-}\gamma = 1$  and for the general relationship (5.3), we find that in the case of the translation sub-group ( $\eta_i = a_i$ )

$$d/dx^0 \int T^{0m} a_m dV = -\xi dS_\alpha T^{\alpha m} a_m$$

Since  $a_m$  is an arbitrary constant vector, this relationship yields

$$d/dx^0 \int T^{0m} dV = -\xi dS_\alpha T^{\alpha m}$$

For an isolated system of interacting fields, the expression on the right-hand side of this relationship vanishes, as a result of which the total 4-momentum of the system is conserved:

$$P^m = \int T^{0m} dV = \text{constante} \quad (5.24)$$

Similarly, at  $\eta_n = \omega_{nm} x^m$  we get

$$d/dx^0 \int dV T^{0m} x^n \omega_{mn} = -\xi dS_\alpha T^{\alpha m} x^n \omega_{mn}$$

Since the constant tensor  $\omega_{mn}$  is antisymmetric, the above leads us to the following integral conservation law for angular momentum

$$d/dx^0 \int dV [T^{0m} x^n - T^{0n} x^m] = -\xi dS_\alpha [T^{\alpha m} x^n - T^{\alpha n} x^m] \quad (5.25)$$

For an isolated system the total angular momentum is conserved because the right-hand side of (5.25) vanishes:

$$M^{mn} = \int dV [T^{0m} x^n - T^{0n} x^m] = \text{constante} \quad (5.26)$$

Only in the pseudo-Euclidean spacetime are there separate laws of conservation for energy-momentum and angular momentum of a closed system.

Note that we can obtain the solution to Killing's equations (5.2) in arbitrary curvilinear coordinates of the pseudo-Euclidean spacetime in view of the tensor nature of  $x^i$  and  $\eta^i$  from the solution (5.23) to these equations in the Cartesian coordinate system. To this end we transfer in (5.23) from Cartesian coordinates  $x^i$  to arbitrary curvilinear coordinates  $x_N^i$  thus:

$$x^i = f^i(x_N)$$

This yields

$$\eta_m^N = \partial f^i / \partial x_N^m \eta^i [x(x_N)]$$

Thus, in an arbitrary curvilinear coordinate system of the pseudo-Euclidean spacetime, the Killing vectors have the form

$$\eta_m^N = \partial f^i(x_N) / \partial x_N^m a^i + \partial f^i(x_N) / \partial x_N^m \omega_{in} f^n(x_N) \quad (5.27)$$

It is not very difficult to generalize Eqs. (5.24)-(5.26) so that they incorporate the case of arbitrary curvilinear coordinates. Proceeding in the same manner as we did above, we arrive at the following expression for the 4-momentum of an isolated system:

$$P^i = \int V - \psi(x_N) dx^1_N dx^2_N dx^3_N \partial^i(x_N) / \partial x^m_N T^{0m}(x_N)$$

The antisymmetric tensor of angular momentum in this case has the form

$$M^{im} = \int V - \psi(x_N) dx^1_N dx^2_N dx^3_N T^{0n}(x_N) [f^m(x_N) \partial^i(x_N) / \partial x^n_N - f^i(x_N) \partial^m(x_N) / \partial x^n_N]$$

Thus, the geometry of spacetime determines the possibility of obtaining integral conservation laws. In the case of four dimensions (the physical spacetime) only spaces with constant curvature possess all ten integral conservation laws; in other spaces the number of these laws is less.

Our analysis demonstrates that if we wish to have the greatest number of conserved quantities, we must reject Riemannian geometry in its general form, and for all fields, including the gravitational, we must select one of the above mentioned geometries of constant curvature as the natural one. Since the existing experimental data on strong, weak, and electromagnetic interactions suggests that for the fields related to these interactions the natural geometry of spacetime is pseudo-Euclidean, we can assume at least at the present level of our knowledge that this geometry is the universal natural geometry for all physical processes, including those involving gravitation.

This assertion constitutes one of the main theses of our approach to the theory of gravitational interaction. It obviously leads to the observance of all laws of conservation of energy-momentum and angular momentum and ensures the existence of all ten integrals of motion for a system consisting of a gravitational field and other material fields.

As we will shortly show, the gravitational field in our framework, as all other physical fields, is characterized by an energy-momentum tensor that contributes to the total tensor of energy-momentum of the system. This constitutes the main difference between our approach and Einstein's. It must also be noted that in the pseudo-Euclidean spacetime the integration of tensor quantities, in addition to its general simplicity, has a well-defined meaning.

Another key issue that emerges in constructing a theory of the gravitational field is the question of the way in which the field interacts with matter. In acting on matter, a gravitational field changes the geometry of matter if it enters into terms in the highest-order derivatives in the equations of motion of the matter. Then the motion of material bodies and other physical fields in the pseudo-Euclidean spacetime under the action of the gravitational field no way be distinguished from their motion in an effective Riemann spacetime.

Experimental data suggests that the action of a gravitational field on matter is universal. This led us to formulate the geometrization principle. Hence, the effective Riemann space-time will be universal for all forms of matter.

The geometrization principle was formulated in Denisov and Logunov, 1982a, 1982c, Denisov, Logunov, and Mestvirishvili, 1981a, and Logunov, Denisov, Vlasov, Mestvirishvili, and Folomeshkin, 1979, but actually the idea was first put forward in

Logunov and Folomeshkin, 1977. The principle means that the description of the motion of matter under the action of a gravitational field in a pseudo-Euclidean spacetime is physically identical to the description of the motion of matter in the appropriate effective Riemann spacetime.

In this approach the gravitational field (as a physical field) is excluded, so to say, from the description of the motion of matter, and the field's energy, figuratively speaking, is spent on forming the effective Riemann spacetime. Thus, the effective Riemann spacetime is a peculiar carrier of energy-momentum. The amount of energy used for creating this spacetime is exactly equal to the amount contained in the gravitational field; hence, the propagation of curvature waves in the Riemann spacetime reflects common energy transfer via gravitational waves in the pseudo-Euclidean spacetime. This means that in our approach the existence of curvature waves in the Riemann spacetime follows directly from the existence of gravitational waves in the sense of Faraday and Maxwell, waves that carry an energy-momentum density.

We note also that when we introduce the geometrization principle, we thereby retain Einstein's idea of the Riemannian geometry of spacetime for matter. This does not mean, however, that we must inevitably return to GR. The general theory of relativity constitutes a partial realization of this idea, rather than the other way round. Hence, the idea of a gravitational field as a physical field that can carry energy changes our conceptions about spacetime and gravity. The relativistic theory of gravitation, which realizes this idea, makes it possible to describe the entire body of data on gravitational experiments, satisfies the correspondence principle, and leads to a number of fundamental corollaries" [9].

## **12. The geometrization principle in RTG.**

The GTR introduced the principle of geometrization (or metric) by explaining gravity as the effect of spacetime geometry on the motion of matter as opposed to all fields in nature such as the weak and strong electromagnetic fields that are physical fields while gravity a metric field. From this principle it is derived that the equations of motion of matter under the action of extended gravity can be represented by equations of motion in a pseudo-Riemannian variety or, what is the same, extended gravity can be described by geodesic motion in a spacetime with constant positive curvature. Thus the metric is responsible for the gravitational interaction, which would cause the acceleration and attraction that is observed in the general gravitational phenomenon.

In RTG also the relationship between matter and spacetime is described by the metric tensor. But the principle of geometrization, on the other hand, is that the Minkowski pseudo-Euclidean space dependent on the energy-impulse tensors of matter and the gravitational field is equal to the effective Riemann spacetime only dependent on the energy-impulse tensor of matter, therefore, in the absence of the gravitational field.

The conservation law of the total energy-impulse tensor,  $t^{\mu\nu}$ , given in Minkowski spacetime, states that the energy-impulse of matter and gravitational field taken together are conserved. Meanwhile, the Riemann spacetime arises as a result of the action of the gravitational field, present in the Minkowski spacetime, in all forms of matter therefore, it is the effective

Riemann spacetime, that is, originated in the gravitational field existing in Minkowsky's pseudo Euclidean spacetime. In contrast to GTR, in RTG no pseudo-impulse-energy tensor can arise, so all non-physical conceptions, derived from the impossibility of locating the gravitational field, are not possible.

“Without loss of generality, let us assume that the tensor density  $\hat{g}^{ik}$  of the metric tensor of the Riemann spacetime is a local function that depends on the density  $\hat{g}^{ik}$  of the metric tensor of the Minkowski spacetime and the density  $\hat{t}^{ik}$  of the gravitational-field tensor. We assume that the material Lagrangian density  $L_M$  is dependent only on the fields  $\Phi_A$ , on their first-order covariant derivatives, and, in view of the geometrization principle, on  $\hat{g}^{ik}$ . We also assume that the gravitational-field Lagrangian density depends on  $\hat{y}^{ik}$ , on the first-order partial derivatives of  $\hat{y}^{ik}$ , on  $\hat{t}^{ik}$  and on the first-order covariant derivatives of  $\hat{t}^{ik}$  with respect to the Minkowski metric. To derive conservation laws we employ the invariance of the action integral under infinitesimal translations of the coordinates. Since for every given Lagrangian density  $L$  the action integral  $J = \int L d^4x$  is a scalar, under an arbitrary infinitesimal coordinate transformation the variation  $\delta J$  vanishes. Let us start by calculating the variation of the material action integral  $J_M = \int L_M d^4x$  brought on by the transformation

$$X'^i = x^i + \xi^i(x) \quad (6.1)$$

where  $\xi^i(x)$  is an infinitesimal 4-vector of displacement:

$$\delta J_M = \int d^4x [\delta L_M / \delta \hat{g}^{mn} \delta_L \hat{g}^{mn} + \delta L_M / \delta \Phi_A \delta_L \Phi_A + \text{div}] = 0 \quad (6.2)$$

Here  $\text{div}$  stands for the divergence terms, which in the present chapter play no role in our discussion.

The Eulerian variation is defined in the usual way:

$$\delta L / \delta \varphi \equiv \partial L / \partial \varphi - \partial_n \partial L / \partial (\partial_n \varphi) + \partial_n \partial_k \partial L / \partial (\partial_n \partial_k \varphi) \dots$$

The variations  $\delta_L \hat{g}^{mn}$  and  $\delta_L \Phi_A$  generated by the coordinate transformation (6.1) can easily be calculated if we employ the transformation laws:

$$\delta_L \hat{g}^{mn} = \hat{g}^{kn} D_k \xi^m + \hat{g}^{km} D_k \xi^n - D_k (\xi^k \hat{g}^{mn}) \quad (6.3)$$

$$\delta_L \Phi_A = \xi^k D_k \Phi_A + F^{B;A;}_k \Phi_B D_n \xi^k \quad (6.4)$$

Here and in what follows the  $D_k$  are the covariant derivatives with respect to the Minkowski metric. Substituting (6.3) and (6.4) into (6.2) and integrating by parts, we get

$$\begin{aligned} \delta J_M = \int d^4x \{ & - \xi^m [D_k (2 \delta L_M / \delta \hat{g}^{mn} \delta_L \hat{g}^{kn}) - D_m (\delta L_M / \delta \hat{g}^{lp} \delta_L \hat{g}^{lp}) \\ & + D_k (\delta L_M / \Phi_A F^{B;A;}_k \Phi_B) + \delta L_M / \Phi_A D_m \Phi_A] + \text{div} \} = 0 \end{aligned}$$

Since the vector  $\xi^m$  is arbitrary, the condition  $\delta J_M = 0$  yields the following strong identity:

$$D_k (2 \delta L_M / \delta \hat{g}^{mn} \delta_L \hat{g}^{kn}) - D_m (\delta L_M / \delta \hat{g}^{lp} \delta_L \hat{g}^{lp}) = - D_k (\delta L_M / \Phi_A F^{B;A;}_k \Phi_B) - \delta L_M / \Phi_A D_m \Phi_A \quad (6.5)$$

which is valid irrespective of whether or not the equations of motion of the fields are valid.

Let us introduce the following notation:

$$T_{mn} = 2 \delta L_M / \delta g^{mn}, T^{mn} = -2 \delta L_M / \delta g_{mn} = g^{mk} g^{np} T_{kp} \quad (6.6a)$$

$$\check{T}_{mn} = 2 \delta L_M / \delta \hat{g}^{mn}, \check{T}^{mn} = -2 \delta L_M / \delta \hat{g}_{mn} = \hat{g}^{mk} \hat{g}^{np} T_{kp} \quad (6.6b)$$

where  $T_{mn}$  is the material energy-momentum tensor density in the Riemann spacetime and is known as the Hilbert-tensor density.

If we allow for (6.6b), we can represent the left-hand side of (6.5) in the following form:

$$D_k (\check{T}_{mn} \hat{g}^{kn}) - \frac{1}{2} \hat{g}^{kp} D_m \check{T}_{kp} = \partial_k (\check{T}_{mn} \hat{g}^{kn}) - \frac{1}{2} \hat{g}^{kp} \partial_m \check{T}_{kp}$$

The right-hand side of this equation can easily be reduced to

$$\partial_k (\check{T}_{mn} \hat{g}^{kn}) - \frac{1}{2} \hat{g}^{kp} \partial_m \check{T}_{kp} = \hat{g}_{mnk} (\check{T}^{kn} - \frac{1}{2} \hat{g}^{kn} \check{T}) \quad (6.7)$$

where  $\check{T} = \hat{g}_{kp} \check{T}^{kp}$  and  $k$  is the symbol of covariant differentiation with respect to the metric of the Riemann spacetime.

On the basis of (6.7) we can now write the strong identity (6.5) in the following form:

$$\hat{g}_{mnk} (\check{T}^{kn} - \frac{1}{2} \hat{g}^{kn} \check{T}) = -D_k (\delta L_M / \Phi_A F^{B_i, k}_{A, m} \Phi_B) - \delta L_M / \Phi_A D_m \Phi_A \quad (6.8)$$

In view of the principle of least action the equations of motion for the material fields have the form

$$\delta L_M / \Phi_A = 0 \quad (6.9)$$

Combining this with (6.8) results in the weak identity

$${}_m (\check{T}^{mn} - \frac{1}{2} \hat{g}^{mn} \check{T}) = 0 \quad (6.10)$$

Note that the energy-momentum tensor density for matter,  $T^{mn}$  in the Riemann spacetime is related to  $\check{T}^{mn}$  in the following manner:

$$\sqrt{-g} T^{mn} = \check{T}^{mn} - \frac{1}{2} \hat{g}^{mn} \check{T} \quad (6.11)$$

Hence, (6.10) results in the following covariant equation of conservation of matter in the Riemann spacetime:

$${}_m T^{mn} = 0 \quad (6.12)$$

Only if the number of equations for a material field is four can we use instead of the equations (6.9) for this field the equivalent equations (6.12). The variation of the action integral (6.2) can be written in the equivalent form

$$\delta J_M = \int d^4x \{ \delta L_M / \delta \Phi'^{mn} \delta_L \Phi'^{mn} + \delta L_M / \delta \check{\gamma}^{mn} \delta_L \check{\gamma}^{mn} + \delta L_M / \delta \Phi_A \delta_L \Phi_A + \text{div} \} = 0 \quad (6.13)$$

where the variations  $\delta_L \Phi'^{mn}$  and  $\delta_L \tilde{\gamma}^{mn}$  generated by the coordinate transformation (6.1) are

$$\delta_L \Phi'^{mn} = \Phi'^{kn} D_k \xi^m + \Phi'^{km} D_k \xi^n - D_k (\xi^k \Phi'^{mn}) \quad (6.14)$$

$$\delta_L \tilde{\gamma}^{mn} = \tilde{\gamma}^{kn} D_k \xi^m + \tilde{\gamma}^{km} D_k \xi^n - \tilde{\gamma}^{mn} D_k \xi^k \quad (6.15)$$

Substituting the expressions for the variations  $\delta_L \Phi'^{mn}$ ,  $\delta_L \tilde{\gamma}^{mn}$  and  $\delta_L \Phi_A$  into (6.13) and integrating by parts, we arrive, in view of the arbitrariness of  $\xi^m$  at the following strong identity:

$$\begin{aligned} & D_k (2 \delta L_M / \delta \Phi'^{mn} \Phi'^{kn}) - D_m (2 \delta L_M / \delta \Phi'^{kp}) \Phi'^{kp} + D_k (2 \delta L_M / \delta \tilde{\gamma}^{mn}) \tilde{\gamma}^{kn} - D_m (\delta L_M / \delta \tilde{\gamma}^{kp}) \tilde{\gamma}^{kp} = \\ & - D_k (\delta L_M / \Phi_A F^{B;A; k}_m \Phi_B) - \delta L_M / \Phi_A D_m \Phi_A \end{aligned} \quad (6.16)$$

which, like (6.5), is valid irrespective of whether or not the equations of the motion of matter and gravitational field are valid.

For an arbitrary Lagrangian we introduce several notations and relationships that will be used later:

$$\dot{t}^{mn} = -2 \delta L / \delta \tilde{\gamma}_{mn}, \quad t^{mn} = -2 \delta L / \delta \tilde{\gamma}_{mn} \quad (6.17a)$$

$$t^{mn} = 1/\sqrt{-\gamma} (\dot{t}^{mn} - \frac{1}{2} \tilde{\gamma}^{mn} \dot{t}) \quad (6.17b)$$

Since  $L_M$  depends, in view of the geometrization principle, on  $\tilde{\gamma}^{mn}$  only through  $\hat{g}^{mn}$ , we can easily find the relationship linking  $\check{T}^{mn}$  and  $\dot{t}_{(M)mn}$

$$\dot{t}_{(M)mn} = 2 \delta L_M / \delta \tilde{\gamma}^{mn} = \check{T}_{hp} \partial \hat{g}^{kp} / \partial \tilde{\gamma}^{mn} \quad (6.18a)$$

where we have allowed for definition (6.6b). Taking into account the identity

$$\partial \hat{g}^{kp} / \partial \tilde{\gamma}^{mn} = - \tilde{\gamma}^{ml} \tilde{\gamma}^{nq} \partial \hat{g}^{kp} / \partial \tilde{\gamma}^{lq}$$

and combining it with (6.17a), we get

$$\dot{t}_{(M)mn} = - \check{T}_{pk} \partial \hat{g}^{pk} / \partial \tilde{\gamma}_{mn} \quad (6.18b)$$

Allowing in (6.18b) by identity (6.6b) and for the fact that

$$- \hat{g}_{lp} \hat{g}_{qk} \partial \hat{g}^{lq} / \partial \tilde{\gamma}_{mn} = \partial \hat{g}_{pk} / \partial \tilde{\gamma}_{mn}$$

we find that (6.18b) yields

$$\dot{t}_{(M)mn} = \check{T}^{pk} \partial \hat{g}_{pk} / \partial \tilde{\gamma}_{mn} \quad (6.18c)$$

Now, if we compare the identities (6.8) and (6.16) and allow for (6.17a), we obtain

$$\begin{aligned} \hat{g}_{mnk} (\check{T}^{kn} - \frac{1}{2} \hat{g}^{kn} \check{T}) &= \tilde{\gamma}_{mn} D_k (\dot{t}_{(M)}^{kn} - \frac{1}{2} \tilde{\gamma}^{kn} \dot{t}_{(M)}) + D_k (2 \delta L_M / \delta \Phi'^{mn} \Phi'^{kn}) \\ &- D_m (\delta L_M / \delta \Phi'^{kp}) \Phi'^{kp} \end{aligned} \quad (6.19)$$



Similarly, from the invariance of the gravitational-field action integral  $J_g = \int L_g d^4x$  under coordinate transformation to (6.1) it follows that

$$\tilde{\gamma}_{mn} D_k (\dot{t}_{(g)}^{kn} - \frac{1}{2} \tilde{\gamma}^{kn} \dot{t}_{(g)}) + D_k (2 \delta L_g / \delta \Phi'^{mn} \Phi'^{kn}) - D_m (\delta L_g / \delta \Phi'^{kp}) \Phi'^{kp} = 0 \quad (6.20)$$

Adding (6.19) to (6.20), we get

$$\hat{g}_{mnk} (\check{T}^{kn} - \frac{1}{2} \hat{g}^{kn} \check{T}) = \tilde{\gamma}_{mn} D_k (\dot{t}^{kn} - \frac{1}{2} \tilde{\gamma}^{kn} \dot{t}) + D_k (2 \delta L / \delta \Phi'^{mn} \Phi'^{kn}) - D_m (\delta L / \delta \Phi'^{kp}) \Phi'^{kp} \quad (6.21)$$

Here and in what follows

$$\dot{t}^{kn} = \dot{t}_{(M)}^{kn} + \dot{t}_{(g)}^{kn} \quad (6.22)$$

Owing to the principle of least action, the equations for the gravitational field assume the form

$$\delta L / \delta \Phi'^{mn} = \delta L_g / \delta \Phi'^{mn} + \delta L_M / \delta \Phi'^{mn} = 0 \quad (6.23)$$

Allowing for these equations, we see that (6.21) yields the most important equality:

$$\hat{g}_{mnk} (\check{T}^{kn} - \frac{1}{2} \hat{g}^{kn} \check{T}) = \tilde{\gamma}_{mn} D_k (\dot{t}^{kn} - \frac{1}{2} \tilde{\gamma}^{kn} \dot{t}) \quad (6.24)$$

Since the density of the total energy-momentum tensor in the Minkowski spacetime is given by the formula

$$\nabla_\gamma t^{kn} = \dot{t}^{kn} - \frac{1}{2} \tilde{\gamma}^{kn} \dot{t} \quad (6.25)$$

combining this expression with (6.11) we find that (6.24) can be written in the following form:

$$D_m t^m_n = {}_m T^m_n \quad (6.26)$$

This formula represents the geometrization principle, namely, that the covariant divergence in the pseudo-Euclidean space of the sum of the tensor densities of energy-momentum of matter and gravitational field taken together is exactly equal to the covariant divergence in the effective Riemann spacetime of only the energy-momentum tensor density of matter. If the equations of motion of matter hold true, we have

$$D_m t^m_n = {}_m T^m_n = 0 \quad (6.27)$$

In our discussion we have assumed that the equations of motion of matter are not corollaries of the equations (6.23) for the gravitational field, since only in this case will the system of equations (6.23), (6.27) be complete for determining the material and gravitational-field variables. The covariant equation of matter conservation in the Riemann spacetime does not provide a clear picture of which quantity is conserved, while the law of conservation of the total energy-momentum tensor  $t^m_n$  in the Minkowski spacetime clearly states that the energy-momentum of matter and gravitational field taken together is conserved. Thus, in the present theory the

Riemann spacetime emerges as a result of the action of the gravitational field on all forms of matter, hence this spacetime is the effective Riemann spacetime of field origin. The Minkowski spacetime finds its precise physical reflection in the laws of conservation of the tensors of energy-momentum and angular momentum of matter and gravitational field taken together.

Since in flat spacetime there are ten Killing vectors, there must be ten conserved integral quantities for a closed system of fields. Also, since the equation that reflects the conservation of the total energy-momentum tensor in the Minkowski spacetime,

$$D_m t^m_n = D_m (t_{(M)}^m_n + t_{(g)}^m_n) = 0 \quad (6.28)$$

is equivalent to the covariant equation representing matter conservation in the Riemann spacetime, and the latter is equivalent to the equations of motion of matter, we can use Eq. (6.28) instead of the equation of motion of matter.

It must be especially noted that both matter and gravitational field are characterized in the given theory by energy-momentum tensors and, therefore, in contrast to GR, there cannot in principle emerge any pseudotensors, with the result that all nonphysical conceptions about the impossibility of localizing the gravitational field are absent from our theory.

If we were to take, following Hilbert and Einstein, the gravitational-field Lagrangian density in a completely geometrized form, that is, depending only on the metric tensor  $g^{ik}$  of the Riemann spacetime and its derivatives,  $L_g = \sqrt{-g} R$  with  $R$  the scalar curvature of the Riemann spacetime, then the energy-momentum tensor density of a free gravitational field in the Minkowski spacetime would, in view of the field equations, vanish everywhere:

$$\delta L_g / \delta \gamma^{mn} = \delta L_g / \delta g^{pk} \delta g^{pk} / \delta \gamma^{mn} = 0 \quad (6.29)$$

Thus, if we take the Minkowski spacetime and a physical tensor field possessing energy and momentum, we cannot in principle build a completely geometrized gravitational-field Lagrangian. Therefore, a theory based on a completely geometrized Lagrangian cannot in principle describe a physical gravitational field in the sense of Faraday and Maxwell in the Minkowski spacetime. It has been stated in the literature (e.g. see Ogievetsky and Polubarinov, 1965a, 1965b) that employing a tensor field with spin 2 in the Minkowski spacetime results unambiguously in a GR gravitational-field Lagrangian equal to  $ft$ . However, such statements carry no physical meaning because the gravitational-field energy-momentum tensor introduced in the argument is zero, as (6.29) clearly shows. Therefore, such research is physically meaningless and the results are erroneous" [9].

### 13. The Basic Identity.

It is essential to RTG to use the “basic identity” in the construction of its equations in such a way that the Riemann metric tensor can be applied with the Minkowsky pseudo Euclidean spacetime and thus satisfy the law of conservation of energy-momentum of the gravitational field along with the rest fields of matter.

“As shown in Barnes, 1965, and Fronsdal, 1958, the symmetric second-rank tensor can be expanded in a direct sum of irreducible representations, one with spin 2, one with spin 1, and two with spin 0:

$$f^i = (P_2 + P_1 + P_0 + P_0')^{lm}_{ik} f^{ik} \quad (7.1)$$

we denote the projection operators, which satisfy the following standard relationships:  $P_3 P_t = \delta^t_s P_t$ , (here there is no summation over  $l$ ),

$$P^{in}_{s;in} = (2s + 1), \quad \Sigma_s P^{lm}_{s;ik} = \frac{1}{2} (\delta^l_i \delta^m_k + \delta^m_i \delta^l_k) \equiv \delta^{lm}_{ik} \quad (7.2)$$

It is convenient to first write the operators  $P_s$ , in the momentum representation, to this end we introduce the following auxiliary (projection) quantities:

$$X_{ik} = 1/\sqrt{3}(\gamma^{ik} - q_i q_k / q^2), \quad Y_{ik} = q_i q_k / q^2 \quad (7.3)$$

It can be demonstrated that the operators  $P_s$ , satisfying (7.2) can be written, via (7.3), in the following form:

$$P^{mi}_{0;ni} = X_{ni} X^{lm}, \quad P^{mi}_{0';ni} = Y_{ni} Y^{ml} \quad (7.4)$$

$$P^{mi}_{1;ni} = \sqrt{3}/2 (X^l_i Y^m_n + X^m_n Y^l_i + X^m_i Y^l_n + X^l_n Y^m_i) \quad (7.5)$$

$$P^{mi}_{2;ni} = 3/2 (X^l_i X^m_n + X^m_i X^l_n) - X_{ni} X^{mi} \quad (7.6)$$

Formulas (7.4)-(7.6) show that the  $P^{mi}_{s;ni}$  are symmetric in the indices (ml) and (nl). In the x-representation the projection operators  $P_s$ , are nonlocal integro differential operators:

$$(P^{mi}_{s;ni} f^{ni}) = \int d^4 y P^{mi}_{s;ni}(x-y) f^{ni}(y)$$

The explicit expressions for  $P^{mi}_{0;ni}(x)$  and  $P^{mi}_{2;ni}(x)$  have the form

$$P^{mi}_{0;ni}(x) = \frac{1}{3} [\gamma^{im} \gamma_{in} \delta(x) + (\gamma^{im} \partial_i \partial_n + \gamma_{in} \partial^i \partial^m) D(x) + \partial_i \partial_n \partial^i \partial^m \Delta(x)] \quad (7.7)$$

$$\begin{aligned} P^{mi}_{2;ni}(x) = & (\delta^{lm}_{in} - \frac{1}{3} \gamma^{im} \gamma_{in}) \delta(x) + 2/3 \partial^i \partial^m \partial_i \partial_n \Delta(x) \\ & + [1/2 (\delta^l_i \partial^m \partial_n + \delta^m_n \partial^l \partial_i + \delta^i_n \partial^m \partial_i + \delta^m_i \partial^l \partial_n) \\ & - \frac{1}{3} (\gamma^{im} \partial_i \partial_n + \gamma_{in} \partial^i \partial^m)] D(x) \end{aligned} \quad (7.8)$$

In both (7.7) and (7.8),  $D(x)$  is the Green function of the wave equation

$$\square D(x) = -\delta(x) \quad (7.9)$$

and

$$\Delta(x) = \int a^4 y D(x-y) D(y)$$

We therefore have the equation

$$\square \Delta(x) = -D(x) \quad (7.10)$$

Using (7.7)-(7.10) we can easily verify that the operators  $P_0$  and  $P_2$ , are conserved, that is, obey the following identities.

$$\begin{aligned} \partial_i P_{0;ni}^{mi}(x) &= \partial^n P_{0;ni}^{mi}(x) \equiv 0 \\ \partial_i P_{2;ni}^{mi}(x) &= \partial^n P_{2;ni}^{mi}(x) \equiv 0 \end{aligned} \quad (7.11)$$

But the operators  $P_1$ , and  $P_0$  do not exhibit this property.

Expansion (7.1) implies that if the tensor field obeys the equation

$$\partial_i f^{im} = 0 \quad (7.12)$$

it does not contain the representations with spins 1 and 0'. This means that such a tensor field describes only spins 2 and 0.

In view of (7.7) and (7,8) it can easily be verified that the operator

$$\begin{aligned} \square(2P_0 - P_2)^{mn}_{;il} &= -(\delta^{mn}_{;il} - \frac{1}{2} \gamma^{mn} \gamma_{il}) \square \delta(x) - (\gamma^{mn} \partial_i \partial_l + \gamma_{il} \partial^m \partial^n) \delta(x) \\ &+ \frac{1}{2} (\delta^n_i \partial^m \partial_l + \delta^m_i \partial^n \partial_l + \delta^n_l \partial^m \partial_i + \delta^m_l \partial^n \partial_i) \delta(x) \end{aligned} \quad (7.13)$$

is the only second-order operator that is local and conserved. Acting with this operator on the function  $\varphi^{il} - \frac{1}{2} \gamma^{il} \varphi$ , where  $\varphi = \gamma^{pq} \varphi^{pq}$  and allowing for (7.7)-(7.10), we find that

$$\begin{aligned} \psi^{mn} &= \int \square y [2P_0(x-y) - P_2(x-y)]^{mn}_{;il} [\varphi^{il}(y) - \frac{1}{2} \gamma^{il} \varphi(y)] d^4 y \\ &= \partial_k \partial_p [\gamma^{nk} \varphi^{pm} + \gamma^{mk} \varphi^{pn} - \gamma^{kp} \varphi^{mn} - \gamma^{mn} \varphi^{kp}] \end{aligned} \quad (7.14)$$

The structure (7.14) for any symmetric tensor field is noteworthy in that it is local and linear, it contains only second-order derivatives, and it satisfies the conservation law, that is, the divergence of  $\psi^{mn}$  is identically nil:

$$\partial_m \psi^{mn} \equiv 0 \quad (7.15)$$

In what follows we will need structure (7.14) written in terms of the covariant derivatives of the metric-tensor density  $\hat{g}^{lm}$  with respect to the Minkowski metric:

$$J^{mn} = D_k D_p [\gamma^{np} \hat{g}^{km} + \gamma^{pm} \hat{g}^{kn} - \gamma^{kp} \hat{g}^{mn} - \gamma^{mn} \hat{g}^{kp}] \quad (7.16)$$

From (7.16) it follows that

$$D_m J^{mn} \equiv 0 \quad (7.17)$$

which we will call the basic identity, since it plays a fundamental role in the construction of RTG " [9].

#### 14. RTG equations.

Because Riemann space-time is necessary to quantitatively describe Mercury's orbit and the deflection of the electromagnetic wave under the action of a strong gravitational field like that of the Sun, but since there cannot be global Cartesian coordinates in this space, which prevent defining the gravitational field as a physical field, equations obtainable by a different methodology than the GTR are required.

The methodological objectives for preparing RTG equations are:

1. Riemannian geometry emerges as a certain effective geometry.
2. Its generation is by the action of a physical gravitational field on matter, in Minkowski's pseudo Euclidean spacetime.

It is essential that the condition is met:

For the construction of an effective Riemannian metric on the Minkowski spacetime variables to have physical meaning, it is required that the gravitational field equations contain the Minkowski  $\gamma^{ik}$  spacetime metric, which determines it as a physical tensor field, as it is not related to the choice coordinate systems. That is, the gravitational field cannot be an island type, as it is in the ordinary Galilean coordinates in an inertial frame of reference, of the Minkowski spacetime of Einstein's SRT, but must be in global Cartesian coordinates, therefore, where the equations can be written in general covariant form. Thus, the gravitational field has an energy-impulse density. Of course, the gravitational field as a Faraday-Maxwell type field in Minkowski spacetime, "as is common practice in elementary particle theory."

Broadly speaking, the steps to obtain the complete system of RTG equations are:

1. Four covariant equations of the gravitational field are obtained, in the Minkowski pseudo Euclidean spacetime.
2. The other ten equations of the gravitational field are obtained in a similar way as in the electromagnetic field, for which the Maxwell electrodynamics is taken as a model. But, while the equations of this field are invariant before gauge transformations, on the other hand, the equations of the gravitational field are different because the source of the same is the energy-impulse tensor of the fields of matter, which is not invariable under the gauge transformations of these fields. A gauge transformation does not modify any physical property and comes from quantum field theory that describes the physical interaction between different material fields. RTG's use of this approach makes the theory of the gravitational field a gauge theory.
3. As the final source of the gravitational field, it takes the energy-impulse tensor of matter and the gravitational field in the Minkowski spacetime, assuming the validity of the conservation law of this tensor and, as a corollary, the validity of the law of covariant conservation for matter in Riemann spacetime. The equations, even for a free gravitational field, are nonlinear, for that reason.
4. The equations of the motion of matter are obtained from the equations of the gravitational field.

5. The Hilbert-Einstein system of equations is incorporated into the system of equations for matter and the gravitational field of RTG, but, maintaining the substantial change of the physical gravitational field of RTG with respect to the metric gravitational field of GRT, since, In the resulting complete system of equations, the Riemann spatiotemporal variables continue to coincide with the variables of the Minkowski pseudo Euclidean spacetime. Thus, these equations of the gravitational field are universal, since they are equations of the gravitational field provided with virtual spin 2 and 0 gravitons; therefore, separating the inertial forces from the gravitational forces.

“Einstein declared that the metric tensor  $g^{ik}$  of the Riemann spacetime characterizes the gravitational field in GR. This, however, was a profound delusion and it must be discarded, since it is impossible to place physical boundary conditions on the behavior of  $g^{ik}$  because their asymptotics depends on the choice of the spatial coordinate system. In this chapter we construct, within the framework of relativity theory and the geometrization principle, the relativistic equations for matter and gravitational field.

The relationship between the effective metric of the field Riemann spacetime and the gravitational field can be chosen, by definition, to be

$$\hat{g}^{ik} = \nu \cdot g^{ik} = \nu \cdot \gamma^{ik} + \nu \cdot \Phi^{ik} \quad (8.1)$$

Hence, Riemannian geometry emerges here as a certain effective geometry, .generated by the action of a physical gravitational field in the Minkowski spacetime on matter. But for this construction of an effective Riemannian metric in the Minkowski spacetime variables to have physical meaning, wo must ensure that the gravitational-field equations contain the Minkowski spacetime metric  $\gamma^{ik}$ . In our theory the tensor  $\Phi^{ik}$  is the field variable of the gravitational field, and the physical boundary conditions must be formulated for this variable. We will assume that the gravitational field in general has only spins 2 and 0. These physical restrictions load in Galilean coordinates to the following four equations for the gravitational field:

$$\partial_i \Phi^{ik} = \partial_i \hat{g}^{ik} = 0 \quad (8.2)$$

The Riemannian geometry of spacetime is determined by fixing the metric-tensor field  $g_{ik}(x)$  in a certain system of coordinate maps. Although de Bondar, 1921, 1926, and Fock, 1939, 1957, 1959, used conditions of the (8.2) type in GR (they called them harmonic conditions), they were not able to show in which spacetime variables these conditions must be written. Nevertheless, Fock, in describing problems of the island type, as much as considered harmonic conditions in terms of global Cartesian coordinates. But where did he find global Cartesian coordinates? They have no place in Riemannian geometry. Intuitively he made a correct move, but he could not comprehend its significance. If he had clearly understood that Eqs. (8.2) are valid only in an inertial reference frame, in Galilean coordinates of the Minkowski spacetime, he could have arrived at the conception of a gravitational field as a physical tensor field in the Minkowski spacetime. Fock focused especially on the importance of harmonic coordinate conditions for the solution of island problems. For instance, he wrote (Fock, 1959): The above remarks concerning the privileged character of the harmonic system

of coordinates should not be understood, in any case, as same kind of prohibition of the use of other coordinate systems. Nothing is more alien to our point of view than such an interpretation. And further: ...the existence of harmonic coordinates, ... although a fact of primary importance in theoretical and practical systems does not exclude, the use of other non-harmonic coordinates. Fock also wrote (Fock, 1939): We believe that the possibility deserves to be noted of introducing, in the old relativity, a fixed inertial coordinate system in a unique manner.

Developing this idea, Fock could probably have arrived at the concept of a gravitational field possessing energy-momentum density. But he did not. Did he attempt to consider the gravitational field as one of the Faraday-Maxwell type in the Minkowski spacetime? No, he was far from this idea and explicitly said so ( Fock, 1939): We mention this only in connection with the wish observed at times (which wo in no way share) to place the theory of gravity into the framework of Euclidean space. In GR, as Fock wrote (Fock, 1959): Gravitational energy can be separated out in the form of additional terms in the energy tensor only in an artificial manner by fixing the coordinate system and reformulating the problem in such a way that the gravitational field is taken to be superimposed on a spacetime of fixed properties, just as is done in Newtonian theory. The additional terms in the energy tensor that correspond to gravitational energy do not possess the property of covariance (i.e. they do not form a tensor). And further: According to the choice of coordinate system the values of these terms at a given spacetime point may prove to be zero or non-zero, which would be impossible for a tensor (This is still not understood by some researchers— The authors). Therefore gravitational energy cannot be localized.

Irrespective of some insights, Fock was deeply averse to both the idea of the Minkowski space-time and the idea of the gravitational field of the Faraday-Maxwell type as playing any role in the theory of gravity. From the standpoint of our theory, Fock in solving island problems unconsciously dealt simply with ordinary Galilean coordinates in an inertial reference frame, and the latter, as is known from the theory of relativity, are preferred, of course. As a result, in his calculations involving island systems, the harmonic conditions emerged not as coordinate conditions, as he believed, but, as we will later see, as field equations in Galilean coordinates of an inertial reference frame.

Thus, Fock considered harmonic conditions only as preferred coordinate conditions and nothing more, and only for problems of the island typo. This is understandable, since lie, like all his great predecessors, was chained to Riemannian geometry, which in principle did not allow for a deeper penetration of the essence of the problem. To take this important step and advance these conditions as universal and covariant, it was necessary to repudiate the ideology of GR, get out of the jungle of Riemannian geometry, extend, contrary to the GR prescription, the principle of relativity to gravitational phenomena, and introduce the idea of a gravitational field as a physical field in the sense of Faraday and Maxwell, that is, possessing energy and momentum. All this has been done in our theory, with an arbitrary choice of system of coordinates, fixed only by the metric tensor  $y^{ik}$  of the Minkowski spacetime, as is common practice

in the theory of elementary particles. Equations (8.2) are universal in our theory, since they are equations governing the gravitational field and have no relation to the choice of the coordinate systems. In the Minkowski spacetime these equations can be written in covariant form as follows:

$$\nabla\text{-}\gamma D_i\Phi^{ik} = D_i\hat{g}^{ik} = 0 \quad (8.3)$$

Only in Cartesian (Galilean) coordinates do the field equations (8.3) assume the form of harmonic conditions. But writing the harmonic conditions within the GR framework in terms of Cartesian coordinates runs contrary to the GR ideology since in the Riemann spacetime there can be no global Cartesian coordinates.

On the basis of the numeral 13, we can say that the field equations (8.3) automatically exclude spins 1 and 0' from the gravitational tensor field. Thus, for the fourteen sought variables describing the gravitational field and matter we have already built four covariant equations (8.3). To construct the other ten, we use a simple but far reaching analogy with the electromagnetic field. Since any vector field  $A^n$  contains spins 1 and 0, it can be expanded in a direct sum of appropriate irreducible representations. This expansion can be realized via the projection operators (7.3) introduced in numeral 13:

$$A^n = X^n_m A^m + Y^n_m A^m \quad (8.4)$$

where operator  $X^n_m$ , is conserved, that is, satisfies the identities

$$\partial_n X^n_m = \partial^n X^n_m \equiv 0 \quad (8.5)$$

while operator  $Y^n_m$ , does not possess this property.

From electrodynamics it is known that the source of an electromagnetic field  $A^n$  conserved electromagnetic current  $J^n$ . Therefore, in constructing the equation of motion of the field it is natural to use the conserved operator  $X^n_m$ . This operator is nonlocal, but on its basis we can build a unique, local, linear, and conserved operator  $\square X^n_m$  that contains only second derivatives. Applying this operator to  $A^m$ , we get an expression which in terms of covariant derivatives has the form

$$\gamma^{mk} D_m D_k A^n - D^n D_m A^m$$

Postulating the equation

$$\gamma^{mk} D_m D_k A^n - D^n D_m A^m = 4\pi J^n \quad (8.6)$$

we arrive at the well-known Maxwell equations.

One the most important features of the electrodynamics equation (8.6) is that it is invariant under the following gauge transformation:

$$A^n \rightarrow A^n + D^n \varphi \quad (8.7)$$

with  $\varphi$  an arbitrary scalar function.



None of the physical quantities is affected by the gauge transformation (8.7). This means that none depends on the presence of spin 0 in the vector field  $A^n$ . Hence, the gauge transformation can be selected such that spin 0 would be excluded once and for all from the vector field. This means introducing the condition

$$D_m A^m = 0 \quad (8.8)$$

Thus, in electrodynamics condition (8.8) can be introduced, but this is not a necessary condition because spin 0 of the vector field has no effect on physical quantities due to gauge invariance.

Allowing for (8.8) in (8.6), we arrive at a system of equations

$$\gamma^{mk} D_m D_k A^n = 4\pi J^n \quad (8.9a)$$

$$D_m A^m = 0 \quad (8.9b)$$

which determines a vector potential  $A^n$  possessing only spin 1.

The Lagrangian formalism that leads to these results is well known. Note that the idea of constructing a theory of interactions of vector fields (both Abelian and non-Abel) based on gauge invariance proved to be extremely fruitful and is being successfully developed.

The problems that we encounter in setting up the remaining equations for a gravitational tensor field are of quite a different nature, since the source of this field, the energy-momentum tensor, is non invariant under gauge transformations of field  $\phi^{ik}$ . We will discuss this aspect in greater detail later. For the present, by analogy with Maxwell's electrodynamics, we will construct the remaining equations for the gravitational tensor field. The second-rank tensor that is conserved is the energy-momentum tensor of matter and gravitational field in the Minkowski spacetime,  $t^{mn}$ . Por lo tanto, es natural tomarlo como la fuente última del campo gravitatorio. Puesto como es establecido en el Numeral 11, el tensor lineal conservado idénticamente más simple en. Hence, it is natural to take it as the ultimate source of the gravitational field. Since, as established in numeral 13, the simplest identically conserved tensor linear in  $g^{mn}$  is  $J^{mn}$  by analogy with electrodynamics we can postulate the validity of the following equations:

$$J^{mn} \equiv D_k D_p (\gamma^{kn} \hat{g}^{pm} + \gamma^{km} \hat{g}^{pn} - \gamma^{kp} \hat{g}^{mn} - \gamma^{mn} \hat{g}^{kp}) = \lambda (t^{mn}_g + t^{mn}_M) \quad (8.10)$$

Generally speaking, such a type of equation presupposes the automatic validity of the law of conservation of the energy-momentum tensor of matter and gravitational field in the Minkowski spacetime,

$$D_m (t^{mn}_g + t^{mn}_M) \equiv D_m t^{mn} = 0 \quad (8.11)$$

and, as a corollary (see Eq. (6.27)), the validity of the covariant conservation law for matter in the Riemann spacetime:

$${}_m T^{mn} = 0 \quad (8.12)$$

The Hilbert energy-momentum tensor  $T^{mn}$  can be specified phenomenologically. In this case Eqs. (8.12) constitute the equations of motion of matter.

Combining (8.3) with (8.10), we get

$$\gamma^{kp} D_k D_p \hat{g}^{mn} = \lambda (t^{mn}_g + t^{mn}_M) \quad (8.13a)$$

$$D_m \hat{g}^{mn} = 0 \quad (8.13b)$$

This system of equations, (8.13a) and (8.13b), is the sought system for RTG.

The role of Eqs. (8.13b) in RTG is essentially different from the role that (8.8) plays in electrodynamics. Indeed, although the left-hand side of (8.10) is invariant under the gauge transformation

$$\hat{g}^{mn} \rightarrow \hat{g}^{mn} + D^m \bar{g}^n + D^n \bar{g}^m - \gamma^{mn} D^k \bar{g}^k \quad (8.14)$$

where  $\bar{g}^n = \nu \cdot \gamma \xi^n$  is the density of an arbitrary 4-vector  $\xi^n(x)$ , in the theory we do not have the arbitrariness of the (8.14) type since the right-hand side of (8.10) is non invariant under transformation (8.14). For this reason, Eqs. (8.3) cannot follow from Eqs. (8.10).

Hence, in RTG Eqs. (8.3) constitute additional independent dynamical equations for the gravitational field rather than coordinate conditions.

The main problem in constructing a theory is to establish whether there exists a Lagrangian density for a gravitational field with spins 2 and 0 that would automatically lead, via the principle of least action, to Eqs. (8.13a). The total Lagrangian density of a gravitational field  $\hat{\phi}^{ik}$  that describes spins 2 and 0 and is quadratic in the first derivatives of the field has the form

$$L_g = a \hat{g}_{km} \hat{g}_{nq} \hat{g}^{lp} D_l \hat{g}^{kq} D_p \hat{g}^{mn} + b \hat{g}_{kq} D_m \hat{g}^{pq} D_p \hat{g}^{km} + c \hat{g}_{km} \hat{g}_{nq} \hat{g}^{lp} D_l \hat{g}^{km} D_p \hat{g}^{nq} \quad (8.15)$$

A characteristic feature of this Lagrangian is that the convolution of covariant derivatives taken with respect to the Minkowski metric is achieved via the effective metric tensor  $\hat{g}^{ik}$  of the Riemann spacetime. It can be shown that this restriction on the gravitational field is a consequence of the geometrization principle and the structure of the gravitational field, which possesses spins 2 and 0.

In view of the principle of least action, the system of equations for the gravitational field assumes the form

$$\delta L_g / \delta \hat{\phi}^{ik} + \delta L_M / \delta \hat{\phi}^{ik} \equiv \delta L_g / \delta \hat{g}^{ik} + \delta L_M / \delta \hat{g}^{ik} = 0 \quad (8.16)$$

where  $L_M$  we have allowed for (8.1), is the material Lagrangian density, and  $L_g$  is specified in (8.15).

To represent the system of equations (8.16) in the form (8.13a) we must select in an unambiguous manner the constants  $a$ ,  $b$ , and  $c$  in the Lagrangian density (8.15). To this end we use formulas (6.17), (6.22), and (6.25) and find for Lagrangian  $L = L_g + L_M$  the

energy-momentum tensor density  $t^{mn}$  for matter and gravitational field in the Minkowski spacetime. Calculating the variation of the total Lagrangian over  $\gamma_{mn}$ , we find that

$$\begin{aligned} t^{mn} = & 2\gamma\gamma(\gamma^{nk}\gamma^{mp} - \frac{1}{2}\gamma^{mn}\gamma^{ph}) \delta L/\delta \hat{g}^{kp} + 2bJ^{mn} + D_p\{(2a + b) [H^{pn}{}_k\gamma^{km} + H^{pm}{}_k\gamma^{km} - \\ & H^{mn}{}_k\gamma^{kp}]\} \\ & -2(a + 2) \gamma^{mn}\hat{g}^{kp}\hat{g}_{iq}D_k\hat{g}^{lq} \end{aligned} \quad (8.17)$$

where

$$H^{pn}{}_k = (\hat{g}^{pl}D_l\hat{g}^{qn} + \hat{g}^{nl}D_l\hat{g}^{pq})\hat{g}_{qk}$$

We see that the equations

$$\begin{aligned} t^{mn} = & 2bJ^{mn} + D_p\{(2a + b) [H^{pn}{}_k\gamma^{km} + H^{pm}{}_k\gamma^{km} - H^{mn}{}_k\gamma^{kp}]\} -2(a + 2) \gamma^{mn}\hat{g}^{kp}\hat{g}_{iq}D_k\hat{g}^{lq} \end{aligned} \quad (8.18)$$

are equivalent to the field equations (8.16). If we wish the condition

$$D_m t^{mn} = 0 \quad (8.19)$$

not to produce any new equation for field  $\Phi^k$ , since this would lead to an over determined system of equations, it is necessary and sufficient that the coefficients  $a$ ,  $b$ , and  $c$  satisfy the following conditions:

$$a = \frac{1}{2}b, \quad c = \frac{1}{4}b \quad (8.20)$$

If the constants are selected in this manner, we arrive at an identity:

$$D_m t^{mn} \equiv 0$$

Thus, the equations of the motion of matter follow directly from the equations for the gravitational field. Allowing for (8.20), we find that (8.18) assumes the form

$$D_p D_k (\gamma^{km}\hat{g}^{pn} + \gamma^{kn}\hat{g}^{pm} - \hat{g}^{mn}\gamma^{kp} - \gamma^{mn}\hat{g}^{kp}) = 1/2b(t^{mn}{}_g + t^{mn}{}_M) \equiv 1/2b t^{mn} \quad (8.21)$$

This coincides with Eqs. (8.10), which were written by analogy with electrodynamics, if we apply  $2b = 1/\lambda$ . Thus, the Lagrangian density that leads us to field equations in the form of (8.21) is

$$L_g = 1/2\lambda [\hat{g}_{kq}D_m\hat{g}^{pq}D_p\hat{g}^{km} - \frac{1}{2}\hat{g}_{km}\hat{g}_{nq}\hat{g}^{lp}D_l\hat{g}^{kq}D_p\hat{g}^{gm} + \frac{1}{4}\hat{g}_{km}\hat{g}_{nq}\hat{g}^{lp}D_l\hat{g}^{km}D_p\hat{g}^{nq}] \quad (8.22)$$

The correspondence principle implies that

$$\lambda = -16\pi \quad (8.23)$$

If we allow for (8.23) in (8.22), we get

$$L_g = 1/32\pi [\hat{G}^l{}_{mn}D_l\hat{g}^{mn} - \hat{g}^{mn}\hat{G}^k{}_{mk}\hat{G}^l{}_{nl}] \quad (8.24)$$

where the third-rank tensor  $\hat{G}^k{}_{lm}$  is defined thus:

$$\hat{G}^k_{lm} = \frac{1}{2} \hat{g}^{pk} (D_m \hat{g}_{lp} + D_l \hat{g}_{mp} - D_p \hat{g}_{lm}) \quad (8.25)$$

We can also write  $L_g$ , in the form

$$L_g = -1/16\pi \sqrt{-g} g^{mn} [G^k_{lm} G^l_{nk} - G^l_{mn} G^k_{lk}] \quad (8.26)$$

The first to consider such a Lagrangian was Rosen, 1940, 1963. The third-rank tensor  $G^k_{lm}$ , in (8.26) is defined as follows:

$$G^k_{lm} = \frac{1}{2} g^{pk} (D_m g_{lp} + D_l g_{mp} - D_p g_{lm}) \quad (8.27)$$

It is easily verified that Lagrangian (8.26) can be transformed into the sum of two terms, one of which does not contain the metric coefficients  $\gamma^{mn}$  and the other, which depends on  $\gamma^{mn}$ , is written in the form of the divergence of a vector and, therefore, does not affect the field equations.

If we allow for Eq. (8.3), the complete system of RTG equations for matter and gravitational field is (see Logunov and Mestvirishvili, 1984, 1985a, 1985b, 1986b, Vlasov and Logunov, 1984, and Vlasov, Logunov, and Mestvirishvili, 1984)

$$\gamma^{pk} D_p D_k \hat{g}^{mn} = 16\pi t^{mn} \quad (8.28)$$

$$D_m \hat{g}^{mn} = 0 \quad (8.29)$$

Obviously, in a Galilean system of coordinates Eqs. (8.28), (8.29) assume the form

$$\square \hat{g}^{mn} = 16\pi t^{mn} \quad (8.28')$$

$$\partial_m \hat{g}^{mn} = 0 \quad (8.29')$$

Equations (8.28) and (8.29) clearly show that the Minkowski spacetime enters into all the gravitational-field equations in an essential way. But this means that it will find its physical reflection not only in the fundamental laws of nature but also in the description of various natural phenomena.

The general-covariant RTG equations (8.28) and (8.29) closely resemble the general-covariant equations of electrodynamics, (8.9a) and (8.9b), in the absence of gravitational fields. In electrodynamics the electromagnetic field is a vector field and its source is the conserved electromagnetic current Equation (8.9b) excludes spin 0 from the vector field. In RTG the gravitational field is a tensor field and the source is the conserved tensor density of the energy-momentum of both matter and gravitational field. For this reason Eq. (8.28) is nonlinear even for a free gravitational field. Equation (8.29) excludes spins 1 and 0' from the tensor field.

The equations of RTG and electrodynamics acquire an especially simple form in the Galilean coordinates in an inertial reference frame. If we were to restrict our discussion to the first system of equations (8.28), the division of the metric of the Riemann spacetime into the metric in the Minkowski spacetime and the gravitational tensor field would be of a purely nominal nature and with no physical meaning. The second system (8.29) of four field equations drastically separates everything that

refers to forces of inertia from everything that refers to the gravitational field. The two systems of equations, (8.28) and (8.29), are general covariant. The behavior of the gravitational field is restricted, as usual, by appropriate physical conditions in a given, say Galilean, system of coordinates. In GR it is impossible to formulate the physical conditions imposed on metric  $g^{mn}$  if one remains within the framework of the Riemann spacetime, since the asymptotic behavior of the metric always depends on the choice of the three-dimensional system of coordinates.

Let us now find the explicit form of the system of equations (8.16). If we take Lagrangian (8.22), it can be demonstrated that

$$\partial L_g / \partial \hat{g}^{mn} = 1/16\pi [G^k_{lm} G^l_{kn} - G^k_{mn} G^l_{kl}]$$

and

$$\partial L_g / \partial D_k \hat{g}^{mn} = 1/16\pi [G^k_{mn} - \frac{1}{2} \delta^k_m G^l_{nl} - \frac{1}{2} \delta^k_n G^l_{ml}]$$

Hence,

$$\partial L_g / \delta \hat{g}^{mn} \equiv \partial L_g / \partial \hat{g}^{mn} - D_k \partial L_g / \partial (D_k \hat{g}^{mn}) = -1/16\pi R_{mn} \quad (8.30)$$

where  $R_{mn}$ , is the second-rank tensor of the curvature of the Riemann spacetime:

$$R_{mn} = D_k G^k_{mn} - D_m G^k_{nl} + G^k_{mn} G^k_{kl} - G^k_{ml} G^l_{nk} \quad (8.31)$$

Since in view of (6.6b) and (6.11) we have

$$2 \delta L_M / \delta \hat{g}^{mn} = 1/\sqrt{-g} (T_{mn} - \frac{1}{2} g_{mn} T) \quad (8.32)$$

Eq. (8.16) yields

$$\sqrt{-g} R_{mn} = 8\pi (T_{mn} - \frac{1}{2} g_{mn} T) \quad (8.33)$$

that is, we have arrived at the system of Hilbert-Einstein equations, the one important difference being that all field variables in the Hilbert-Einstein equations in our theory depend on universal spatial-temporal coordinates in the Minkowski spacetime. In an inertial reference frame these universal coordinates can be chosen to be Galilean. It must be emphasized that the system of equations (8.28) does not coincide with the system of Hilbert-Einstein equations (8.33). Only if the general-covariant equations (8.29) hold true does the system of Hilbert-Einstein equations, formally written in GR in the variables of the Minkowski spacetime, reduce to the system of equations (8.28), and these depend essentially on the metric tensor of the Minkowski spacetime.

It has long been known (see Rosen, 1940, 1963, and Tolman, 1934) that Lagrangian (8.26) leads to system (8.33). We have shown, however, that for a gravitational field with spins 2 and 0 the gravitational-field Lagrangian density (8.22) is the only one that leads to a self-consistent system of equations for matter and field, (8.28) and (8.29). This means that RTG equations are the only simplest second-order equations that can exist.

In view of the importance of the equivalence of Eqs. (8.28) and (8.33) in the Minkowski variables, we can give another variant of the proof of the above statement based on direct calculations of the tensor densities  $t^{mn}_g$  and  $t^{mn}_{M,,}$ , provided that (8.29) is valid.

If we take formulas (6.17) and the Lagrangian density (8.22) and allow for (8.1), we will find that the gravitational-field energy-momentum tensor density in the Minkowski spacetime is

$$t^{mn}_g = -1/16\pi J^m - \sqrt{-g}/8\pi (\gamma^{mp}\gamma^{nk} - 1/2\gamma^{mn}\gamma^{pk})R_{pk} \quad (8.34)$$

We see that the second-rank curvature tensor  $R_{pk}$  of the Riemann spacetime has emerged automatically. Similarly, using formulas (6.17) and (8.1) and the definition (6.6a) of the Hilbert-tensor density, we arrive at the following formula for the material energy-momentum tensor density in the Minkowski spacetime:

$$t^{mn}_M = (\sqrt{-g})^{1/2} (\gamma^{mp}\gamma^{nk} - 1/2\gamma^{mn}\gamma^{pk}) (T_{pk} - 1/2 g_{pk}T) \quad (8.35)$$

Substituting (8.34) and (8.35) into the field equations (8.10), we get

$$(\gamma^{mp}\gamma^{nk} - 1/2\gamma^{nm}\gamma^{pk}) [R_{pk} - 8\pi/\sqrt{-g} (T_{pk} - 1/2 g_{pk}T)] = 0,$$

which leads us to the system of equations for the gravitational field in the form of (8.33).

The complete system of equations for matter and gravitational field, (8.28) and (8.29), is equivalent to the following system of equations:

$$\sqrt{-g} R_{mn} = 8\pi (T_{mn} - 1/2 g_{mn}T) \quad (8.36)$$

$$D_m \hat{g}^{mn} = 0 \quad (8.37)$$

Thus, although in RTG the complete system of equations (8.36) and (8.37) does contain the system of Hilbert-Einstein equations, the content of the latter changes substantially\*, since the spatial-temporal variables now coincide with the variables of the Minkowski spacetime. We must again emphasize that Eqs. (8.37) are universal, since they are field equations describing gravitational fields with spins 2 and 0; they unambiguously separate forces of inertia from gravitational fields. Within the framework of GR this is impossible to do in principle. The choice of the reference frame (or system of coordinates) is fixed by the metric tensor of the Minkowski spacetime, while Eqs. (8.37) lay no restrictions on the choice of the coordinate system.

\* Equations (8.36) do not contain metric  $\gamma_{ik}$ , and it is meaningless to speak of  $\gamma_{ik}$  in GR. This implies that the statement of Zel'dovich and Grishchuk, 1986, that GR can be constructed on the basis of the Minkowski spacetime is erroneous.

Note that some aspects of the theory of gravitation in the Minkowski spacetime have been considered in Gupta, 1952, Kohler, 1952, 1953, 1954, Papapetrou, 1948, Pugachev, 1958, 1959, 1964, Rosen, 1940, 1963, and Thirring, 1961. However, even scientists who were on the right track at the beginning failed to understand this and

took a different direction in building the theory of gravitation, a direction that has not led to a complete theory.

In conclusion, one remark is in order. The system (8.3) whose validity we have postulated does not follow from the principle of least action. Therefore, in applying this principle to Lagrangian (8.15), we were forced to allow for Eqs. (8.3) by introducing in the integrand in the action integral a term of the form  $\eta_m D_n \hat{g}^{mn}$ , where  $\eta_m$ , are Lagrange's multipliers" [9].

## 15. Gravitational waves in RTG.

Since RTG defines the gravitational field as a physical field, of static type, therefore, composed of virtual gravitons of spin 2 and 0, this field must radiate gravitational waves that will correspond to its dynamic state, composed of real gravitons, with mass not zero, which will propagate in a vacuum, like electromagnetic waves, and consequently will be detectable, although, due to its low mass of  $4.5 \cdot 10^{-66}$  g, it is undetectable with our current technological scope.

Let us point out that the false gravitational waves detected by LIGO are below  $10^{-54}$  g [12], [13], close to the electromagnetic spectrum. Of course, such radiation would have been easily detected and the graviton would be as handle as the photon. As this is not the case, it is clear that this radiation is not gravitational. Tom Van Flandern and the author have argued that the waves radiated by binary pulsars are not gravitational waves, but some form of electromagnetism. Said radiation is the residue that remains unexplained once all the known electromagnetic mechanical effects are included, causing energy losses that can reappear in the form of radiation. Such residual for the Hulse-Taylor binary pulsar, PSR B1913 + 16, coincides with the orbital decay rate predicted by GTR, behind Einstein's back, although, the value of this rat is above, approximately, at 0.3% of the predicted value. In the GTR gravitational radiation estimation equations the actual graviton with mass 0 is assumed to match the forecast exactly with the observed value. But when the orbital decay rats of the binary pulsars PSR B1913 + 16 and PSR B1534 + 12 are combined, it is obtained that the mass of the real graviton is not zero but maximum less than  $1.35342 \cdot 10^{-52}$  grams, with 90% trustworthy. This upper limit for the mass of the real graviton was calculated, in 2002, by Lee Samuel Finn and Patrick J. Sutton of the "Center for Gravitational Wave Physics", of the State University of Pennsylvania, USA. The value of the assumed mass of the real graviton less than  $1.35342 \cdot 10^{-52}$  grams is very close to the value of the upper limit of the mass of the real photon which is less than  $10^{-51}$  grams, according to its 2003 calculation, conducted by Jun Luo and colleagues at the Huazhong University of Science and Technology in Wuhan, China. And very far from the upper limit value of the mass of the graviton less than  $4.5 \cdot 10^{-66}$  grams, estimated by SS Gershtein, AA Logunov and MA Mestvirishvili, in 1997, based on the observed parameters of the expansion of the Universe, and which is consistent with the value less than  $0.5 \cdot 10^{-65}$  grams estimated by K. Staniukovich and M. Vasiliev, around 1968, based on the Einstein relation  $E = m \cdot c^2$ . Therefore, in reality the radiation from the binary stars may be some electromagnetic radiation, as Tom Van Flandern has argued, although with the detection of LIGO the wave would be quadrupole. Anyway, the mass of that false graviton-photon would be about  $10^{12}$  stronger than the mass of the true graviton estimated by RTG. It is obvious that

if it were true that the mass of the graviton were of the order of the mass of the photon, we would not exist, since the gravitational force coming from such a mass would have prevented our appearance, because the Lorentz force of the static gravitational field would be close to that of the static electromagnetic field, which does not correspond to our well-established knowledge of the extreme weakness of gravity.

However, that in RTG, the graviton has a mass at rest, since spin 0, no “phantom” states of negative energy flow are generated, which would be non-physical states, when their effects in the Solar System are interpreted, due to the causality condition, according to which it is restricted that the cone of the Riemann spacetime must be inside the cone of the Minkowski spacetime, since in RTG there are two cones of causality.

"We start from the existence of a free gravitational field: gravitational waves, as an objective physical reality similar to electromagnetic waves in a vacuum.

For the simplicity and precision of our analysis, we consider a weak plane gravitational wave in vacuum with amplitude  $a^{\mu\nu}(k)$ , which propagates along the Z axis.

$$\Phi^{\mu\nu} = a^{\mu\nu}(k) \cos kx, \quad (1)$$

where  $k_\nu = (\omega, 0, 0, -q)$ ,  $q^2 = 1 - m^2 / \omega^2$ , and  $m$  is the mass of the graviton.

We use the system of conventions units  $G = \hbar = c = 1$ . In a vacuum, the basic RTG equations in linear approximation and in an inertial frame with Galilean coordinates take the following form

$$\Phi^{\mu\nu} + m^2 \Phi^{\mu\nu} = 0, \quad (2)$$

$$\partial_\nu \Phi^{\mu\nu} = 0 \quad (3)$$

Wave (1) is a solution to these equations. A weak gravitational field  $\Phi^{\mu\nu}$  produces an effective Riemannian space with the following metric tensor

$$g_{\mu\nu} = \gamma_{\mu\nu} - \Phi_{\mu\nu} + 1/2 \gamma_{\mu\nu} \Phi, \quad \Phi_{\mu\nu} \gamma^{\mu\nu} = \Phi$$

the tensor  $g^{\mu\nu}$  is given by the analogous expression

$$g^{\mu\nu} = \gamma^{\mu\nu} + \Phi^{\mu\nu} - 1/2 \gamma^{\mu\nu} \Phi \quad (4)$$

From the above it follows that the scalar curvature of the effective Riemannian space  $R$  is

$$R = 1/2 m^2 \Phi.$$

But it happens so that it does not influence the energy flow, as we will see next. The Minkowski spatial interval in an inertial frame with Galilean coordinates is

$$d\sigma^2 = \gamma_{\mu\nu} dx^\mu dx^\nu = dt^2 - dx^2 - dy^2 - dz^2 \quad (5)$$

Since RTG treats the gravitational field as a physical tensor field that propagates in Minkowski space, the cone of causality in the actual Riemannian space that arises must



not come out of the cone of causality in Minkowski space. Just this is RTG causality principle. According to this principle, the light and time-like geodesics of the effective Riemannian space that is produced by the physical field must not go outside the limits of the Minkowski spatial cone. Only this physical requirement should obtain the proper mathematical formulation.

The terms with second derivatives on spacetime coordinates appear in the hyperbolic dynamic equations of the gravitational field in RTG as follows

$$g^{\mu\nu} \partial^2 \Phi^{\alpha\beta} / \partial \Phi_{x^\mu} \Phi_{x^\nu} \quad (6)$$

The characteristic equation for gravitational equations is provided only by higher order derived terms (6)

$$g^{\mu\nu} \partial S / \partial x^\mu \partial S / \partial x^\nu = 0 \quad (7)$$

This equation determines the wavefront of the field, if the graviton has no mass at rest. The characteristics determine the cone of causality of the effective Riemannian space. Each term with a second derivative of (6) has the corresponding term in the characteristics (7). If any term with a second derivative is absent in (6), then there will be no corresponding term in (7).

The temporal geodetic lines corresponding to (7) are given by the Hamilton-Jacobi equations

$$g^{\mu\nu} \partial S / \partial x^\mu \partial S / \partial x^\nu = 1 \quad (8)$$

The total set of geodetic lines in correspondence with (6) is determined by the following equations

$$0$$

$$g^{\mu\nu} \partial S / \partial x^\mu \partial S / \partial x^\nu = 1$$

$$-1$$

where the first equation gives the isotropic geodesic lines, the second temporal geodesics, while the third gives spatial geodesics.

Therefore, based on the equations. (7) and (8) isotropic and temporal geodetic lines, corresponding to the equation. (6), satisfy the following inequality

$$g^{\mu\nu} \partial S / \partial x^\mu \partial S / \partial x^\nu \geq 0 \quad (9)$$

This inequality can be written as follows

$$g_{\alpha\beta} p^\alpha p^\beta \geq 0 \quad (10)$$

where the counter variant  $p^\alpha$  is

$$p^\alpha = g^{\alpha\mu} \partial S / \partial x^\mu \quad (11)$$

To provide that the cone of causality of the effective Riemannian space is within the cone of causality of the Minkowski space, it is necessary and sufficient to satisfy the following inequality

$$\gamma_{\alpha\beta} p^\alpha p^\beta \geq 0 \quad (12)$$

The inequalities (10) and (12) can be written in a way directly connected to the geodetic movements (7) and (8) that are in exact correspondence with (6)

$$g^{\mu\nu} p_\mu p_\nu \geq 0 \quad (13)$$

$$\gamma_{\alpha\beta} g^{\alpha\mu} g^{\beta\nu} p_\mu p_\nu \geq 0 \quad (14)$$

where the covariant vector  $p_\nu$  is

$$p_\nu = \partial S / \partial x^\nu \quad (15)$$

The causality conditions (13) and (14) impose defined rigid constraints on the solutions of the gravitational field equations. Only solutions that satisfy inequalities (13) and (14) have a physical meaning in theory. The inequalities (13) and (14) are directly connected with the hyperbolic equations for the gravitational field, since they are derived from the second structure of derivatives (6) of the dynamic equations. Only this mathematical formulation of the causality principle guarantees the position of the Riemann cone of causality within the cone of causality of the Minkowski space, in correspondence with the dynamic structure (6).

Previously in [4] we have not recognized this fact of necessity to establish the direct correspondence of the principle of causality with the dynamic system of hyperbolic equations. In the case of the static system, the gravitational field equations are not hyperbolic and, therefore, such direct correspondence is absent. But in that case, the causality condition can be used in the form of inequalities (10) and (12). Taking into account

$$\hat{g}^{\mu\nu} = \tilde{\gamma}^{\mu\nu} + \check{\Phi}^{\mu\nu},$$

where

$$\hat{g}^{\mu\nu} = \nu - g g^{\mu\nu}, \quad \tilde{\gamma}^{\mu\nu} = \nu - \gamma \gamma^{\mu\nu}, \quad \check{\Phi}^{\mu\nu} = \nu - \gamma \Phi^{\mu\nu},$$

The inequalities (13) and (14) in inertial frame with Galilean coordinates take the following form

$$(\gamma^{\mu\nu} + \Phi^{\mu\nu}) p_\mu p_\nu \geq 0, \quad (16)$$

$$\gamma^{\alpha\beta} (\gamma^{\alpha\mu} + \Phi^{\alpha\mu}) (\gamma^{\beta\nu} + \Phi^{\beta\nu}) p_\mu p_\nu \geq 0 \quad (17)$$

For motion (1) the following characteristic equation is valid

$$g^{\mu\nu} p_\mu p_\nu = g^{00} (p_0)^2 + 2g^{03} p_0 p_3 + g^{33} (p_3)^2 = 0.$$

For the weak gravitational field and the special movement (1) along the inequality of the Z axis (16) it is true if the value of x defined as

$$x = p_3/p_0,$$

is limited by the following inequalities

$$x_1 \leq x \leq x_2, \quad (18)$$

where

$$x_1 = \Phi^{03} - 1 - 1/2 (\Phi^{00} + \Phi^{33}), \quad (19)$$

$$x_2 = \Phi^{03} + 1 + 1/2 (\Phi^{00} + \Phi^{33}).$$

So we have defined the set of temporal vectors that lie within the causality cone determined by the characteristics at the base of (6) for the movement (1), which leads to the metric (4). The inequality (17) will be fulfilled if

$$x'_1 \leq x \leq x'_2,$$

where

$$x'_1 = 2\Phi^{03} - 1 - \Phi^{00} - \Phi^{33}, \quad (20)$$

$$x'_2 = 2\Phi^{03} + 1 + \Phi^{00} + \Phi^{33}.$$

To provide the position of the effective Riemannian causality cone within the Minkowski causality cone, it is necessary and sufficient to satisfy the following inequalities

$$x'_1 \leq x_1, x_2 \leq x'_2 \quad (21)$$

Based on (21) and taking into account (19) and (20) we obtain

$$\Phi^{00} \pm 2\Phi^{03} + \Phi^{33} \geq 0 \quad (22)$$

From equations (3) we find for solution (1):

$$\Phi^{00} = q\Phi^{03}, \Phi^{33} = q\Phi^{03} \quad (23)$$

After substituting these equations in (22) we obtain

$$(q \pm 1)^2 \Phi^{03} \geq 0 \quad (24)$$

It follows from these inequalities for wave (1) that

$$\Phi^{03} \equiv 0, \quad (25)$$

and therefore, based on the equations. (23), the following equations take place

$$\Phi^{00} \equiv 0, \Phi^{33} \equiv 0 \quad (26)$$

In RTG, the causality principle selects the physical solution of the gravitational equations. From (25) and (26) it follows that the longitudinal-longitudinal components are absent in the wave solution (1). For this reason alone there is no term like

$$R\Phi^{03} = 1/2m^2 \Phi\Phi^{03},$$

in the energy flow, this term is identically zero. When the graviton mass is zero, the equations. (25), (26) as a rule are derived from gauge transformations. Here they follow the principle of causality.

This alone provides the positivity of the energy flux in RTG in the case of the non-zero graviton mass.

In the quadratic approximation RTG considered in the Galilean coordinates, the energy flow is determined, by means of the quantity of tensor

$$t_g^{\alpha\lambda} = 1/32\pi \gamma^{\epsilon\alpha}\gamma^{\lambda\beta}(\partial_\alpha \Phi_{\epsilon\nu}^{\tau} \cdot \partial_\beta \Phi_{\tau}^{\nu} - 1/2\partial_\alpha \Phi \cdot \partial_\beta \Phi) \quad (27)$$

The raising and lowering of the indices for  $\Phi^{\mu\nu}$  is carried out by means of the metric tensor  $\gamma_{\mu\nu}$ . According to the equation. (3), the following relationships take place for solution (1):

$$a^{10} = qa^{13} \quad (28)$$

$$a^{20} = qa^{23}.$$

Taking into account (1), and also Equations (25), (26) and (28), based on the equation. (27) we obtain for wave (1) after averaging over time

$$t_g^{03} = 1/32 \pi q\omega^2 \{ (a^{21})^2 + 1/4 (a^{11} - a^{22})^2 + m^2/\omega^2 [(a^{13})^2 + (a^{23})^2] \} \quad (29)$$

From this it follows that only the transverse-transverse components are present in the flux density for wave (1), and also longitudinal transverse  $a^1_3, a^2_3, a^1_0, a^2_0$ . The latter are multiplied by  $m^2/\omega^2$  in the energy flow (29). Longitudinal-longitudinal components are absent in wave (1). It should be noted that according to RTG it is possible to provide a continuous transformation to the zero graviton mass in this problem.

Therefore, from (29) it follows that the presence of a non-zero mass of graviton does not lead to the appearance of non-physical "phantom" states in RTG. The "phantom" states also do not appear in RTG when explaining the effects of the solar system. Here is a continuous transformation to the zero graviton mass at the distance from the source  $r \gg r_g = 2M$ . The mass of graviton arises in RTG with necessity when we begin to treat the gravitational field as physical in Minkowski space" [14].

## 16. Conclusions.

The relativistic theory of gravitation is far superior to "general relativity" for the following reasons:

1. It generalizes that the physical laws are fulfilled in the same way, regardless of the frame of reference where they are applied, while Einstein was unable to do so, since he sought to generalize Galileo's principle through the equivalence principle that leads to all kinds of movement it is relative and equivalent.
2. It separates the gravitational forces from the inertial forces; therefore, the gravitational motion of the motion of inertia, while for Einstein they are equivalent, without him being able to demonstrate it, since in extended gravity they cease to be.
3. It is a gauge theory, therefore, compatible with the theories of quantum physics while Einstein's is not; Einsteinians' assimilation of the cosmological constant as the graviton is spurious.
4. Gravity is the effect of gravitational forces, while for Einstein gravity is the effect of Riemann spacetime geometry, that is, as a  $g_{\mu\nu}$  effect, both as homogeneous gravity, when  $G_{uv} = 0$ , or as extended gravity, when  $G_{uv} > 0$ .
5. The energy and momentum laws are conserved, while in Einstein no; in return, the Einsteinians present the non-localizable energy of the gravitational metric field.
6. There are gravitational waves, that is, the dynamic state produced by the static gravitational field while for Einstein it is impossible for a metric field to produce them, although, for the Einsteinians yes, but forced to destroy its metric character by conferring substantiality to spacetime.
7. There are no singularities, absent from description within current physics, that is, collapsed matter without spacetime, while for Einsteinians they are, deriving them from the Grossmann-Einstein-Hilbert equations, although, Einstein never agreed.

However, the field equations for RTG and GTR are similar [15]:

$$\text{RTG} \quad \hat{R}^{\mu\nu} - \frac{1}{2}\hat{R}g^{\mu\nu} = 8\pi G/c^4 \frac{1}{\sqrt{-g}} T^{\mu\nu}$$

$$\text{GTR} \quad R^{\mu\nu} - \frac{1}{2}Rg^{\mu\nu} = 8\pi G/c^4 T^{\mu\nu}$$

Where:

$$\hat{R}^{\mu\nu} = R^{\mu\nu} - (Gm_g)^2 / 2c^4 [g^{\mu\nu} - g^{\mu\alpha}g^{\nu\beta}\gamma_{\alpha\beta}], m_g \text{ is the mass of the graviton.}$$

$$\hat{R} = g_{\mu\nu}\hat{R}^{\mu\nu}, \text{ is the analog of the Ricci scalar curvature in RTG.}$$

$R^{\mu\nu}$ , is the Ricci curvature tensor associated with the metric tensor  $g^{\mu\nu}$ .

$$R = g_{\mu\nu}R^{\mu\nu}, \text{ is the Ricci scalar curvature.}$$

Solutions in empty regions

$$\text{RTG} \quad \hat{R}^{\mu\nu} = 8\pi G / c^4 \frac{1}{\sqrt{-g}} [T^{\mu\nu} - \frac{1}{2}Tg^{\mu\nu}]$$

$$\text{GTR} \quad R^{\mu\nu} = 8\pi G / c^4 [T^{\mu\nu} - \frac{1}{2}Tg^{\mu\nu}]$$

In a region of spacetime absent of fermionic matter the second member of the two previous equations tends to zero due to the smallness of the mass of the graviton, which Logunov and Mestvirishvili estimate on the order of  $mg = 4.5 \cdot 10^{-66}$  g.

The reason is that both systems of equations describe the gravity in the Riemann spacetime, although, the GTR in its generic formulation while RTG in the effective Riemann resulting from the physical processes that occur in the pseudo Euclidean spacetime of Minkowski subject to the universal force of gravity.

In terms of the author's reading of this paper on RTG, the identity between the pseudo-Euclidean space and the effective Riemann spacetime would really be due to the independence of the physical phenomenon with respect to its geometry. And according to his conception of spacetime as the structural property of matter [1], he stresses that the need for such an identity in RTG as in the Riemann spacetime of GTR, is that otherwise, his equations would not give the orbits of the planets corrected for their anomalies with respect to celestial mechanics, based on Newton's equations for gravity, nor the deflection undergone by the propagation of the electromagnetic wave, both effects that would be the curvature of the interacting quantum vacuum with very strong gravity of large stellar structures, as in our system: it is the Sun. Therefore, such a curvature effect would be external to the gravitational phenomenon, although, caused by the gravity that curves the quantum vacuum, the medium in which electromagnetic waves propagate and stars move and, also, rotate. For RTG and GTR, that medium is spacetime, definitely in open and irreconcilable contradiction with its alleged relational conception of spacetime, since, due to this repeated scientific prejudice, spacetime has to be substantial, as the relativists have said to be "Einsteinians" Assume in front of its gravitational waves or internal and external gravitomagnetism, that as a frame drag it would really be the drag of the quantum vacuum. RTG equations, although they come from a gauge theory, presuppose that the spacetime curves and the GTR equations integrate the entire gravitational phenomenon as a metric effect of the spacetime that curves, in the regions of strong gravity, therefore both systems of equations, are similar and give consistent results with the observations obtained from celestial mechanics. There is no doubt that in celestial mechanics there is the effect of "something that curves", for the author, what curves would be the quantum vacuum, properly speaking, the geometry of its spacetime, as a structural property of dynamic matter.

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